Leading Power Quark Mass Effects in p_T-Spectra at the LHC

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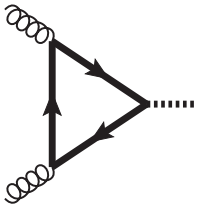
In collaboration with Piotr Pietrulewicz

Alternative title (more precise):

Bottom Mass Effects in p_T-Spectrum of Gluon Fusion Higgs Production at $\mathcal{O}\left[(m_b/m_H)^0\right]$ and low p_T

<u>Not in this talk:</u>

- Quark initiated processes (DY, W/Z production) $\longrightarrow$ talk by Daniel Samitz
- Finite top mass $(\sim m_H)$ effects $\longrightarrow$ [Grazzini, Sargsyan '13] [Lindert, Melnikov, Tancredi, Wever '17]
- Subleading power in m_b/m_H $\longrightarrow$ [Melinkov, Penin '16] [Penin, Liu '17]
- NNLO cross section $\longrightarrow$ [Boughezal, Caola, Melnikov, Petriello, Schulze '15] [Boughezal, Focke, Giele, Liu, Petriello '15][Caola, Melnikov, Schulze '15]



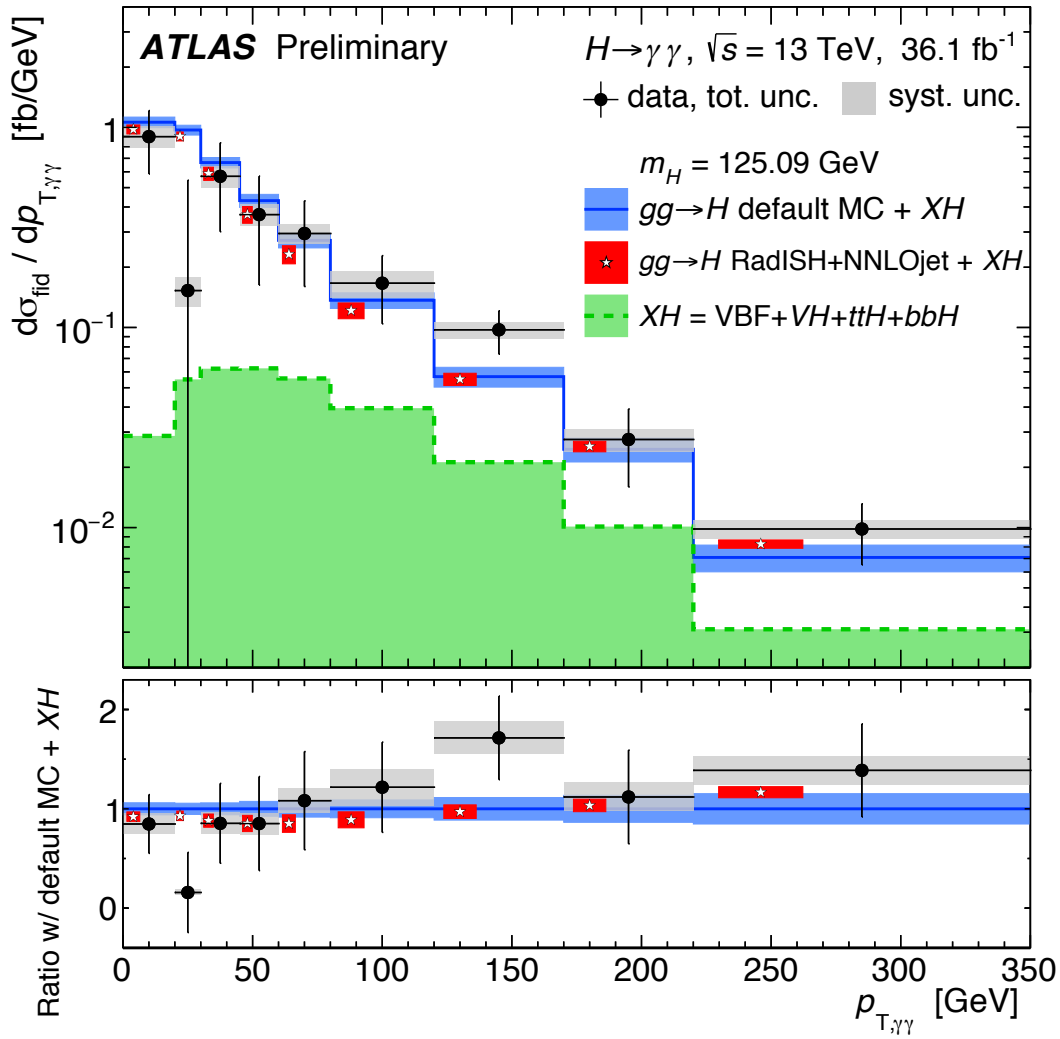
Outline

- Motivation
- Factorization at low q_T
- Calculation of TMD beam functions
- Results
- Summary/Outlook

Motivation

Higgs production @ LHC
p_T distribution:

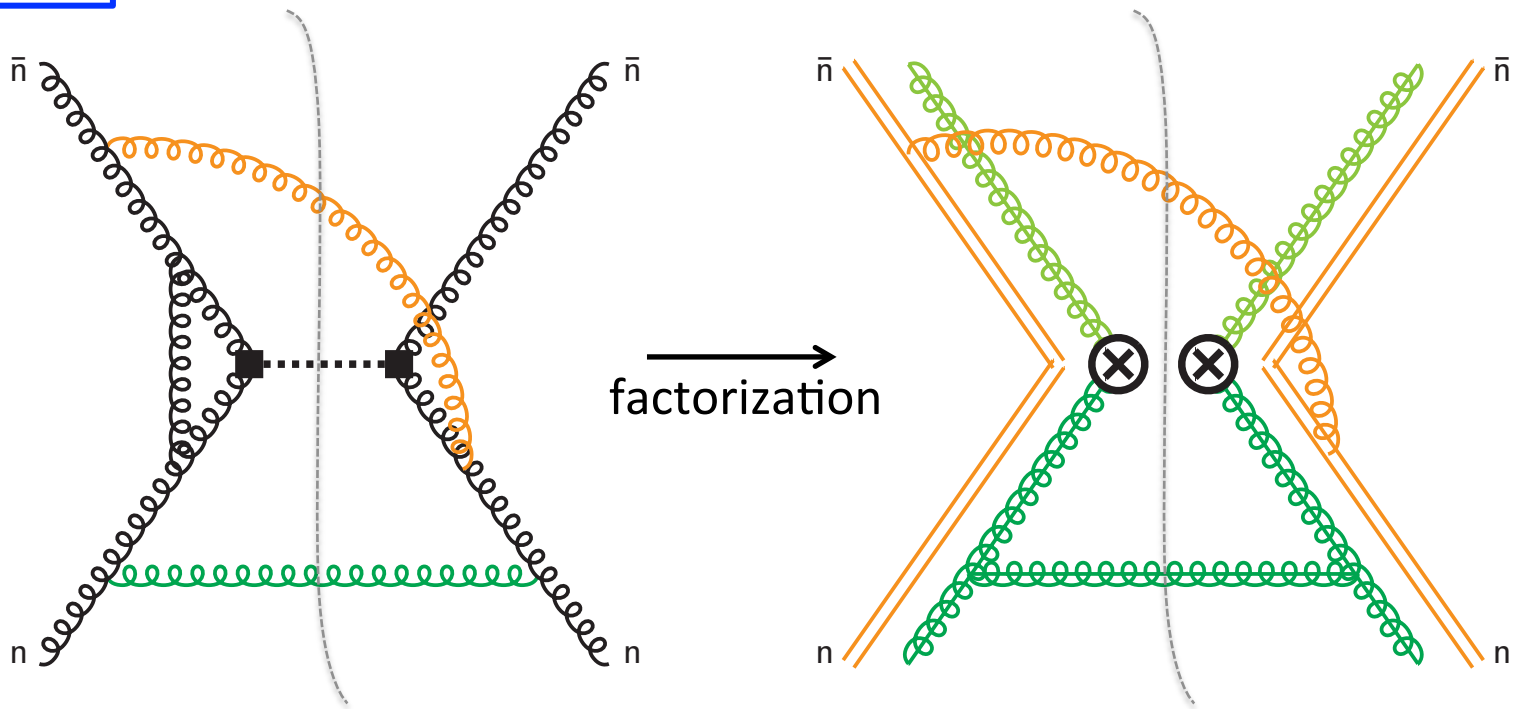
- Gluon fusion dominant
- Study ggH coupling
  [Grazzini, Ilnicka, Spira, Wiesemann '16]
- Large logs in peak region
  $\longrightarrow$ Resummation!
- Precision observable
  N^{3}LL correction $\lesssim 10\%$
  [Bizon, Monni, Re, Rottoli, Torielli '16]
- No systematic description of bottom mass effects yet



Factorization at low q_T

$\boxed{m = 0}$ $\quad n_f = 5$

[Collins, Soper, Sterman 1984]
[Becher, Neubert '11; Echevarria, Idilbi, Scimemi '12; Chiu, Jain, Neill, Rothstein '12]



$$\frac{d\sigma}{dq_T} = H(Q) \times \left[B_g(x_a, q_T) \otimes B_g(x_b, q_T) \otimes S(q_T) \right] + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

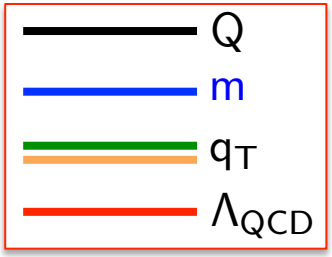
PDF

beam function: $\mathcal{I}_{gi}(x, q_T) \otimes f_i(x) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{q_T}\right)$

soft function

Factorization at low q_T

$$\boxed{q_T \ll m \ll Q}$$



number of active flavors

$$\frac{d\sigma}{dq_T} = \underbrace{H^{(5)}(Q) \times H_m(m)}_{H_c(m) \times H_c(m) \times H_s(m)} \times \left[B_g^{(4)}(x_a, q_T) \otimes B_g^{(4)}(x_b, q_T) \otimes S^{(4)}(q_T) \right] + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

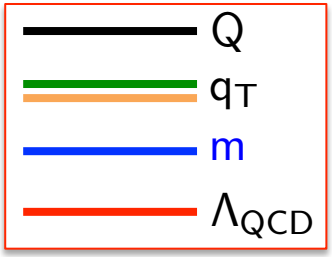
mass mode matching fcts. for soft/collinear mass modes with $p^2 \sim m^2$

Ingredients & anomalous dimensions known at two loops $\longrightarrow$ NNLL' precision

- ✓ massless $B_g(x, q_T)$, $S(q_T)$ [Catani, Grazzini '11] [Gehrmann, Luebbert, Yang '14] [Luebbert, Oredsson, MS '16] [Echevarria, Scimemi, Vladimirov '15,16]
- ✓ massless $H(Q)$ (QCD form factor) [Lee, Smirnov, Smirnov '10] [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
- ✓ $H_s(m)$ (qq channel + Casimir scaling) [Hoang, Pathak, Pietrulewicz, Stewart '15]
- ✓ $H_c(m)$ extracted from consistency with DIS ($x \to 1$) and PDF matching [Hoang, Pietrulewicz, Samitz '15]

Factorization at low q_T

$$m \ll q_T \ll Q$$



$$\frac{d\sigma}{dq_T} = H^{(5)}(Q) \times \left[B_g^{(5)}(x_a, q_T) \otimes B_g^{(5)}(x_b, q_T) \otimes S^{(5)}(q_T) \right] + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

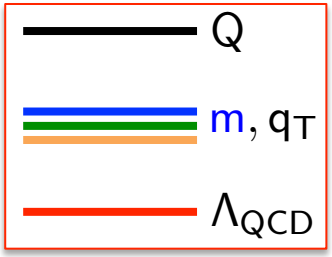
$$\mathcal{I}_{gi}^{(5)}(x, q_T) \otimes \left[\mathcal{M}_{ij}(x, m) \otimes f_j^{(4)}(x) \right] + \mathcal{O}\left(\frac{m^2}{q_T^2}, \frac{\Lambda_{QCD}^2}{m^2}\right)$$

Ingredients & anomalous dimensions known at two loops $\longrightarrow$ NNLL' precision

- ✓ massless $H(Q)$, $\mathcal{I}_{gi}(x, q_T)$, $S(q_T)$
- ✓ massive matching coefficient $\mathcal{M}_{ij}(x, m)$ for PDF [Buza, Matiounine, Smith, van Neerven, 1998]

Factorization at low q_T

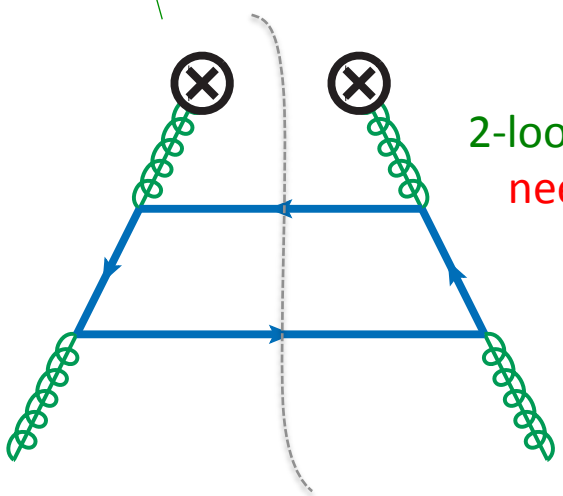
$q_T \sim m \ll Q$ (peak region)



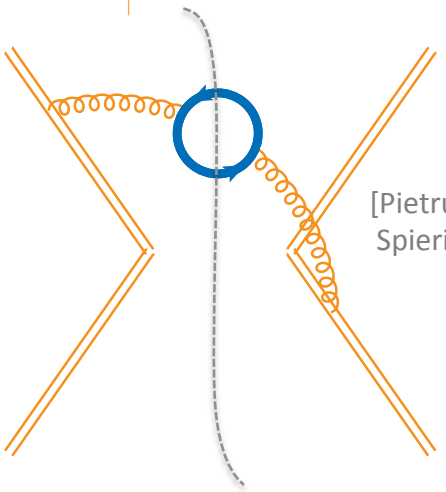
$$\frac{d\sigma}{dq_T} = H^{(5)}(Q) \times \left[B_g^{(5)}(x_a, q_T, m) \otimes B_g^{(5)}(x_b, q_T, m) \otimes S^{(5)}(q_T, m) \right] + \mathcal{O}\left(\frac{q_T}{Q}\right)$$

$$\mathcal{I}_{gi}^{(5)}(x, q_T, m) \otimes f_i^{(4)}(x) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m^2}\right)$$

2-loop soft function known ✓



2-loop beam function needed for NNLL' precision!



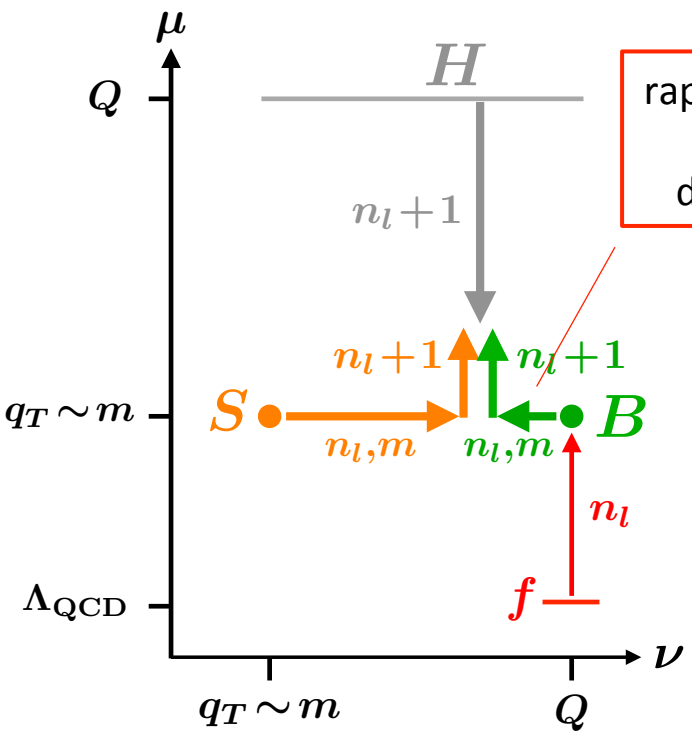
[Pietrulewicz, Samitz, Spiering, Tackmann '17]

Factorization at low q_T

$q_T \sim m \ll Q$ (peak region)

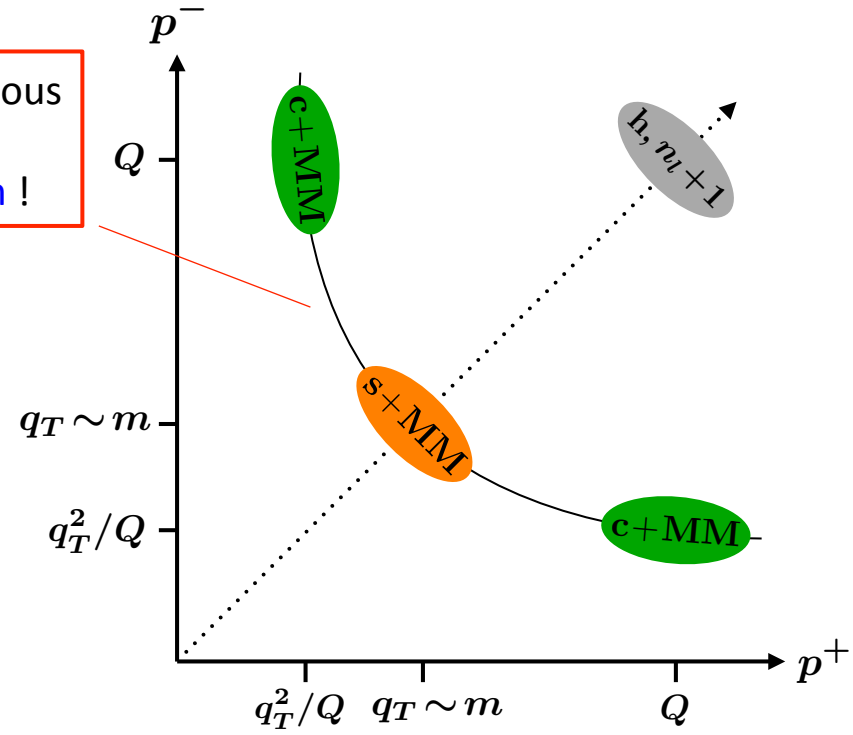
[Pietrulewicz, Samitz, Spiering, Tackmann '17]

RG running:



rapidity anomalous dimension depends on m !

SCET modes:



(R)RGEs resum large logs $\sim \log^n\left(\frac{q_T^2}{Q^2}\right), \log^n\left(\frac{m^2}{Q^2}\right), \log^n\left(\frac{\Lambda_{QCD}^2}{q_T^2}\right), \log^n\left(\frac{\Lambda_{QCD}^2}{m^2}\right)$

Calculation of TMD beam functions

Generic gluon beam function in SCET: (lightlike beam directions: n, $\bar{n}$)

$$B_g^{\mu\nu} = -\omega \left\langle P_n \middle| \mathrm{Tr}\left\{ \mathcal{B}_n^{\mu}(0)\, \widehat{\mathcal{M}}\, \delta(\omega - \bar{n}\cdot\hat{\mathcal{P}})\, \mathcal{B}_n^{\nu}(0) \right\} \middle| P_n \right\rangle$$

$\omega \equiv x\sqrt{s}$

proton state

measurement operator

n-collinear gluon field:

$$\mathcal{B}_n^{\mu}(y) = \frac{1}{g}\left[W_n^{\dagger}(y)\, iD_{n\perp}^{\mu} W_n(y)\right]$$

collinear Wilson line

Calculation of TMD beam functions

Generic gluon beam function in SCET: (lightlike beam directions: n, $\bar{\mathsf{n}}$)

$$\mathsf{B}_{\mathsf{g}}^{\mu\nu} = -\omega \left\langle \mathsf{P}_{\mathsf{n}} \middle| \mathsf{Tr}\left\{ \mathcal{B}_{\mathsf{n}}^{\mu}(0)\, \widehat{\mathcal{M}}\, \delta(\omega - \bar{\mathsf{n}}\cdot\hat{\mathcal{P}})\, \mathcal{B}_{\mathsf{n}}^{\nu}(0) \right\} \middle| \mathsf{P}_{\mathsf{n}} \right\rangle$$

Typical measurement operators:

inclusive: $\widehat{\mathcal{M}} = \mathbb{1}$ (PDF)

N-jettiness: $\widehat{\mathcal{M}} = \delta(\tau - \mathsf{n}\cdot\hat{\mathsf{p}})$

transverse momentum: $\boxed{\widehat{\mathcal{M}} = \delta^{(2)}(\mathsf{q}_{\mathsf{T}} - \hat{\mathsf{p}}_{\mathsf{T}})}$ ("TMDPDF")

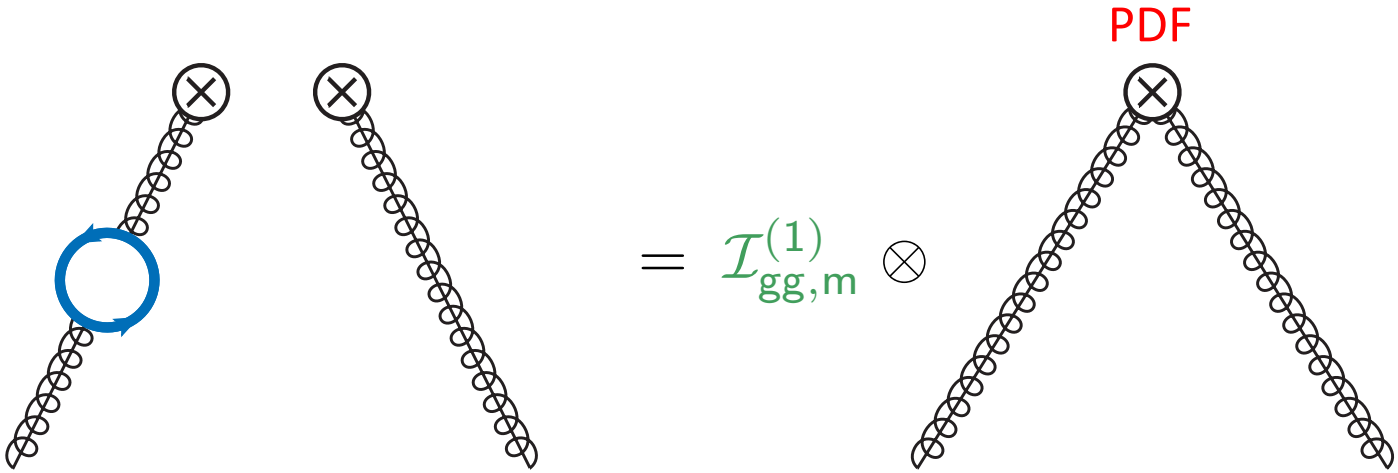
Calculation of TMD beam functions

OPE: $B_g^{\mu\nu}(x, q_T, m) = \mathcal{I}_{gi}^{\mu\nu}(x, q_T, m) \otimes f_i(x) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m^2}, \frac{\Lambda_{QCD}^2}{q_T^2}\right)$

partonic: $B_{g/j}^{\mu\nu}(x, q_T, m) = \mathcal{I}_{gi}^{\mu\nu}(x, q_T, m) \otimes f_{i/j}(x)$

Two orthogonal tensor structures. For NNLL' sufficient to calculate $\mathcal{I}_{gi} \equiv \mathcal{I}_{gi,\mu}^{\mu}$

1-loop matching:



$= \mathcal{I}_{gg,m}^{(1)} \otimes$

$\overline{\mathrm{MS}}$ renormalization $\longrightarrow$ $\mathcal{I}_{gg}^{(1)}(x, q_T, m, \mu) = \frac{\alpha_s}{3\pi} T_F\, \delta(1-x)\, \delta^{(2)}(q_T)\, \ln\left(\frac{m^2}{\mu^2}\right)$

Calculation of TMD beam functions

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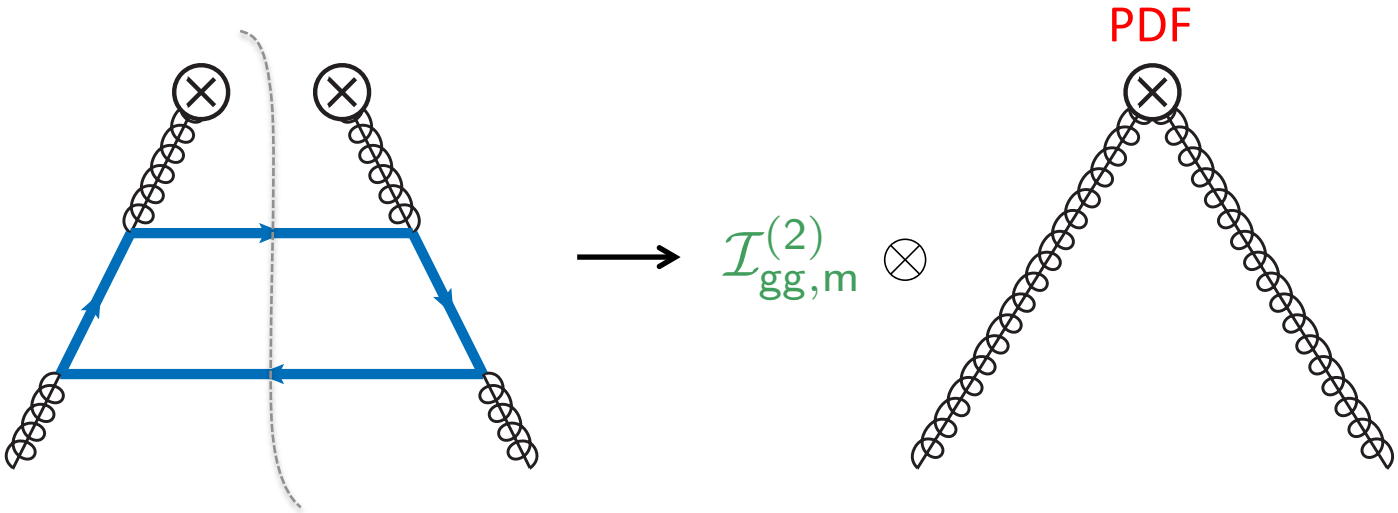
2-loop matching:



Calculation of TMD beam functions

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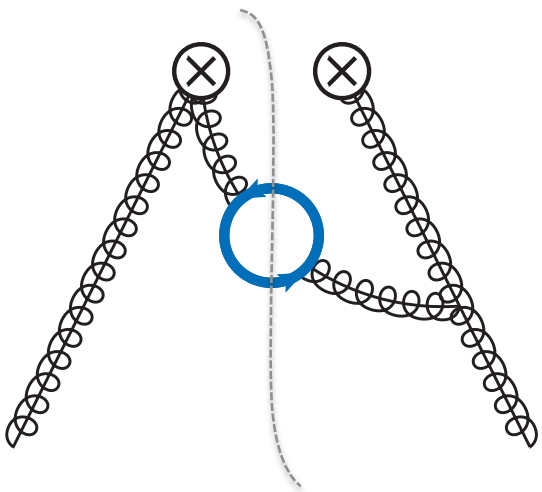

$$\propto \frac{1}{1-x}$$

rapidity divergence for $(x \to 1)$!

Calculation of TMD beam functions

OPE: $B_g^{\mu\nu}(x, q_T, m) = \mathcal{I}_{gi}^{\mu\nu}(x, q_T, m) \otimes f_i(x) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m^2}, \frac{\Lambda_{QCD}^2}{q_T^2}\right)$

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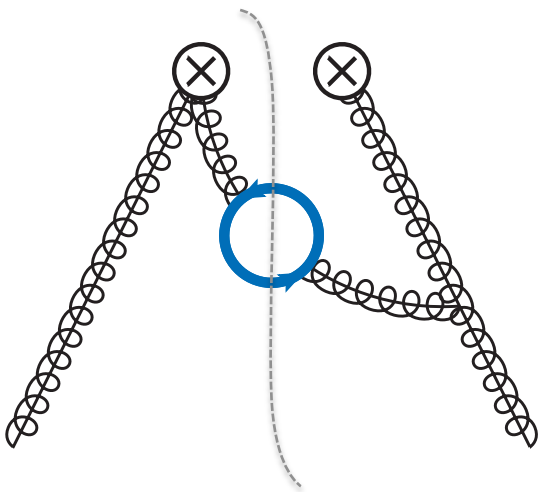

$$\propto \frac{1}{1-x} (1-x)^{-\eta} \left(\frac{Q}{\nu}\right)^{-\eta}$$

rapidity regulator

[Chiu, Jain, Neill, Rothstein '12]

Calculation of TMD beam functions

OPE: $B_g^{\mu\nu}(x, q_T, m) = \mathcal{I}_{gi}^{\mu\nu}(x, q_T, m) \otimes f_i(x) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m^2}, \frac{\Lambda_{QCD}^2}{q_T^2}\right)$

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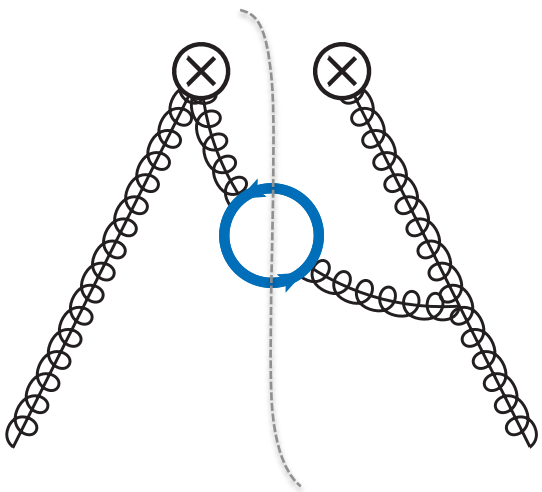
Two orthogonal tensor structures. For NNLL' sufficient to calculate $\mathcal{I}_{gi} \equiv \mathcal{I}_{gi,\mu}^{\mu}$

2-loop matching:



$$\propto \underbrace{\frac{1}{1-x}\,(1-x)^{-\eta}\left(\frac{Q}{\nu}\right)^{-\eta}}_{\left[-\frac{1}{\eta}+\log\left(\frac{Q}{\nu}\right)\right]\delta(1-x)+\left[\frac{1}{1-x}\right]_{+}+\ldots}$$

Calculation of TMD beam functions

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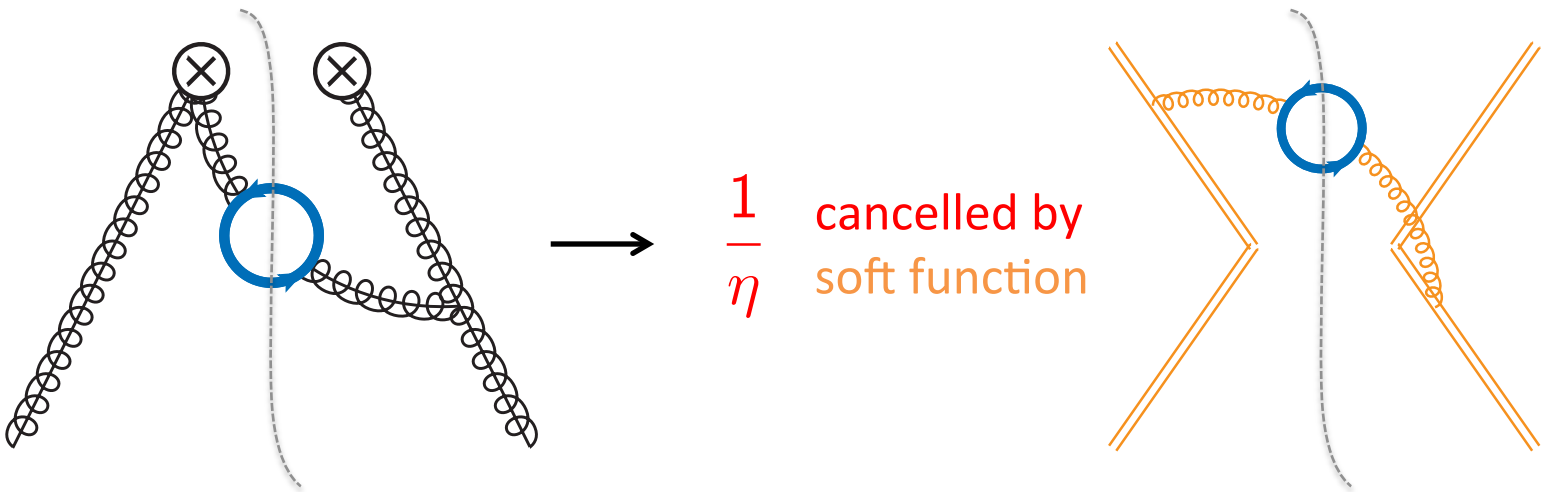
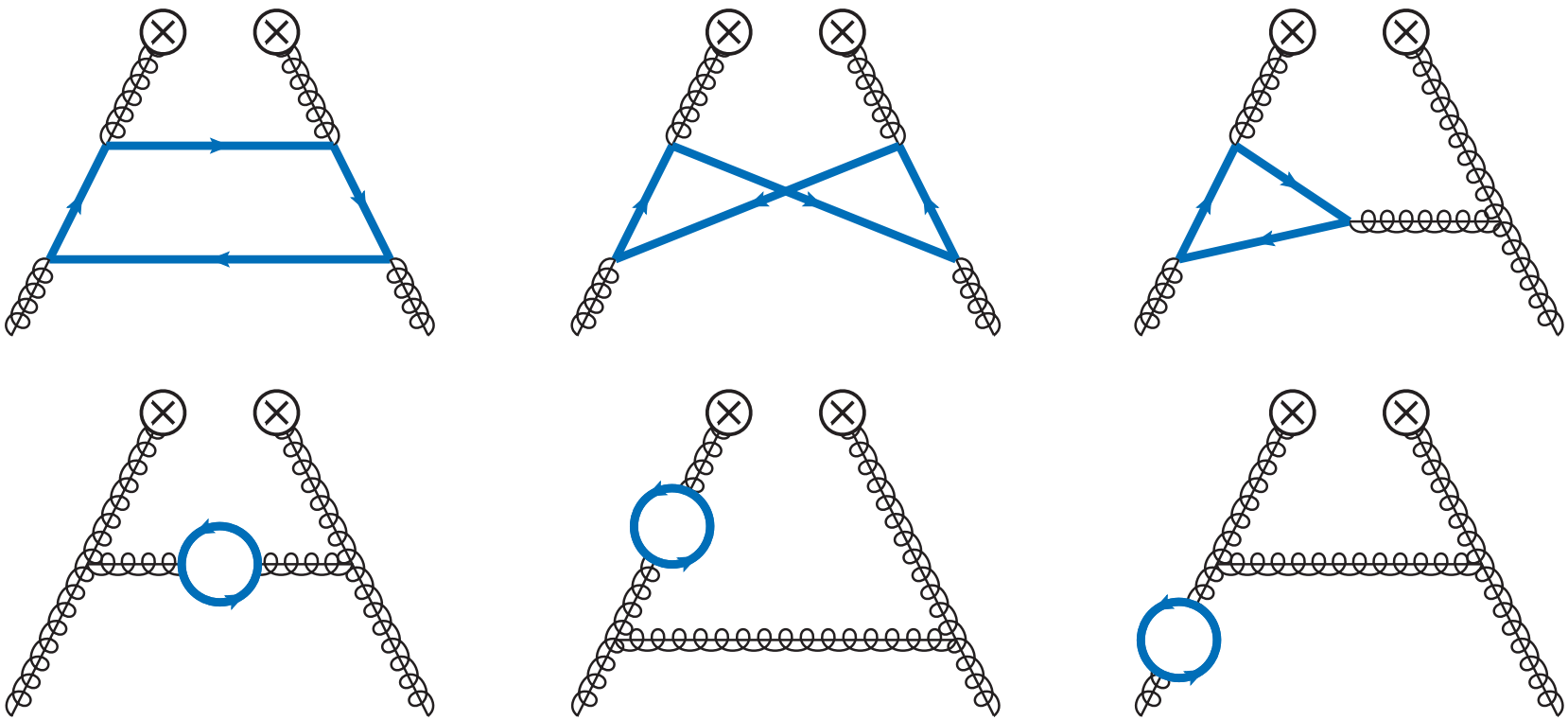


Calculation of TMD beam functions

2-loop matching: gg − channel diagrams (real-real + real-virtual)

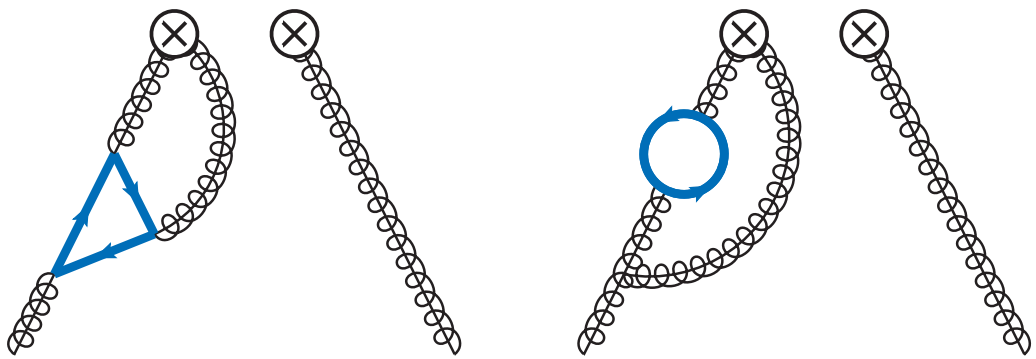


+ diagrams with gluons attached to Wilson lines

Consider all possible cuts!

Calculation of TMD beam functions

2-loop matching: gg − channel purely virtual diagrams



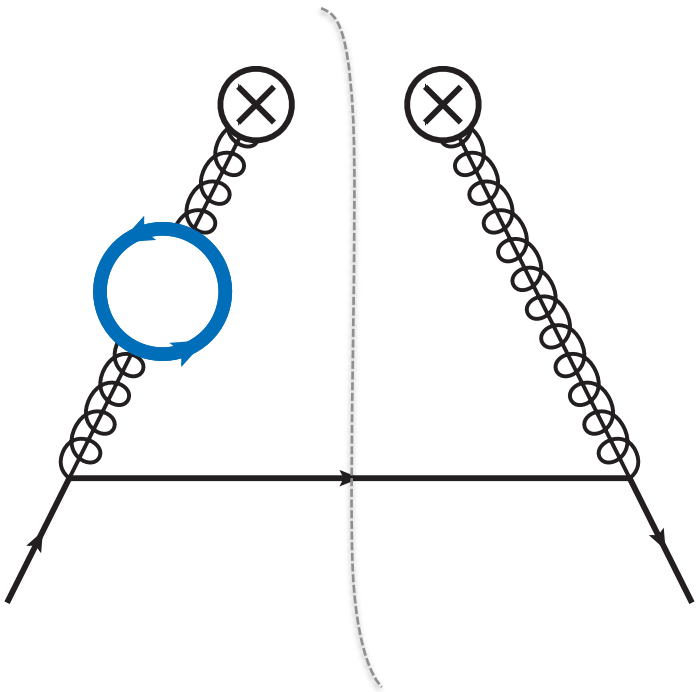
+ self energy diagrams

Same as in massive PDF matching:

$$f_i^{(5)}(x, m) = \mathcal{M}_{ij}(x, m) \otimes f_j^{(4)}(x) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m^2}\right)$$

Calculation of TMD beam functions

2-loop matching: gq − channel diagram (real-virtual)



Results

splitting function

$$\mathcal{I}^{(2)}_{gq,m}(q_T, m, z, \mu) = \frac{\alpha_s^2 C_F T_F}{16\pi^2}\,\frac{1}{\pi}\,\frac{p_{gq}(z)}{q_T^2}\left\{\frac{16}{3}\sqrt{1+\frac{4m^2}{q_T^2}(1-z)}\left[1-2\frac{m^2}{q_T^2}(1-z)\right]\right.$$

$$\left.\times\ln\left(\frac{\sqrt{1+\frac{4m^2}{q_T^2}(1-z)}+1}{\sqrt{1+\frac{4m^2}{q_T^2}(1-z)}-1}\right)-\frac{80}{9}+\frac{64}{3}\frac{m^2}{q_T^2}(1-z)\right\}+\frac{\alpha_s T_F}{4\pi}\,\frac{8}{3}L_m\,\mathcal{I}^{(1)}_{gq}(q_T, z, \mu)$$

$$\mathcal{I}^{(2)}_{gg,m}(q_T, m, z, \mu, \nu) = \frac{\alpha_s^2 T_F^2}{16\pi^2}\,\frac{16}{9}L_m^2\,\delta^{(2)}(q_T)\,\delta(1-z)+\frac{\alpha_s T_F}{4\pi}\,\frac{8}{3}L_m\,\mathcal{I}^{(1)}_{gg}(q_T, z, \mu)$$

$$+\frac{\alpha_s^2 C_F T_F}{16\pi^2}\left\{\delta^{(2)}(q_T)\,\delta(1-z)\left[4L_m-15\right]+\text{regular terms}\right\},$$

$$+\frac{\alpha_s^2 C_A T_F}{16\pi^2}\left\{\delta^{(2)}(q_T)\,\delta(1-z)\left[\left(\frac{8}{3}L_m^2+\frac{80}{9}L_m+\frac{224}{27}\right)\ln\frac{\nu}{Q}+\frac{16}{3}L_m+\frac{10}{9}\right]\right.$$

$$\left.+\mathcal{L}_0(q_T,\mu)\left[-\frac{32}{3}L_m\ln\frac{\nu}{\omega}\,\delta(1-z)+\frac{16}{3}L_m p_{gg}(z)\right]+\text{regular terms in } q_T\right\}$$

$$\mathcal{L}_0(q_T,\mu)\equiv\frac{1}{\pi}\frac{1}{\mu^2}\left[\frac{\mu^2}{q_T^2}\right]_+ \qquad L_m\equiv\ln\left(\frac{m^2}{\mu^2}\right)$$

Results

Cross checks:

- ✓ Check small mass limit:

$$\mathcal{I}^{(5)}_{gi}(x, q_T, m, \mu, \nu) = \mathcal{I}^{(5)}_{gi}(x, q_T, \mu, \nu) \otimes \underbrace{\mathcal{M}_{ij}(x, m, \mu)}_{\text{massive PDF matching}} + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

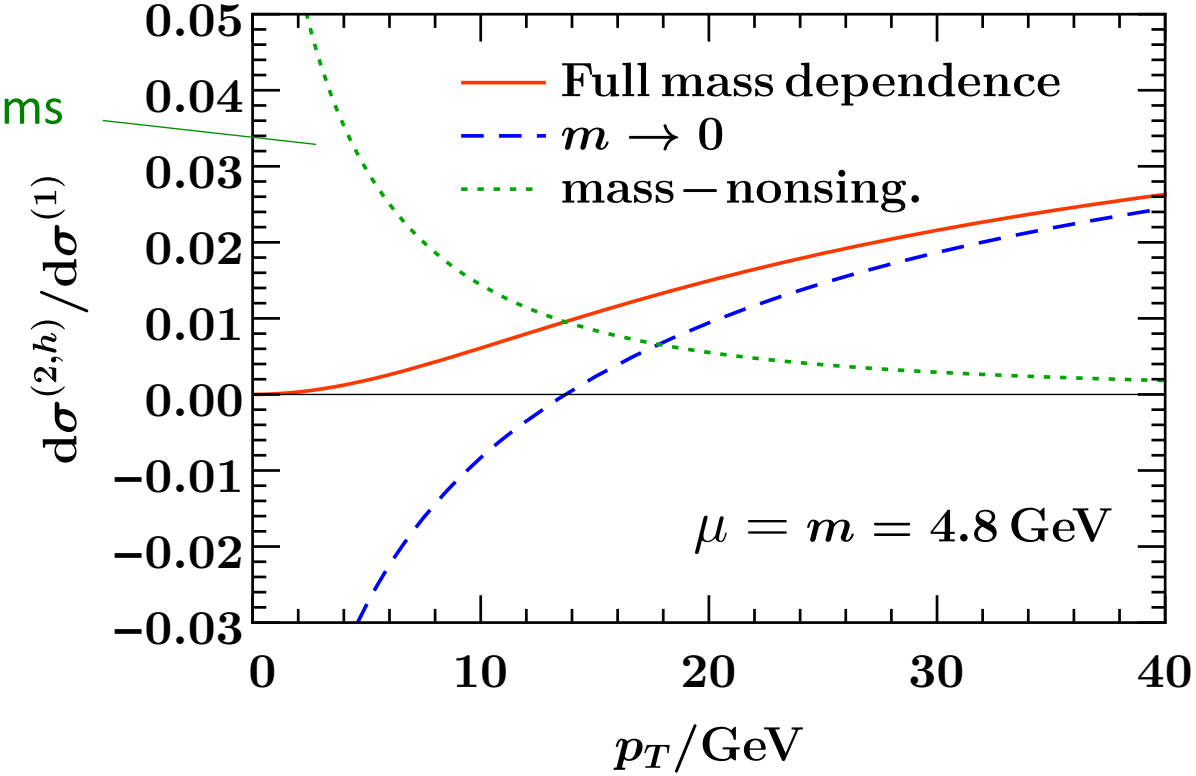
- ✓ RG consistency:
  μ ,ν dependent terms fixed by known anomalous dimensions

- Gauge invariance (general covariant gauge) $\longrightarrow$ WIP

Results

Preliminary FO spectrum, relative effect of massive corrections:

$\left(\frac{m}{q_T}\right)^n$ terms



⟶ Few percent effect! Full analysis including resummation to be done ...

Summary/Outlook

- ✓ Calculated massive quark effects in TMD gluon beam function @ 2loops

$$B_g^{\mu\nu}(x, q_T, m, \mu, \nu)$$

- ✓ Now all ingredients for NNLL' Higgs transverse momentum spectrum in gluon fusion known

- ✓ Fixed order bottom mass effects at small q_T (peak) at the percent level

Outlook:

- Full-fledged analysis including resummation & matching to NNLO