

Humboldt-Universität zu Berlin  
Institute of Physics  
Quantum Field and String Theory Group



# Symmetries of the S-matrix

Josua Faller  
[josua.faller@physik.hu-berlin.de](mailto:josua.faller@physik.hu-berlin.de)

supervised by Jan Plefka

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# Motivation and Outline



- ▶ Importance of symmetries
- ▶ Restrictions on possible symmetries for physical theories

- ▶ Simplifies calculations, e.g. Kepler problem
- ▶ Symmetries form a paradigm in current theoretical physics
  - ▶ Physical theories are dictated by symmetries
  - ▶ e.g. General Relativity (diffeomorphism invariance), Gauge Theories (invariance under gauge transformations)

Hence:

- ▶ Better understanding of symmetries will lead to a deeper understanding of possible theories

## Properties of a symmetry generator

1. A symmetry transformation should not change the outcome of a measurement
2. A symmetry generator should act on incoming and outgoing particles the same way
3. A symmetry generator should act on multiparticle states locally

## Coleman-Mandula Theorem

Coleman, Mandula (1967)

If a theory conserves momentum, energy and rotation in space-time (Poincaré symmetry), any additional conserved quantity is independent of these space-time transformations.

# Coleman-Mandula Theorem



## Properties of a symmetry generator $B_I$

1. A symmetry transformation should not change the outcome of a measurement, i.e.  $B_I = B_I^\dagger$
2. A symmetry generator should act on incoming and outgoing particles the same way, i.e.  $[S, B_I] = 0$
3. A symmetry generator should act on multiparticle states locally, i.e.  $B_I^{\text{two particle}} |p_1, p_2\rangle = (B_I^{(1)} |p_1\rangle) |p_2\rangle + |p_1\rangle (B_I^{(2)} |p_2\rangle)$

## Coleman-Mandula Theorem

Coleman, Mandula (1967)

If a theory conserves momentum, energy ( $P_\mu$ ) and rotation ( $M_{\mu\nu}$ ) in space-time (Poincaré symmetry), any additional conserved quantity ( $B_I$ ) is independent of these space-time transformations.

$$[P_\mu, B_I] = [M_{\mu\nu}, B_I] = 0$$

## Properties of a symmetry generator

1. A symmetry transformation should not change the outcome of a measurement
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## Coleman-Mandula Theorem

Coleman, Mandula (1967)

For the massless case: Each additional conserved quantity commutes with the conformal algebra

- ▶ A new type of symmetry with many nice properties has been proposed: Supersymmetry

Therefore: Most general form of (super-)symmetries which commute with the S-matrix?

## HLS-Theorem

Haag-Łopuszański Sohnius (1974)

By considering fermionic conserved quantities and the most general symmetry group for massless particles is superconformal symmetry

- ▶ The inclusion of fermionic generators has been the generalization of the approach of Coleman and Mandula

## Non-local symmetries:

- ▶ At tree level superconformal Yang-Mills  $\mathcal{N} = 4$  possesses a (non-local) symmetry: Yangian symmetry

Drummond, Henn, Plefka (2009)

- ▶ The generators of this Yangian symmetry commute with the S-matrix

Therefore: Most general form of non-local symmetries which commute with the S-matrix?



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Therefore: Most general form of non-local symmetries which commute with the S-matrix?

$$J_A^{\text{two particle}} |p_1, p_2\rangle = \left( J_A^{(1)} |p_1\rangle \right) |p_2\rangle + |p_1\rangle \left( J_A^{(2)} |p_2\rangle \right) + f_A^{BC} \sum_{i < j} l_B^i |p_1\rangle l_C^j |p_2\rangle$$

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Yangian algebra of the Lie algebra of the superconformal group:

- ▶ Infinitely many generators;  $I_A, J_A, \dots$
- ▶  $I_A$  are generators of the superconformal symmetry
- ▶ Commutation relations:

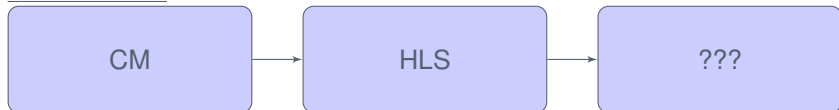
$$[I_A, I_B] = f_{AB}^C I_C, \quad [I_A, J_B] = f_{AB}^C J_C$$

- ▶ Consistency for 2- and 3-particle states imposes non-trivial restrictions: Serre relations

## Starting point:

- Symmetry group includes conformal group and the generators commute with S-matrix

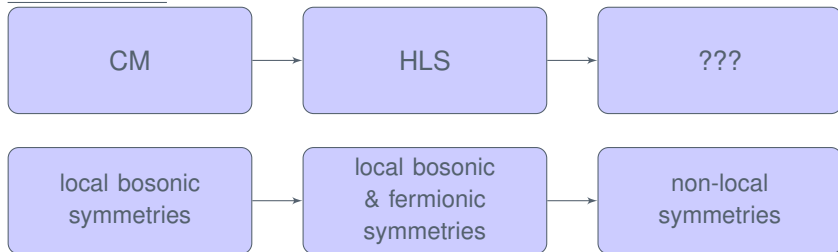
## General idea:



## Starting point:

- Symmetry group includes conformal group and the generators commute with S-matrix

## General idea:



# The End



# Thank you for your attention