

Factorization and Resummation for Massive Quark Effects in Exclusive Drell-Yan

Daniel Samitz

(University of Vienna)

based on JHEP **08** (2017) 114 [arXiv:1703.09702]

in collaboration with Piotr Pietrulewicz, Anne Spiering and Frank J. Tackmann

RADCOR 2017, St.Gilgen
26 September 2017



universität
wien

$\int dk \prod$ Doktoratskolleg
Particles and Interactions

Outline

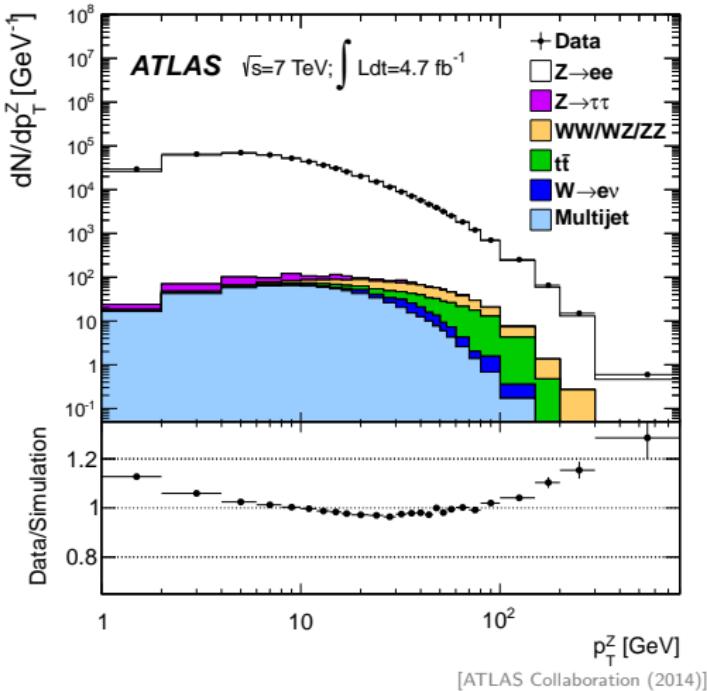
- ① Introduction
- ② Factorization for the p_T spectrum
- ③ Modes and resummation
- ④ Outlook: effects on W boson mass measurements
- ⑤ Conclusions

Outline

- ① Introduction
- ② Factorization for the p_T spectrum
- ③ Modes and resummation
- ④ Outlook: effects on W boson mass measurements
- ⑤ Conclusions

Motivation

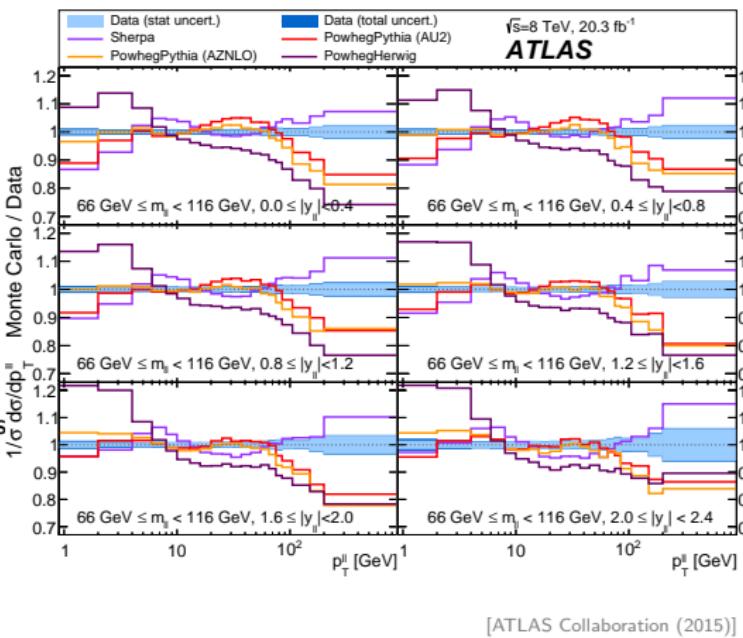
- p_T spectrum of Z-boson measured with high precision
- NNLL' analyses available
- no systematic theoretical description of b-quark mass effects yet can be relevant in m_W measurements
- discrepancies between MC and experiment in low p_T region



goal: systematic treatment of quark mass effects at NNLL' accuracy
for transverse momentum (SCET_{II}) and beam thrust (SCET_{I}) spectrum

Motivation

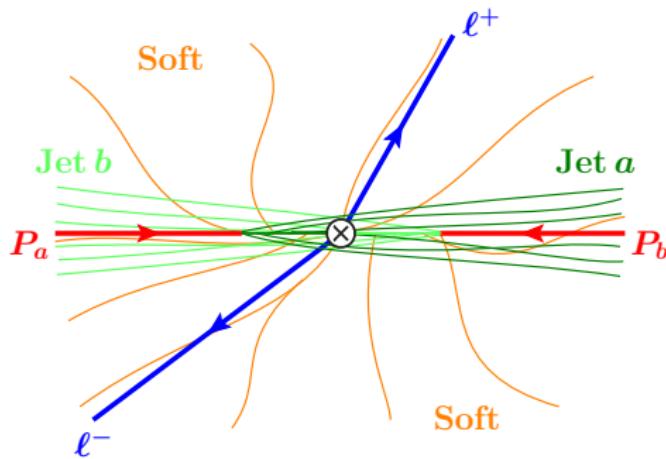
- p_T spectrum of Z-boson measured with high precision
- NNLL' analyses available
- no systematic theoretical description of b-quark mass effects yet can be relevant in m_W measurements
- discrepancies between MC and experiment in low p_T region



[ATLAS Collaboration (2015)]

goal: systematic treatment of quark mass effects at NNLL' accuracy
for transverse momentum (SCET_{II}) and beam thrust (SCET_I) spectrum

Drell-Yan in the low p_T region



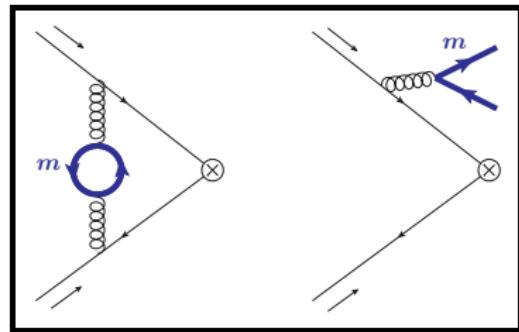
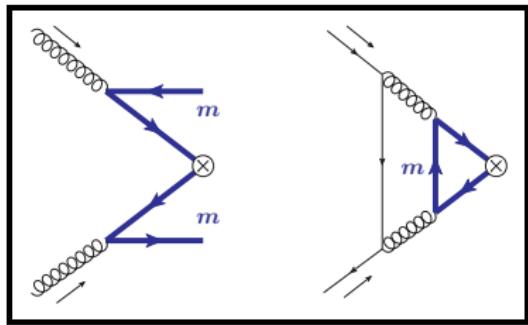
- Drell-Yan + 0 jets
- transverse momentum as jet veto to restrict hadronic final state
- requires resummation of logarithms for $p_T \ll Q$

from: I.W.Stewart,F.J.Tackmann,W.J.Waalewijn, *Phys. Rev.* D81 (2010) 094035

- $p_T = |\sum_i \vec{p}_{T,i}| = |\vec{p}_{T,\ell^+} + \vec{p}_{T,\ell^-}|$
- 0-jet limit for $p_T \ll Q$
- 2 scales (SCET_{II}): hard $\sim Q$, collinear/soft $\sim p_T$
- rapidity logarithms

Massive Quarks in Drell-Yan

primary and secondary **massive** quarks.



- primary: massive quarks go into hard interaction
- secondary: massive quark corrections to light quark induced processes
- both start at $\mathcal{O}(\alpha_s^2)$, relevant for NNLL' resummation
- different hierarchies between p_T and m possible:

$$m \ll p_T$$

$$p_T \sim m$$

$$p_T \ll m$$

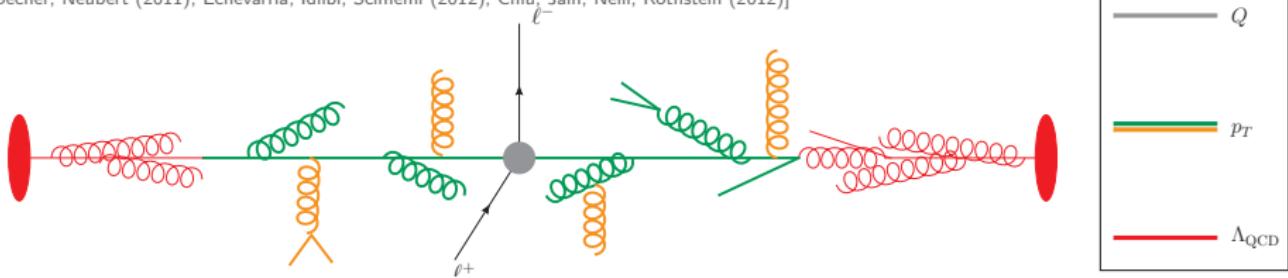
Outline

- ① Introduction
- ② Factorization for the p_T spectrum
- ③ Modes and resummation
- ④ Outlook: effects on W boson mass measurements
- ⑤ Conclusions

Massless Factorization for p_T -spectrum in Drell-Yan

[Collins, Soper, Sterman (1985)]

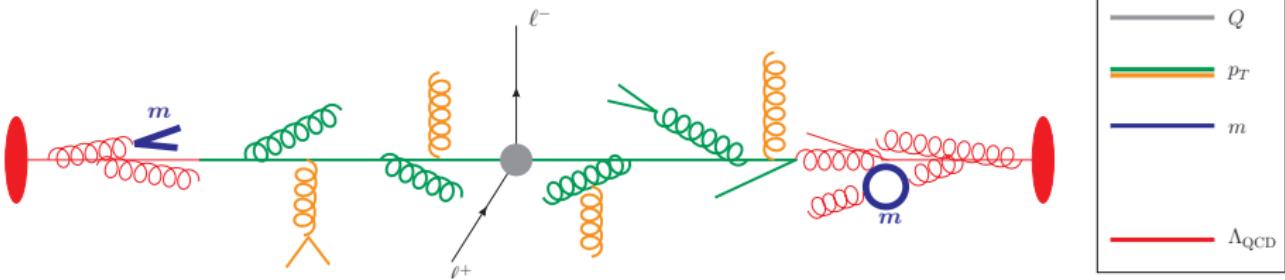
[Becher, Neubert (2011); Echevarria, Idilbi, Scimemi (2012); Chiu, Jain, Neill, Rothstein (2012)]



$$\frac{d\sigma}{dp_T} = \sum_{i \in \{q\}} H_i^{(n_f)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{ij}^{(n_f)}(p_T, x) \otimes f_j^{(n_f)}(x) \right]^2 \otimes_{\perp} \mathcal{S}^{(n_f)}(p_T) + \mathcal{O}\left(\frac{p_T}{Q}, \frac{\Lambda_{\text{QCD}}^2}{p_T^2}\right)$$

- H : hard function, scale Q
- \mathcal{I} : beam function, scale p_T
- \mathcal{S} : soft function, scale p_T
- f : PDF, scale Λ_{QCD}

Factorization for $m \ll p_T$



$$\frac{d\sigma}{dp_T} = \sum_{i \in \{q, Q\}} H_i^{(5)}(Q) \times \left[\sum_{j \in \{q, Q, g\}} \mathcal{I}_{ij}^{(5)}(p_T, x) \otimes f_j^{(5)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{p_T^2}\right)$

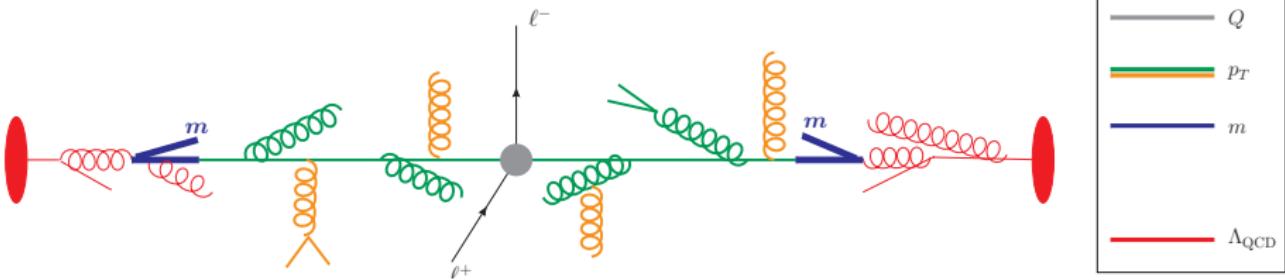
- matching: $f_j^{(5)}(x) = \sum_k \mathcal{M}_{jk}(x, m) \otimes f_k^{(4)}(x)$

- secondary \mathcal{M}_{qg} and primary \mathcal{M}_{Qg} heavy quarks

[M. Buza, Y. Matinounine, J. Smith, W. van. Neerven (1998)]

- hard, **beam** and **soft** functions with **5** massless flavors

Factorization for $m \ll p_T$



$$\frac{d\sigma}{dp_T} = \sum_{i \in \{q, Q\}} H_i^{(5)}(Q) \times \left[\sum_{j \in \{q, Q, g\}} \mathcal{I}_{ij}^{(5)}(p_T, x) \otimes f_j^{(5)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{p_T^2}\right)$

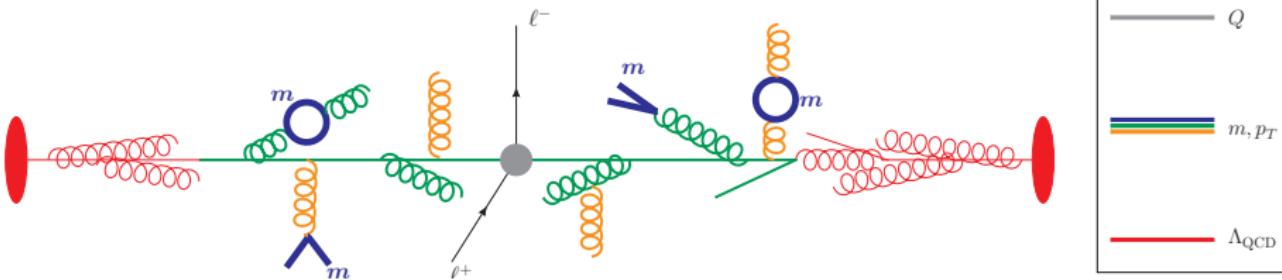
- matching: $f_j^{(5)}(x) = \sum_k \mathcal{M}_{jk}(x, m) \otimes f_k^{(4)}(x)$

- secondary \mathcal{M}_{qq} and primary \mathcal{M}_{Qg} heavy quarks

[M. Buza, Y. Matinounine, J. Smith, W. van. Neerven (1998)]

- hard, **beam** and **soft** functions with **5** massless flavors

Factorization for $p_T \sim m$

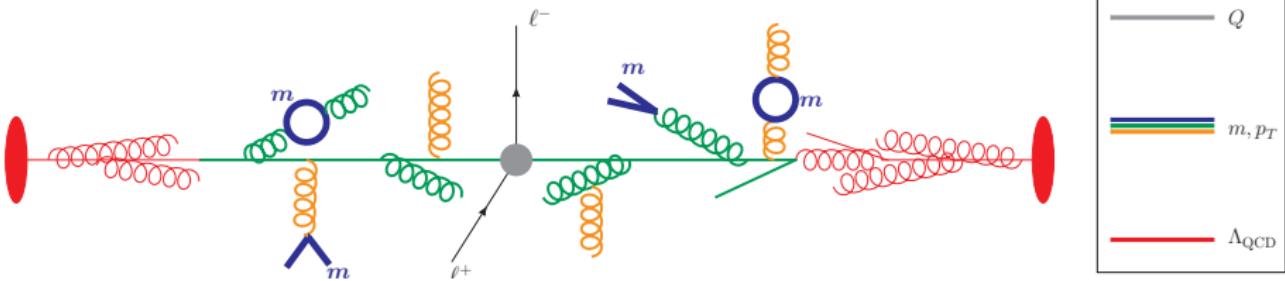


$$\frac{d\sigma_{\text{sec}}}{dp_T} = \sum_{i \in \{q\}} H_i^{(5)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{ij}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$

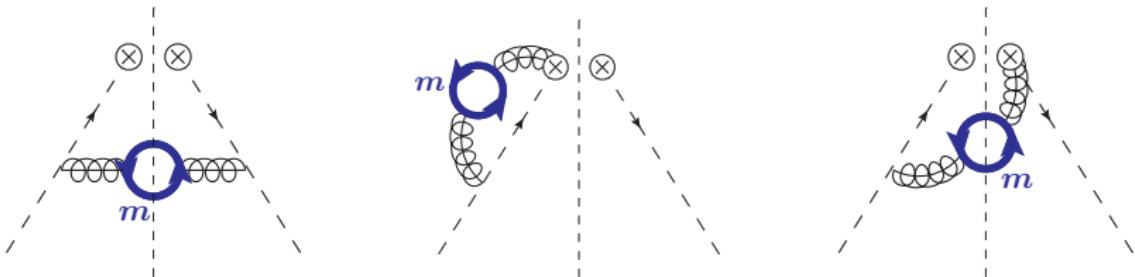
- secondary **massive** quarks in beam function \mathcal{I}_{qq} and **soft function S**

Factorization for $p_T \sim m$

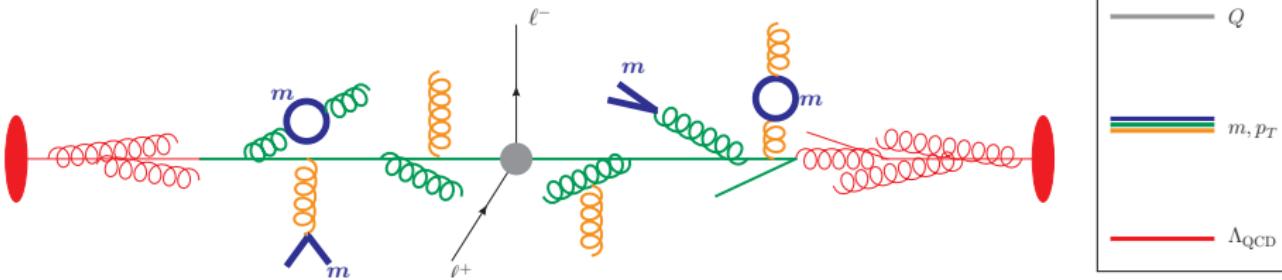


$$\frac{d\sigma_{\text{sec}}}{dp_T} = \sum_{i \in \{q\}} H_i^{(5)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{ij}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$

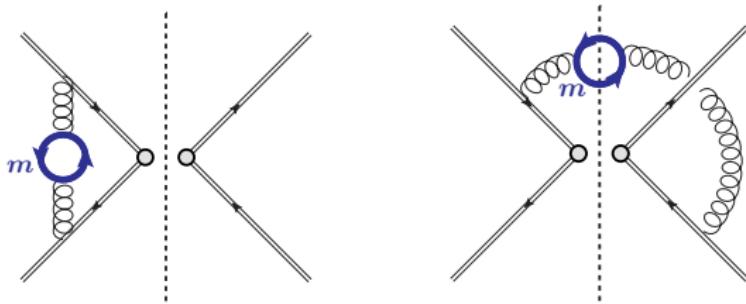


Factorization for $p_T \sim m$

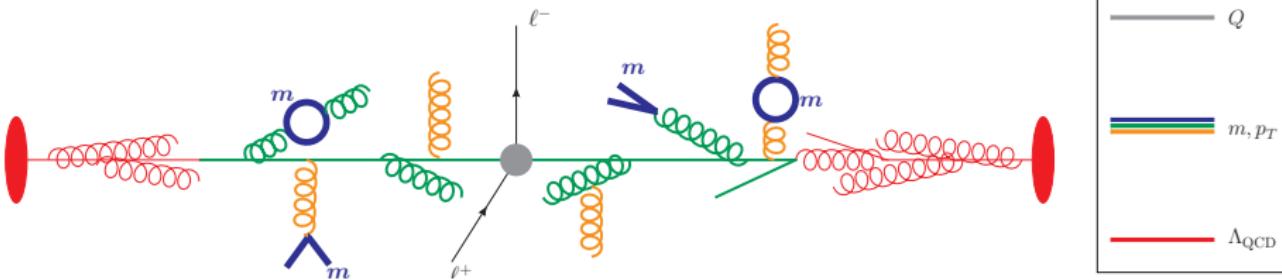


$$\frac{d\sigma_{\text{sec}}}{dp_T} = \sum_{i \in \{q\}} H_i^{(5)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{ij}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$



Factorization for $p_T \sim m$

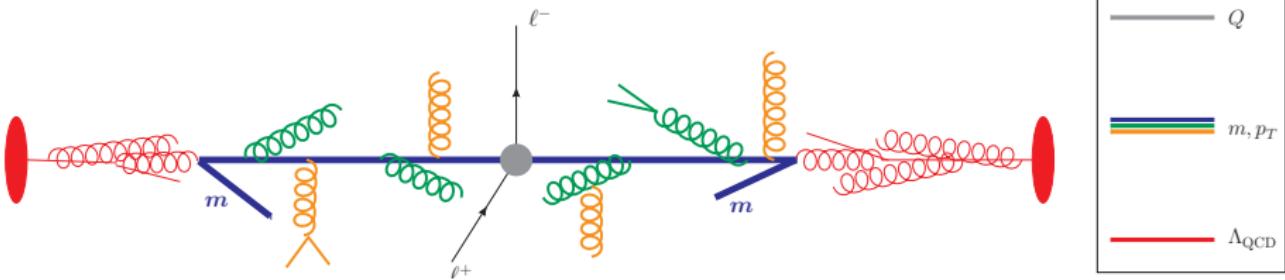


$$\frac{d\sigma_{\text{sec}}}{dp_T} = \sum_{i \in \{q\}} H_i^{(5)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{ij}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$

- secondary **massive** quarks in beam function \mathcal{I}_{qq} and **soft function S**

Factorization for $p_T \sim m$

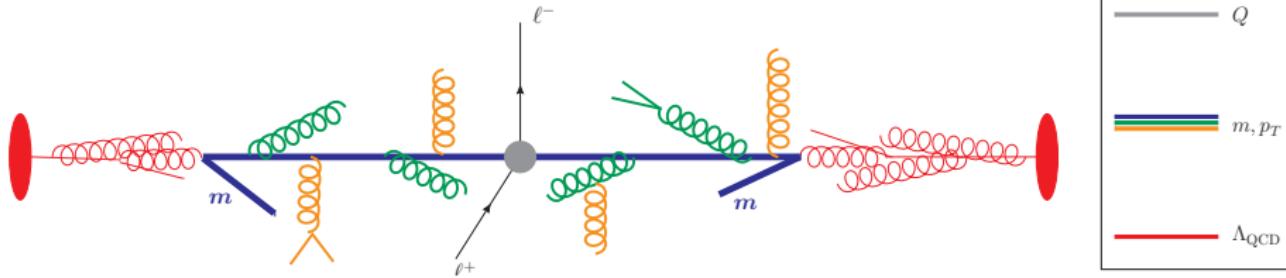


$$\frac{d\sigma_{\text{prim}}}{dp_T} = H_Q^{(5)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{Qj}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$

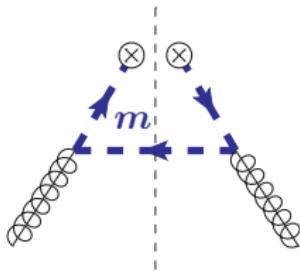
- secondary **massive** quarks in beam function \mathcal{I}_{qq} and soft function S
- primary **massive** quark beam function \mathcal{I}_{Qg}
[A. Balyaev, P. Nadolsky, C.-P. Yuan (2005); S. Berge, P. Nadolsky, F. Olness (2005)]

Factorization for $p_T \sim m$

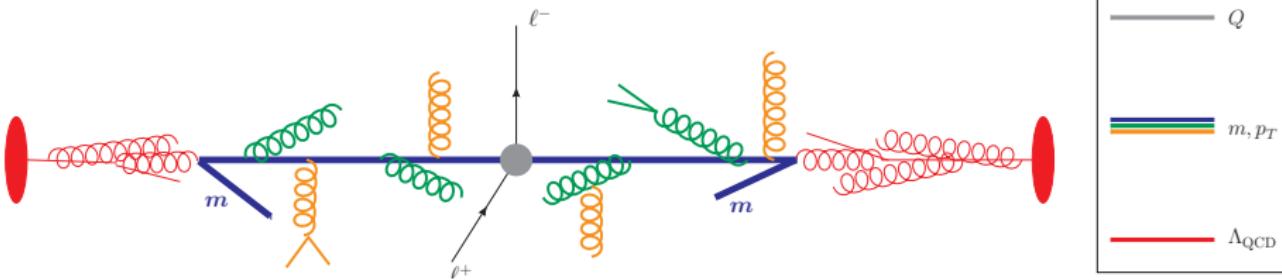


$$\frac{d\sigma_{\text{prim}}}{dp_T} = H_Q^{(5)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{Qj}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$



Factorization for $p_T \sim m$

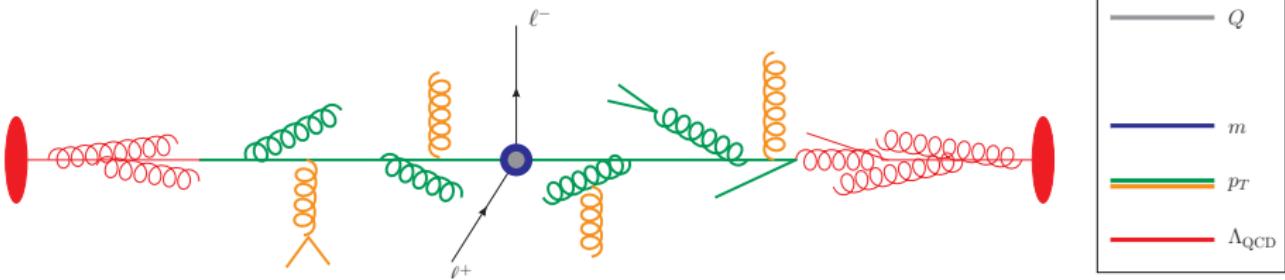


$$\frac{d\sigma_{\text{prim}}}{dp_T} = H_Q^{(5)}(Q) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{Qj}^{(5)}(p_T, x, m) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(5)}(p_T, m)$$

Power Corrections: $\mathcal{O}\left(\frac{m^2}{Q^2}\right)$

- secondary **massive** quarks in beam function \mathcal{I}_{qq} and soft function S
- primary **massive** quark beam function \mathcal{I}_{Qg}
[A. Balyaev, P. Nadolsky, C.-P. Yuan (2005); S. Berge, P. Nadolsky, F. Olness (2005)]
- **PDF** with 4 and hard function with 5 massless flavors

Factorization for $p_T \ll m$



$$\frac{d\sigma}{dp_T} = \sum_{i \in \{q\}} H_i^{(5)}(Q) \times H_m(m) \times \left[\sum_{j \in \{q,g\}} \mathcal{I}_{ij}^{(4)}(p_T, x) \otimes f_j^{(4)}(x) \right]^2 \otimes_{\perp} S^{(4)}(p_T)$$

Power Corrections: $\mathcal{O}\left(\frac{p_T^2}{m^2}, \frac{m^2}{Q^2}\right)$

- mass mode matching function H_m from integrating out the heavy flavor from the current
[S. Gritschacher, A. Hoang, I. Jemos , V. Mateu, P. Pietrulewicz (2014)]
- beam function, soft function, PDF with 4 massless flavors,
hard function with 5 massless flavors

Outline

- ① Introduction
- ② Factorization for the p_T spectrum
- ③ Modes and resummation
- ④ Outlook: effects on W boson mass measurements
- ⑤ Conclusions

Massless Factorization for p_T

- hard matching coefficient

$$J_{\text{QCD}}^\mu = C \times J_{\text{SCET}}^\mu$$

- measurement function

$$\mathcal{M}(\vec{p}_T) = \delta^{(2)}(\vec{p}_T - \vec{\mathcal{P}}_\perp)$$

- beam function

$$B_{ij}(\vec{p}_T, \frac{\omega}{p_-}) = \langle j | \bar{\chi} \mathcal{M}(\vec{p}_T) \frac{\not{\ell}}{2} [\delta(\omega - \mathcal{P}_n) \chi] | j \rangle$$

$$B_{ij}(\vec{p}_T, x) = \sum_k \mathcal{I}_{ik}(\vec{p}_T, x) \otimes f_{k/j}(x)$$

$$f_{i/j}(\frac{\omega}{p_-}) = \langle j | \bar{\chi} \frac{\not{\ell}}{2} [\delta(\omega - \mathcal{P}_n) \chi] | j \rangle$$

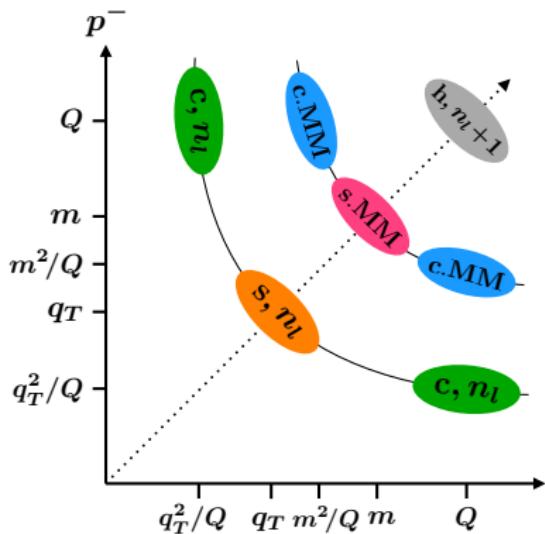
- soft function

$$S(\vec{p}_T) = \frac{1}{N_c} \text{tr} \langle 0 | \overline{T} [S_n^\dagger S_{\bar{n}}] \mathcal{M}(\vec{p}_T) T [S_{\bar{n}}^\dagger S_n] | 0 \rangle$$

SCET_{\parallel} - rapidity divergences cancel between soft and beam functions.

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l)} \times \left[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T}{Q}\right)$$

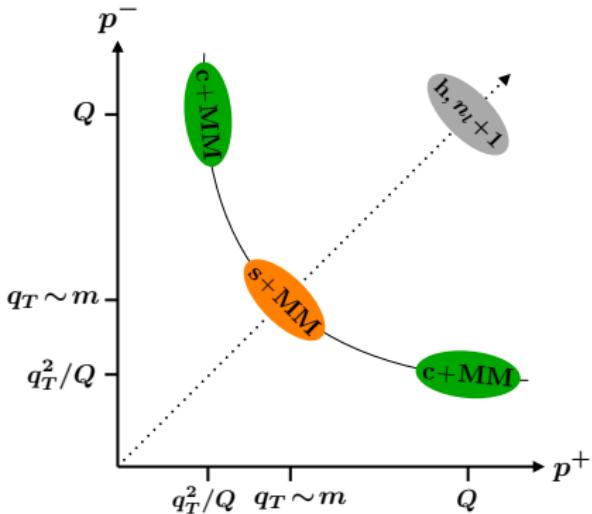
$$p_T \ll m$$



- integrate out massive flavor in SCET current
⇒ mass mode matching functions
- $$(J_{\text{SCET}}^{(n_l+1)})^\mu = C_c(m) \times C_{\bar{c}}(m) \times C_s(m) \times (J_{\text{SCET}}^{(n_l)})^\mu$$
- [S. Gritschacher, A. Hoang, I. Jemos, V. Mateu, P. Pietrulewicz (2014)]
[A. Hoang, P. Pietrulewicz, D.S. (2016)]
- rapidity divergences cancel between these matching coefficients
⇒ rapidity RGE to resum associated logs
 - hard function with $(n_l + 1)$ massless flavors
 - beam and soft function with (n_l) massless flavors

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_c(m) \times H_{\bar{c}}(m) \times H_s(m) \times \left[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right)$$

$$p_T \sim m$$



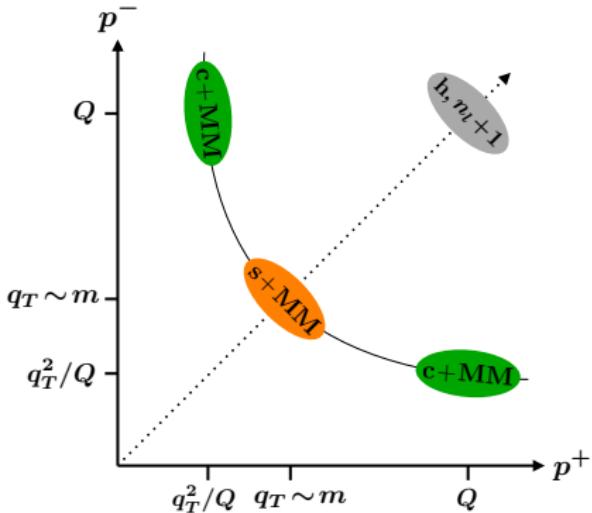
- mass scale coincides with beam/soft scale
⇒ massive beam and soft functions
- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

$$i \in \{q, Q, g\} \quad k \in \{q, g\}$$
- secondary massive quarks in soft function
- secondary massive quarks change rapidity RGE
- hard function with $(n_l + 1)$ massless flavors

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)}(m)$$

$$p_T \sim m$$



- mass scale coincides with beam/soft scale
⇒ massive beam and soft functions
- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

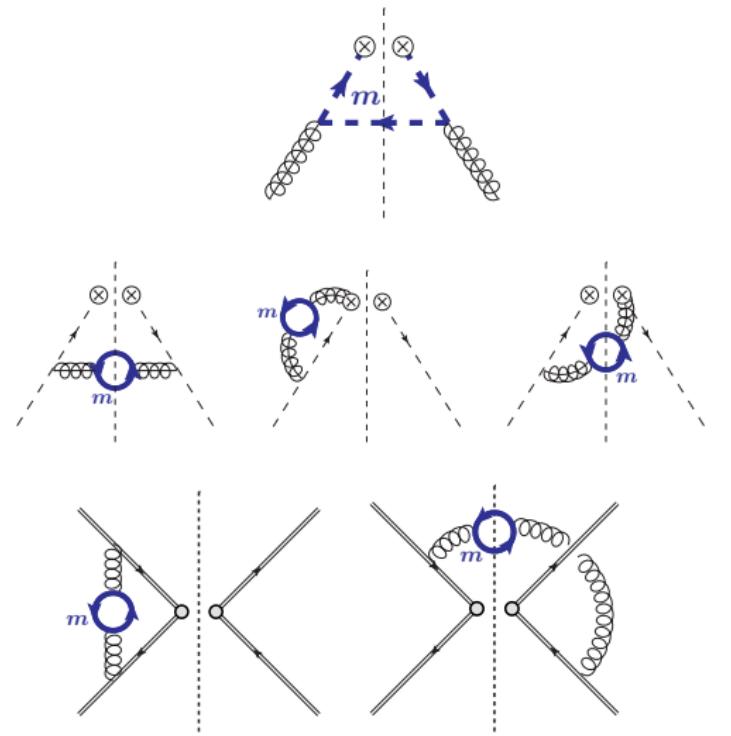
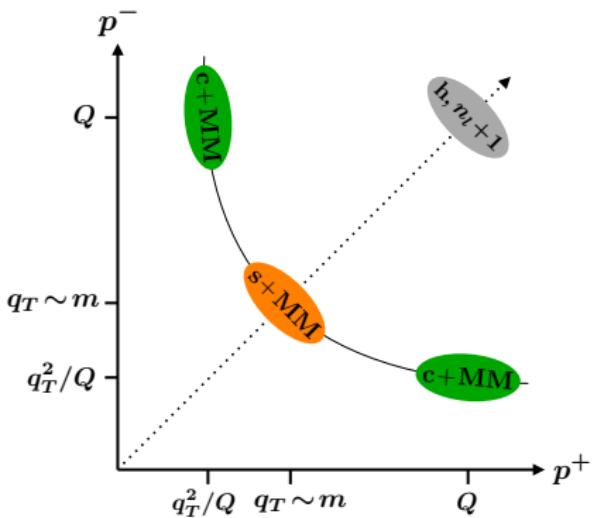
new

$$i \in \{q, Q, g\} \quad k \in \{q, g\}$$

- secondary massive quarks in soft function
- secondary massive quarks change rapidity RGE
- hard function with $(n_l + 1)$ massless flavors

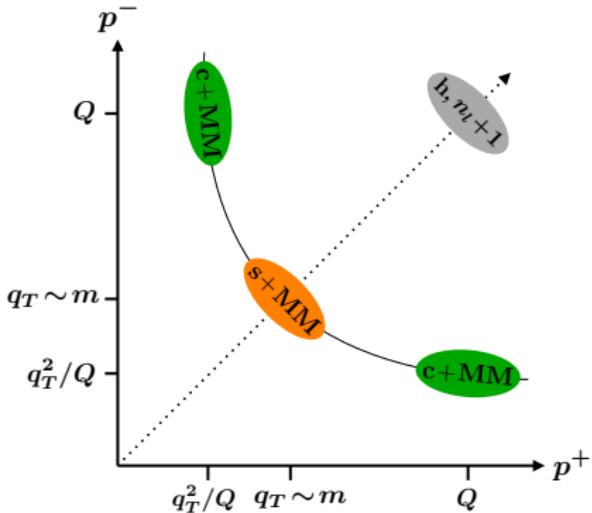
$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)}(m)$$

$$p_T \sim m$$



$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{L}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)}(m)$$

$$p_T \sim m$$



- mass scale coincides with beam/soft scale
⇒ massive beam and soft functions
- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

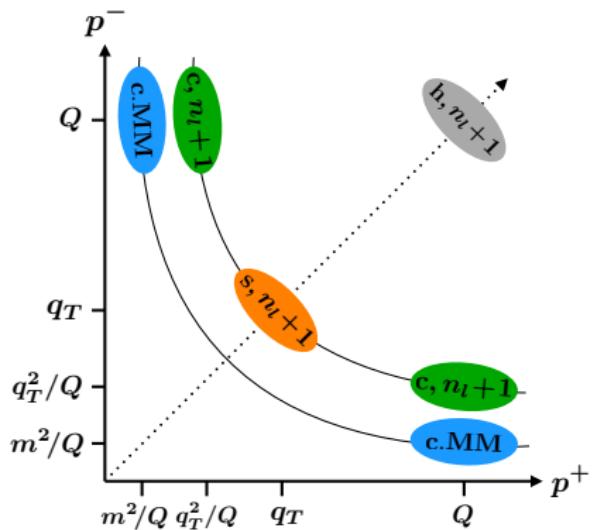
new

$$i \in \{q, Q, g\} \quad k \in \{q, g\}$$

- secondary massive quarks in soft function
- secondary massive quarks change rapidity RGE
- hard function with $(n_l + 1)$ massless flavors

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)}(m)$$

$$m \ll p_T$$



- matching in the PDF evolution

$$f_{i/j}^{(n_l+1)}(m) = \sum_k \mathcal{M}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

$$i \in \{q, Q, g\} \quad k \in \{q, g\}$$

[M. Buza, Y. Matiounine, J. Smith, W. van Neerven (1998)]

- beam/soft function with $(n_l + 1)$ massless flavors

$$B_{ij}^{(n_l+1)} = \sum_k \mathcal{I}_{ik} \otimes f_{kj}^{(n_l+1)}$$

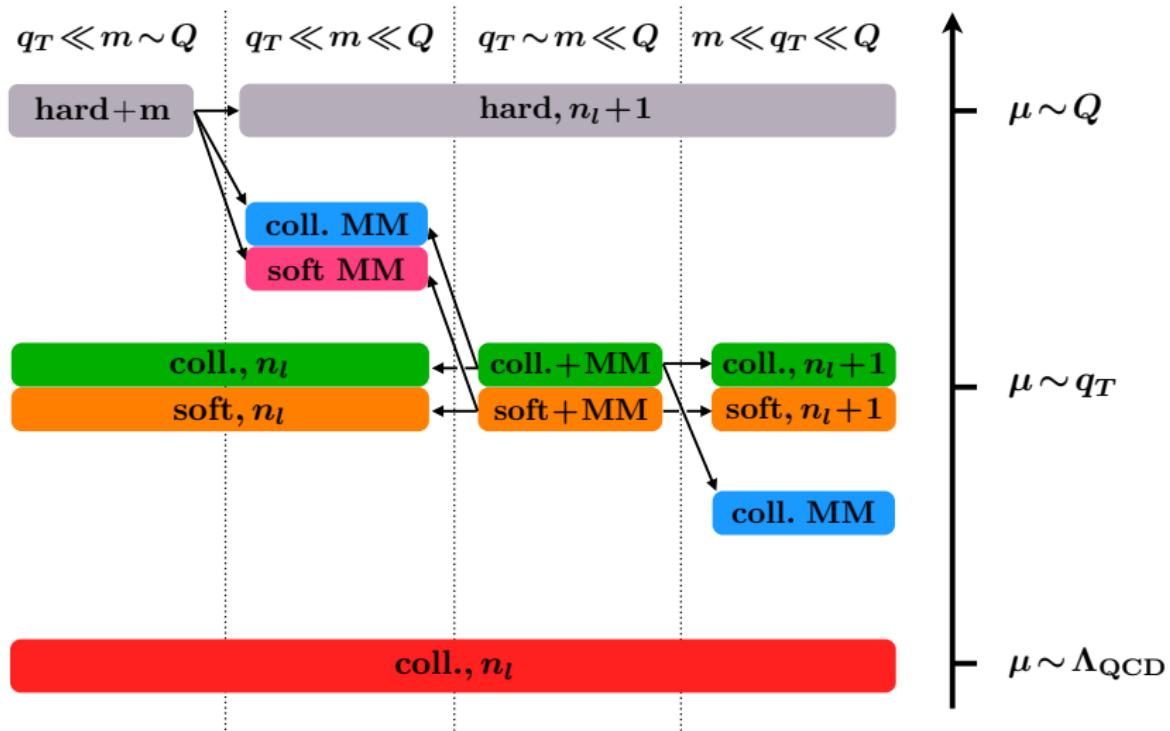
$$i \in \{q, Q, g\} \quad k \in \{q, Q, g\}$$

- no rapidity divergences in PDFs

- hard function with $(n_l + 1)$ massless flavors

$$\frac{d\sigma}{dp_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{p_T^2}\right)$$

Summary of all Modes



Relations between Hierarchies

- components for the different hierarchies are related.

- beam function matching coefficients:

$$\mathcal{I}_{ik}(m) = \mathcal{I}_{ik}^{(n_l)} \times H_c(m) \times \left[1 + \mathcal{O}\left(\frac{p_T^2}{m^2}\right) \right]$$

$$\mathcal{I}_{ik}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \times \left[1 + \mathcal{O}\left(\frac{m^2}{p_T^2}\right) \right] \quad j \in \{q, Q, g\}$$

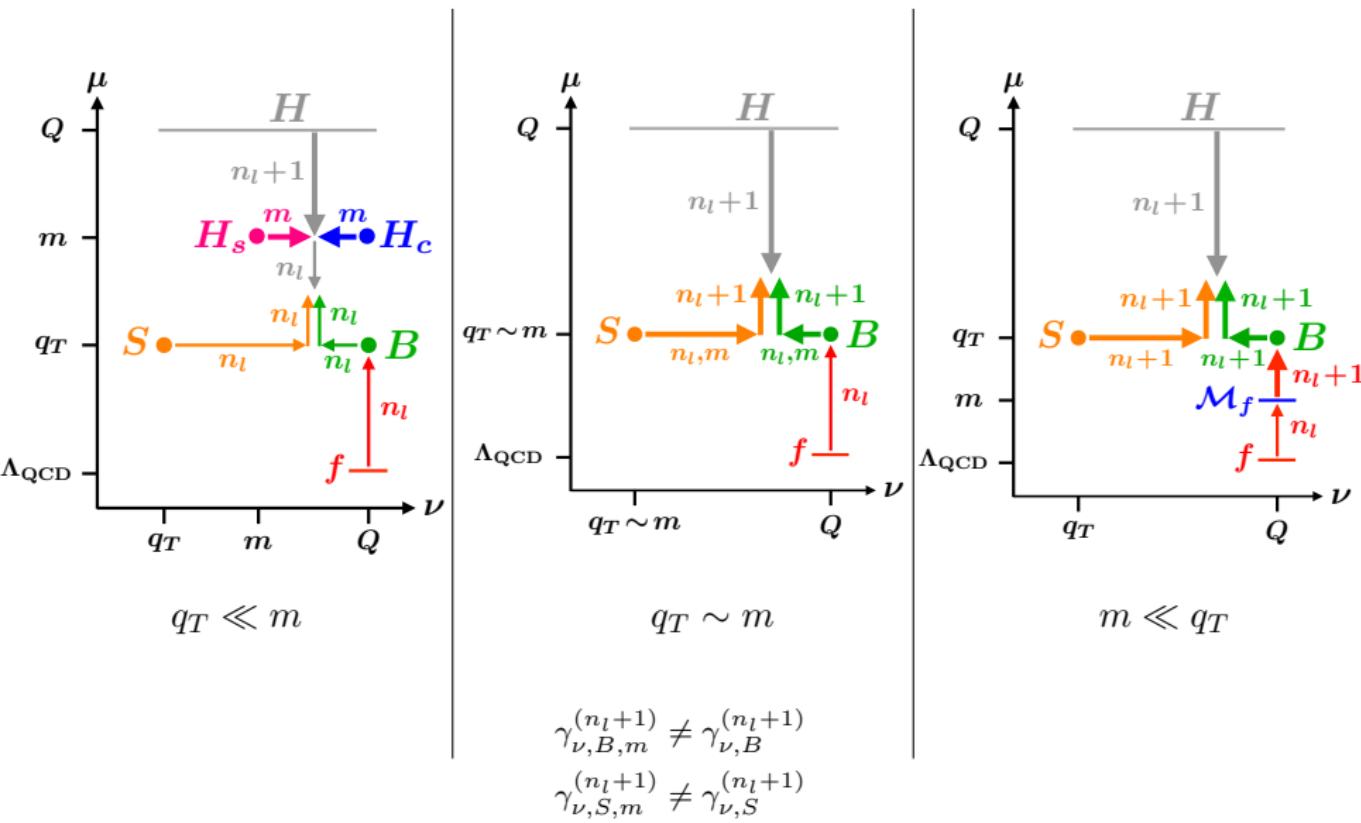
- soft function:

$$S^{(n_l+1)}(m) = S^{(n_l)} \times H_s(m) \times \left[1 + \mathcal{O}\left(\frac{p_T^2}{m^2}\right) \right]$$

$$S^{(n_l+1)}(m) = S^{(n_l+1)} \times \left[1 + \mathcal{O}\left(\frac{m^2}{p_T^2}\right) \right]$$

- can be used to systematically include all power corrections.

Renormalization Group Evolution



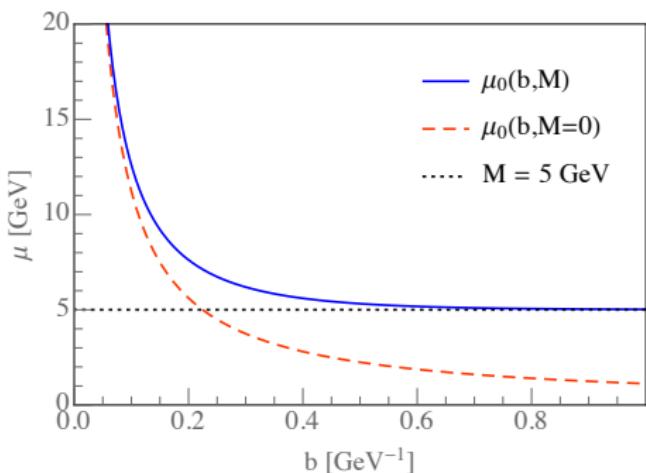
Rapidity Logs from Massive Flavors

- secondary massive quarks lead to mass dependent rapidity anomalous dimension $\tilde{\gamma}_\nu(b, m, \mu)$ (here in b space)

$$\tilde{\gamma}_\nu(b, m, \mu) \xrightarrow{b \rightarrow 0} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C_{FTF} \left(\frac{16}{3} L_b^2 + \frac{160}{9} L_b + \frac{224}{27} \right) \quad L_b = \ln\left(\frac{b^2 \mu^2 e^{2\gamma_E}}{4}\right)$$

$$\tilde{\gamma}_\nu(b, m, \mu) \xrightarrow{b \rightarrow \infty} \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 C_{FTF} \left(-\frac{16}{3} L_m^2 - \frac{160}{9} L_m - \frac{224}{27} \right) \quad L_m = \ln \frac{m^2}{\mu^2}$$

- large logarithms in $\tilde{\gamma}_\nu$ can be avoided e.g. by b space setting of μ



$$\begin{aligned}\mu_0(b, m) &\xrightarrow{b \rightarrow 0} \frac{2e^{-\gamma_E}}{b} \\ \mu_0(b, m) &\xrightarrow{b \rightarrow \infty} m\end{aligned}$$

⇒ mass introduces IR cutoff

⇒ no non-pert. regime for $b \rightarrow \infty$

Outline

- ① Introduction
- ② Factorization for the p_T spectrum
- ③ Modes and resummation
- ④ Outlook: effects on W boson mass measurements
- ⑤ Conclusions

W boson mass measurements

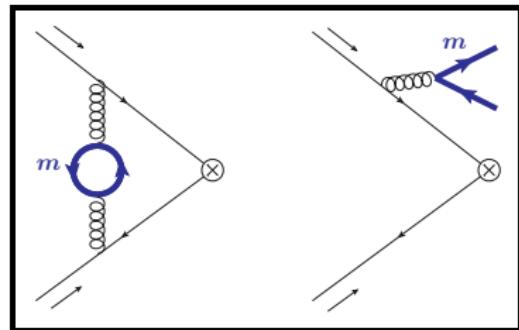
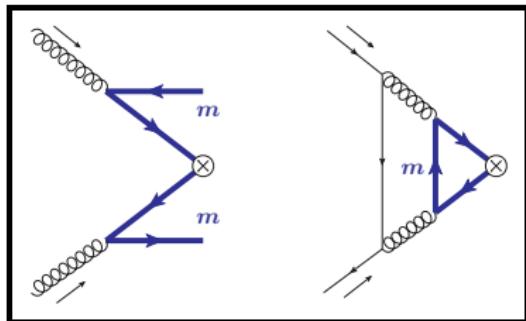
- experimentalists rely on ratio of W and Z boson spectrum

$$\left[\frac{d\sigma^W}{dq_T} \right]_{\text{prediction}} = \left[\frac{d\sigma^W}{dq_T} \times \left(\frac{d\sigma^Z}{dq_T} \right)^{-1} \right]_{\text{theory}} \times \left[\frac{d\sigma^Z}{dq_T} \right]_{\text{measured}}$$

- Z boson spectrum measured with very high accuracy
- many things cancel to large extent in the ratio
- every difference between Z and W production can become relevant

Massive Quarks in Z production

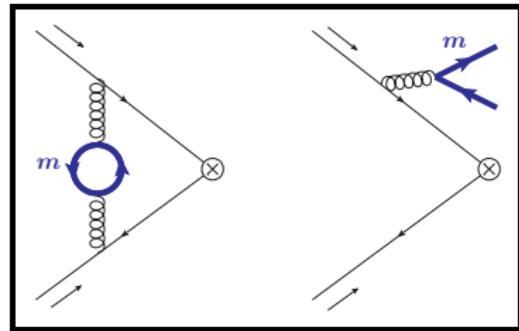
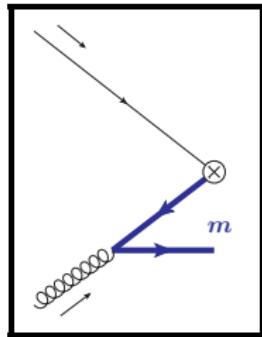
primary and secondary **massive** quarks.



- primary: massive quarks go into hard interaction
- secondary: massive quark corrections to light quark induced processes
- both start at $\mathcal{O}(\alpha_s^2)$, relevant for NNLL' resummation

Massive Quarks in W production

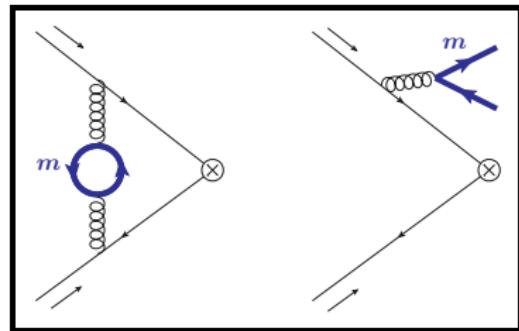
primary and secondary **massive** quarks.



- secondary massive quark effects the same as for Z production
- primary charm quarks already contribute at $\mathcal{O}(\alpha_s)$
- primary bottom quarks CKM suppressed

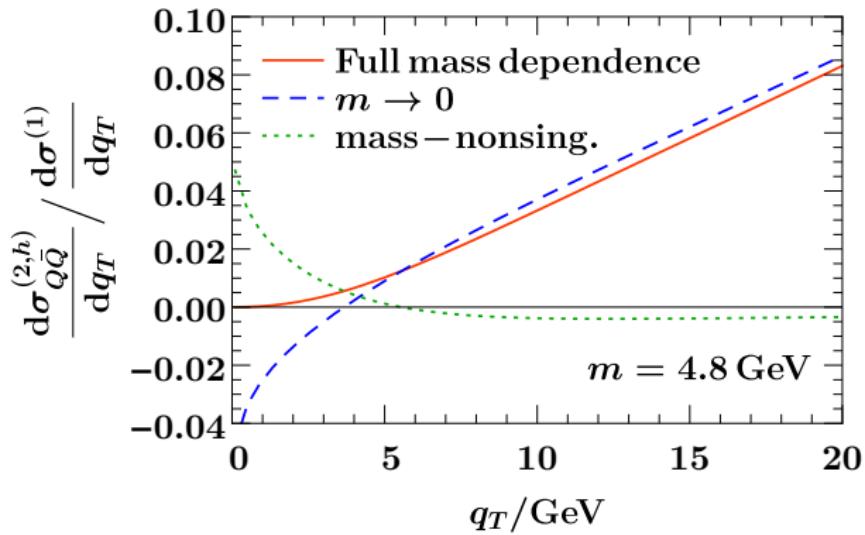
Massive Quarks in W production

primary and secondary **massive** quarks.



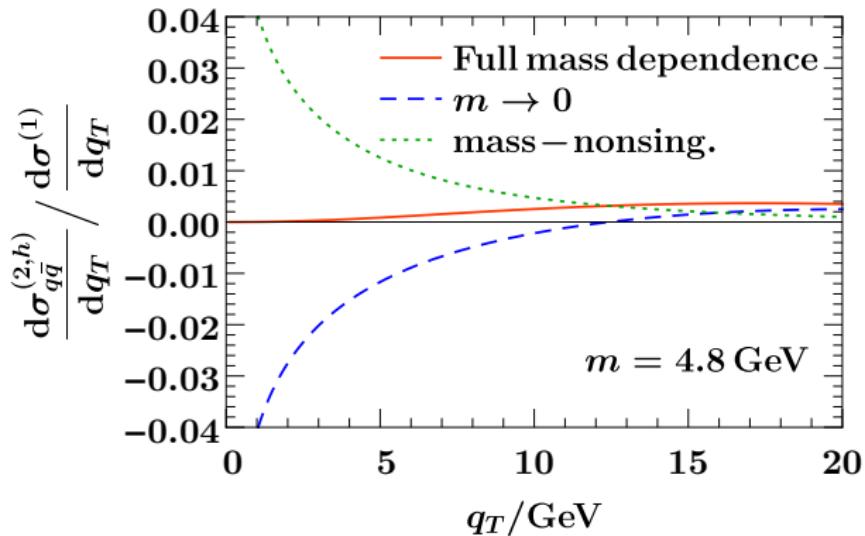
- secondary massive quark effects the same as for Z production
- primary charm quarks already contribute at $\mathcal{O}(\alpha_s)$
- primary bottom quarks CKM suppressed

Mass Effects - Primary



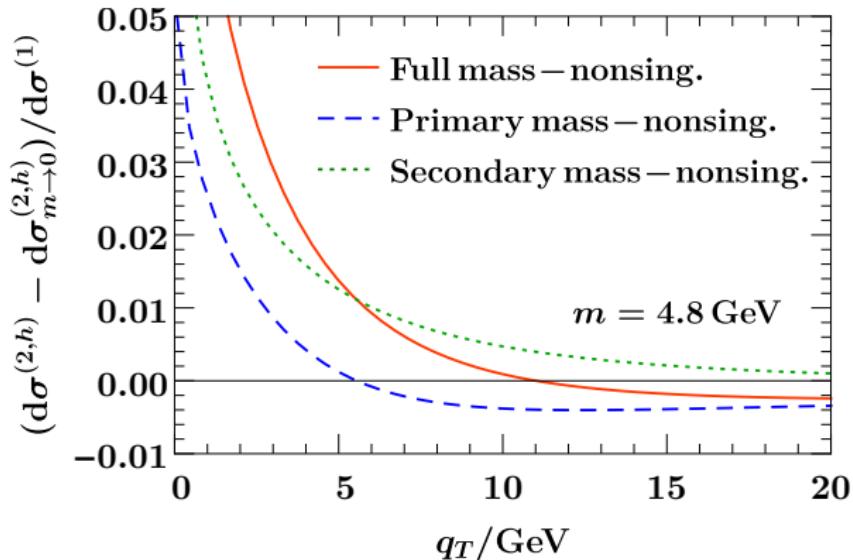
- $\mathcal{O}(\alpha_s^2)$ contributions from primary massive quarks at fixed order, normalized to the LO spectrum
- $E_{\text{cm}} = 13 \text{ TeV}$, $m_b = 4.8 \text{ GeV}$, $Q = m_Z$, $Y = 0$, MSTW NLO PDFs

Mass Effects - Secondary



- $\mathcal{O}(\alpha_s^2)$ contributions from secondary massive quarks at fixed order, normalized to the LO spectrum
- $E_{\text{cm}} = 13 \text{ TeV}$, $m_b = 4.8 \text{ GeV}$, $Q = m_Z$, $Y = 0$, MSTW NLO PDFs

Mass Effects



- $\mathcal{O}(\alpha_s^2)$ quark mass corrections to the Z-boson p_T -spectrum at fixed order, normalized to the LO spectrum
- $E_{\text{cm}} = 13 \text{ TeV}$, $m_b = 4.8 \text{ GeV}$, $Q = m_Z$, $Y = 0$, MSTW NLO PDFs

Outline

- ① Introduction
- ② Factorization for the p_T spectrum
- ③ Modes and resummation
- ④ Outlook: effects on W boson mass measurements
- ⑤ Conclusions

Outlook and Conclusions

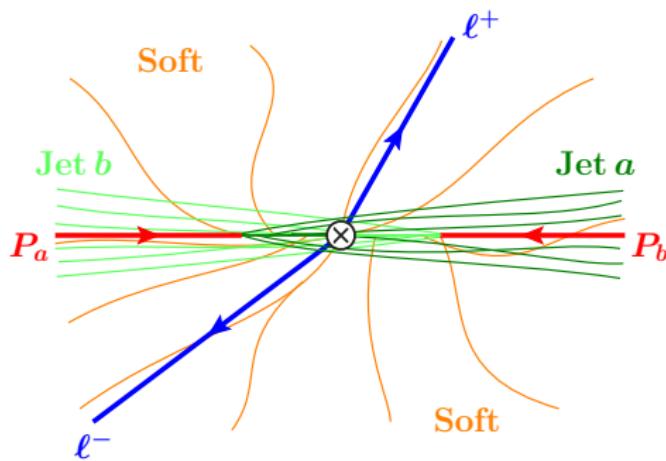
- included massive quarks into the factorization theorem for Drell-Yan + 0 jets for a SCET_{\perp} and a SCET_{\parallel} jet veto
- resummation of all mass related logarithms at NNLL' accuracy (one and two loop beam and soft functions with massive quarks)
- different structure of rapidity logarithms due to secondary massive quarks
- relevant for precision measurements of W boson mass
- parts of this also relevant for other processes, e.g. $b\bar{b} \rightarrow H$

Outlook and Conclusions

- included massive quarks into the factorization theorem for Drell-Yan + 0 jets for a SCET_\perp and a SCET_\parallel jet veto
- resummation of all mass related logarithms at NNLL' accuracy (one and two loop beam and soft functions with massive quarks)
- different structure of rapidity logarithms due to secondary massive quarks
- relevant for precision measurements of W boson mass
- parts of this also relevant for other processes, e.g. $b\bar{b} \rightarrow H$

Thank you for your attention!

Beam Thrust

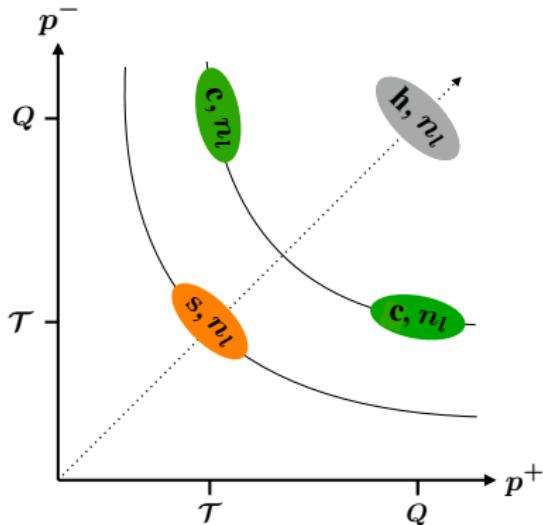


- beam thrust as jet veto to restrict hadronic final state
- requires resummation of logarithms for $T \ll Q$

from: I.W.Stewart,F.J.Tackmann,W.J.Waalewijn, *Phys. Rev.* D81 (2010) 094035

- $\mathcal{T} = \sum_i \min\{n_a \cdot p_i, n_b \cdot p_i\}$ $n_{a,b}^\mu = (1, \pm \hat{z})$
- 0-jet limit for $\mathcal{T} \ll Q$
- closely related to 0-jettines τ_0 : $\mathcal{T} = \tau_0 + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right)$
- 3 scales (SCET_I): hard $\sim Q$, collinear $\sim \sqrt{QT}$, soft $\sim \mathcal{T}$
- no rapidity logarithms (in the massless case)

Massless Factorization for \mathcal{T}



(non-pert. coll.-modes (PDF) not shown)

- hard matching coefficient

$$J_{\text{QCD}}^\mu = C \times J_{\text{SCET}}^\mu$$

- measurement function

$$\mathcal{M}_a(k, \pm) = \delta(k - \hat{p}_a^\pm)$$

- beam function

$$B_{ij}(\omega b, \frac{\omega}{p^-}) = \langle j | \bar{\chi} \mathcal{M}(b, +) \frac{\not{p}}{2} [\delta(\omega - \mathcal{P}_n) \chi] | j \rangle$$

$$B_{ij}(t, x) = \sum_k \mathcal{I}_{ik}(t, x) \otimes f_{k/j}(x)$$

$$f_{i/j}(\frac{\omega}{p^-}) = \langle j | \bar{\chi} \frac{\not{p}}{2} [\delta(\omega - \mathcal{P}_n) \chi] | j \rangle$$

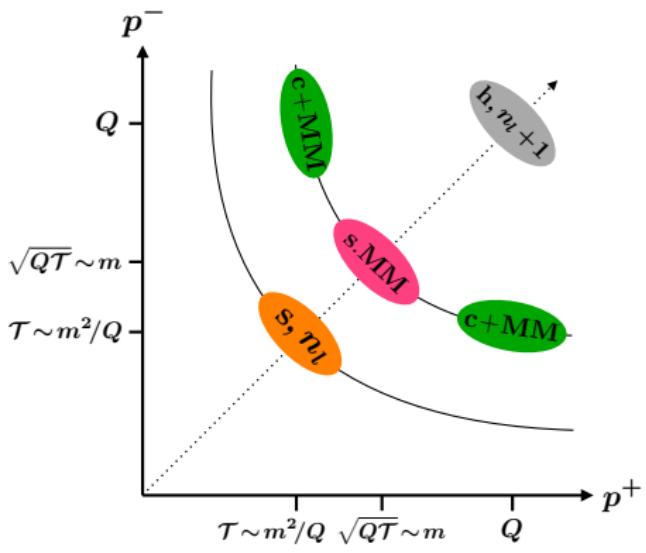
- (u)soft function

$$S(k_a, k_b) = \frac{1}{N_c} \text{tr} \langle 0 | \overline{\text{T}}[Y_n^\dagger Y_{\bar{n}}] \mathcal{M}_a(k_a, +) \mathcal{M}_b(k_b, -) \text{T}[Y_{\bar{n}}^\dagger Y_n] | 0 \rangle$$

SCET_I - no rapidity divergences.

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l)} \times \left[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\mathcal{T}}{Q}\right)$$

$$\sqrt{Q\tau} \sim m$$



- prim. and sec. massive quarks in beam function

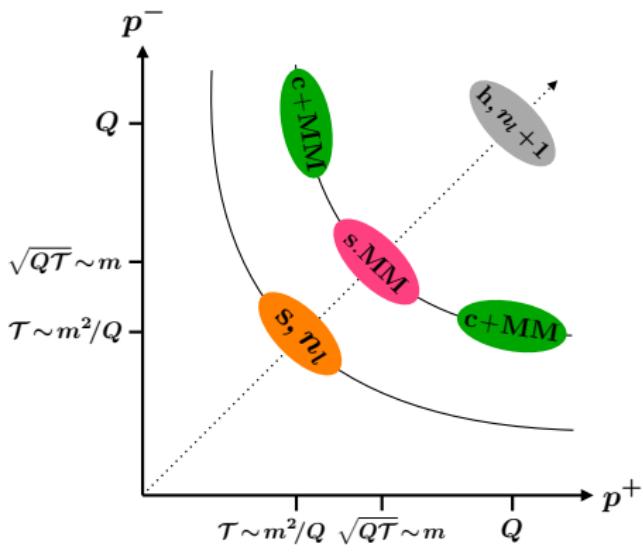
$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

$$i \in \{q, Q, g\} \quad k \in \{q, g\}$$

- secondary massive quarks introduce rapidity divergences in beam function
- mass scale still above soft scale
⇒ soft MM integrated out in current ⇒ H_s
- resummation of rapidity logs between massive beam function and H_s
- hard function with $(n_l + 1)$ massless flavors
- soft function with (n_l) massless flavors

$$\frac{d\sigma}{d\tau} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\tau^2}{m^2}\right)$$

$$\sqrt{Q\tau} \sim m$$



- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

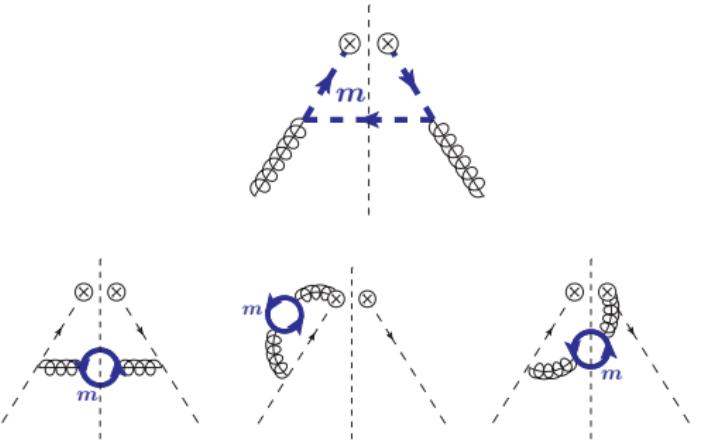
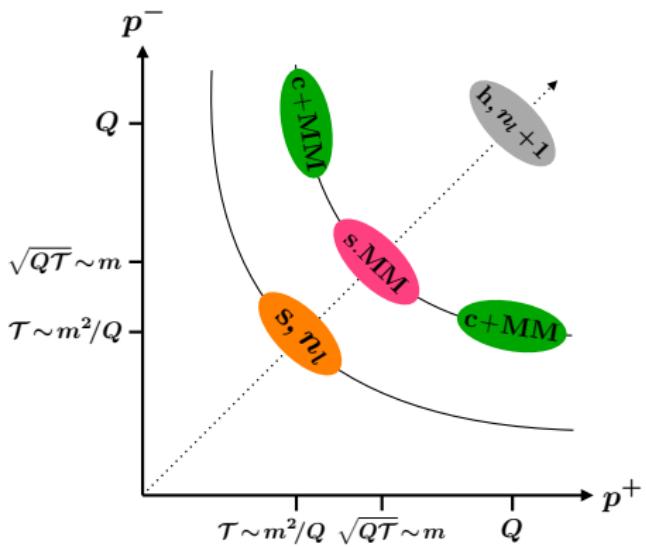
new

$$i \in \{q, Q, g\} \quad k \in \{q, g\}$$

- secondary massive quarks introduce rapidity divergences in beam function
- mass scale still above soft scale
⇒ soft MM integrated out in current ⇒ H_s
- resummation of rapidity logs between massive beam function and H_s
- hard function with $(n_l + 1)$ massless flavors
- soft function with (n_l) massless flavors

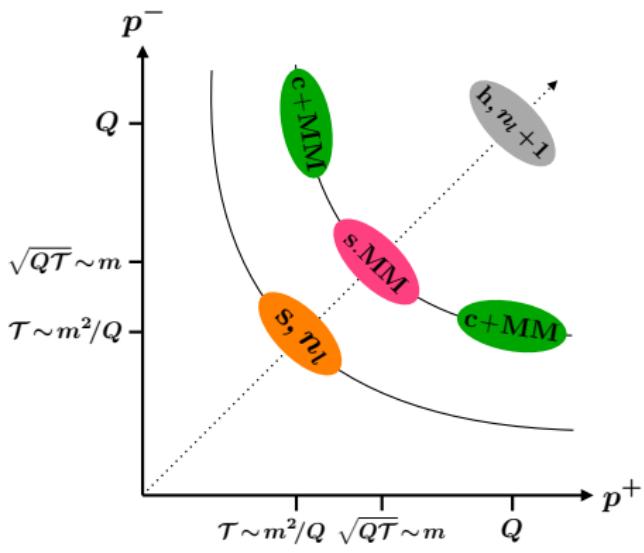
$$\frac{d\sigma}{d\tau} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\tau^2}{m^2}\right)$$

$$\sqrt{Q\tau} \sim m$$



$$\frac{d\sigma}{d\tau} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\tau^2}{m^2}\right)$$

$$\sqrt{Q\tau} \sim m$$



- prim. and sec. massive quarks in beam function

$$B_{ij}^{(n_l+1)}(m) = \sum_k \mathcal{I}_{ik}(m) \otimes f_{k/j}^{(n_l)}$$

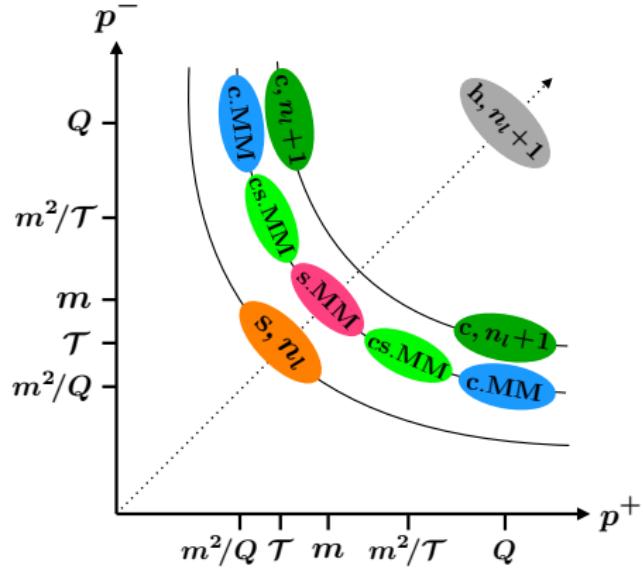
new

$$i \in \{q, Q, g\} \quad k \in \{q, g\}$$

- secondary massive quarks introduce rapidity divergences in beam function
- mass scale still above soft scale
⇒ soft MM integrated out in current ⇒ H_s
- resummation of rapidity logs between massive beam function and H_s
- hard function with $(n_l + 1)$ massless flavors
- soft function with (n_l) massless flavors

$$\frac{d\sigma}{d\tau} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{\tau^2}{m^2}\right)$$

$$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$$



- collinear-soft function S_c

$$S_c(\ell) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T} [X_n^\dagger V_n] \mathcal{M}(\ell, +) T [V_n^\dagger X_n] | 0 \rangle$$

- X_n, V_n : boosted Wilson lines of collinear-soft gluon fields

$$A_c \sim (Q, \mathcal{T}, \sqrt{Q\mathcal{T}}) \quad Q\mathcal{T}$$

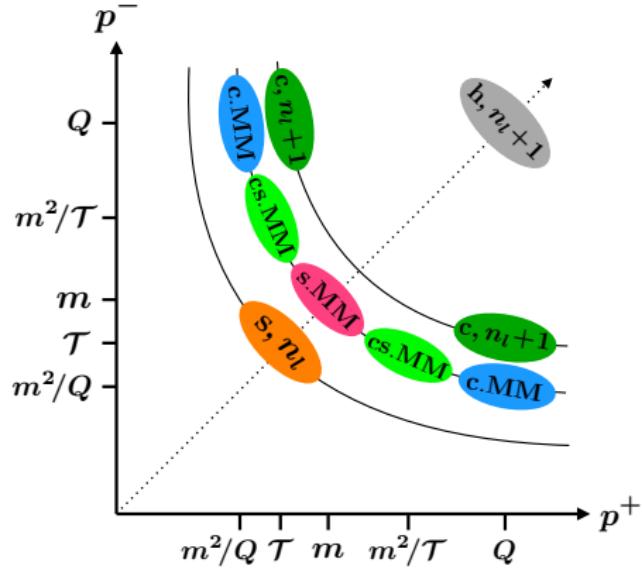
$$A_{cs} \sim \left(\frac{m^2}{\mathcal{T}}, \mathcal{T}, m \right) \quad m^2$$

$$A_{us} \sim (\mathcal{T}, \mathcal{T}, \mathcal{T}) \quad \mathcal{T}^2$$

- vanishes for purely massless contributions
- rapidity divergences for massive quarks

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S_c(m) \otimes S^{(n_l)} \otimes S_{\bar{c}}(m)$$

$$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$$



- collinear-soft function S_c

$$S_c(\ell) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T} [X_n^\dagger V_n] \mathcal{M}(\ell, +) T [V_n^\dagger X_n] | 0 \rangle$$

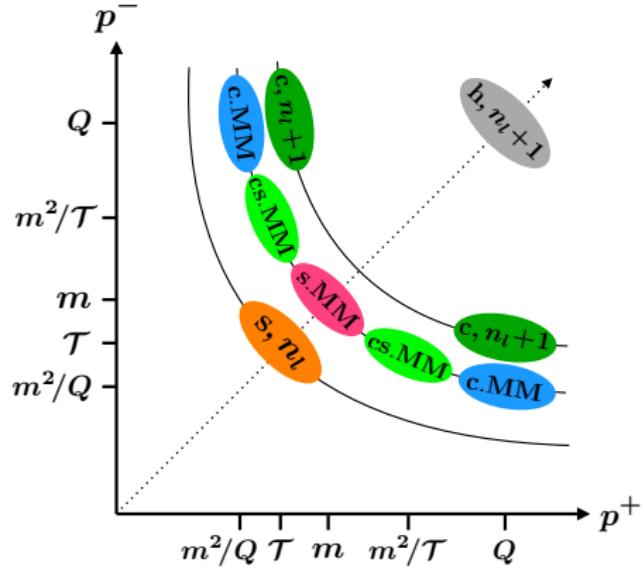
- X_n, V_n : boosted Wilson lines of collinear-soft gluon fields

$$\begin{aligned} A_c &\sim (Q, \mathcal{T}, \sqrt{Q\mathcal{T}}) & Q\mathcal{T} \\ A_{cs} &\sim \left(\frac{m^2}{\mathcal{T}}, \mathcal{T}, m \right) & m^2 \\ A_{us} &\sim (\mathcal{T}, \mathcal{T}, \mathcal{T}) & \mathcal{T}^2 \end{aligned}$$

- vanishes for purely massless contributions
- rapidity divergences for massive quarks

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S_c(m) \otimes S^{(n_l)} \otimes S_{\bar{c}}(m)$$

$$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$$



- collinear-soft function S_c

$$S_c(\ell) = \frac{1}{N_c} \text{tr} \langle 0 | \bar{T} [X_n^\dagger V_n] \mathcal{M}(\ell, +) T [V_n^\dagger X_n] | 0 \rangle$$

new

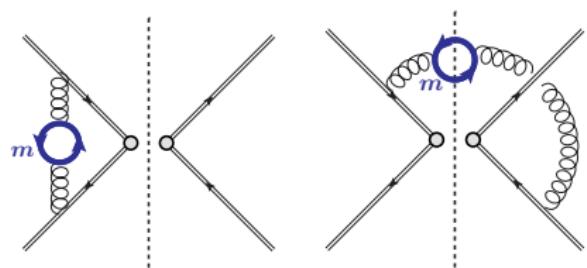
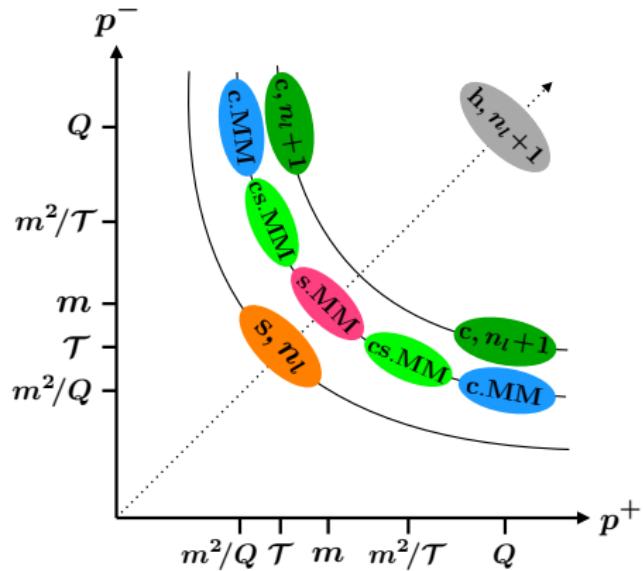
- X_n, V_n : boosted Wilson lines of collinear-soft gluon fields

$$\begin{aligned} A_c &\sim (Q, \mathcal{T}, \sqrt{Q\mathcal{T}}) & Q\mathcal{T} \\ A_{cs} &\sim \left(\frac{m^2}{\mathcal{T}}, \mathcal{T}, m \right) & m^2 \\ A_{us} &\sim (\mathcal{T}, \mathcal{T}, \mathcal{T}) & \mathcal{T}^2 \end{aligned}$$

- vanishes for purely massless contributions
- rapidity divergences for massive quarks

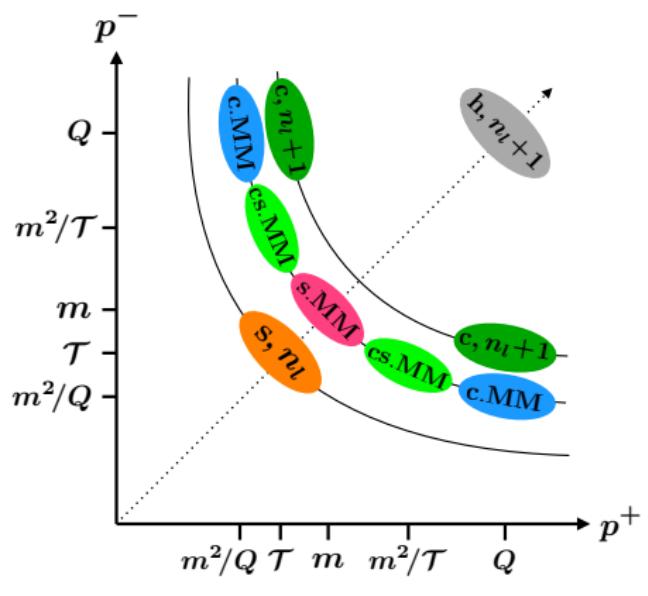
$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S_c(m) \otimes S^{(n_l)} \otimes S_{\bar{c}}(m)$$

$$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$$



$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times H_s(m) \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S_c(m) \otimes S^{(n_l)} \otimes S_{\bar{c}}(m)$$

$$\mathcal{T} \ll m \ll \sqrt{Q\mathcal{T}}$$



- collinear soft function S_c
$$S_c(\ell) = \frac{1}{N_c} \text{tr} \langle 0 | \overline{T}[X_n^\dagger V_n] \mathcal{M}(\ell, +) T[V_n^\dagger X_n] | 0 \rangle$$

new

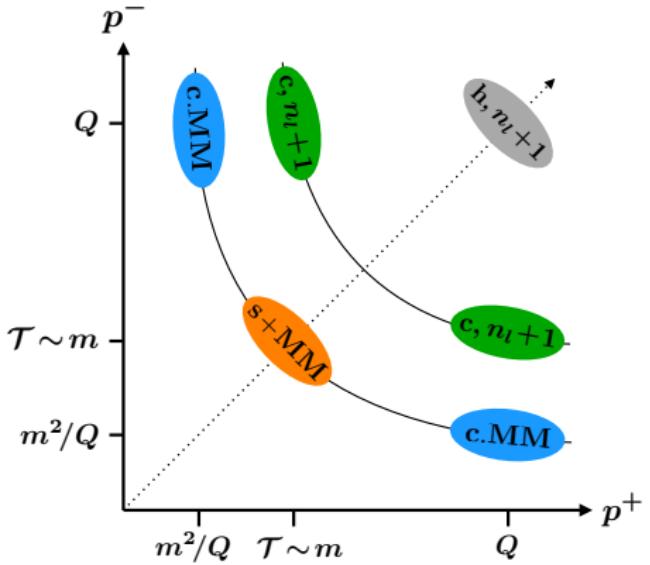
 - matching in PDFs: $(n_l + 1) \rightarrow (n_l)$
 - soft mass mode matching function H_s
 - resummation of rapidity logs between S_c and H_s
 - hard/beam function with $(n_l + 1)$ massless flavors
 - soft function with (n_l) massless flavors

results with csoft function equivalent to threshold corrections in previous works

[S. Gritschacher, A. Hoang, I. Jemos, V. Mateu, P. Pietrulewicz (2014); A. Hoang, P. Pietrulewicz, D.S. (2016)]

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \textcolor{blue}{H_s(m)} \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes \textcolor{blue}{S_c(m)} \otimes S^{(n_l)} \otimes \textcolor{blue}{S_{\bar{c}}(m)}$$

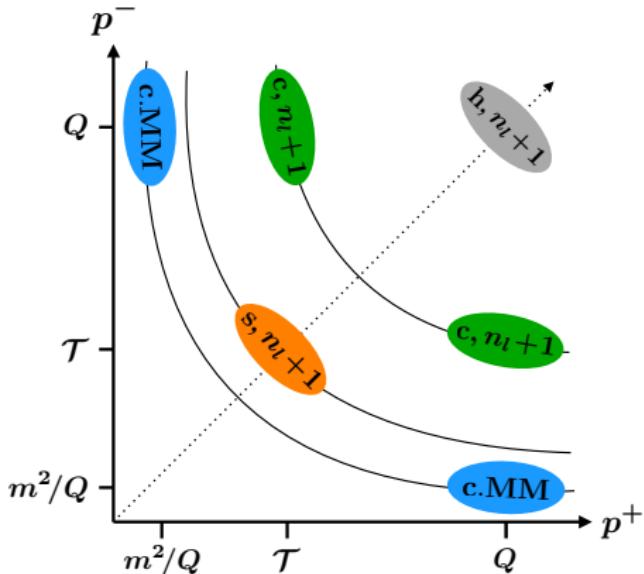
$$\mathcal{T} \sim m$$



- soft function with secondary massive quarks
[S. Gritschacher, A. Hoang, I. Jemos, P. Pietrulewicz (2014)]
- matching in PDFs: $(n_l + 1) \rightarrow (n_l)$
- hard/beam function with $(n_l + 1)$ massless flavors
- no rapidity logarithms

$$\frac{d\sigma}{d\mathcal{T}} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)}(m) + \mathcal{O}\left(\frac{m^2}{Q\mathcal{T}}\right)$$

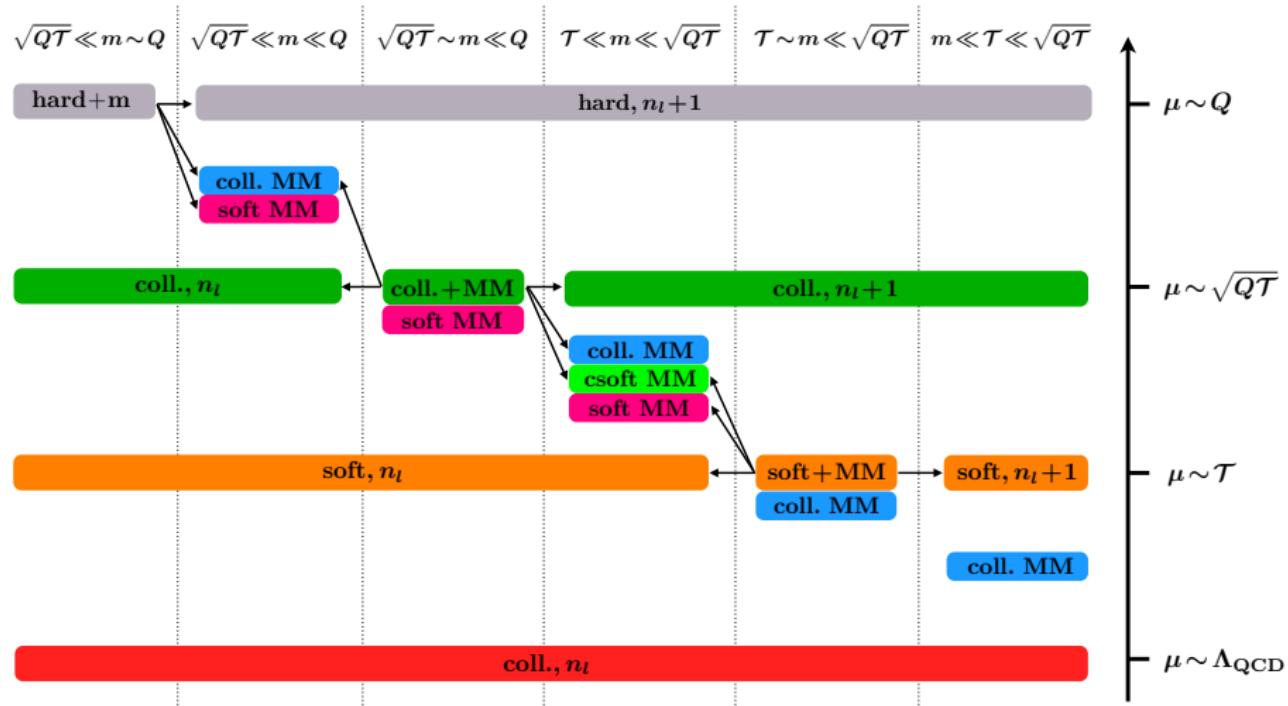
$$m \ll T$$



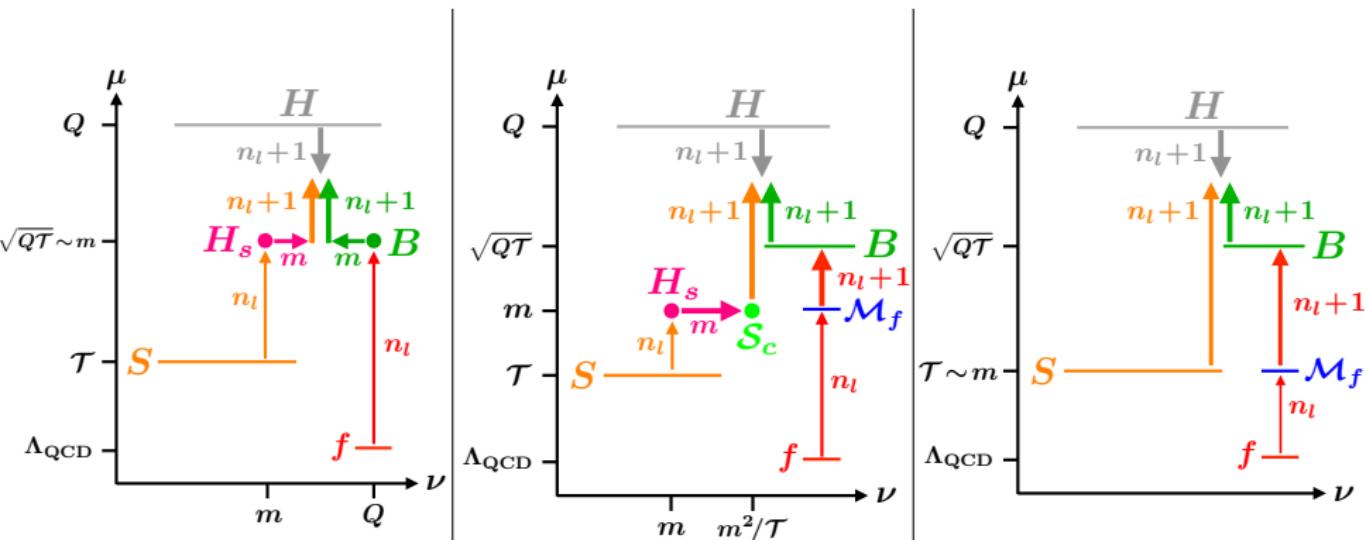
- soft function with $(n_l + 1)$ massless flavors
- matching in PDFs $(n_l + 1) \rightarrow (n_l)$
- hard/beam function with $(n_l + 1)$ massless flavors
- no rapidity logarithms

$$\frac{d\sigma}{dT} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_{k/P}^{(n_l)} \right]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{T^2}\right)$$

Summary of all Modes



Renormalization Group Evolution



$$\sqrt{Q\tau} \sim m$$

$$\tau \ll m \ll \sqrt{Q\tau}$$

$$\tau \sim m$$

$$\gamma_{B,m}^{(n_l+1)} \neq \gamma_B^{(n_l+1)}$$

$$\gamma_{H_s} + \gamma_{B,m}^{(n_l+1)} + \gamma_S^{(n_l)} = \gamma_B^{(n_l+1)} + \gamma_S^{(n_l+1)}$$

$$\gamma_{\nu,B,m} = \gamma_{\nu,S_c} = \gamma_{\nu,H_c}$$