Electroweak and non-resonant corrections to top-pair production near threshold at NNLO

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Based on work in collaboration with
M. Beneke, A. Maier, J. Piclum and P. Ruiz-Femenía

Top threshold scan

Top threshold scan at a future lepton collider (ILC/CLIC/FCC-ee)

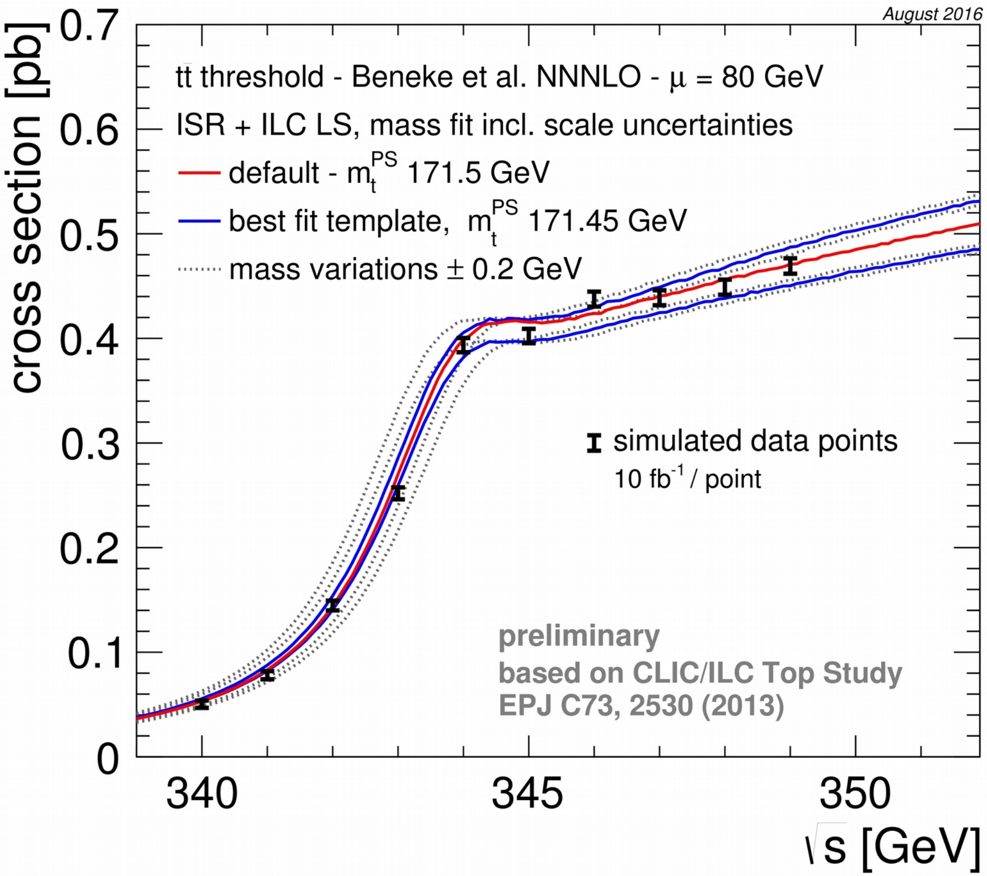
- Extremely precise determination of the top-quark mass

$$\delta m_t^{\overline{\mathrm{MS}}} \sim 50\,\mathrm{MeV}$$

- High sensitivity to Γ_t and α_s
- Possibility to measure top Yukawa coupling

Requires precise theory predictions

- Known at NNNLO in QCD, scale uncertainty of 3%
- NLO non-QCD effects up to 15%
- Full NNLO necessary



[Simon '16]

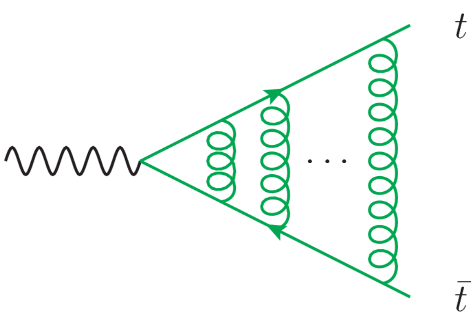
Top quarks near threshold

Near threshold tops are non-relativistic with velocity $v \sim \alpha_s$

- Multiple scales are relevant

hard scale	m_t	mass
soft scale	$\mathrm{m}_t\, v$	momentum
ultrasoft scale	$\mathrm{m}_t\, v^2$	energy

- Coulomb singularities $(\alpha_s/v)^n$ from n exchanges of potential gluons



$$\mathrm{k}^0 \sim m_t\, v^2, \ \mathbf{k} \sim m_t\, v$$

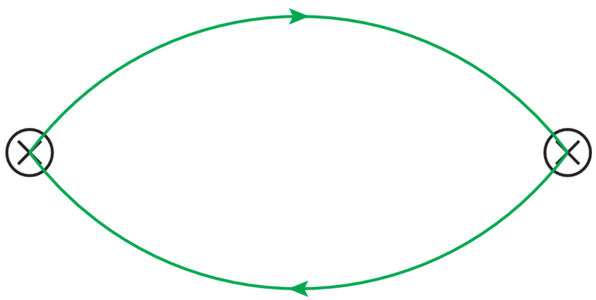
- Conventional perturbation theory in α_s fails
- Coulomb singularities must be resummed to all orders
- Done with potential non-relativistic QCD (PNRQCD)

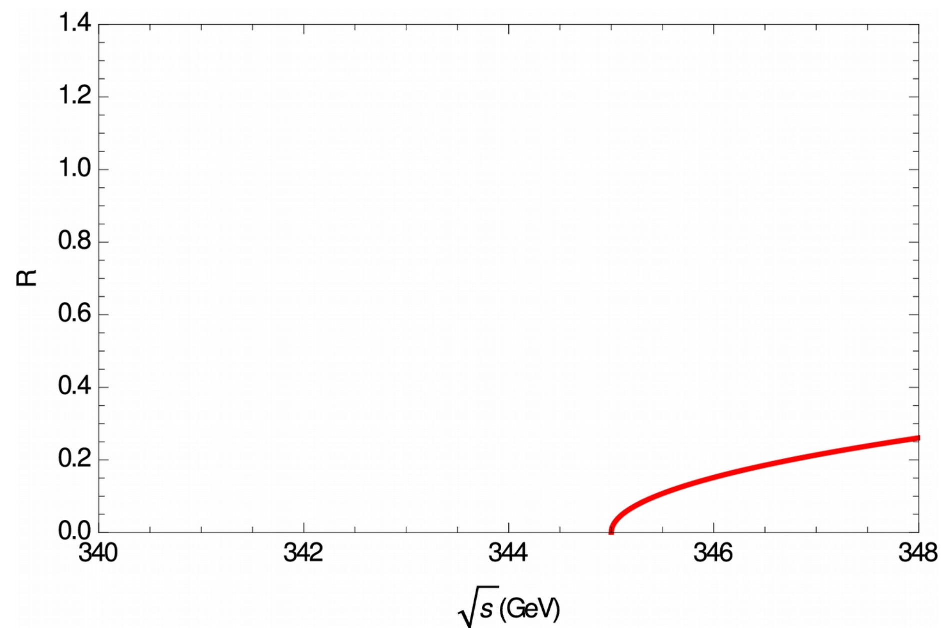
[Pineda, Soto '98; Beneke, Signer, Smirnov '99; Brambilla, Pineda, Soto, Vairo '00; Beneke, Kiyo, Schuller '13]

QCD cross section

Normalized cross section: $R(s) = \frac{\sigma(e^+e^- \to t\bar{t}X)}{\sigma_0(e^+e^- \to \mu^+\mu^-)} = 12\pi e_t^2 f(s)\ \mathrm{Im}\left[\Pi^{(v)}(s)\right]$

Born cross section:





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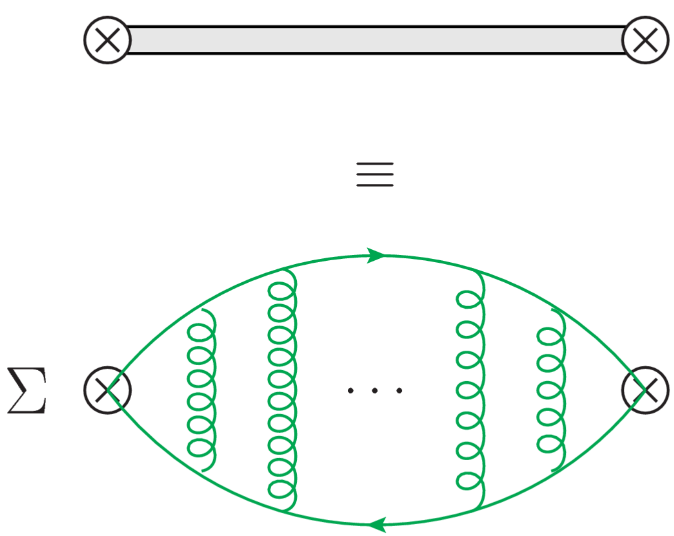
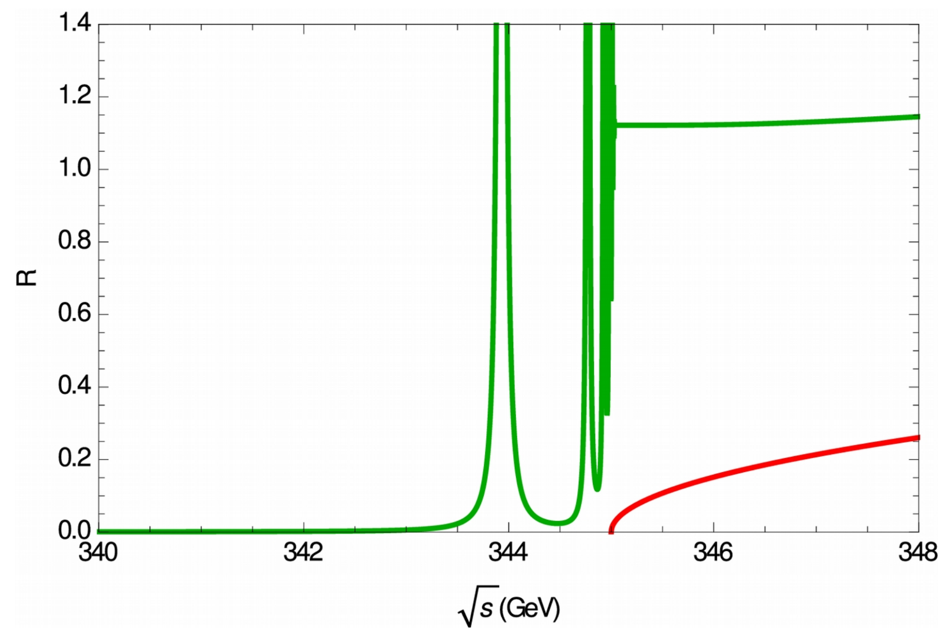
Resummed cross section at LO:



$$\Gamma_t = 0$$



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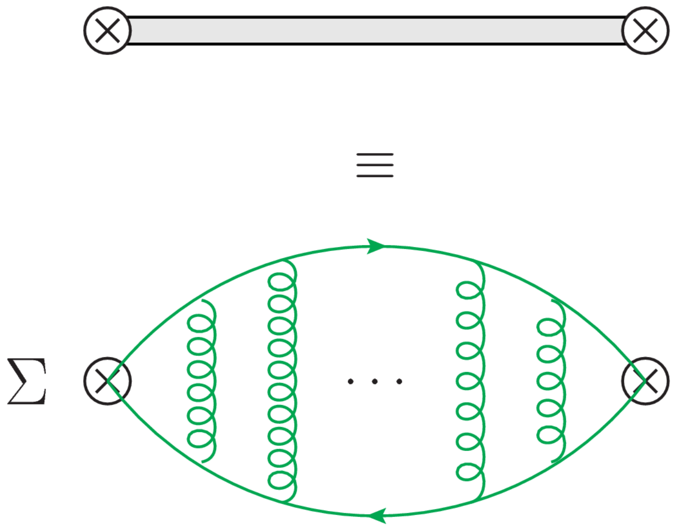
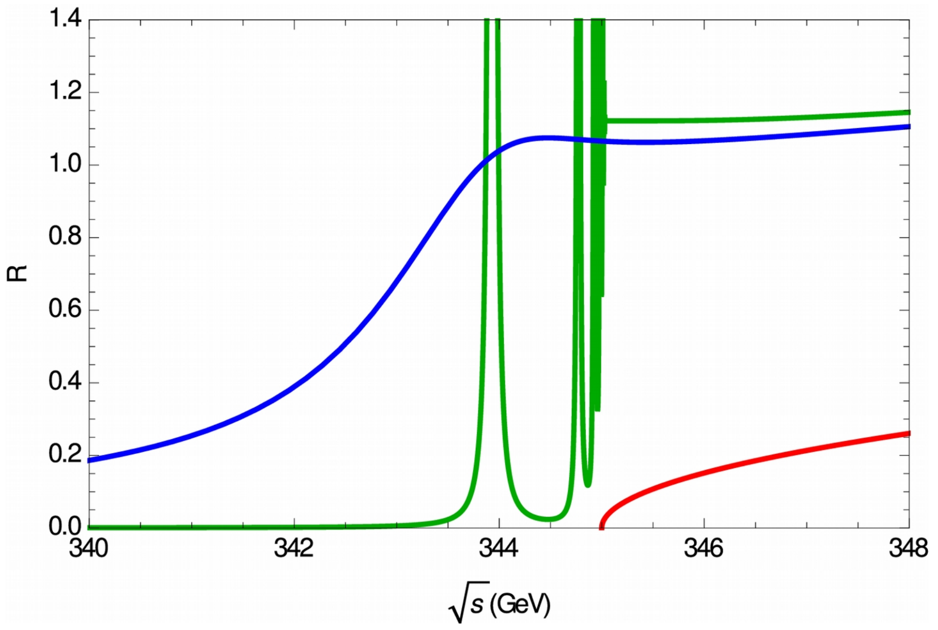
Resummed cross section at LO:

$\Gamma_t \neq 0$





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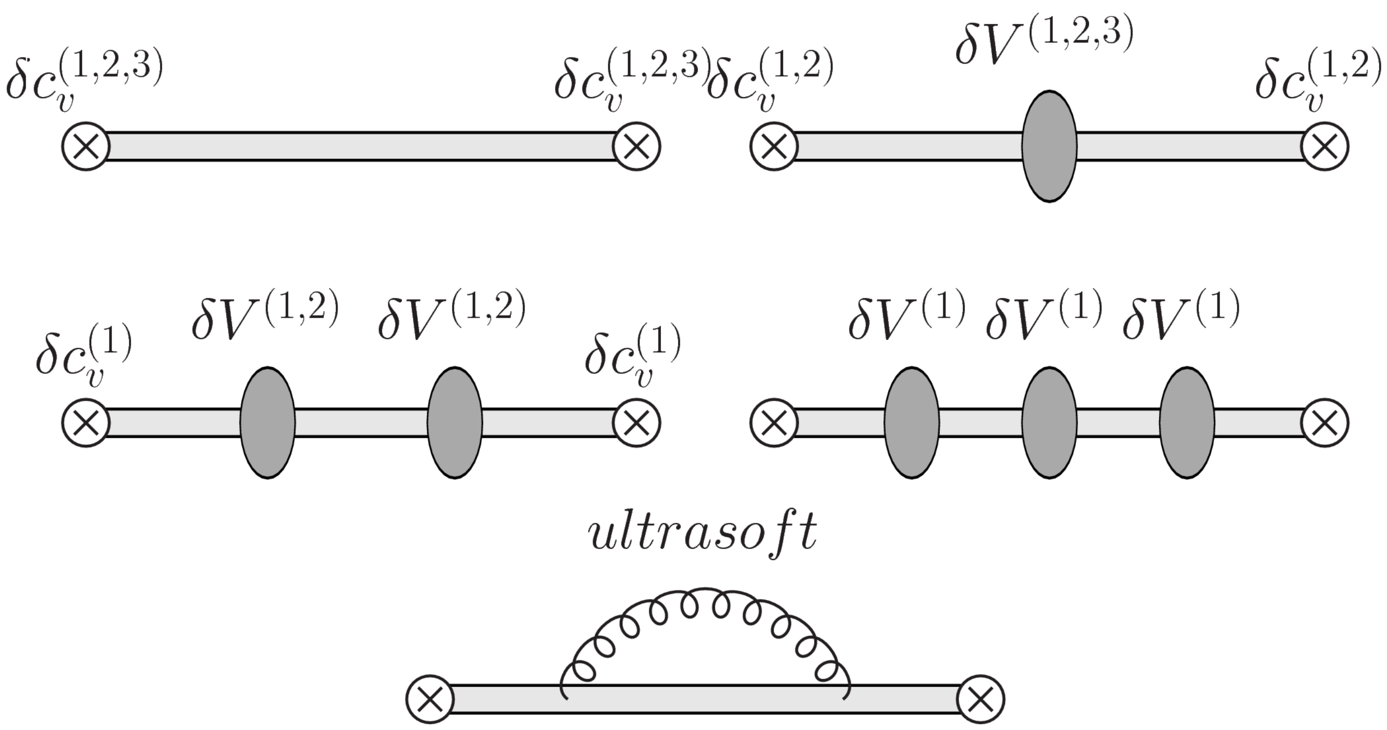
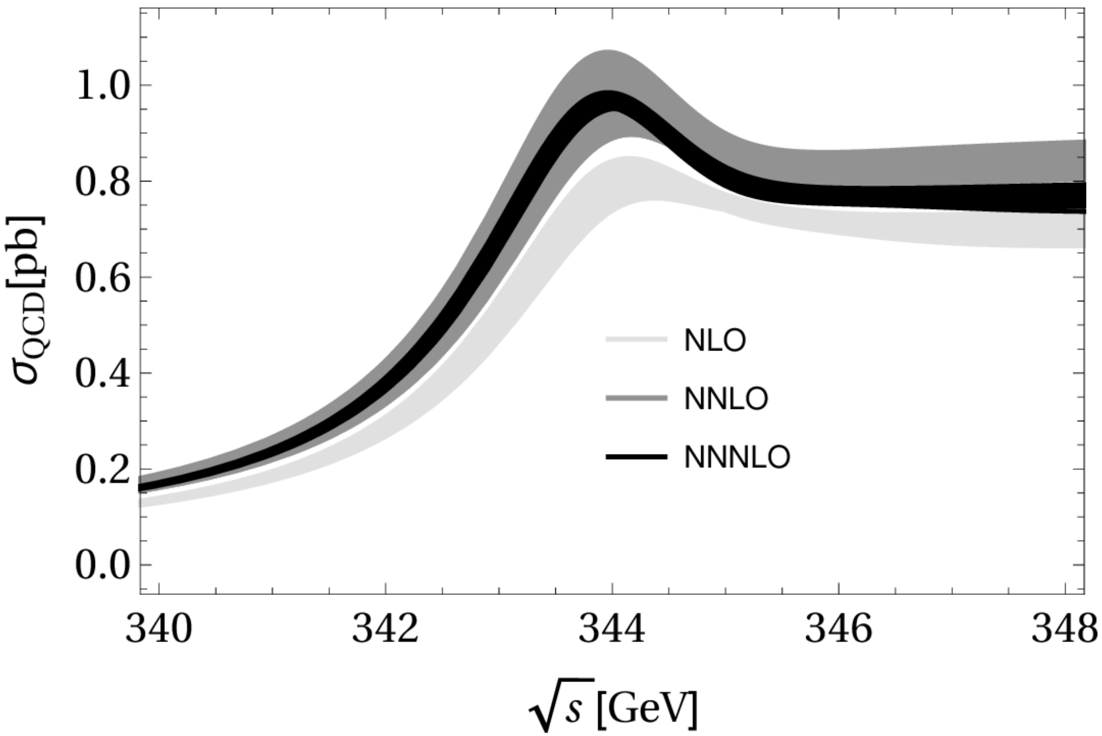
Resummed cross section at NNNLO:



NNNLO QCD cross section



- NNNLO S-wave: [Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser '15]
- NLO P-wave: [Beneke, Piclum, TR '13]
- QQbar_Threshold code: [Beneke, Kiyo, Maier, Piclum '16; Beneke, Maier, TR, Ruiz-Femenía 17??.?????]

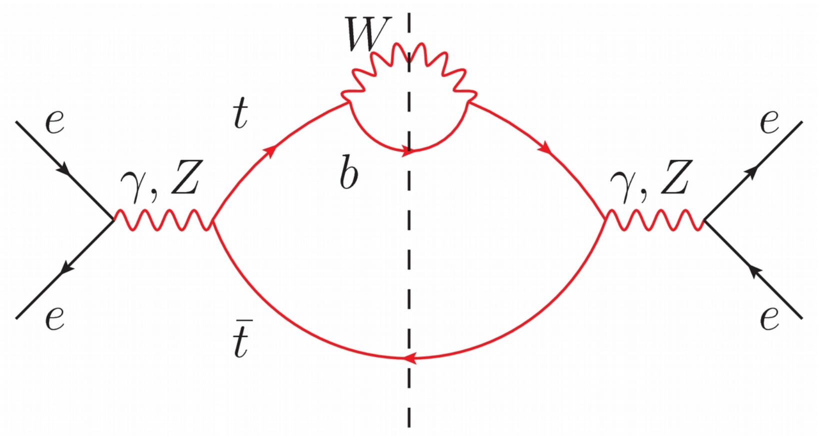
- Stabilization of perturbative expansion at NNNLO
- 3% uncertainty due to scale variation from 50 to 350 GeV
- Similar conclusions at NNLL (5% uncertainty) [Hoang, Stahlhofen '13]

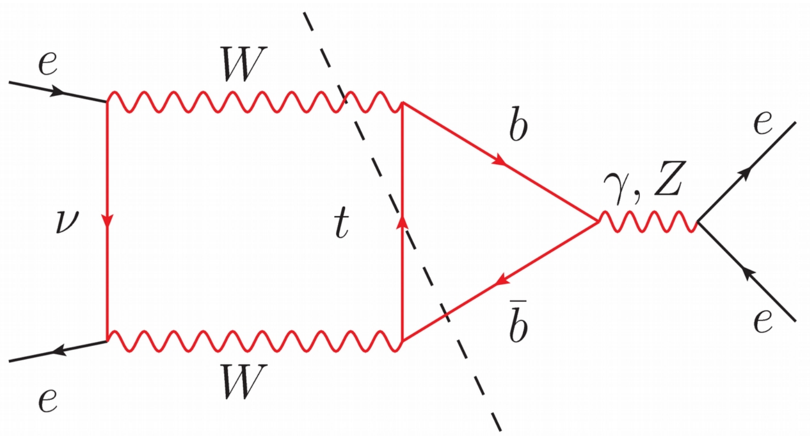
Non-QCD effects

The physical final state is $\mathrm{W}^+W^-b\bar{b}$

- $\Gamma_t \sim m_t\alpha \sim m_t v^2$ is not suppressed with respect to the usoft scale
- Narrow width approximation is unphysical!
- Top decay modifies cross section in non-perturbative way

Top instability implies existence of contributions to the cross section from hard subgraphs that connect to the initial and final state





Generalization of the EFT setup

Contributions can be organized systematically within Unstable Particle Effective Theory [Beneke, Chapovsky, Signer, Zanderighi '03-04]

$$\sigma \sim \mathrm{Im}\Bigg\{ \sum_{k,l} C_p^{(k)} C_p^{(l)} \int d^4x \left\langle e^+e^- \middle| \mathrm{T}[i\mathcal{O}_p^{(k)\dagger}(0) i\mathcal{O}_p^{(l)}(x)] \middle| e^+e^- \right\rangle_{\mathrm{EFT}} + \sum_k C_{4e}^{(k)} \left\langle e^+e^- \middle| i\mathcal{O}_{4e}^{(k)}(0) \middle| e^+e^- \right\rangle_{\mathrm{EFT}} \Bigg\}$$

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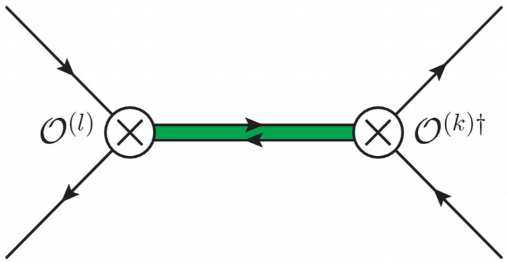
Resonant contribution involving non-rel. tops. Width resummed into propagators $E \to E + i\Gamma_t$



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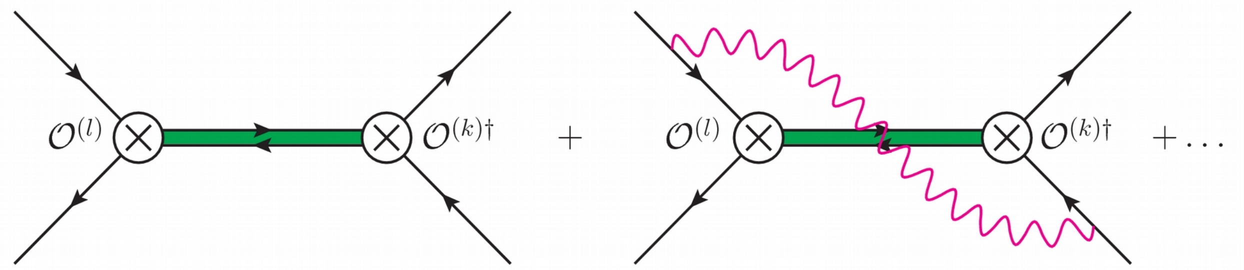

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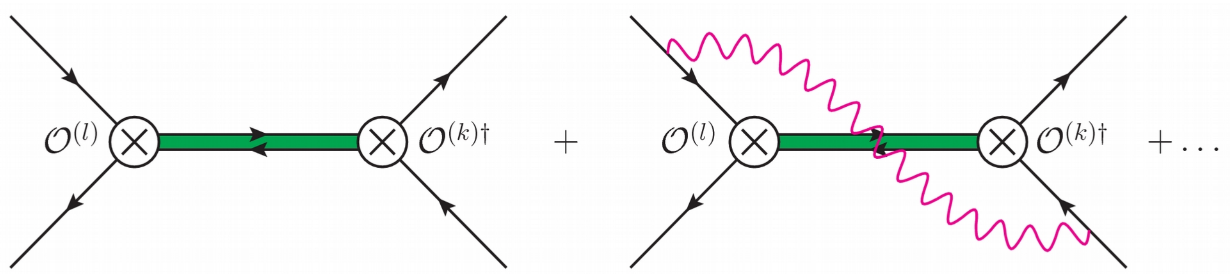

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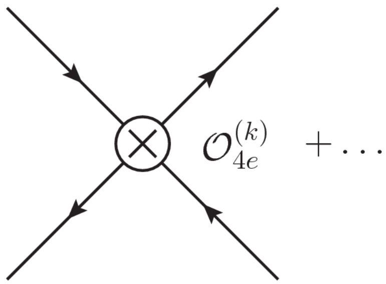
Non-resonant contribution from $W^+W^-b\bar{b}$ production in hard process



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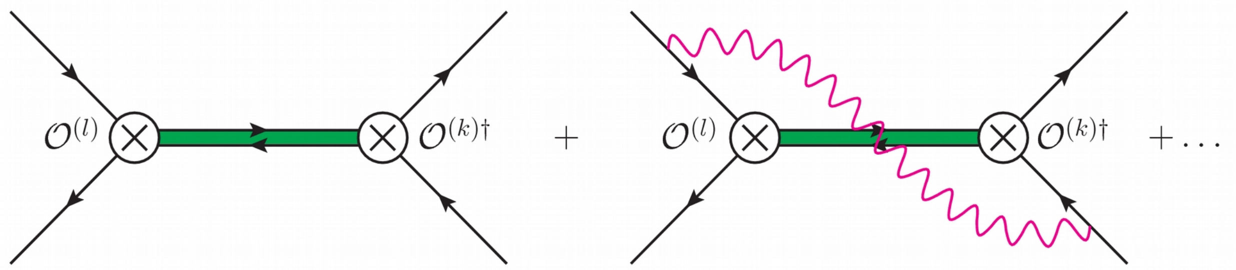

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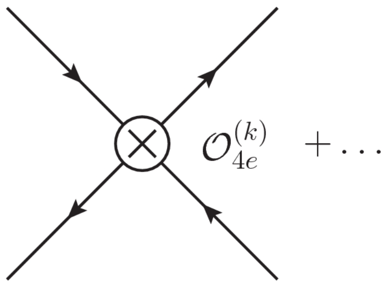
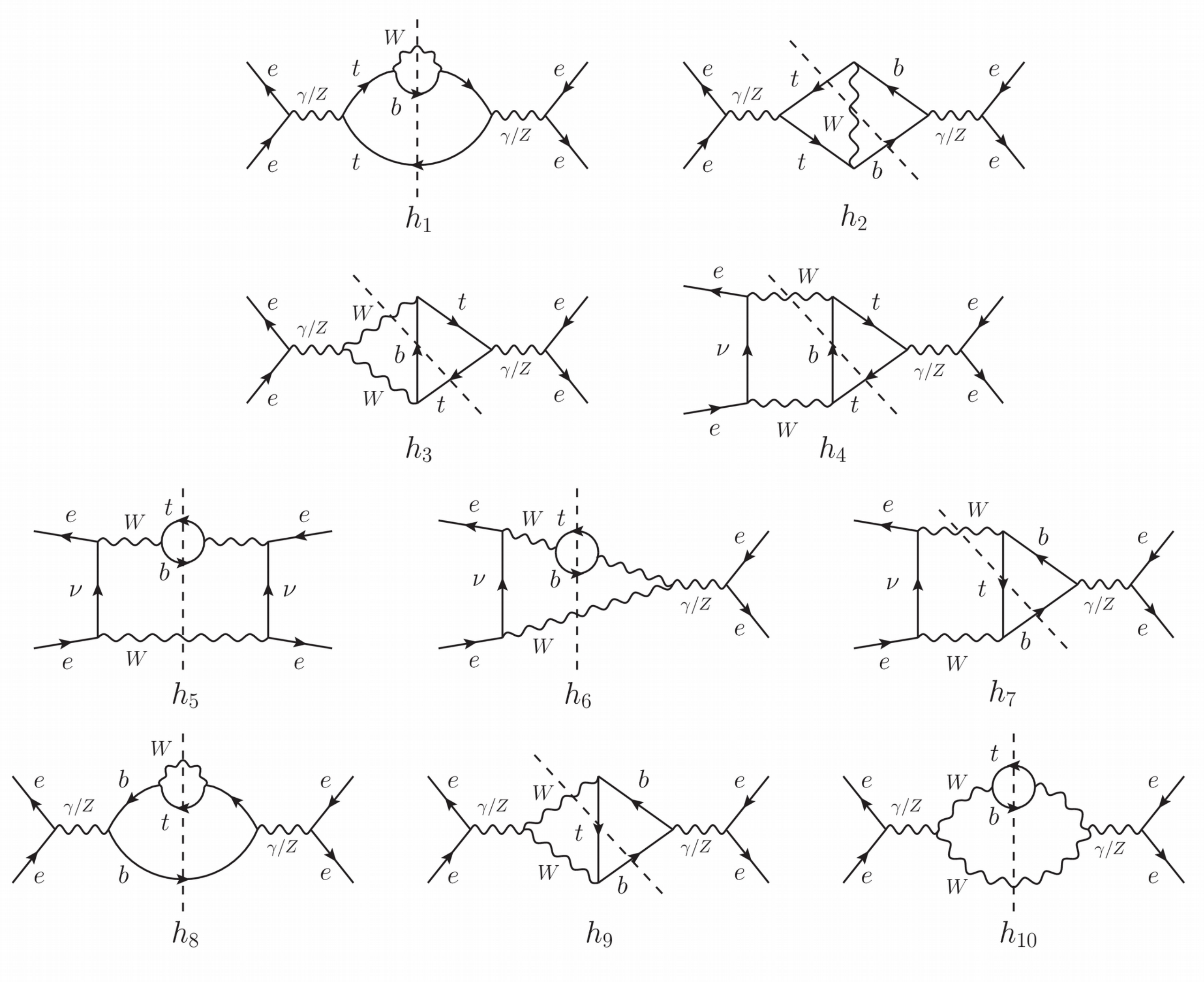


Both parts contain spurious divergences! Only the sum is finite. Calculations must be done in the same regularization scheme.

Endpoint divergences

Non-resonant part
at NLO



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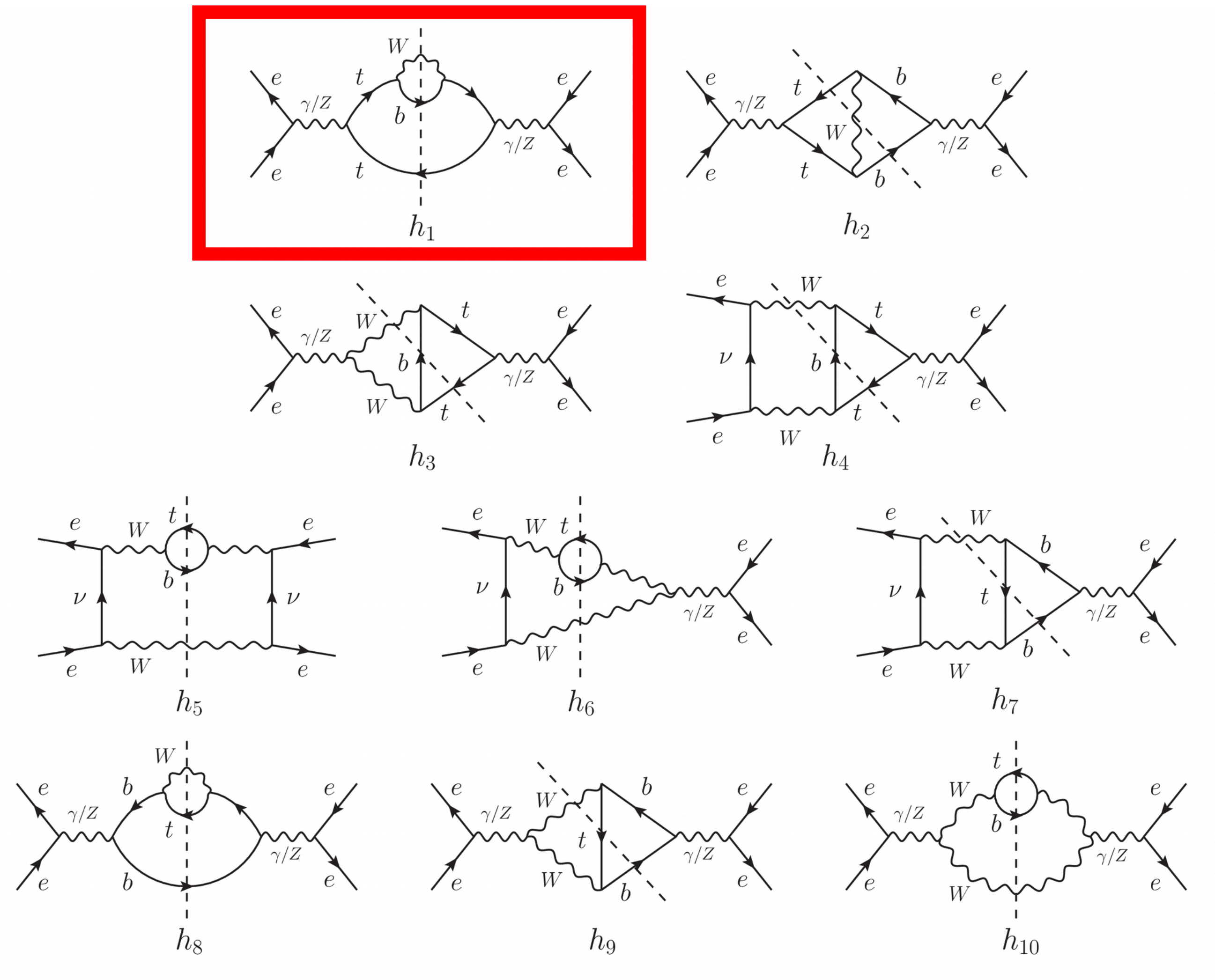

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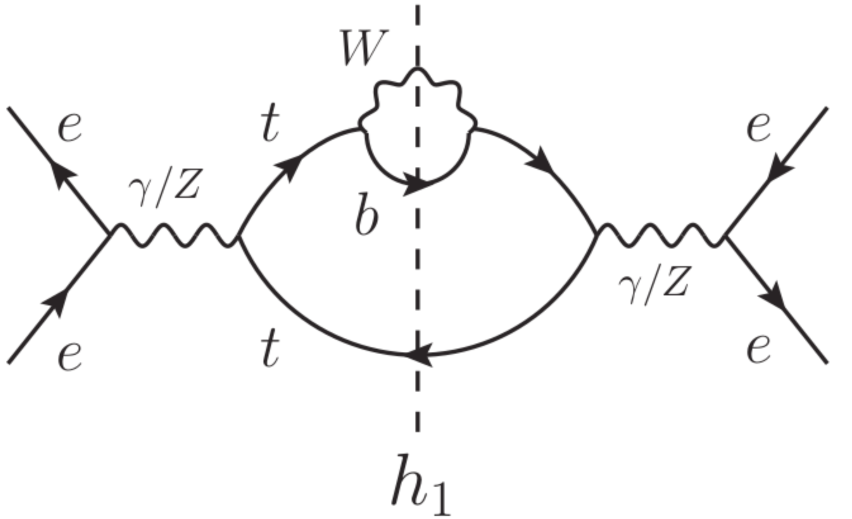

$\mathrm{t} = \mathrm{p}_t^2/m_t^2$

$$\int_y^1 dt g_i(t) \equiv \int d\mathrm{LIPS}_{e^+e^-\to\bar{t}W^+b}\, f_i(p_{e^+}, p_{e^-}, p_{\bar{t}}, p_{W^+}, p_b)\, \theta\big((p_{W^+}+p_b)^2 - y m_t^2\big)$$
$$= \frac{m_t^2}{2\pi}\int_y^1 dt \int d\mathrm{LIPS}_{e^+e^-\to t\bar{t}} \int d\mathrm{LIPS}_{t\to W^+b}\, f_i(p_{e^+}, p_{e^-}, p_{\bar{t}}, p_{W^+}, p_b),$$

Endpoint divergences

Non-resonant part
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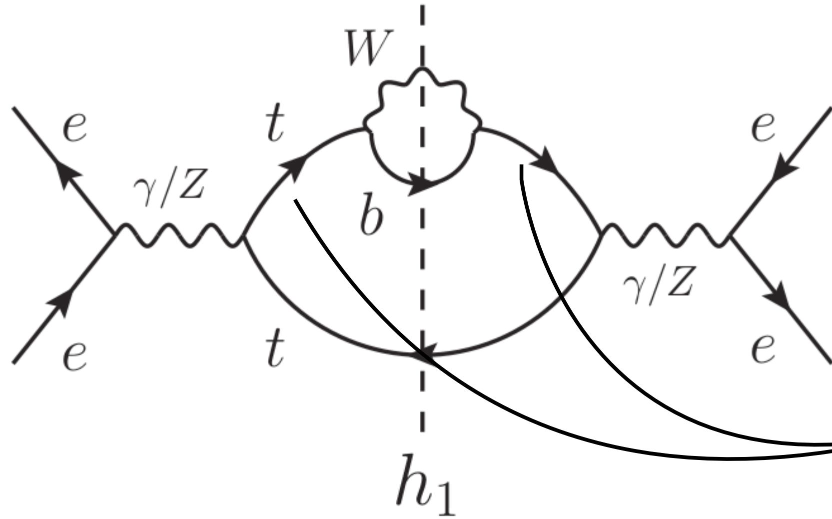

Phase-space
factor

$$g_1(t \to 1) \sim \frac{(1-t)^{1/2-\epsilon}}{(1-t)^2}$$

$$t = p_t^2/m_t^2$$

$$\int_y^1 dt g_i(t) \equiv \int d\mathrm{LIPS}_{e^+e^-\to \bar{t}W^+b}\, f_i(p_{e^+}, p_{e^-}, p_{\bar{t}}, p_{W^+}, p_b)\,\theta\big((p_{W^+}+p_b)^2 - y m_t^2\big)$$
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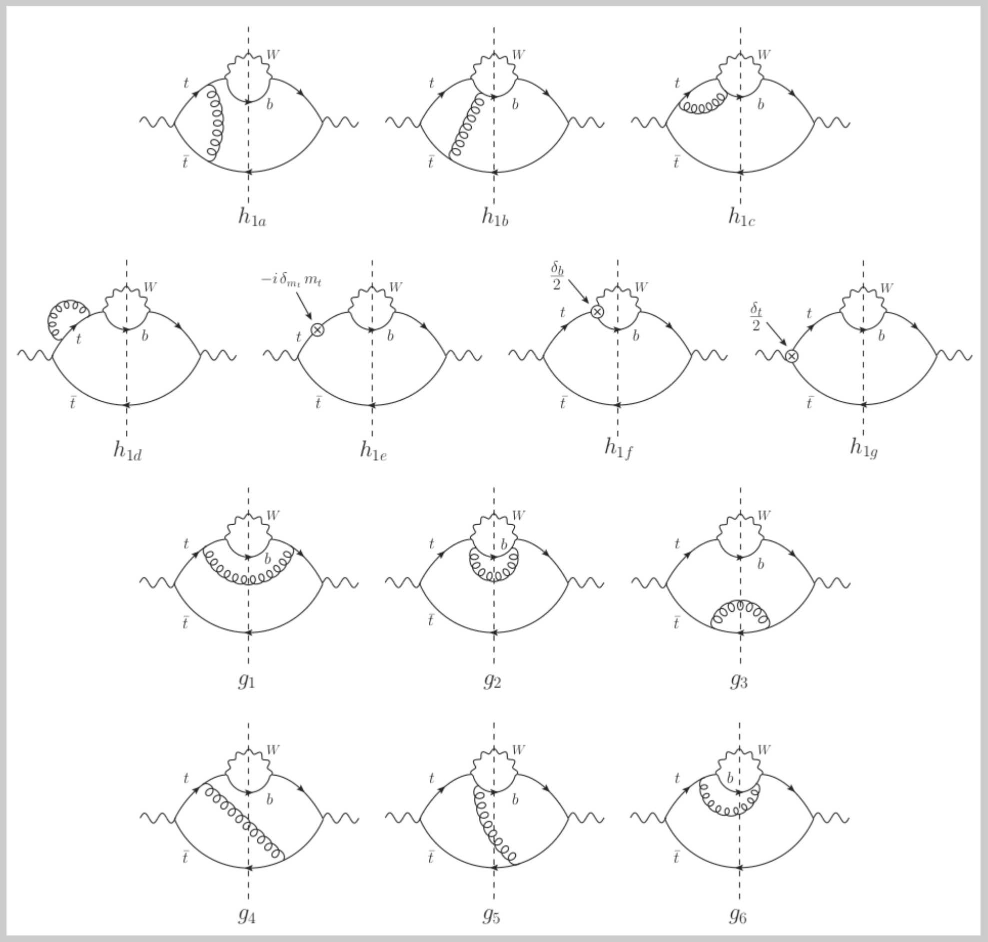
Endpoint divergence for $p_t^2 \to m_t^2$, i.e. $t \to 1$.
Must be regularized dimensionally for consistency with resonant part.
Other diagrams are finite because they contain at most one top propagator.

NNLO non-resonant contribution

Endpoint divergent diagrams identified in [Jantzen, Ruiz-Femenía '13]



'Squared contribution': Gluon corrections to h1, endpoint divergent but UV & IR finite

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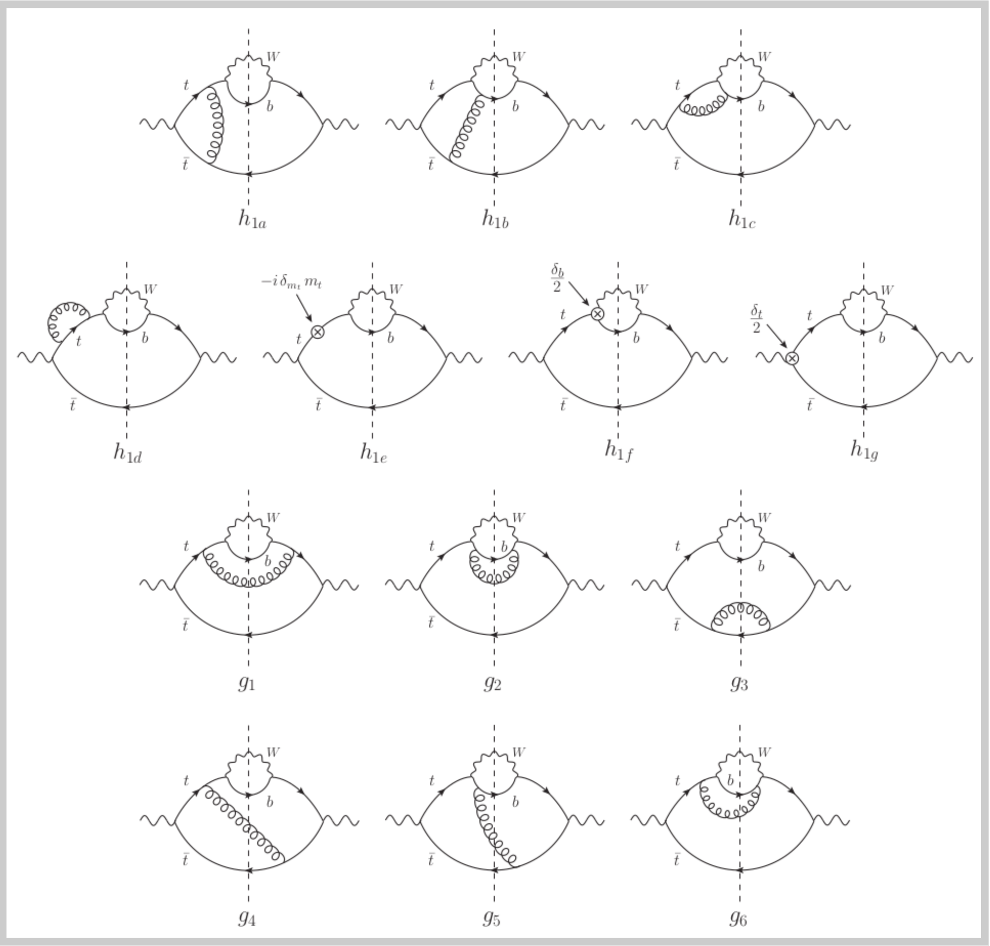

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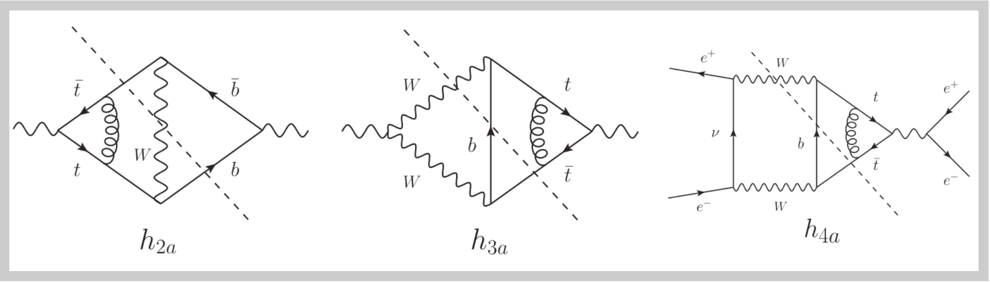

'Interference contribution': endpoint & UV divergent

NNLO non-resonant contribution

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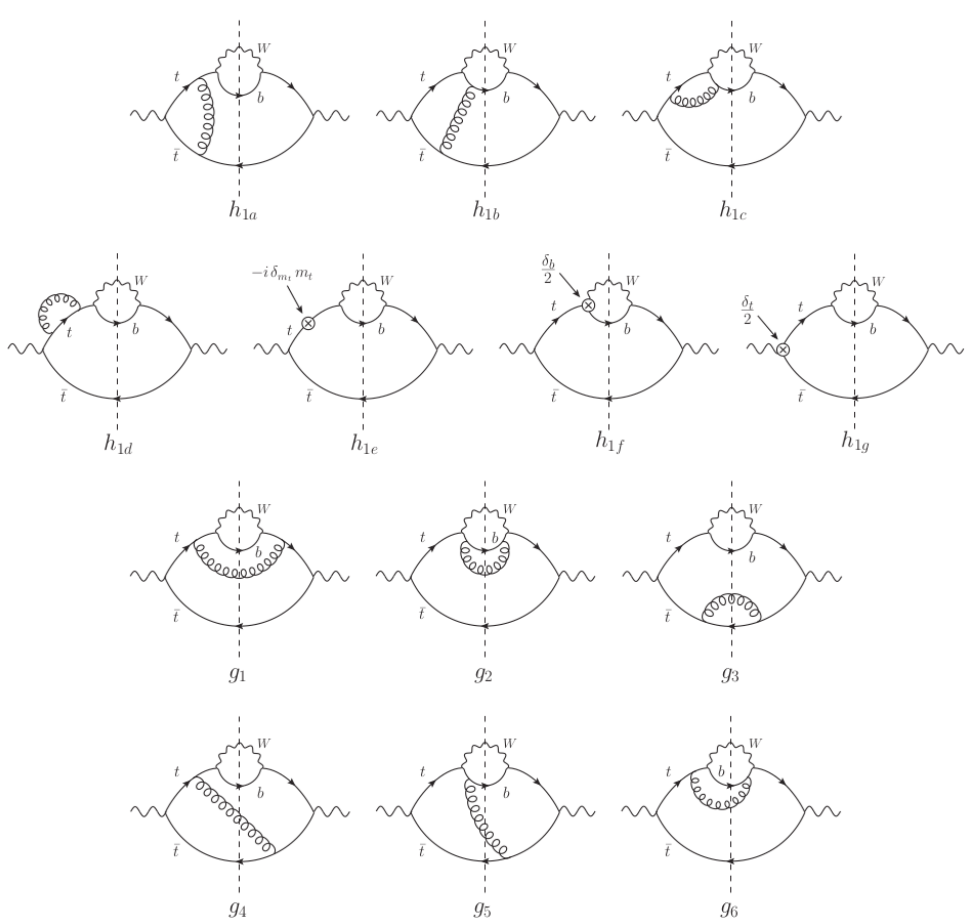

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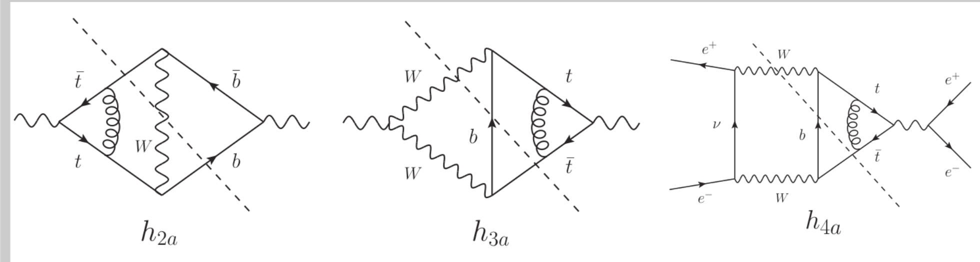


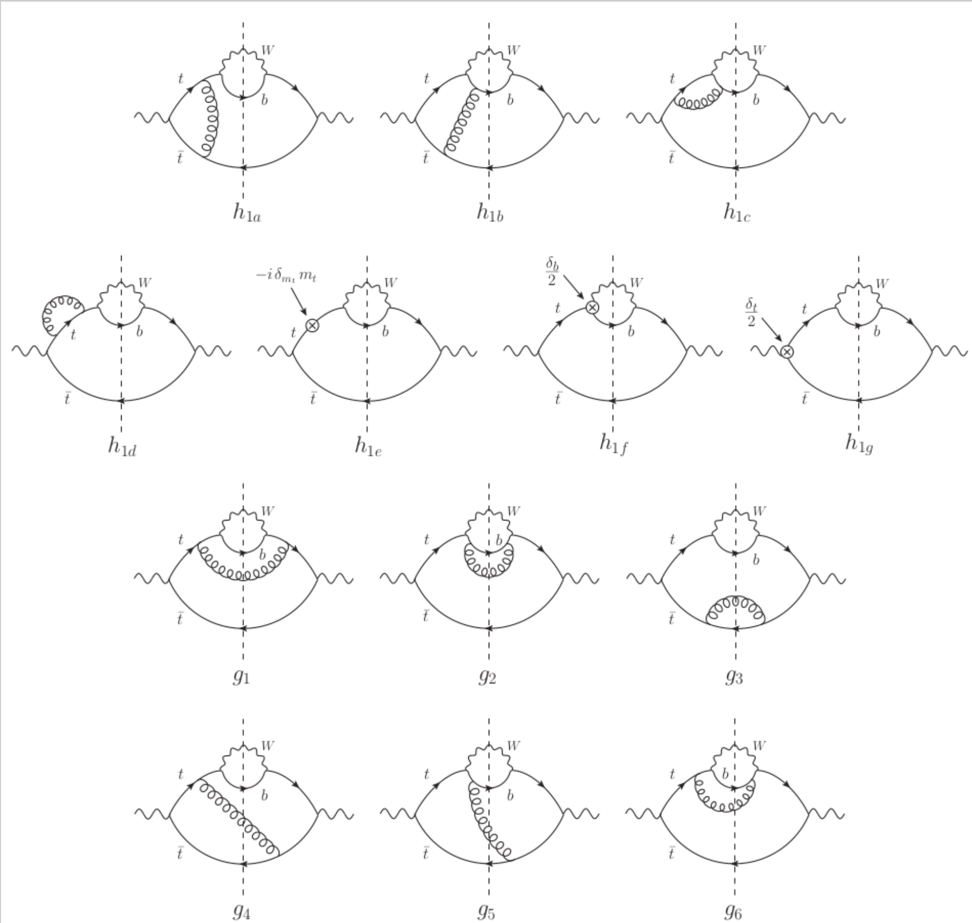
'Interference contribution': endpoint & UV divergent

+ O(100) endpoint finite diagrams (not drawn)

'Automated contribution': endpoint finite but UV divergent, computed with automated tools (MadGraph)

NNLO non-resonant contribution

Endpoint divergent diagrams identified in [Jantzen, Ruiz-Femenía '13]



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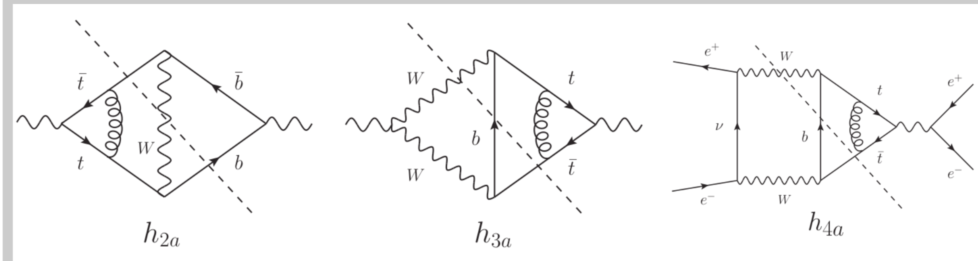

'Interference contribution': endpoint & UV divergent

Cancels in sum

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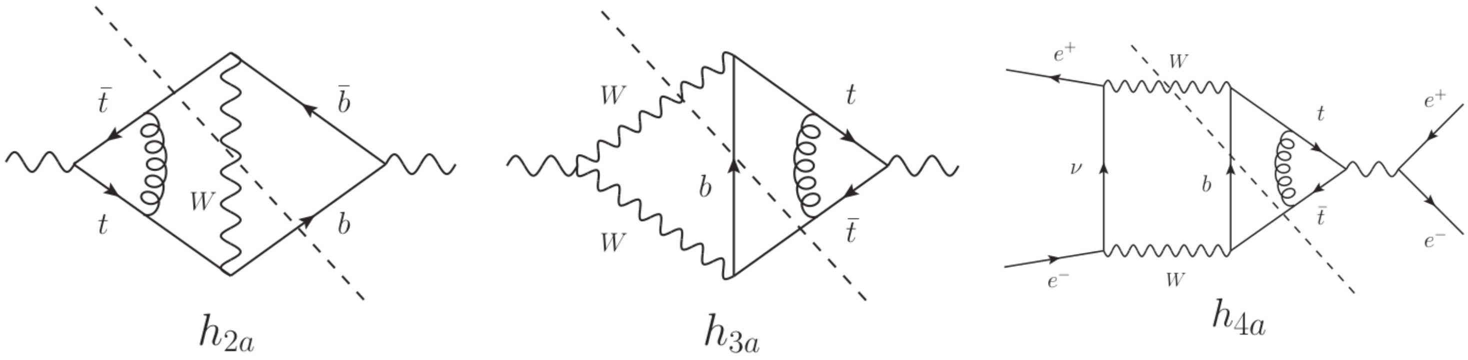
'Interference contribution'



Split endpoint and UV divergences:

Subtraction terms: [Jantzen, Ruiz-Femenía '13; Beneke, Maier, TR, Ruiz-Femenía 17??.?????]

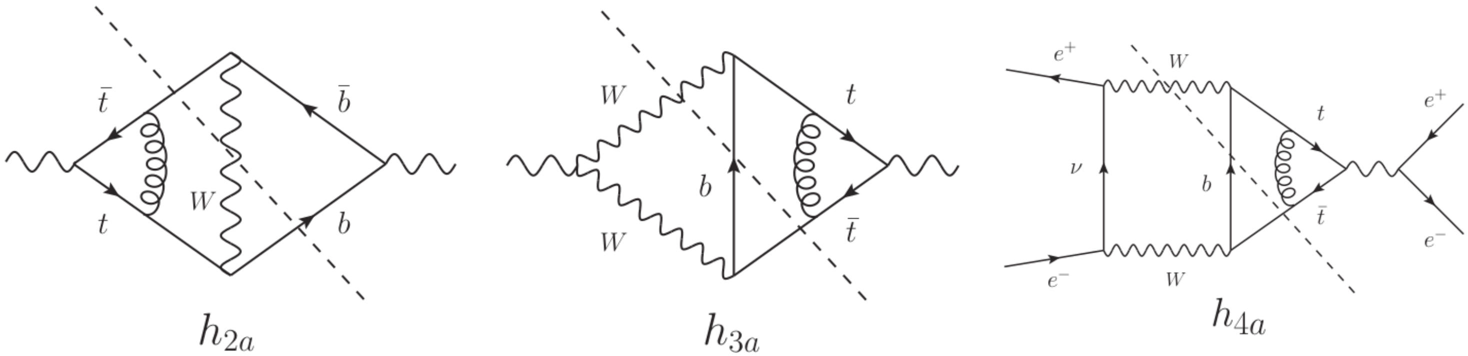
$$\int_y^1 dt\, g_{ia}(t) = \int_y^1 dt \left[g_{ia}(t) - \sum_n \frac{\hat{g}_{ia}^{(1,n)}}{(1-t)^{1+n\epsilon}} \right] + \sum_n \frac{\hat{g}_{ia}^{(1,n)} (1-y)^{-n\epsilon}}{-n\epsilon}$$

$$\sigma_{\text{int}} = \sigma_{\text{int}}^{(\text{EP fin})} + \sigma_{\text{int}}^{(\text{EP div})}$$

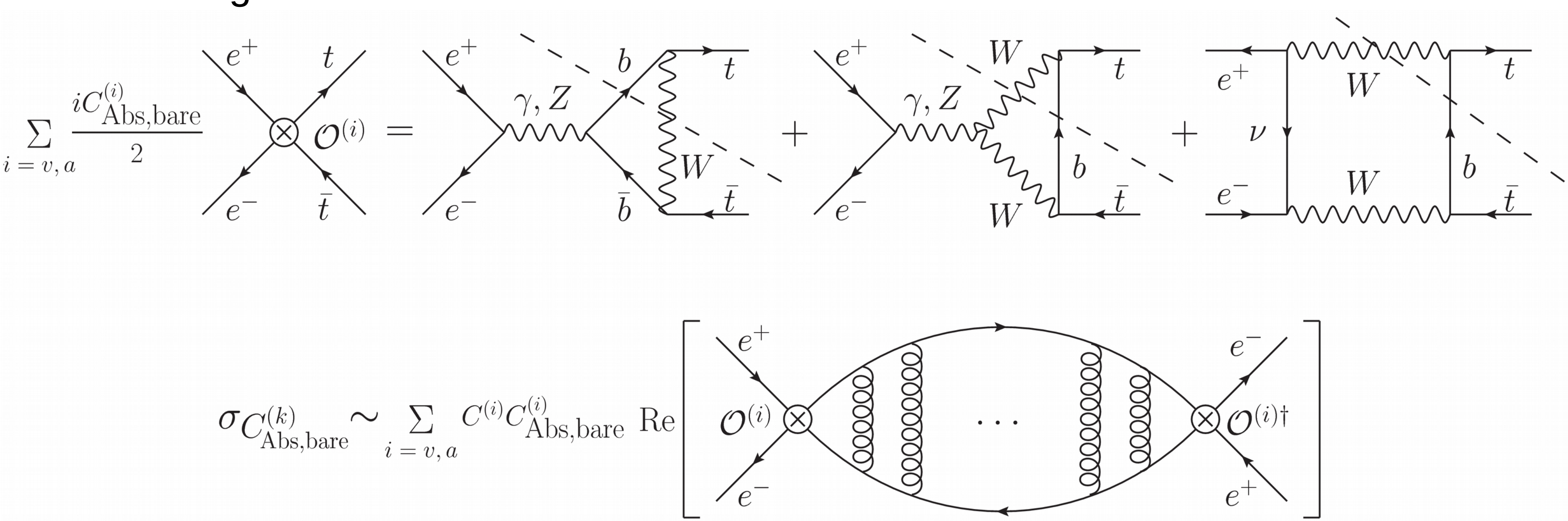
EP finite but
UV divergent

EP divergent
but UV finite

'Interference contribution'



Cancellation of endpoint divergences with contribution from absorptive matching coefficients



Organization of the computation

Split cross section into three separately finite parts (I), (II) and (III):

$$\sigma^{\mathrm{NNLO}} = \underbrace{\left[\sigma_{\mathrm{sq}} + \sigma_{\mathrm{res,\ rest}}\right]}_{(\mathrm{I})} + \underbrace{\left[\sigma_{\mathrm{int}}^{(\mathrm{EP\ div})} + \sigma_{C^{(k)}_{\mathrm{Abs,bare}}}\right]}_{(\mathrm{II})} + \underbrace{\left[\sigma_{\mathrm{int}}^{(\mathrm{EP\ fin})} + \sigma_{\mathrm{aut}}\right]}_{(\mathrm{III})}$$

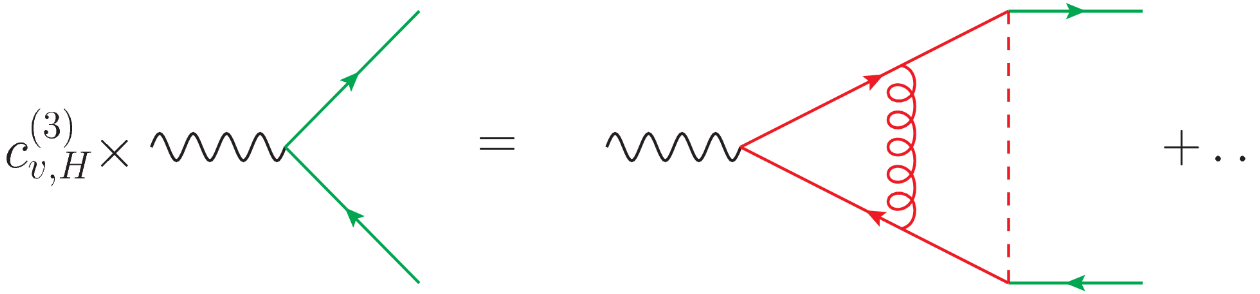
- (I): computational scheme for 'squared contribution' fixed by existing QCD results (Dim reg with NDR for γ^5)
- (II): Use freedom of scheme choice to simplify calculation (some parts done in four dimensions)
- (III): Endpoint finite part of 'interference contribution' must be computed consistent with MadGraph

[Beneke, Maier, TR, Ruiz-Femenía 17??.?????]

Resonant EW corrections

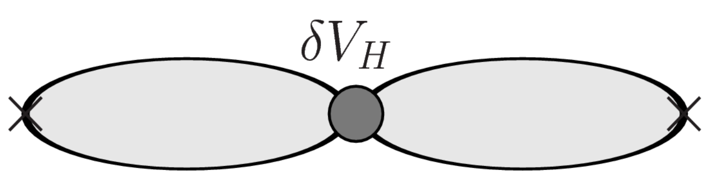
- Higgs contributions up to NNNLO

  - Hard matching coefficients [Eiras, Steinhauser '06]



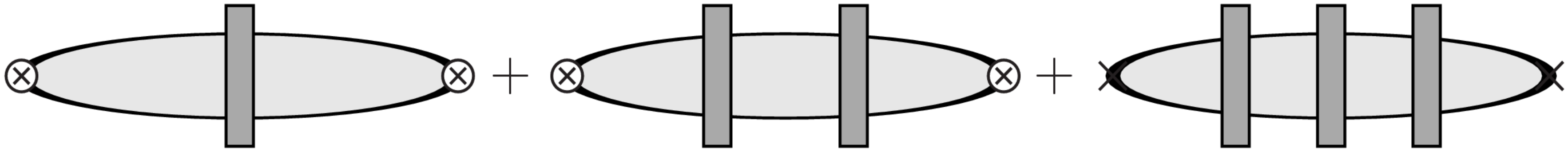
  - Local (not Yukawa) Higgs potential [Beneke, Maier, Piclum, TR '15]

$$\frac{1}{\mathbf{q}^2 + m_H^2} \sim \frac{1}{m_H^2} + \mathcal{O}\left(\frac{\mathbf{q}^2}{m_H^2} \sim v^2\right) \quad \xrightarrow{\text{FT}} \quad \frac{\delta^{(3)}(\mathbf{r})}{m_H^2}$$



Resonant EW corrections

- Higgs contributions up to NNNLO
- QED Coulomb potential



[Beneke, Maier, Piclum, TR '15]

Resonant EW corrections

- Higgs contributions up to NNNLO
- QED Coulomb potential
- Width corrections

$$\Gamma_n = 2\Gamma_t - \frac{\Gamma_t \alpha_s^2 C_F^2}{4n^2} + \dots$$

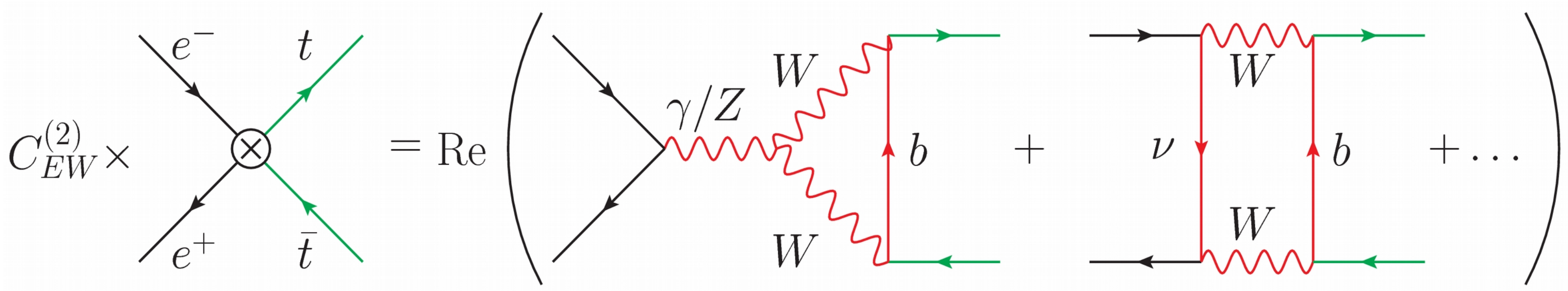
toponium width

time dilatation effects

[Hoang, Reisser, Ruiz-Femenía '10; Beneke, Maier, TR, Ruiz-Femenía 17??.?????]

Resonant EW corrections

- Higgs contributions up to NNLO
- QED Coulomb potential
- Width corrections
- Hard matching coefficients



[Grzadkowski, Kühn, Krawczyk, Stuart '87; Guth, Kühn '92; Hoang, Reißer '04-06; Beneke, Maier, TR, Ruiz-Femenía 17??.?????]

Resonant EW corrections

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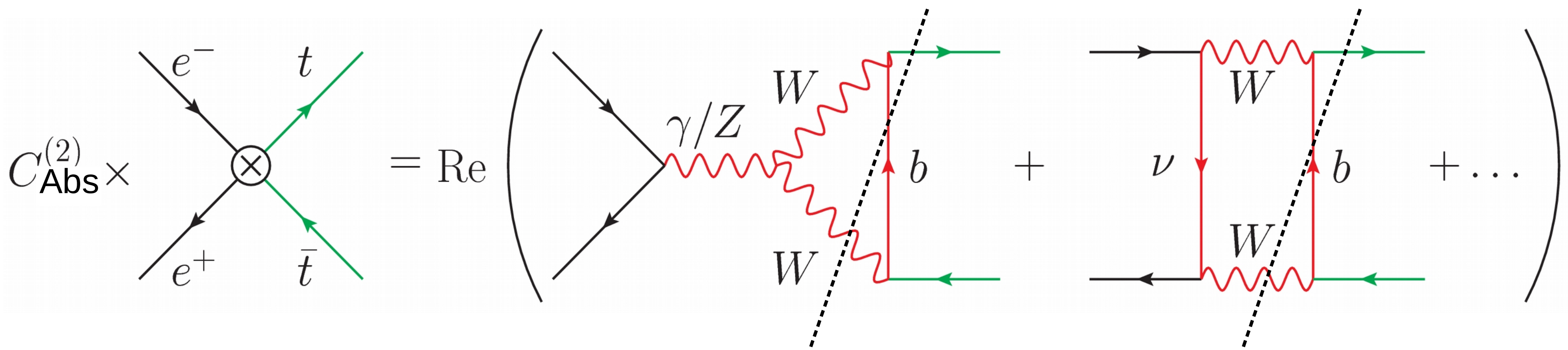
- QED Coulomb potential
- Width corrections
- Hard matching coefficients
- Absorptive part of hard matching coefficients



[Grzadkowski, Kühn, Krawczyk, Stuart '87; Guth, Kühn '92; Hoang, Reißer '04-06; Beneke, Maier, TR, Ruiz-Femenía 17??.?????]

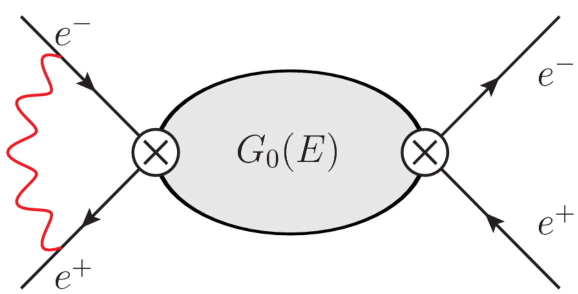
Resonant EW corrections

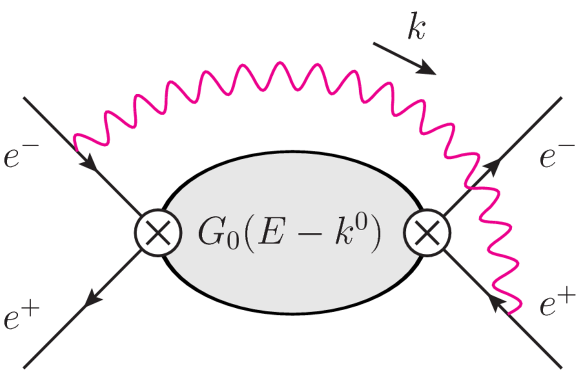
- Higgs contributions up to NNLO
- QED Coulomb potential
- Width corrections
- Hard matching coefficients
- Absorptive part of hard matching coefficients
- Initial state radiation

  - Leading logs: electron structure functions [Fadin, Khoze '87]

$$\sigma_{\text{w. ISR}}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}^{\text{LL}}(x_1)\Gamma_{ee}^{\text{LL}}(x_2)\sigma^{\text{conv}}(x_1 x_2 s)$$

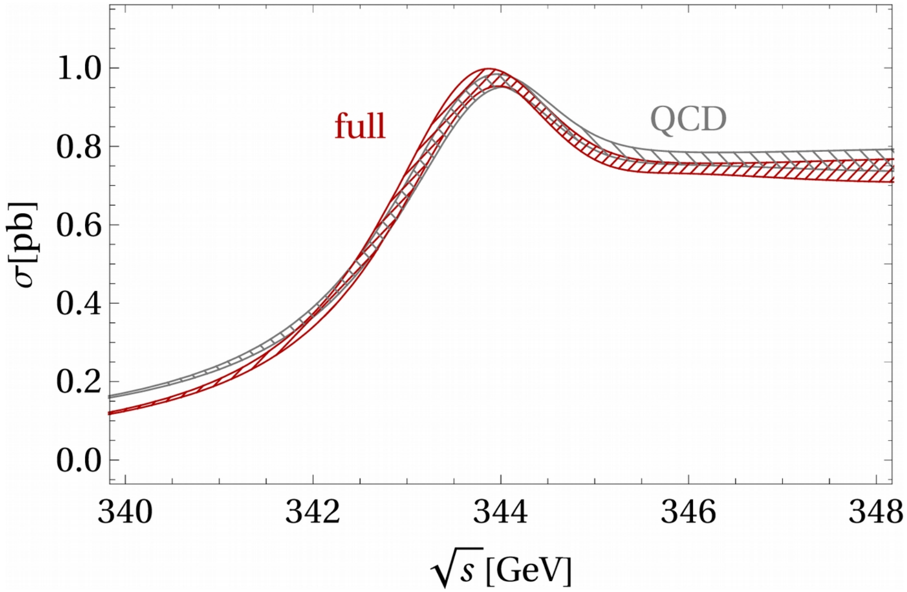
  - NNLO fixed order:

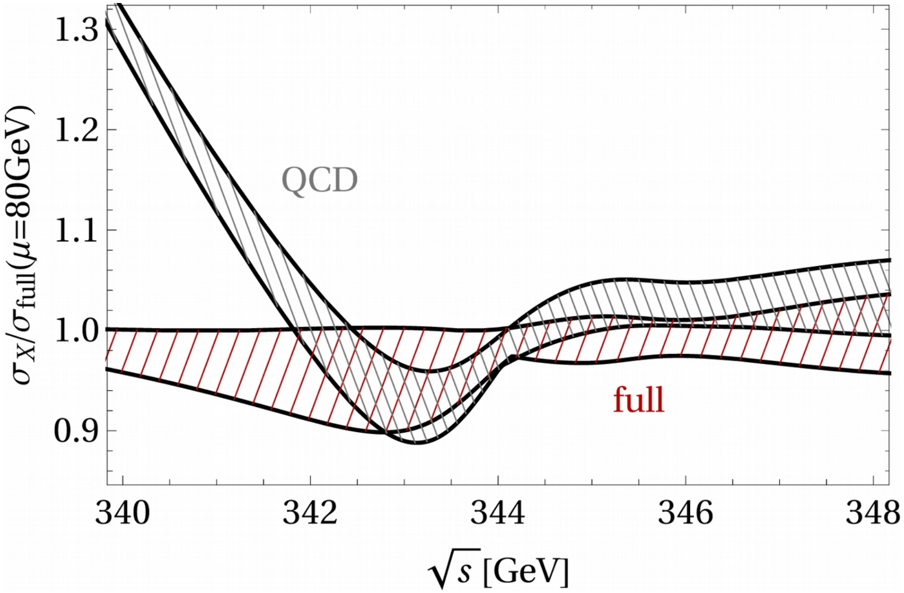




[Beneke, Maier, TR, Ruiz-Femenía 17??.?????]

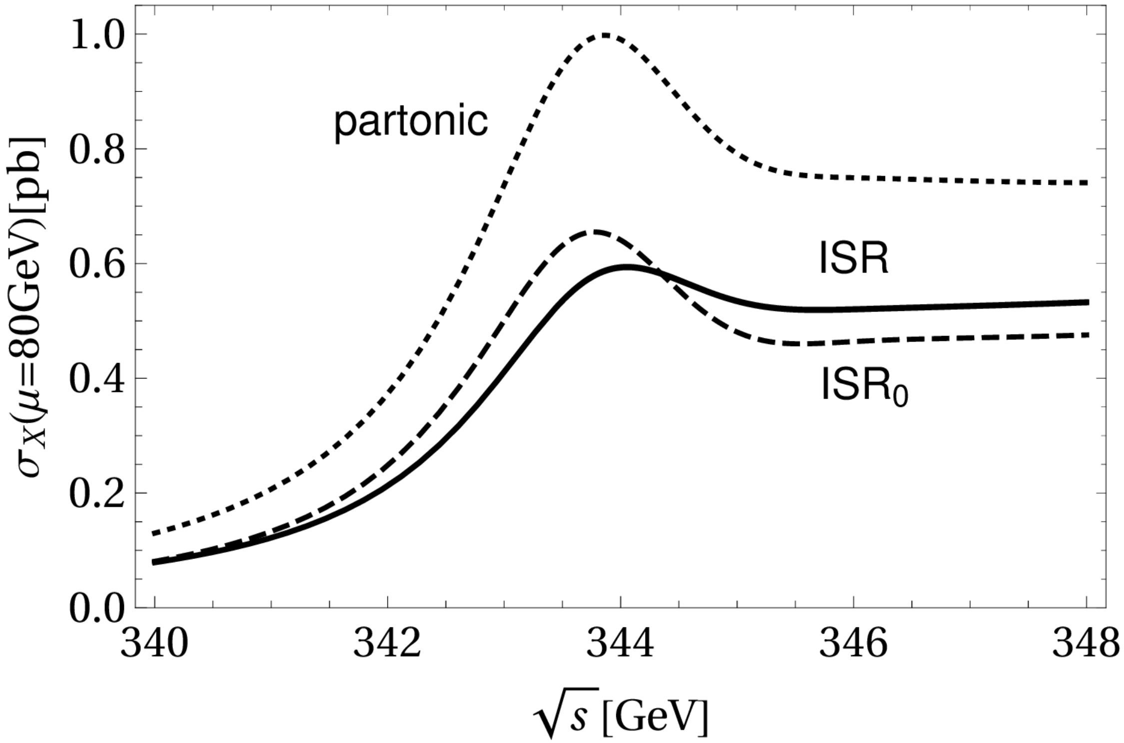
Non-QCD effects





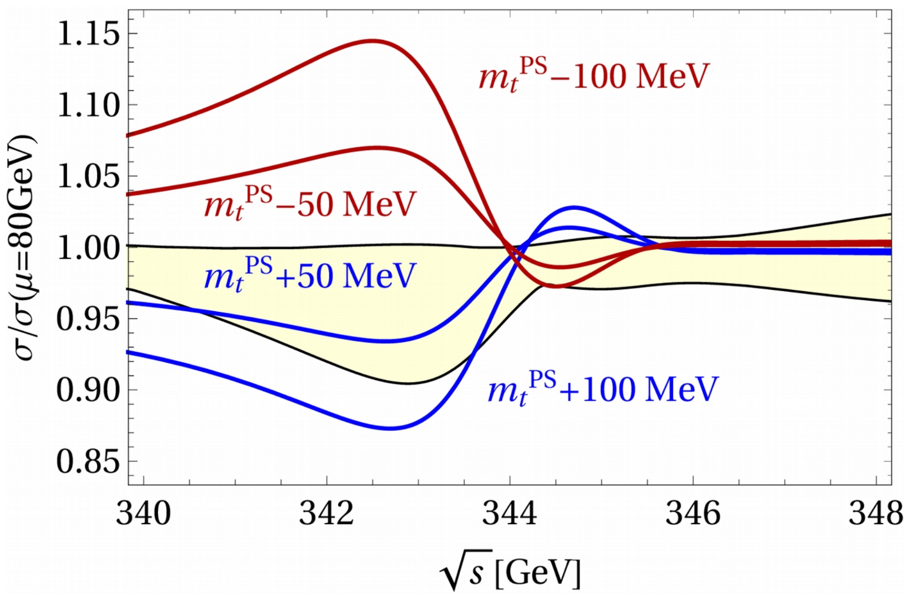
- Uncertainty due to renormalization scale variation between 50 and 350 GeV
- Effects significantly larger than QCD uncertainty
- Shape changes particularly in the important region at and below threshold

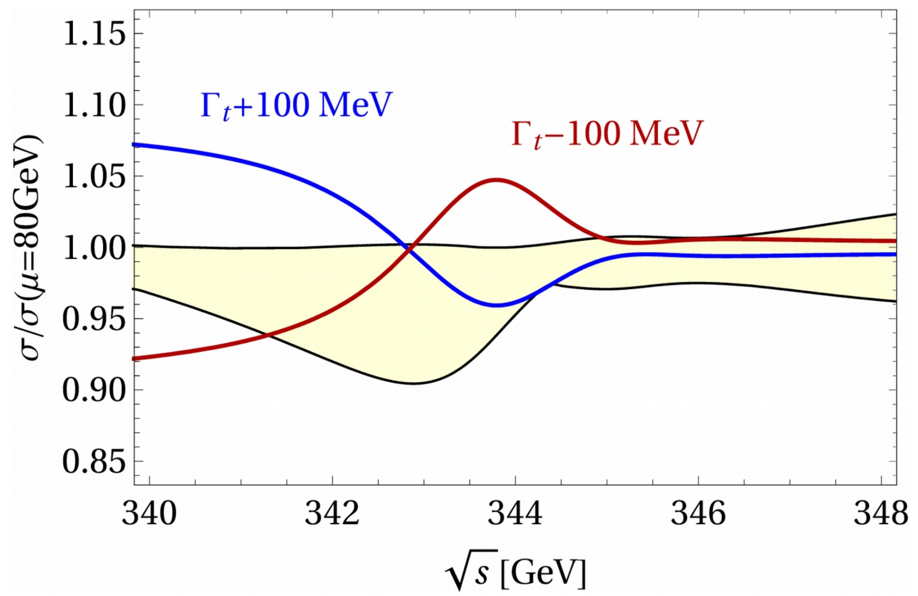
Initial state radiation



- ISR reduces cross section by 30-45 %
- NLL precision is a must for a lepton collider (not just for ttbar)

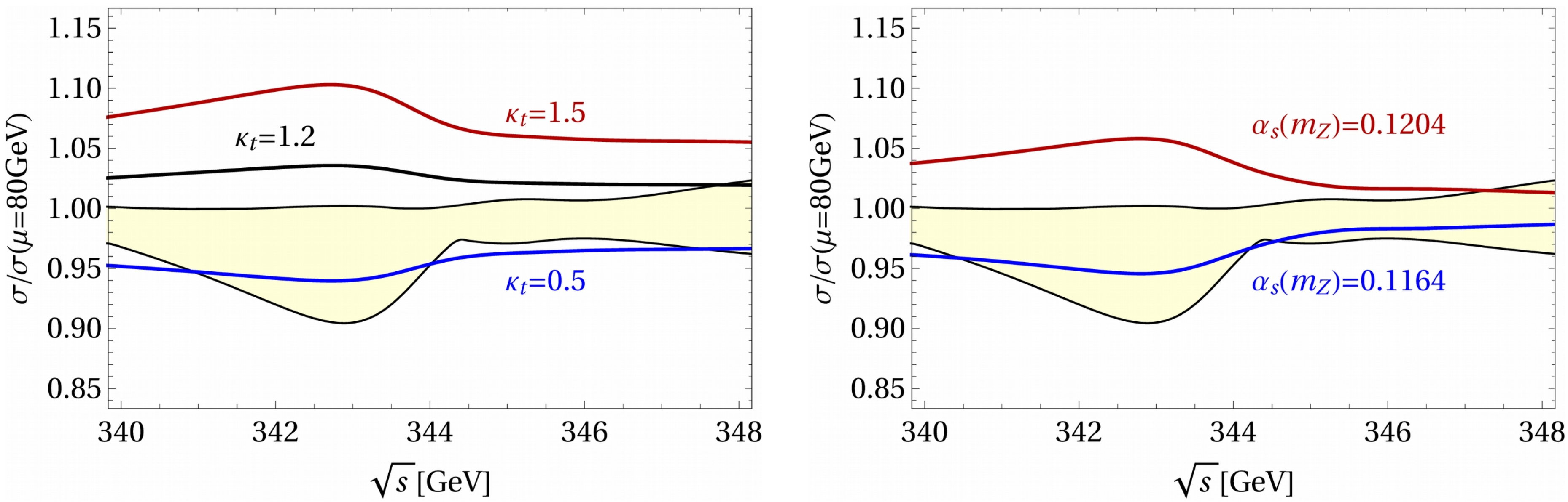
Parameter sensitivity





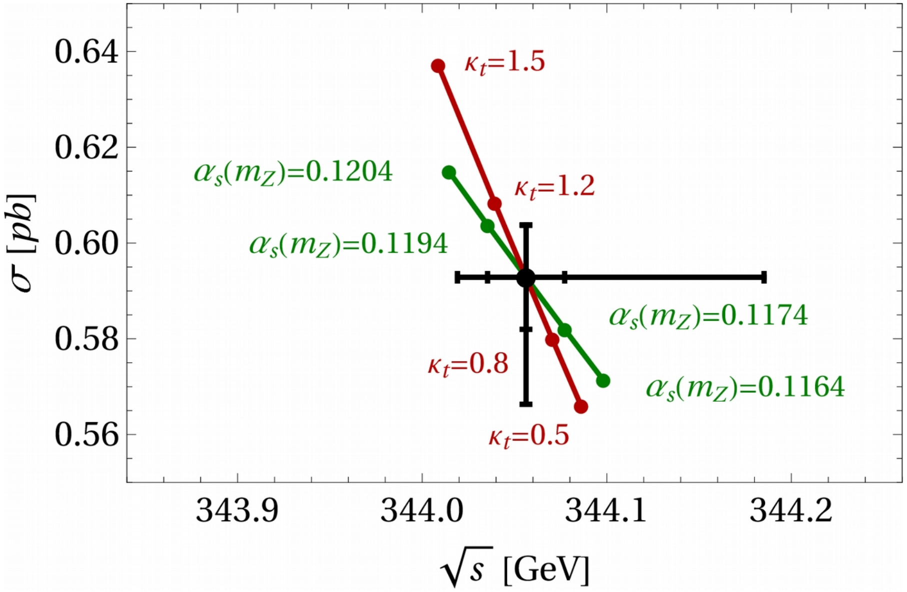
- Estimate theory uncertainty by determining what parameter shift is needed to obtain curves outside the scale variation band
- Naive expectation: $\delta m_t^{\mathrm{PS}} \approx 40\ \mathrm{MeV}$ and $\delta\Gamma_t \approx 60\ \mathrm{MeV}$
- Full simulation: theory uncertainty $\delta m_t^{\mathrm{PS}} \approx 40\ \mathrm{MeV}$ [Simon '16]
- Statistical uncertainty: $\delta m_t^{\mathrm{PS}} = 18\ \mathrm{MeV}$ (ILC) [Simon '16]

Parameter sensitivity



- Consider rescaling of top Yukawa coupling $y_t = \kappa_t y_t^{\mathrm{SM}}$
- Naive expectation: $\delta\kappa_t \approx^{+20}_{-25}$ % and $\delta\alpha_s \approx 0.0015$
- Effects from variation of Yukawa coupling and strong coupling very similar
- Need full simulation to see how well they can be disentangled

Parameter sensitivity



Peak position and height

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Conclusions

- QCD uncertainty under control, non-QCD effects are important
- NNNLO QCD + NNLO SM + NNNLO Higgs known and available in QQbar_Threshold (soon)
- Ultraprecise measurement of m_t and Γ_t from threshold scan
- Sensitive to top Yukawa and strong coupling

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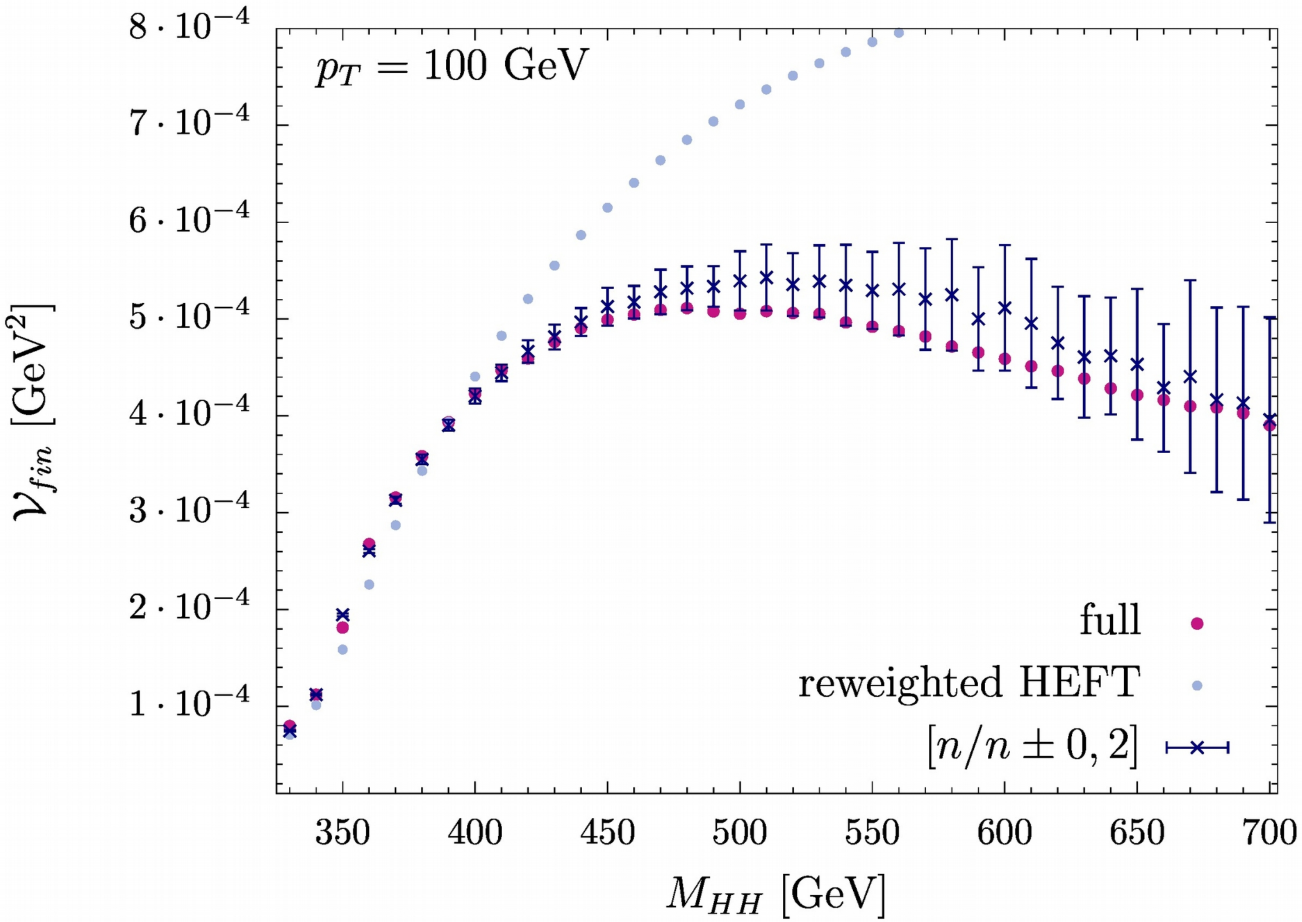
Outlook

- NNNLO + NNLL QCD accuracy
- NNNLO resonant contributions
- NLL ISR
- Other applications of formalism

Application of formalism: Higgs pair production



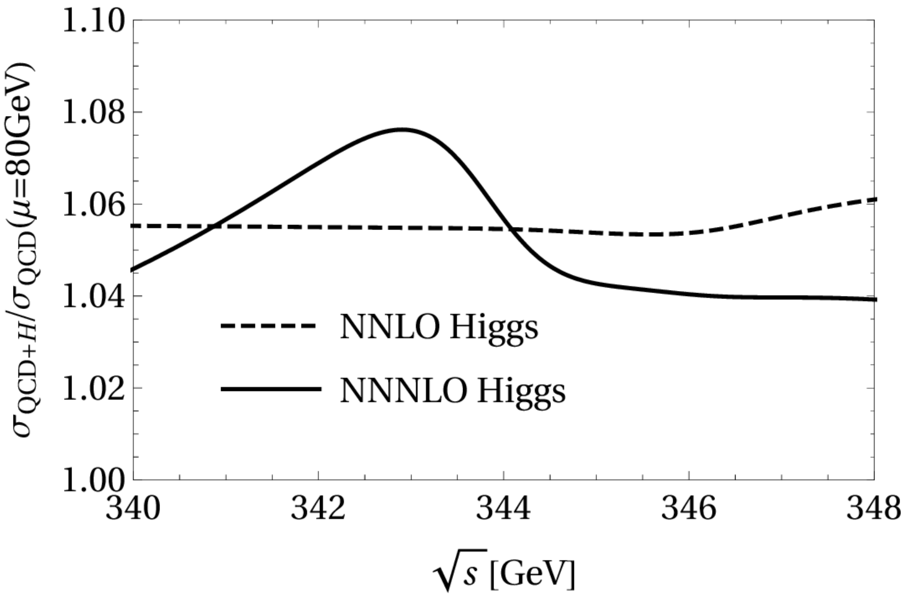
[Gröber, Maier, TR 1709.07799]

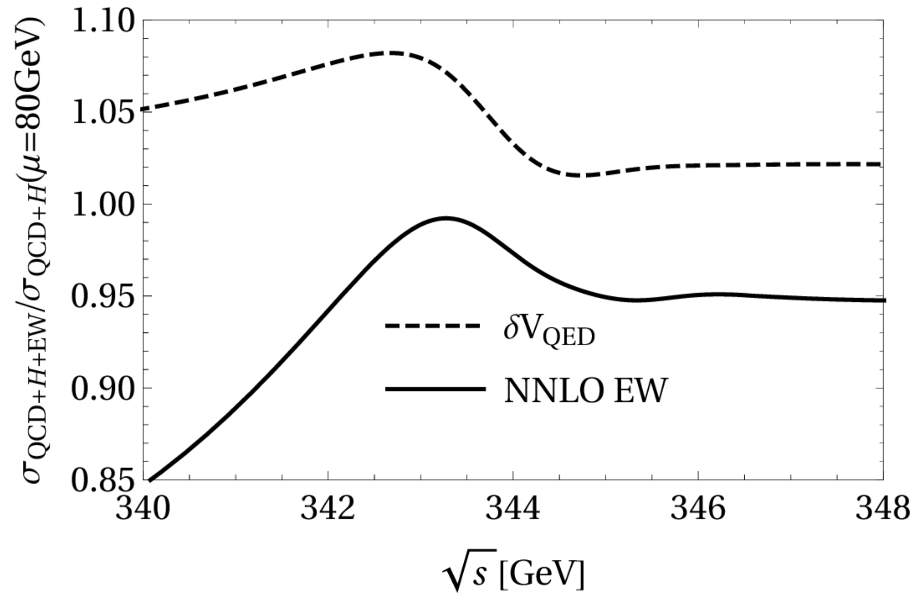
- Reconstructed top-quark mass dependence of 2-loop ggHH amplitude with Pade approximants based on LME and of top threshold expansion
- Can be applied at NNLO and for other gluon-fusion processes

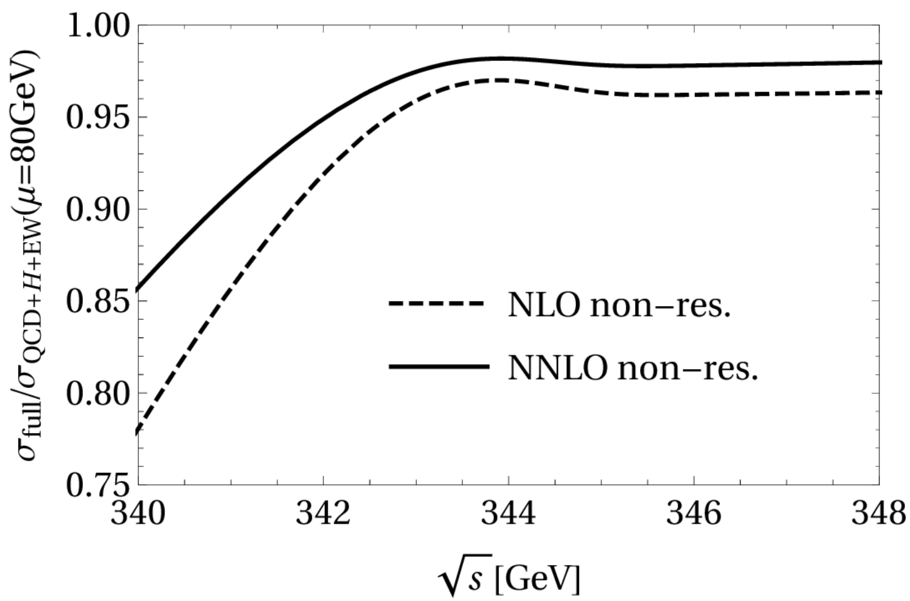
Thank you!

Backup

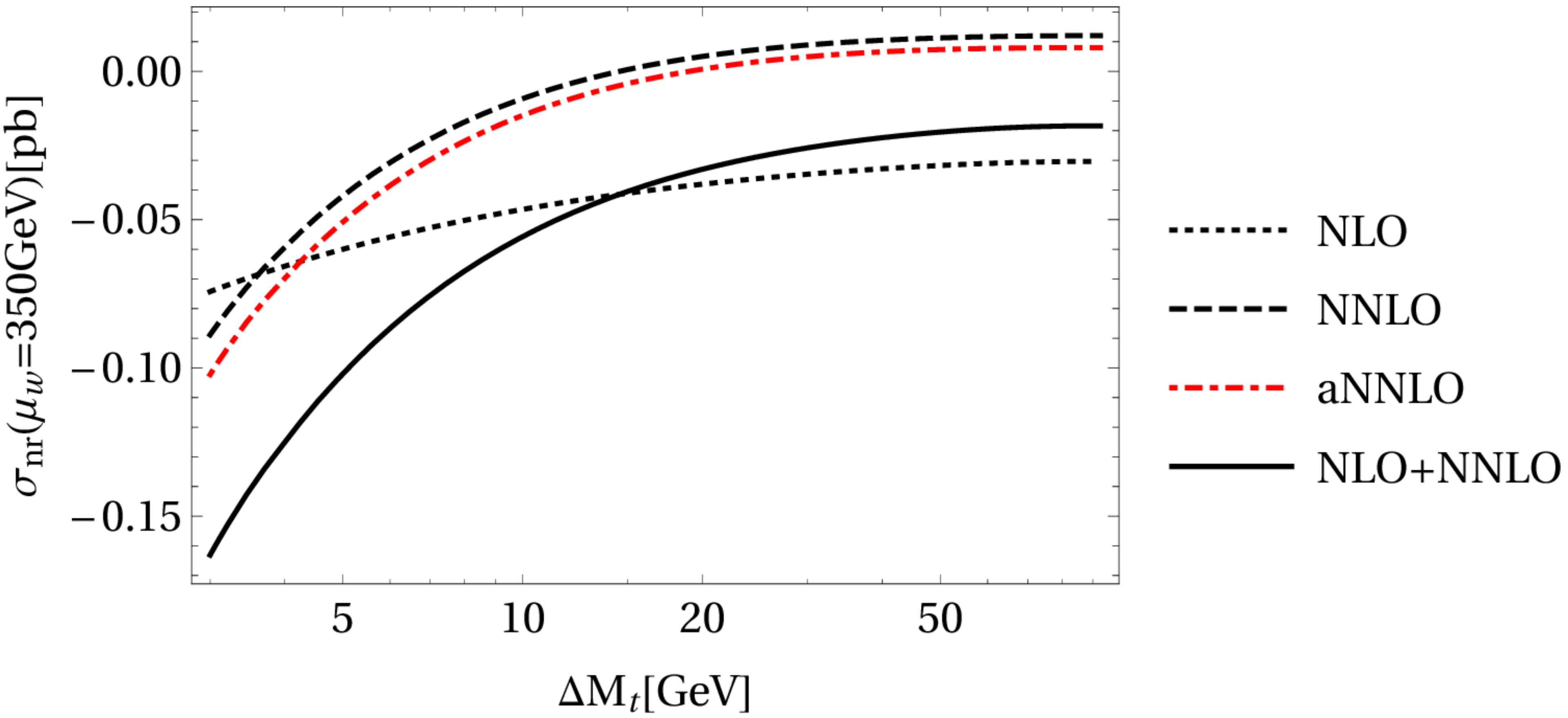
Non-QCD effects







Top invariant mass cuts



$$(\mathrm{m}_t - \Delta M_t)^2 \leq p_{t,\bar{t}}^2 \leq (m_t + \Delta M_t)^2$$

Consistency check