An Introduction to the Quantum Spectral Curve

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Why should you pay attention?

Investigate the most fundamental object in any QFT:

The two point correlation function

$$\langle\Omega|\, O_1(x)O_2(y)\, |\Omega\rangle$$

Leads to other interesting quantities like Scattering Amplitudes, Form Factors, etc.

N=4 SYM

Conformal symmetry $\rightarrow$ one parameter Δ

Dimension of an Operator

$$\langle\Omega|\, O_1(x) O_2(y)\, |\Omega\rangle = \frac{\delta_{1,2}}{|x-y|^{2\Delta}}$$

aim

Find the dimension of operators

An easy example

The harmonic oscillator

$$\frac{-\hbar^2}{2m}\Psi''(x) + \frac{m\omega^2 x^2}{2}\Psi(x) = E\Psi(x)$$

or equivalently

$$p^2 - i\hbar p' = 2m(E - V) \quad \text{where} \quad p = \frac{\hbar}{i}\frac{\Psi'}{\Psi}$$

How do you solve this?

The harmonic oscillator

$$p^2 - i\hbar p' = 2m(E - V)$$

Analyticity

Assume the function can be expanded in powers of x.

Immediately leads to the following Ansatz

$$p = im\omega x + C + \frac{\hbar}{i}\sum_{i=1}^{N}\frac{1}{x - x_i}$$

The harmonic oscillator

$$p^2 - i\hbar p' = 2m(E - V)$$

Asymptotics

Look at the behaviour as $x \to \infty$

Comparing the constant terms:

$$2\hbar m\omega N + \hbar m\omega = 2mE$$

Energy

$$E = (N + \frac{1}{2})\hbar\omega \quad C = 0$$

The harmonic oscillator

We can extract two more things:

x_i

We require the poles to cancel which gives equations for the x_i

$$x_i = \frac{\hbar}{2m\omega} \sum_{j \neq i} \frac{1}{x_j - x_i}$$

Ψ

$$\Psi = exp(-\frac{m\omega x^2}{2\hbar}) \prod_{i=1}^{N} (x - x_i)$$

Generalization to N=4 SYM

Q-functions

Now have 256 functions: Q_ϕ, Q_8, $Q_{1,5,7}$, $Q_{1,2,3,4,5,6,7,8}$

What happened to the Schrödinger equation?

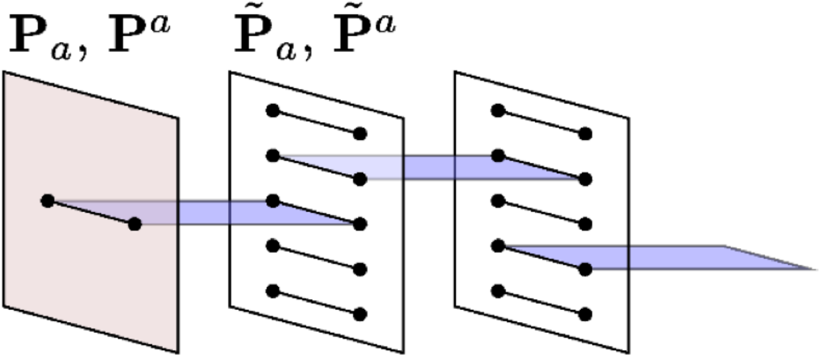
QQ-relations

The equivalent of the Schrödinger equation are the so called QQ-relations, for example

$$Q_{1,2}(u) = Q_1(u+\frac{i}{2})Q_2(u-\frac{i}{2}) - Q_1(u-\frac{i}{2})Q_2(u+\frac{i}{2})$$

Analytic behaviour

The analytic behaviour of the Q-functions:



[**Gromov Kazakov Laurent Volin**]

Assymptotics

Quantum numbers of the operator $\rightarrow$ Assymptotics of the Q funtions.

Dimension

The dimension Δ enters here!

This is the entire Quantum spectral curve!

Summary and Outlook

Two steps:

1. Make an Ansatz for the simplest Q-functions
2. Plug into the QQ-relations to determine unfixed parameters

What else should be done:

- Find the dimensions of various operators
- Evaluate the QSC for non-physical Quantum numbers
- Find the Operatorial picture
- Investigate symmetries of the Q-functions/operators