Humboldt-Universität zu Berlin
Institute of Physics
Quantum Field and String Theory Group

Symmetries of the S-matrix

Josua Faller
josua.faller@physik.hu-berlin.de

supervised by Jan Plefka

September 19, 2016

Motivation and Outline

- Importance of symmetries
- Restrictions on possible symmetries for physical theories

- Simplifies calculations, e.g. Kepler problem
- Symmetries form a paradigm in current theoretical physics
  - Physical theories are dictated by symmetries
  - e.g. General Relativity (diffeomorpism invariance), Gauge Theories (invariance under gauge transformations)

Hence:

- Better understanding of symmetries will lead to a deeper understanding of possible theories

Coleman-Mandula Theorem

Properties of a symmetry generator

1. A symmetry transformation should not change the outcome of a measurement
2. A symmetry generator should act on incoming and outgoing particles the same way
3. A symmetry generator should act on multiparticle states locally

Coleman-Mandula Theorem `Coleman, Mandula (1967)`

If a theory conserves momentum, energy and rotation in space-time (Poincaré symmetry), any additional conserved quantity is independent of these space-time transformations.

Coleman-Mandula Theorem

Properties of a symmetry generator B_l

1. A symmetry transformation should not change the outcome of a measurement, i.e. $B_l = B_l^\dagger$
2. A symmetry generator should act on incoming and outgoing particles the same way, i.e. $[S, B_l] = 0$
3. A symmetry generator should act on multiparticle states locally, i.e. $B_l^{\text{two particle}} |p_1, p_2\rangle = \left(B_l^{(1)} |p_1\rangle\right) |p_2\rangle + |p_1\rangle \left(B_l^{(2)} |p_2\rangle\right)$

Coleman-Mandula Theorem

Coleman, Mandula (1967)

If a theory conserves momentum, energy (P_μ) and rotation ($M_{\mu\nu}$) in space-time (Poincaré symmetry), any additional conserved quantity (B_l) is independent of these space-time transformations.

$$[P_\mu, B_l] = [M_{\mu\nu}, B_l] = 0$$

Coleman-Mandula Theorem

Properties of a symmetry generator

1. A symmetry transformation should not change the outcome of a measurement
2. A symmetry generator should act on incoming and outgoing particles the same way
3. A symmetry generator should act on multiparticle states locally

Coleman-Mandula Theorem Coleman, Mandula (1967)

For the massless case: Each additional conserved quantity commutes with the conformal algebra

Haag-Łopuszaṅski Sohnius (HLS)

- A new type of symmetry with many nice properties has been proposed: Supersymmetry

Therefore: Most general form of (super-)symmetries which commute with the S-matrix?

HLS-Theorem `Haag-Łopuszaṅski Sohnius (1974)`

By considering fermionic conserved quantities and the most general symmetry group for massless particles is superconformal symmetry

- The inclusion of fermionic generators has been the generalization of the approach of Coleman and Mandula

Further Generalization

Non-local symmetries:

- At tree level superconformal Yang-Mills $\mathcal{N} = 4$ possesses a (non-local) symmetry: Yangian symmetry

  `Drummond, Henn, Plefka (2009)`

- The generators of this Yangian symmetry commute with the S-matrix

Therefore: Most general form of non-local symmetries which commute with the S-matrix?

Further Generalization

Non-local symmetries:

- At tree level superconformal Yang-Mills $\mathcal{N} = 4$ possesses a (non-local) symmetry: Yangian symmetry

  Drummond, Henn, Plefka (2009)

- The generators of this Yangian symmetry commute with the S-matrix

Therefore: Most general form of non-local symmetries which commute with the S-matrix?

$$J_A^{\text{two particle}} \left| p_1, p_2 \right\rangle = \left(J_A^{(1)} \left| p_1 \right\rangle \right) \left| p_2 \right\rangle + \left| p_1 \right\rangle \left(J_A^{(2)} \left| p_2 \right\rangle \right) + f_A{}^{BC} \sum_{i<j} I_B^i \left| p_1 \right\rangle I_C^j \left| p_2 \right\rangle$$

Further Generalization

Non-local symmetries:

- At tree level superconformal Yang-Mills $\mathcal{N} = 4$ possesses a (non-local) symmetry: Yangian symmetry

  Drummond, Henn, Plefka (2009)

- The generators of this Yangian symmetry commute with the S-matrix

Therefore: Most general form of non-local symmetries which commute with the S-matrix?

$$J_A^{\text{two particle}} \left|p_1, p_2\right\rangle = \left(J_A^{(1)} \left|p_1\right\rangle\right) \left|p_2\right\rangle + \left|p_1\right\rangle \left(J_A^{(2)} \left|p_2\right\rangle\right) + f_A{}^{BC} \sum_{i<j} I_B^i \left|p_1\right\rangle I_C^j \left|p_2\right\rangle$$

Yangian Symmetry

Yangian algebra of the Lie algebra of the superconformal group:

- Infinitely many generators; $I_A, \; J_A, \ldots$
- I_A are generators of the superconformal symmetry
- Commutation relations:

$$[I_A, I_B] = f_{AB}{}^{C} I_C, \qquad [I_A, J_B] = f_{AB}{}^{C} J_C$$

- Consistency for 2- and 3-particle states imposes non-trivial restrictions: Serre relations

Conclusion

Starting point:

- Symmetry group includes conformal group and the generators commute with S-matrix

General idea:

CM → HLS → ???

Conclusion

Starting point:

- Symmetry group includes conformal group and the generators commute with S-matrix

General idea:

CM → HLS → ???

local bosonic symmetries → local bosonic & fermionic symmetries → non-local symmetries

Thank you for your attention