Theoretical Introduction to LHC Physics

GRK Autumn Block Course

Berlin – September 2016

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bibliography

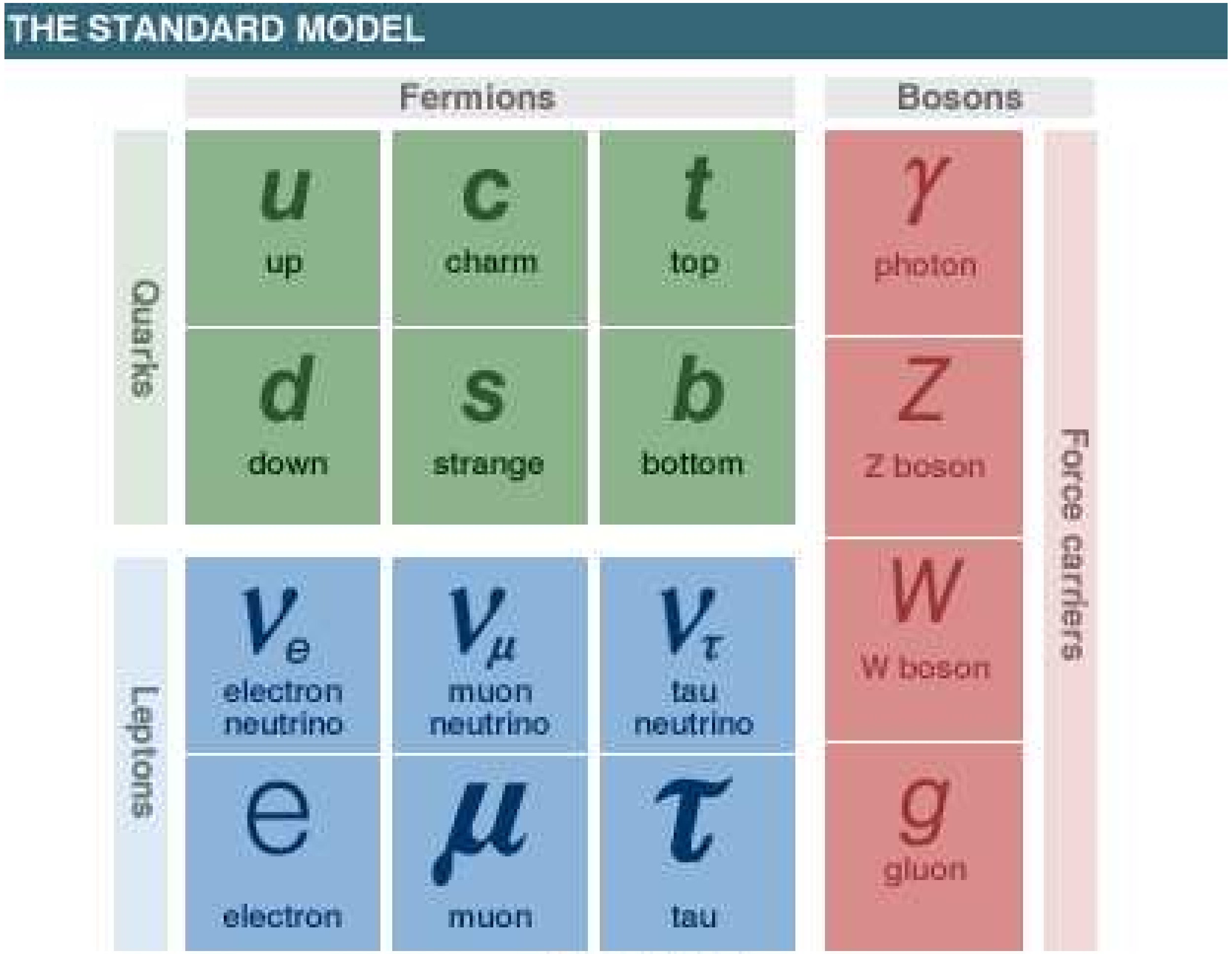
these lectures are based on many references, including:

- lectures at the Maria Laach School
  (in particular those by A. Denner, M. Krämer,
  M. Mühlleitner, L. Reina)
- lectures on specific topics (G. Salam, G. Zanderighi)
- textbooks:
  - Ellis, Sterling, Webber: *QCD and collider physics*
  - Quigg: *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*
  - Muta: *Foundations of Quantum Chromodynamics*
  - Schwartz: *Quantum Field Theory and the Standard Model*
  - Halzen, Martin: *Quarks and Leptons*

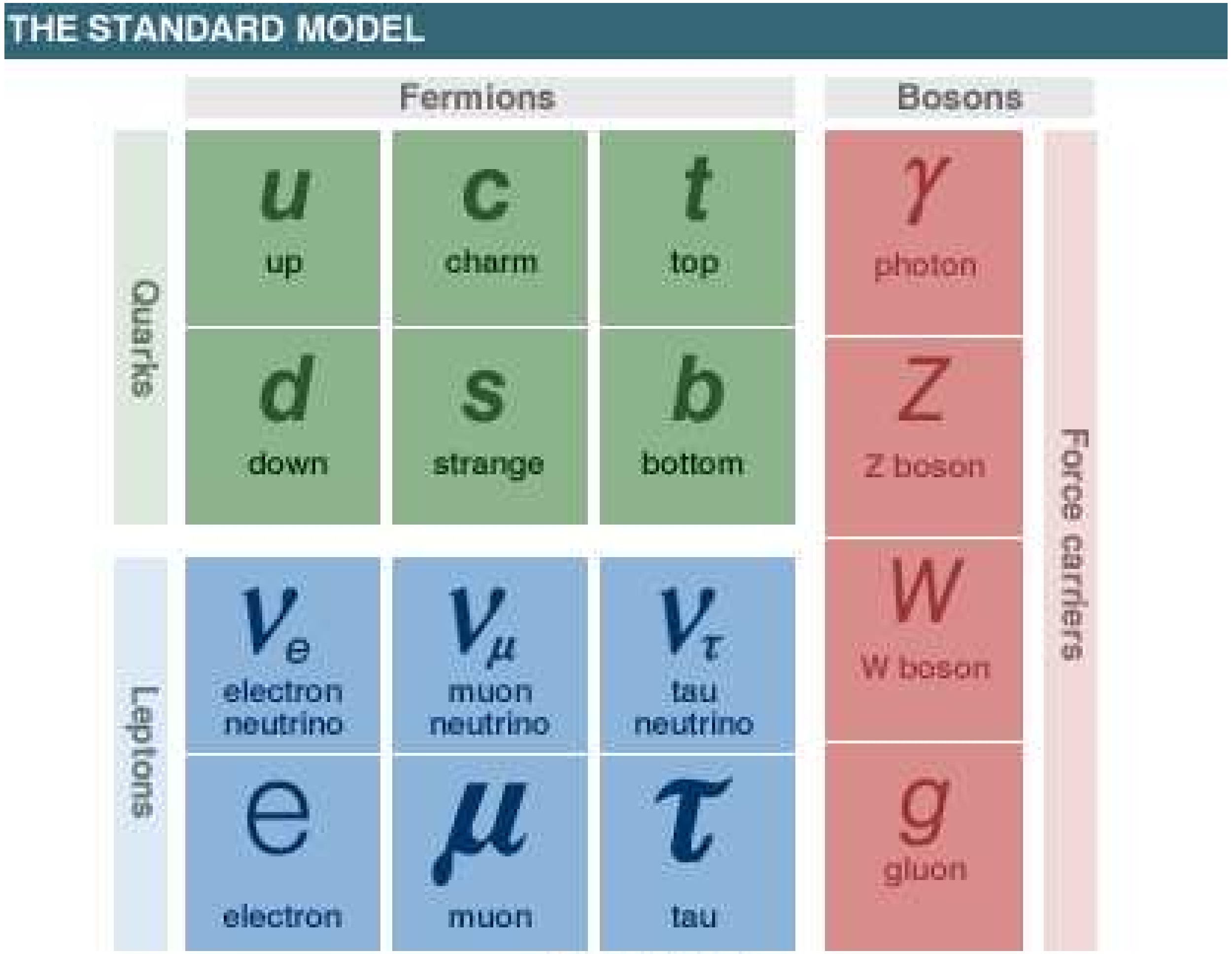
outline

- the Standard Model of elementary particles (SM)
  - local gauge theories
  - electroweak symmetry breaking
- precision calculations for hadron colliders
  - fixed-order perturbation theory
  - beyond fixed order: parton shower simulations
- physics at the LHC
  - electroweak processes
  - Higgs physics
- summary & conclusions

the 20^{th} century picture of elementary particles



the 20^{th} century picture of elementary particles



electromagnetism

$U(1)_{\mathrm{EM}}$

↑

interactions described by
local gauge theories

↓

quantum chromodynamics

$SU(3)_{\mathrm{color}}$

the concept of gauge transformations

electrodynamics: physics of the $\vec{E}$ and $\vec{B}$ fields is described by Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = \rho, \qquad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0,$$
$$\vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}.$$

- alternative notation: em. fields $\vec{E}, \vec{B} \longleftrightarrow$ scalar and vector potential ϕ, $\vec{A}$

$$\vec{B} = \vec{\nabla} \times \vec{A}, \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi$$

- changing ϕ, $\vec{A}$ in a specific way

$$\begin{aligned} \vec{A} &\rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\chi \\ \phi &\rightarrow \phi' = \phi - \partial\chi/\partial t \end{aligned}$$

$\rightarrow$ no impact on $\vec{E}$, $\vec{B}$

the concept of gauge transformations

- changing $\phi, \vec{A}$ in a specific way

$$\begin{aligned}\vec{A} &\to \vec{A}' = \vec{A} + \vec{\nabla}\chi \\ \phi &\to \phi' = \phi - \partial\chi/\partial t\end{aligned}$$

$\to$ no impact on $\vec{E}, \vec{B}$:

$$\begin{aligned}\vec{B} \to \vec{B}' &= \vec{\nabla} \times (\vec{A} + \vec{\nabla}\chi) = \vec{\nabla} \times \vec{A} \\ \vec{E} \to \vec{E}' &= -\frac{\partial(\vec{A} + \vec{\nabla}\chi)}{\partial t} - \vec{\nabla}(\phi - \partial\chi/\partial t) \\ &= -\frac{\partial \vec{A}}{\partial t} - \frac{\partial(\vec{\nabla}\chi)}{\partial t} - \vec{\nabla}\phi + \vec{\nabla}(\partial\chi/\partial t) \\ &= -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi\end{aligned}$$

☞ gauge transformation: change fields in a well-defined manner such that physics does not change

Maxwell's equations in covariant form

- more compact: covariant notation with

$$A^\mu = (\phi, \vec{A})\,, \quad j^\mu = (\rho, \vec{j})\,,$$

$\longrightarrow$ Maxwell's equations:

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = j^\mu\,,$$

$\longrightarrow$ gauge transformation:

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \chi$$

- alternative: introduce field-strength tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu\,,$$

$\longrightarrow$ Maxwell's equations:

$$\partial_\mu F^{\mu\nu} = j^\nu$$

Quantum Electrodynamics (QED)

interactions of charged particles (e.g. electrons) with photons described by:

$$\begin{aligned}\mathcal{L}_{\mathrm{QED}} &= \mathcal{L}_{\mathrm{Dirac}} + \mathcal{L}_{\mathrm{Maxwell}} + \mathcal{L}_{\mathrm{interaction}} \\ &= \bar{\psi}\,(i\not{\partial} - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + e\bar{\psi}\gamma^{\mu}\psi A_{\mu} \\ &= \bar{\psi}\,(i\,\not{D} - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

crucial property: $\mathcal{L}_{\mathrm{QED}}$ is invariant under a local gauge transformation:

$$\psi(x) \to \psi' = e^{i\alpha(x)}\psi(x)\,, \quad A_{\mu} \to A_{\mu} + \frac{1}{e}\partial_{\mu}\alpha$$

$\to$ redefine lepton and photon fields at every point in space-time without changing the physics content of the theory

nota bene: only works, if ψ and A_{μ} are transformed together!

Quantum Electrodynamics (QED)

requirement of local gauge invariance restricts

form of possible contributions to Lagrangian

example: transformation properties of photon mass term:

$$\begin{aligned}
m^2 A_\mu A^\mu \rightarrow m^2 A'_\mu A'^\mu &= m^2 \left(A_\mu + \frac{1}{e}\partial_\mu \alpha\right)\left(A^\mu + \frac{1}{e}\partial^\mu \alpha\right) \\
&= m^2 \left(A_\mu A^\mu + \frac{1}{e}(\partial_\mu \alpha) A^\mu + \right. \\
&\qquad \left. \frac{1}{e} A_\mu (\partial^\mu \alpha) + \frac{1}{e^2}(\partial_\mu \alpha)(\partial^\mu \alpha)\right) \\
&\neq m^2 A_\mu A^\mu
\end{aligned}$$

☞ local gauge invariance violated

Quantum Chromodynamics (QCD)

theory that describes interactions of quarks and gluons

$\longrightarrow$ many similarities with QED, but also some differences:

- quarks are a bit like leptons, but there are three of each type
- gluons are a bit like photons, but there are eight of them
- gluons interact with themselves
- the QCD coupling g_s is larger than the QED one

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_i^{(f)} \left(i \not{D}_{ij} - m_f \delta_{ij} \right) \psi_j^{(f)} - \frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

f: quark flavor
i, j, a: color indices

covariant derivative:

$$D^\mu_{ij} = \partial^\mu \delta_{ij} + i g_s t^a_{ij} A^\mu_a$$

field-strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu$$

the gauge group of QCD

the gauge group of QCD is the special unitary group $SU(N)$ with $N = 3$;

the fundamental representation of SU(N) has $N^2 - 1$ generators $t^a = \frac{1}{2}\lambda^a$ formed by $N \times N$ traceless Hermitian matrices:

$$U = e^{i\theta_a(x)t^a}, \quad a = 1, \ldots, N^2 - 1$$

with the Gell-Mann matrices λ^a:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}.$$

the gauge group of QCD

important group property: commutator of two infinitesimal transformations:

$$
\begin{aligned}
[U(\delta_1), U(\delta_2)] &= U(\delta_1)U(\delta_2) - U(\delta_2)U(\delta_1) \\
&= (i\delta_1^a)(i\delta_1^b)[t^a, t^b] + \mathcal{O}(\delta^3)
\end{aligned}
$$

with $[t^a, t^b] = if^{abc}t_c$ (f^{abc} . . . structure constants of the group)

two matrices do not commute $\rightarrow$ transformations do not commute

(group is called non-Abelian)

compare:

- QED: Abelian gauge group U(1) $\rightarrow$ transformations commute
- 3-dim rotations described by SO(3) group

$\rightarrow$ transformations do not commute

gauge invariance of QCD

local SU(3) transformations include

- gauge transformation of the quark field

$$\psi \to \psi' = U(x)\psi$$

- gauge transformations of the gluon field strength

$$t^a F^a_{\mu\nu} \;\to\; t^a F'^a_{\mu\nu} = U(x) t^a F^a_{\mu\nu} U^{-1}(x)$$

- the covariant derivative transforms “with the field” as

$$D_\mu \psi \to D'_\mu \psi' = U(x) D_\mu \psi$$

☞ the QCD Lagrangian is indeed gauge invariant:

$$-\frac{1}{4} F'^a_{\mu\nu} F'^{\mu\nu}_a \;=\; -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

$$\sum_f \bar{\psi}'^{(f)}_i \left(i \not{D}'_{ij} - m\delta_{ij} \right) \psi'^{(f)}_j \;=\; \sum_{f=1}^{N_f} \bar{\psi}^{(f)}_i \left(i \not{D}_{ij} - m\delta_{ij} \right) \psi^{(f)}_j$$

electroweak interactions

theorist's postulate: description by local gauge theory, but. . .

✔ experimental fact:

the mediators of the weak force
($W^{\pm}$ and Z bosons) are massive!

✘ theoretical problem:

explicit mass terms for gauge bosons violate
local gauge invariance of the Lagrangian

. . . but:

✔ experimental fact:

mediators of the weak force
($W^\pm$ and Z bosons)
are massive!

✘ theoretical problem:

explicit mass terms in Lagrangian
violate local gauge invariance

❖ the solution:

spontaneous symmetry breaking

spontaneous breaking of local gauge symmetry

basic concept:

gauge boson sector of the SM: $\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}}$

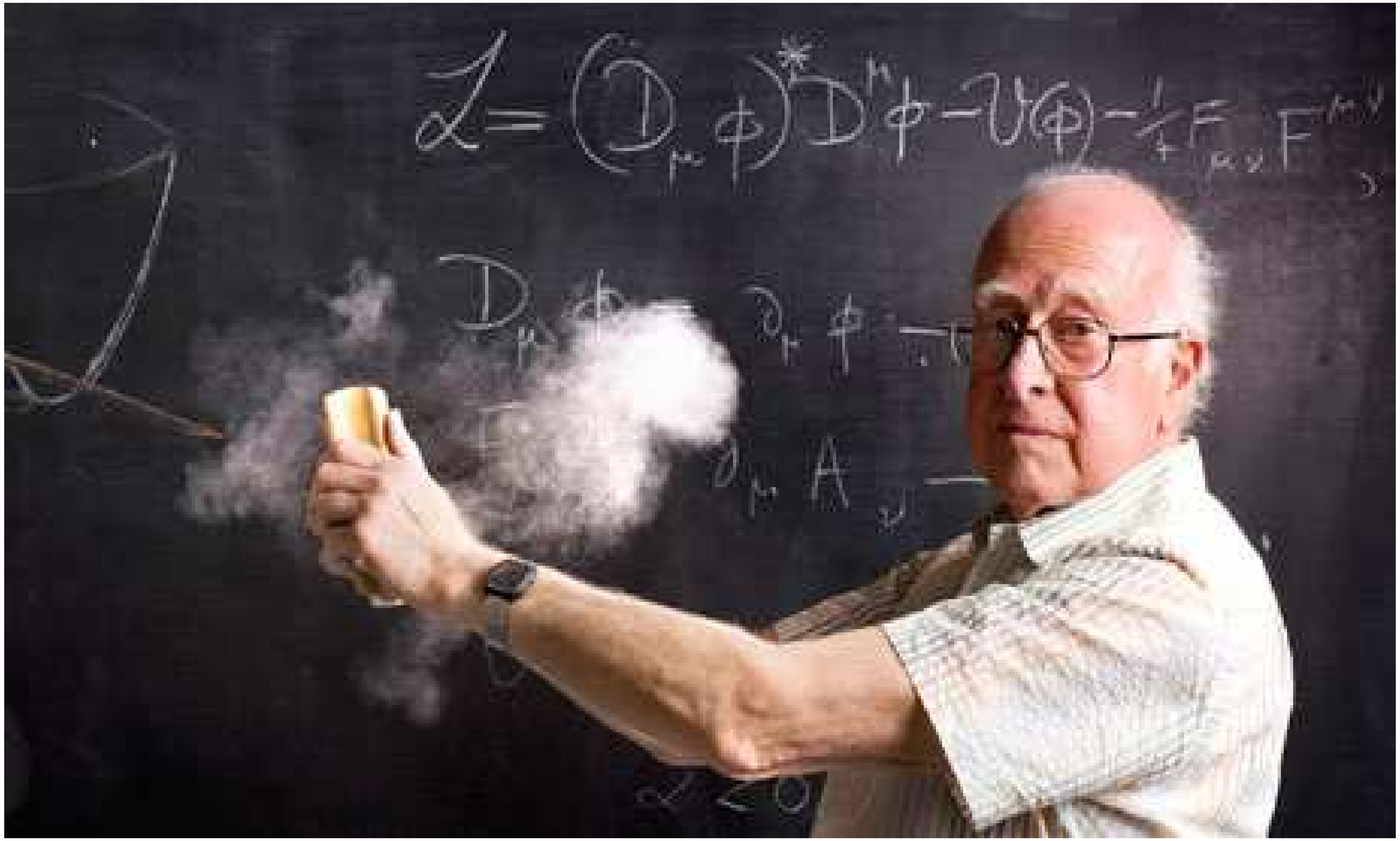
$\mathcal{L}_{\text{gauge}}$ → gauge fields ($W^{\pm}$, Z, γ)

$\mathcal{L}_{\text{Higgs}}$ → extra massive, neutral, scalar field

– full Lagrangian invariant
– vacuum state not invariant under electroweak symmetry

☞ symmetry is spontaneously broken!

more details on spontaneous symmetry breaking



spontaneous symmetry breaking: Abelian gauge theory

recall U(1) local gauge theory with a spin-1 gauge field A_μ:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- explicit mass term of the form $m^2 A_\mu A^\mu$ violates gauge invariance

  $\rightarrow$ local gauge invariance a priori implies massless gauge boson

- how can we incorporate massive gauge bosons in the theory?

use a trick: add complex scalar field ϕ with charge $-e$:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi)$$

with $V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$,

$$D_\mu = \partial_\mu - ieA_\mu$$

spontaneous symmetry breaking: Abelian gauge theory

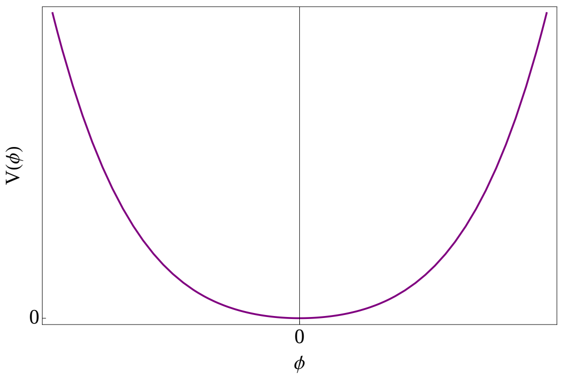
$$\mathcal{L} \;=\; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi)\,, \quad \text{with } D_\mu = \partial_\mu - ieA_\mu$$

$$V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$$

$\mu^2 > 0$:

unique minimum at $\phi = 0$

QED with massless gauge field ($m_A = 0$) and additional scalar field ($m_\phi = \mu$)

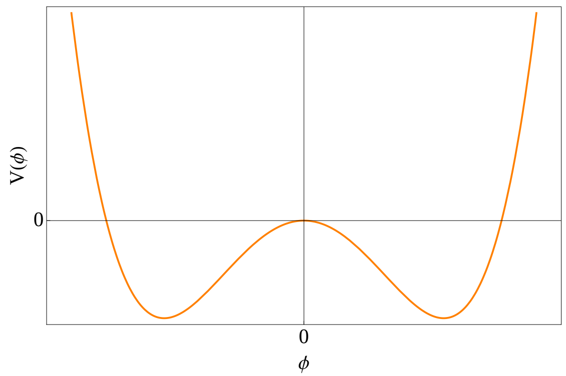


$\mu^2 < 0$:

degenerate minima at

$$|\phi| = \sqrt{-\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$$

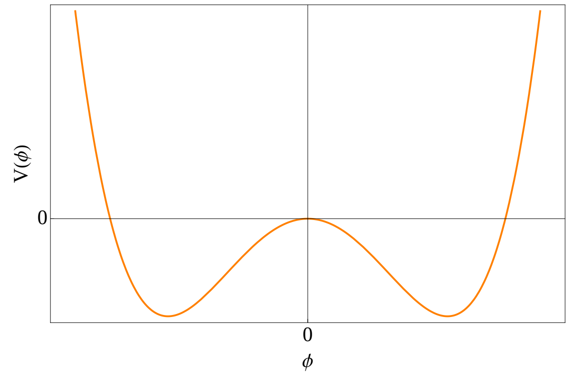
(phase arbitrary)



spontaneous symmetry breaking: Abelian gauge theory

$$\mathcal{L} \;=\; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi)\,, \quad \text{with } V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$$

$\mu^2 < 0$: minima at $|\phi| = \sqrt{-\frac{\mu^2}{2\lambda}} = \frac{v}{\sqrt{2}}$



expand ϕ around vacuum expectation value v :

$$\phi = \frac{1}{2}(v + H + i\chi)$$

☞ $$\mathcal{L} \;=\; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu H\partial^\mu H + \partial_\mu\chi\partial^\mu\chi + e^2v^2A_\mu A^\mu + evA^\mu\partial_\mu\chi$$
$$-eA^\mu(\chi\partial_\mu H - H\partial_\mu\chi) + \frac{1}{2}A_\mu A^\mu(H^2 + \chi^2) - V(\phi)$$

spontaneous symmetry breaking: Abelian gauge theory

$$\begin{aligned}\mathcal{L} \;=\;& -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu H \partial^\mu H + \partial_\mu \chi \partial^\mu \chi + e^2 v^2 A_\mu A^\mu + evA^\mu \partial_\mu \chi \\ & -eA^\mu(\chi\partial_\mu H - H\partial_\mu \chi) + \frac{1}{2}A_\mu A^\mu (H^2 + \chi^2) - V\left((v + H + i\chi)/2\right)\end{aligned}$$

photon of mass $m_A = ev$

scalar field H with $m_H^2 = -2\mu^2 > 0$

massless scalar field χ (Goldstone boson)

- the mixed $(A - \chi)$ propagator can be removed by a gauge transformation:

$A_\mu \to A_\mu - \frac{1}{ev}\partial_\mu \chi$ and $\phi \to e^{-i\chi/v}\phi$ (unitary gauge)

$\to$ the field χ has been absorbed by a redefinition of A

("χ has been eaten" to give mass to the photon)

spontaneous symmetry breaking: Abelian gauge theory

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu H \partial^\mu H + \partial_\mu \chi \partial^\mu \chi + e^2 v^2 A_\mu A^\mu + evA^\mu \partial_\mu \chi \\ & -eA^\mu(\chi \partial_\mu H - H\partial_\mu \chi) + \frac{1}{2}A_\mu A^\mu (H^2 + \chi^2) - V\left((v + H + i\chi)/2\right)\end{aligned}$$

photon of mass $m_A = ev$

scalar field H with
$m_H^2 = -2\mu^2 > 0$

massless scalar field χ
(Goldstone boson)

✧ balance of degrees of freedom:

before symmetry breaking:
massless gauge boson (2 d.o.f.) and complex scalar (2 d.o.f.) = 4 total

after symmetry breaking:
massive gauge boson (3 d.o.f.) and physical scalar (1 d.o.f.) = 4 total ✔

electroweak symmetry breaking (EWSB)

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

GLOBAL CONSERVATION LAWS AND MASSLESS PARTICLES*

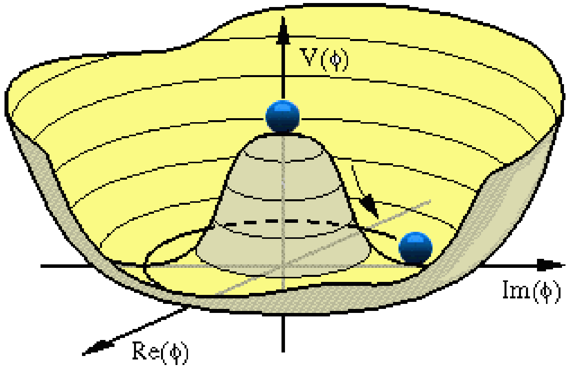
G. S. Guralnik,† C. R. Hagen,‡ and T. W. B. Kibble
Department of Physics, Imperial College, London, England
(Received 12 October 1964)

Physical Review Letters (1964)

spontaneous symmetry breaking in the SM

- add complex scalar isodoublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$



- scalar potential of the complex field:

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \ \lambda > 0$$

- for $\mu^2 < 0$: minimum of the potential at $|\Phi| = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$

☞ specific choice of phase breaks gauge invariance spontaneously;

typically choose: $\langle \Phi_0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

the Higgs sector of the SM

- Higgs field in unitary gauge: $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$

- Higgs Lagrangian:

$$\mathcal{L}_{\mathrm{Higgs}} = \frac{1}{2} \partial_\mu H \partial^\mu H - \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} \lambda H^4$$

  - ☞ Higgs mass $m_H = \sqrt{2}\mu = \sqrt{2\lambda} v$
  - ☞ vacuum expectation value $\leftrightarrow$ weak parameters $\frac{g^2}{8 m_W^2} = \frac{1}{2v^2}$
  - ☞ Higgs self couplings in the SM

uniquely determined by the Higgs mass

generation of gauge-boson masses

...proceeds via the kinetic term of the scalar doublet

$$\mathcal{L}_{\text{kin}} = (D_\mu \Phi)(D^\mu \Phi^\dagger)\,, \quad \text{with} \;\; D_\mu = \partial_\mu + \frac{ig}{2}\sigma^i W^i_\mu + \frac{ig'}{2} B_\mu$$

σ_i ... Pauli matrices

g, g' ... gauge couplings

W_i^μ, B_μ ... gauge fields

with $W^\pm_\mu = W_1^\mu \pm W_2^\mu$

covariant derivative of the underlying $SU(2) \times U(1)$ gauge theory

expand Φ about its vacuum expectation value in unitary gauge:

$$\rightarrow D_\mu \Phi = \frac{1}{\sqrt{2}} \left[\partial_\mu + \tfrac{ig}{2} \begin{pmatrix} W^3_\mu & \sqrt{2} W^-_\mu \\ \sqrt{2} W^+_\mu & -W^3_\mu \end{pmatrix} + \tfrac{ig'}{2} B_\mu \right] \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

generation of gauge-boson masses

$$\rightarrow \; |D_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu H)^2 + \frac{g^2 v^2}{4} W^{+\mu} W^-_\mu + \frac{v^2}{8}\left(g W^3_\mu - g' B_\mu\right)^2 + \text{ interaction terms}$$

- propagator for W^3 and B fields not diagonal $\rightarrow$ introduce new fields:

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

using the weak mixing angle

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}},$$

generation of gauge-boson masses

$$\rightarrow \quad |D_\mu\Phi|^2 = \frac{1}{2}(\partial_\mu H)^2 + \frac{g^2v^2}{4}W^{+\mu}W^-_\mu + \frac{v^2}{8}\left(gW^3_\mu - g'B_\mu\right)^2 + \text{ interaction terms}$$

✔ massive gauge bosons:

- $Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gW^3_\mu - g'B_\mu\right)$, with mass $m_Z = \frac{v}{2}\sqrt{g^2+g'^2}$

- $W^\pm_\mu$ with mass $m_{W^\pm} = \frac{gv}{2}$

✔ orthogonal superposition to Z boson:

massless photon $A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gW^3_\mu + g'B_\mu\right)$

generation of fermion masses

...generated via Yukawa interactions; e.g. for electrons

$$\mathcal{L}^{\mathrm{e}}_{\mathrm{Yuk}} \;=\; -G_e \bar{e}^i_L \Phi_i e_R + h.c. = -\frac{G_e}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}^T \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R + h.c.$$

✥ electron mass term

$$\mathcal{L}^{\mathrm{e,mass}}_{\mathrm{Yuk}} \;=\; -\frac{G_e v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) = -\frac{G_e v}{\sqrt{2}} \bar{e}e = -m_e \bar{e}e$$

☞ Yukawa coupling G_e related to electron mass m_e via

$$G_e = \frac{\sqrt{2} m_e}{v} = g \frac{m_e}{\sqrt{2} m_W}$$

✥ interaction between electron and Higgs boson

$$\mathcal{L}^{\mathrm{e,int}}_{\mathrm{Yuk}} \;=\; = -\frac{G_e v}{\sqrt{2}} \bar{e}He = -g \frac{m_e}{\sqrt{2} m_W} \bar{e}He$$

☞ proportional to the mass of the electron!

generation of quark masses

. . . also generated via Yukawa interactions; e.g. for the first generation:

$$\mathcal{L}_{\text{Yuk}}^{\text{q}} = -G_d \bar{q}_L^i \Phi_i d_R + h.c. - G_u \varepsilon_{ij} \bar{q}_L^i \Phi^{\dagger j} u_R + h.c.$$

d-quark mass:

$$m_d = \frac{G_d}{\sqrt{2}} v = \sqrt{2} \frac{G_d m_W}{g}$$

u-quark mass:

$$m_u = \frac{G_u}{\sqrt{2}} v = \sqrt{2} \frac{G_u m_W}{g}$$

- interaction between the quarks and the Higgs boson

$$\mathcal{L}_{\text{Yuk}}^{\text{q,int}} = -g \frac{m_d}{\sqrt{2} m_W} \bar{d} H d - g \frac{m_u}{\sqrt{2} m_W} \bar{u} H u$$

- note: adding more generations introduces mixing in the Yukawa interactions

the Standard Model with one family

$$\mathcal{L}_{\mathrm{SM},1} = \sum_{\text{gauge bosons}} -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{fermions}} i\bar{\psi}\gamma^{\mu} D_{\mu}\psi + \mathcal{L}_{\mathrm{Yuk}} + |D_{\mu}\Phi|^2 - V(\Phi)$$

with $F_{\mu\nu} = -\frac{1}{ig}[D_\mu, D_\nu]$ and $D_\mu = \partial_\mu + \frac{ig}{2}\sigma^i W^i_\mu + ig' Y B_\mu + \frac{ig_s}{2} T^a G^a_\mu$

- $F_{\mu\nu}F^{\mu\nu}$ term generates

  interactions among the gauge bosons, e.g.:

$$W^i_{\mu\nu} W^{i\,\mu\nu} \to g\varepsilon_{ijk}(\partial_\mu W^i_\nu) W^{j\,\mu} W^{k\,\nu} - \frac{1}{4}\varepsilon_{ijk}\varepsilon_{ilm} W^j_\mu W^k_\nu W^{l\,\mu} W^{m\,\nu}$$

the Standard Model with one family

$$\mathcal{L}_{\mathrm{SM},1} = \sum_{\text{gauge bosons}} -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{\text{fermions}} i\bar{\psi}\gamma^\mu D_\mu\psi + \mathcal{L}_{\mathrm{Yuk}} + |D_\mu\Phi|^2 - V(\Phi)$$

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- $i\bar{\psi}\gamma^\mu D_\mu\psi$ term generates

  interactions among fermions and gauge bosons, e.g.:

$$\begin{aligned} & i\bar{\ell}_L\gamma^\mu D_\mu \ell_L + i\bar{e}_R\gamma^\mu D_\mu e_R \\ = \; & -\frac{g}{2\sqrt{2}}\bar{\nu}\gamma^\mu(1-\gamma_5)eW_\mu^- + h.c. \; + g\sin\theta_W \bar{e}\gamma^\mu e A_\mu \\ & -\frac{g}{4\cos\theta_W}\bar{\nu}\gamma^\mu(1-\gamma_5)\nu Z_\mu + \frac{g}{4\cos\theta_W}\bar{e}\left[\gamma^\mu(1-\gamma_5) - 4\sin^2\theta_W\gamma^\mu\right]eZ_\mu \end{aligned}$$

parameters of the Standard Model

- free parameters of the $SU(2)_L \times U(1)_Y$ part of the SM with one generation of leptons:
  - the two gauge couplings g and g'
  - the two parameters μ and λ of the scalar potential $V(\phi)$
  - the Yukawa couplings G_f

- more convenient: replace by parameters which can be measured accurately, e.g.

$$\{g, g', \mu, \lambda, G_f\} \to \{e, \sin\theta_W, m_H, m_W, m_f\}$$

these are related to original parameters via

$$\tan\theta_W = \frac{g'}{g}\,, e = g\sin\theta_W\,, m_H = \sqrt{2}\mu\,,$$
$$m_W = \frac{g}{2\sqrt{\lambda}}\,, m_f = G_f\frac{\mu}{\sqrt{\lambda}}\,.$$

parameters of the Standard Model

- more convenient: replace by parameters which can be measured accurately, e.g.

$$\{g, g', \mu, \lambda, G_f\} \to \{e, \sin\theta_W, m_H, m_W, m_f\}$$

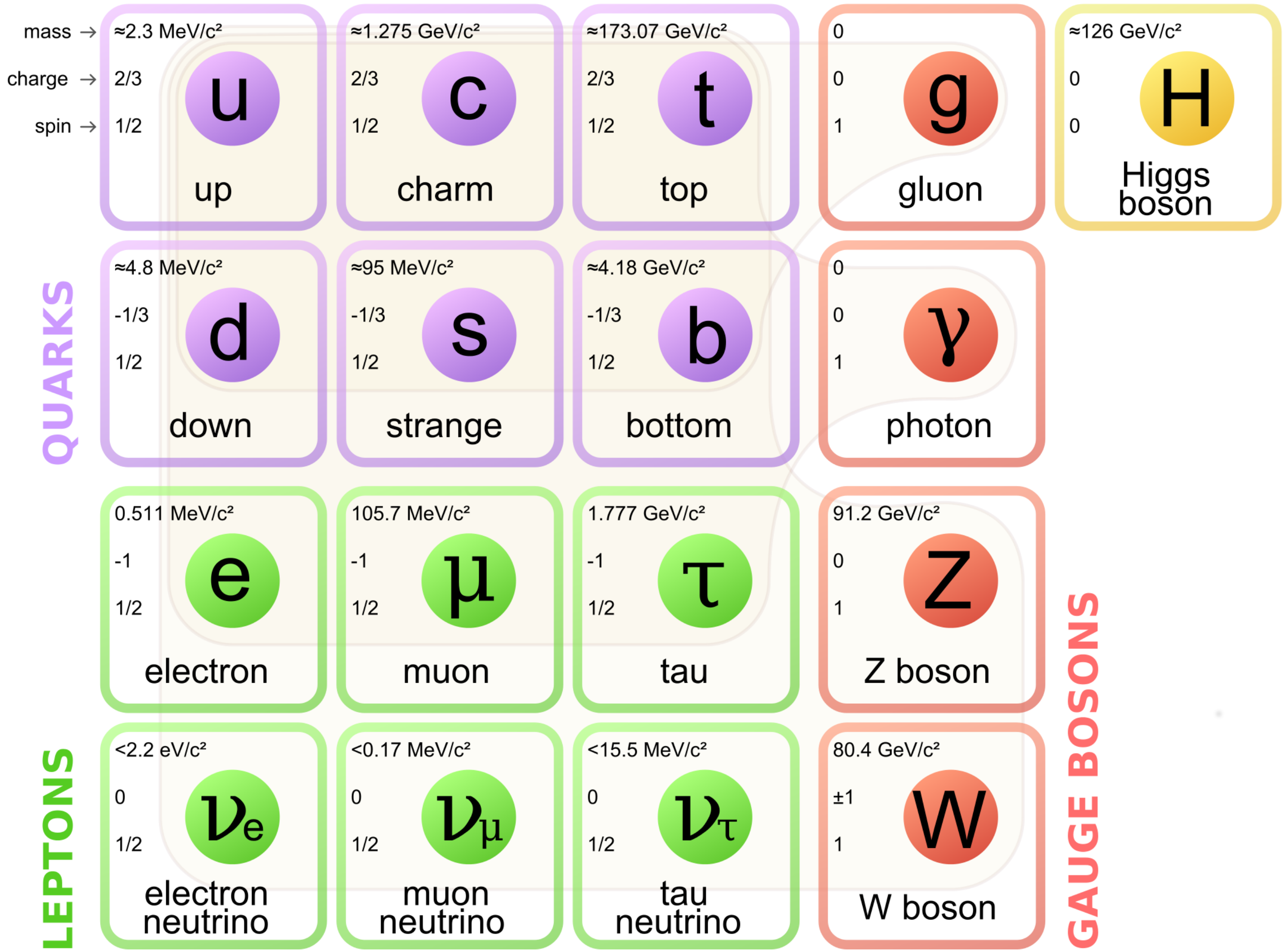
- other parameters are predictions,

  e.g. the Z-boson mass m_Z or the Fermi constant G_F:

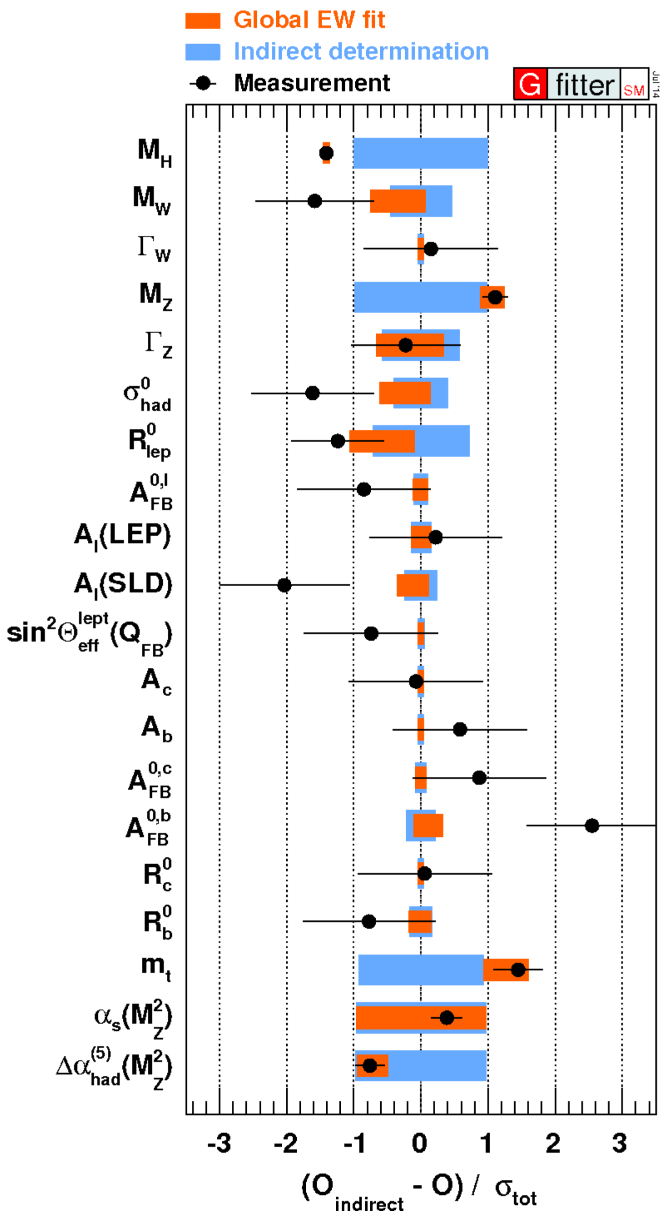
$$m_Z = \frac{m_W}{\cos\theta_W} \quad \text{and} \quad G_F = \frac{e^2}{4\sqrt{2}m_W^2 \sin^2\theta_W}$$

- full SM with three generations: additional parameters are needed for fermion masses and mixing angles between the generations

the full picture (?)



global electroweak fit



calculate all precision observables in the SM including higher order corrections in terms of

$$\alpha(m_Z), G_F, m_Z, m_f, m_{\text{top}}, m_H, \alpha_s$$

and determine parameters by fit to all EW precision data

here:

comparing fit results (orange bars) with indirect determinations (blue bars) and direct measurements (data points)

precision tests of the Standard Model

powerful tool for testing the SM to high accuracy:
precision electroweak measurements

very accurate results provided by:

- LEP (Large Electron Positron collider at CERN),

  run 1: $\sqrt{s} = m_Z$,
  run 2: $\sqrt{s} \lesssim 200$ GeV

- SLC (Standford Linear Collider, $\sqrt{s} = m_Z$)

☞ allow to test the SM at the percent level!

to achieve this precision need to
include quantum corrections in predictions

extra gain: indirect sensitivity to energy scales beyond direct reach

the theorist's task

provide precise predictions for experimentally accessible observables

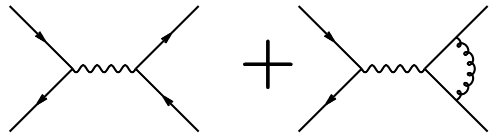
as pre-requisites for

- accurate determination of physics parameters (couplings, masses, . . .)
- discovery of new particles and physics scenarios

hard scattering: the perturbative approach

high energies: (ideally) series expansion in coupling parameter

$$\sigma = \sum_{n=n_0}^{N} \alpha^n \sigma^{(n)} + \mathcal{O}(\alpha^{N+1})$$



$+\ldots$

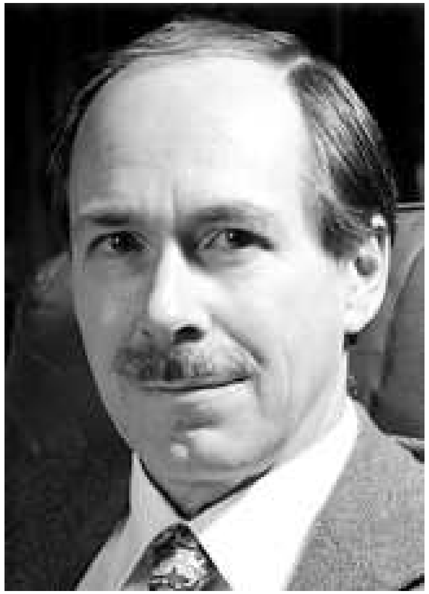
truncation at fixed order α^N ($\rightarrow$ LO, NLO, ...)

order N provided by theoretician *("# of loops")* depends on:

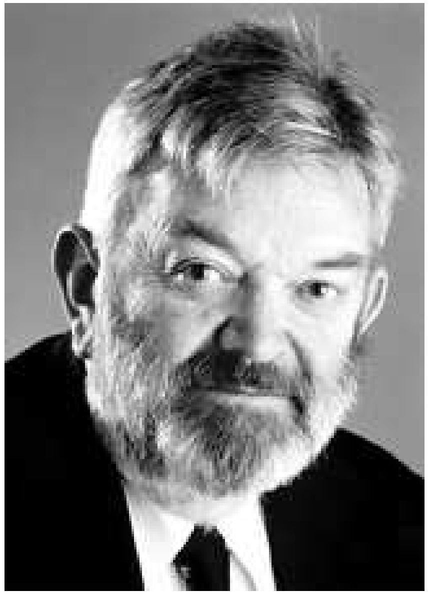
- complexity of the problem
  - kinematic properties of the reaction
  - multiplicity of the final state *("# of legs")*
  - mass scales of involved particles
  - ...
- accuracy which can be achieved in experiment
- computational skills of the perturbationist

renormalizability of the SM

Gerardus 't Hooft



Martinus J. G. Veltman



The Nobel Price in Physics 1999:

"for elucidating the quantum structure of electroweak interactions in physics"

renormalizability of the SM

the Standard Model is renormalizable

$\longrightarrow$ observables can be calculated from few input parameters,

in principle to arbitrarily high precision

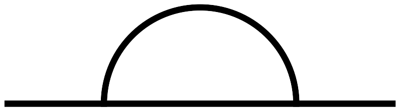
but:

- radiative corrections sensitive to highest momentum scales
- large corrections
- sensitive to unknown physics

renormalizability of the SM

but:

- radiative corrections sensitive to highest momentum scales
- large corrections
- sensitive to unknown physics



example: corrections to mass parameter m

$$\delta m \sim \int^{\Lambda_{\mathrm{cut}}} \frac{d^4k}{k^4} \sim \int^{\Lambda_{\mathrm{cut}}} \frac{dk}{k} \sim \ln \Lambda_{\mathrm{cut}}^2$$

Λ_{cut} . . . energy up to which the SM is valid

($\Lambda_{\mathrm{cut}} = M_{\mathrm{Planck}} \approx 10^{19}$ GeV?)

renormalizability of the SM

✘ problem: radiative corrections are large

✔ solution: absorb large corrections (here $\sim \ln \Lambda_{\rm cut}$) into
redefinition of the parameters of the theory:

- physical couplings: $g = g_0 + \delta g$
- physical mass: $m = m_0 + \delta m$

$g_0, m_0 \ldots$ "bare" parameters of $\mathcal{L}$
$\delta g, \delta m \ldots$ contain the large corrections $\sim \ln \Lambda_{\rm cut}$

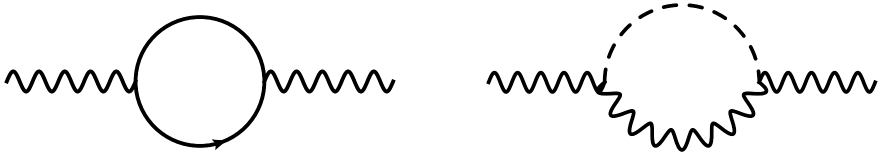
renormalizable theories: all UV divergences can be absored into the redefinition of couplings and masses

$\to$ physical observables are independent of $\Lambda_{\rm cut}$

indirect searches

quantum corrections to precision observables $\rightarrow$ indirect access to high mass scales

e.g., the W boson mass:



calculate m_W from m_Z and G_F including quantum corrections:

$$\frac{m_W^2}{m_Z^2}\left(1-\frac{m_W^2}{m_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F m_Z^2(1-\Delta r)}$$

with quantum corrections $\Delta r = \Delta\alpha - \cot\theta_W \Delta\rho^{\text{top}} + \Delta r^{\text{Higgs}} + \ldots$

leading top-quark contribution:
quadratic in m_{top}:

$$\Delta\rho^{\text{top}} = \frac{3G_F m_{\text{top}}^2}{8\pi^2\sqrt{2}} + \ldots$$

Higgs contribution: screened
$\rightarrow$ only logarithmic dependence on m_H:

$$\Delta r^{\text{Higgs}} = \frac{G_F m_W^2}{8\pi^2\sqrt{2}}\frac{1+9\sin^2\theta_W}{3\cos^2\theta_W}\ln\left(\frac{m_H^2}{m_W^2}\right) + \ldots$$

indirect searches for the top quark

indirect searches for top quark work rather well

(recall: top mass enters precision observables quadratically)

historically (around 2000):

direct observation: $\boldsymbol{m_{\mathrm{top}} = 172.7 \pm 2.9}$ GeV (CDF and D0)

indirect observation: $\boldsymbol{m_{\mathrm{top}} = 179.4 \pm 11}$ GeV (LEP and SLD)

more recent (PDG 2015): best limits come from the LHC

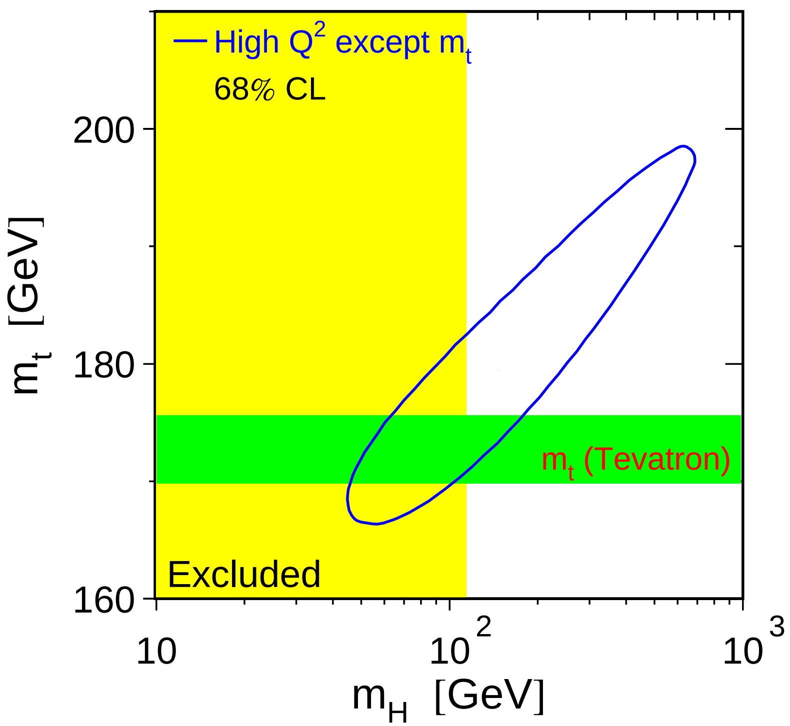
ATLAS: $\boldsymbol{m_{\mathrm{top}} = 172.99 \pm 0.48}$ (stat.) $\boldsymbol{\pm 0.78}$ (syst.) GeV

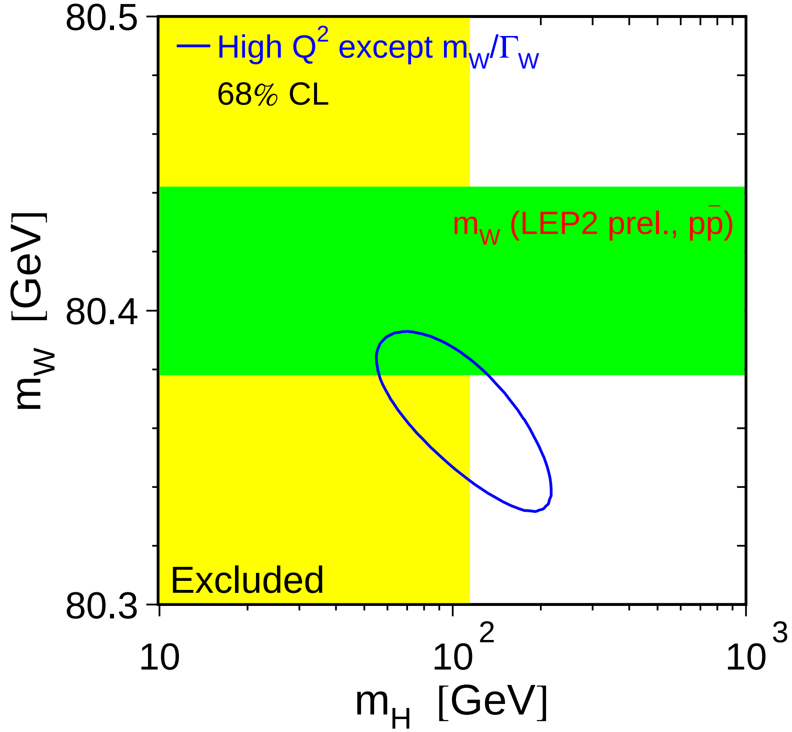
CMS: $\boldsymbol{m_{\mathrm{top}} = 172.32 \pm 0.25}$ (stat.) $\boldsymbol{\pm 0.59}$ (syst.) GeV

indirect searches for the Higgs boson

indirect searches for the Higgs boson are harder
because of logarithmic Higgs mass dependence

LEPEWWG (2005)



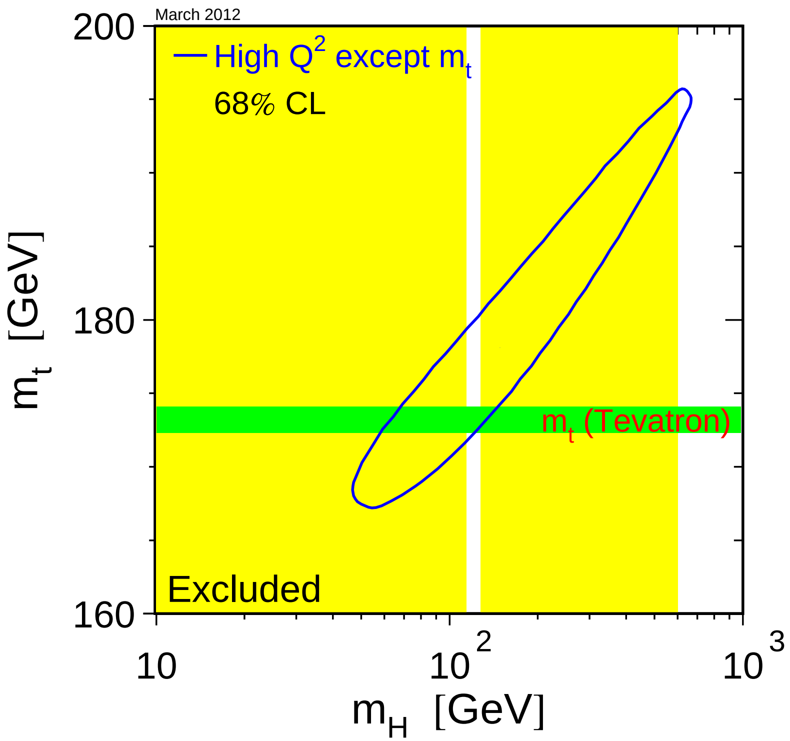


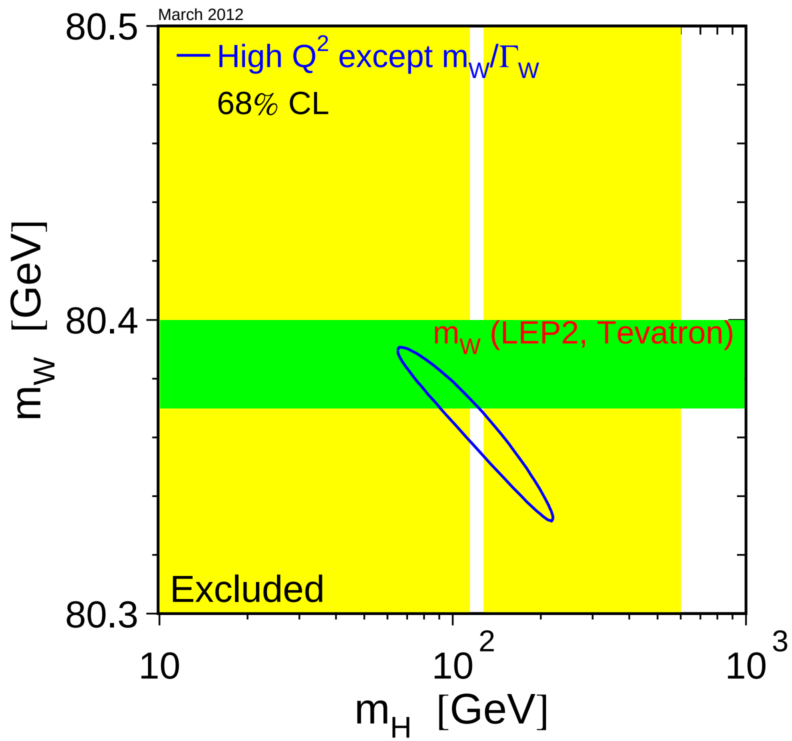
data consistent with SM; fits to EW data $\rightarrow$ $m_H < 219$ GeV

indirect searches for the Higgs boson

indirect searches for the Higgs boson are harder
because of logarithmic Higgs mass dependence

LEPEWWG (winter 2012)





direct searches at Tevatron exclude large parameter range!

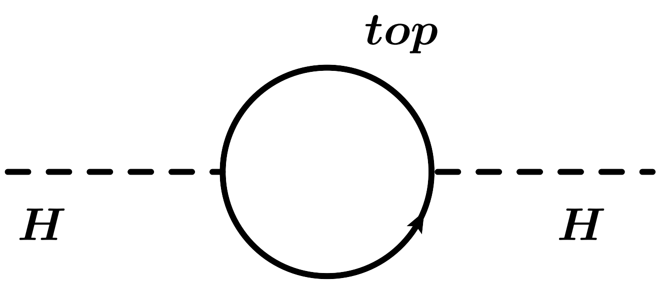
the hierarchy problem

Higgs boson is light and weakly interacting;

but why is $m_H \ll M_{\mathrm{Planck}}$?

quantum corrections to Higgs boson mass

are quadratically divergent:

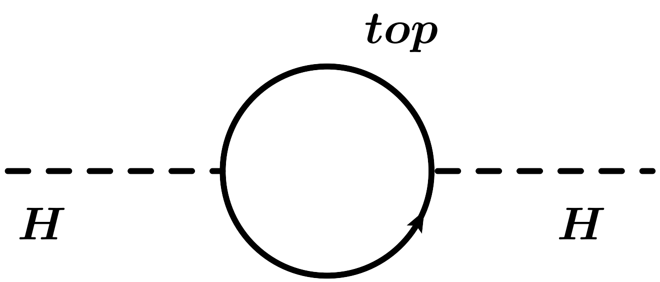


$$\delta m_H^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_{\mathrm{top}}^2 \Lambda^2$$

Λ . . . cutoff scale up to which the SM is valid

(need Λ of $\mathcal{O}$(1 TeV) to avoid unnaturally large corrections)

the hierarchy problem



$$\delta m_H^2 \sim \frac{3G_F}{\sqrt{2}\pi^2} m_{\text{top}}^2 \Lambda^2$$

Λ . . . cutoff scale up to which the SM is valid

(need Λ of $\mathcal{O}$(1 TeV) to avoid unnaturally large corrections)

☞ need new physics to stabilize the hierarchy $M_{\text{Planck}} \gg m_H$ which decouples from electroweak precision tests

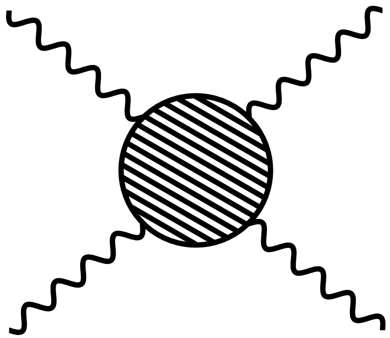
some popular candidates:

✔ supersymmetry, extra dimensions

✘ techicolor, little Higgs models

theoretical bounds from perturbative unitarity

can we employ the requirement of unitarity in processes with massive gauge bosons to constrain the weak sector?



most sensitive to the mechanism of electroweak symmetry breaking:

longitudinal modes of the $W^{\pm}$ and Z bosons

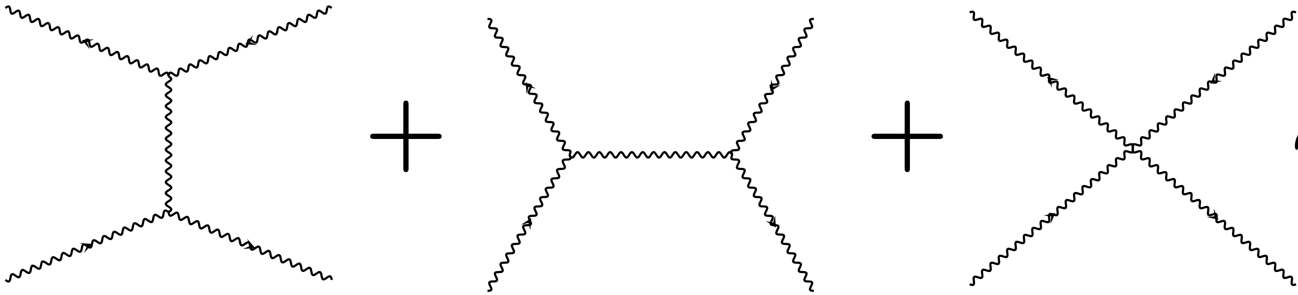
$\rightarrow$ consider longitudinal gauge boson scattering:

$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$$

theoretical bounds from perturbative unitarity

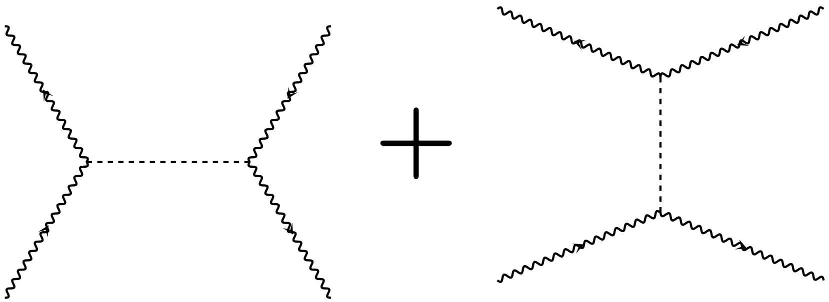
$W_L^+ W_L^- \to W_L^+ W_L^-$ with $\varepsilon_L^\mu \sim \frac{\sqrt{s}}{M_W}$

$\mathcal{M} =$



$\sim \frac{s}{M_W^2}$

growth violates unitarity $\to$ need:



Higgs with $M_H \lesssim 1$ TeV
or new physics at TeV scale

needed: high-energy hadron colliders

Superconducting Super Collider (SSC)

location: Texas, USA

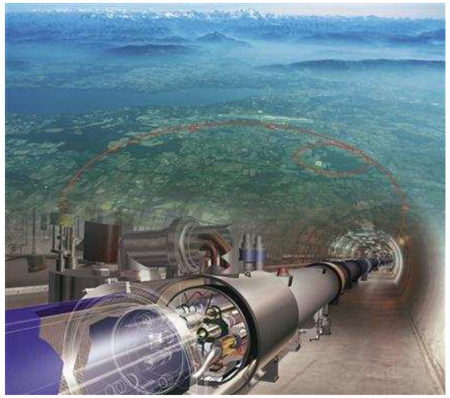
design energy: 40 TeV

Tevatron

location: Fermilab, USA

energy: 2 TeV

Large Hadron Collider (LHC)



location: CERN, Switzerland

design energy: 14 TeV

needed: high-energy hadron colliders

Superconducting Super Collider (SSC)

location: Texas, USA

design energy: 40 TeV

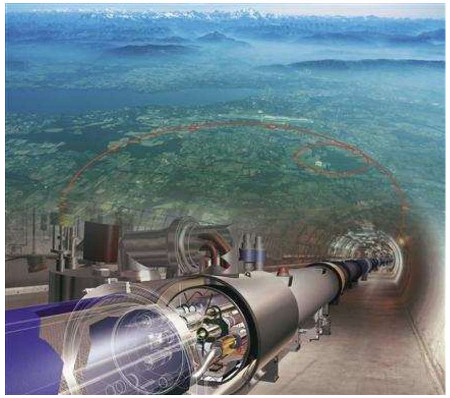
cancelled: 1993

Tevatron

location: Fermilab, USA

energy: 2 TeV

Large Hadron Collider (LHC)



location: CERN, Switzerland

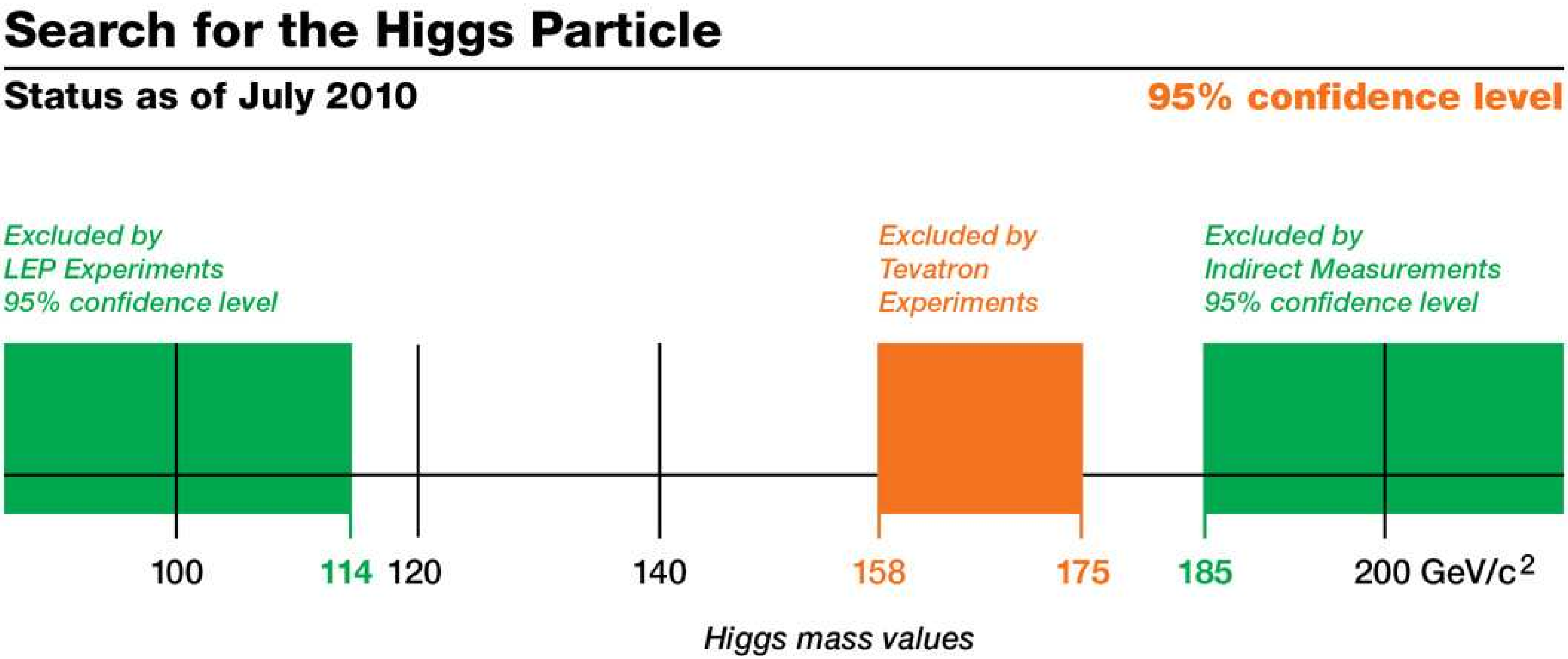
design energy: 14 TeV

the first hadron collider at the Terascale

the Tevatron at Fermilab:

high energy synchrotron
with proton–anti-proton collisions
at c.m.s. energy
$\sqrt{S} \simeq 2$ TeV

combined experimental bounds on the Higgs mass



the world's largest hadron collider . . .

. . . the Large Hadron Collider (LHC) at CERN

the world's largest hadron collider . . .

. . . smashes proton or heavy-ion beams

tunnel crosses border between France and Switzerland

circumference: 27 km

more than 1600 superconducting magnets

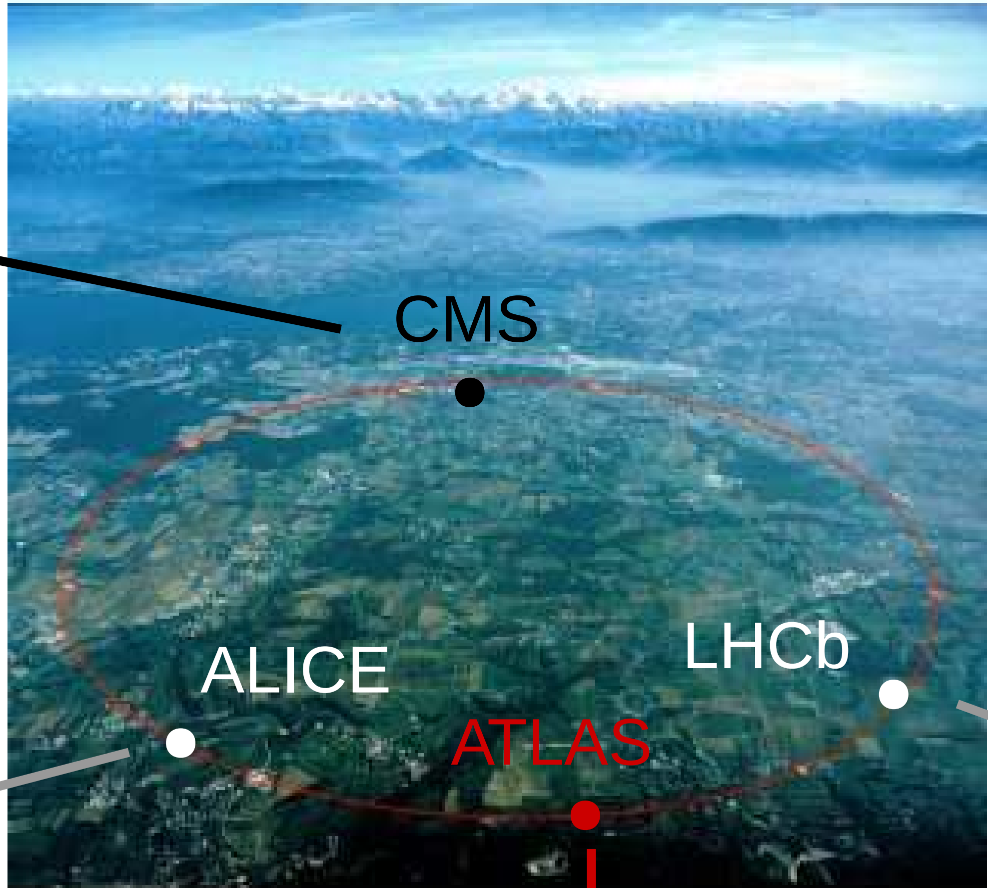
96 tons of liquid helium to keep operating temperature of -271.25° C

design energy $\sqrt{S} = 14$ TeV
(3 m/s slower than the speed of light)

interactions between colliding beams: every 25 ns

the world’s largest hadron collider . . .

. . . and its four major experiments . . .



Compact Muon Solenoid

A Large Ion Collider Experiment

LHC-beauty

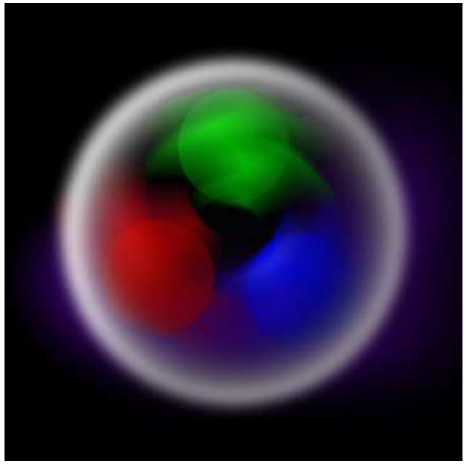
A Toroidal LHC Apparatus

how to calculate cross sections for the LHC

- high energies $\longrightarrow$ can calculate QCD processes perturbatively
- EW coupling: sufficiently small for perturbation theory
- Feynman rules $\longrightarrow$ in principle calculate
  any process at any order in perturbation theory
- but: perturbative calculations for quarks and gluons

  ☞ have to connect

  partons $\leftrightarrow$ protons

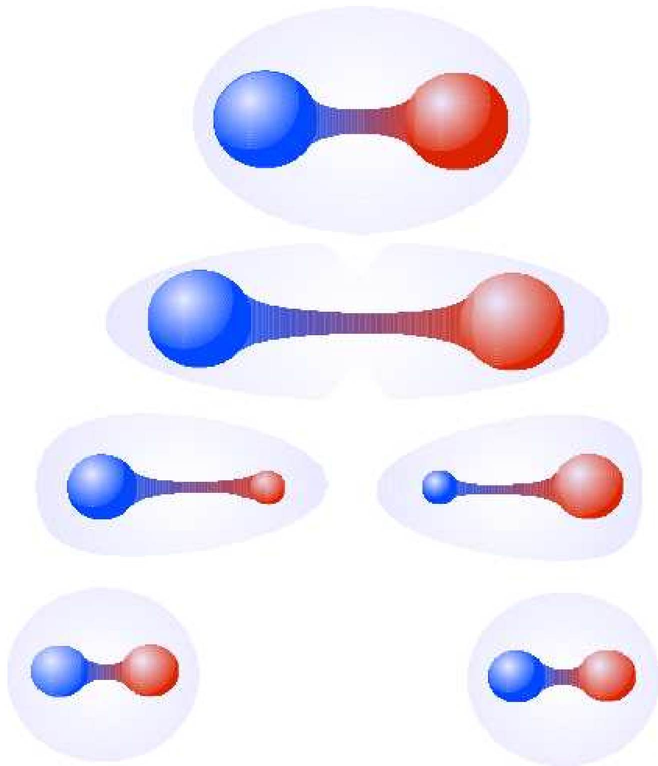


confinement

quarks and gluons appear only in bound states (hadrons):

$$|\text{meson}\rangle \sim \delta_{ij}|q_i\bar{q}_j'\rangle\,, \quad |\text{baryon}\rangle \sim \epsilon_{ijk}|q_i q_j^{'} q_k^{''}\rangle$$

$\rightarrow$ hadrons are color singlets!



- quarks linked by "spring" that breaks when they move apart
- at small distances perturbation theory breaks down

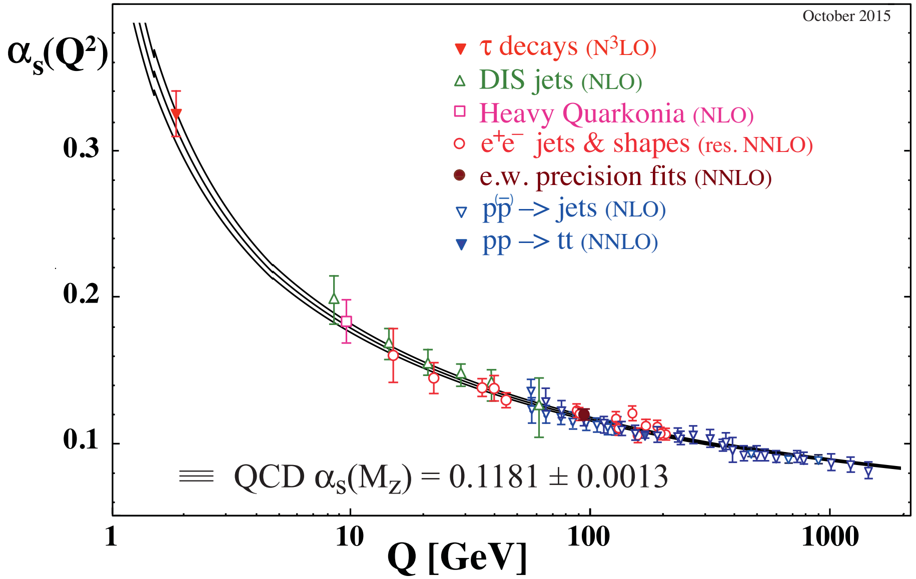
☞ no rigorous theoretical understanding of confinement as of yet

asymptotic freedom

prerequisite for perturbative calculations in QCD:

strong coupling α_s depends on energy scale Q

$$\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 + \frac{33-2N_f}{12}\frac{\alpha_s(Q_0^2)}{\pi}\ln\frac{Q^2}{Q_0^2}}$$



Q_0 ... reference scale

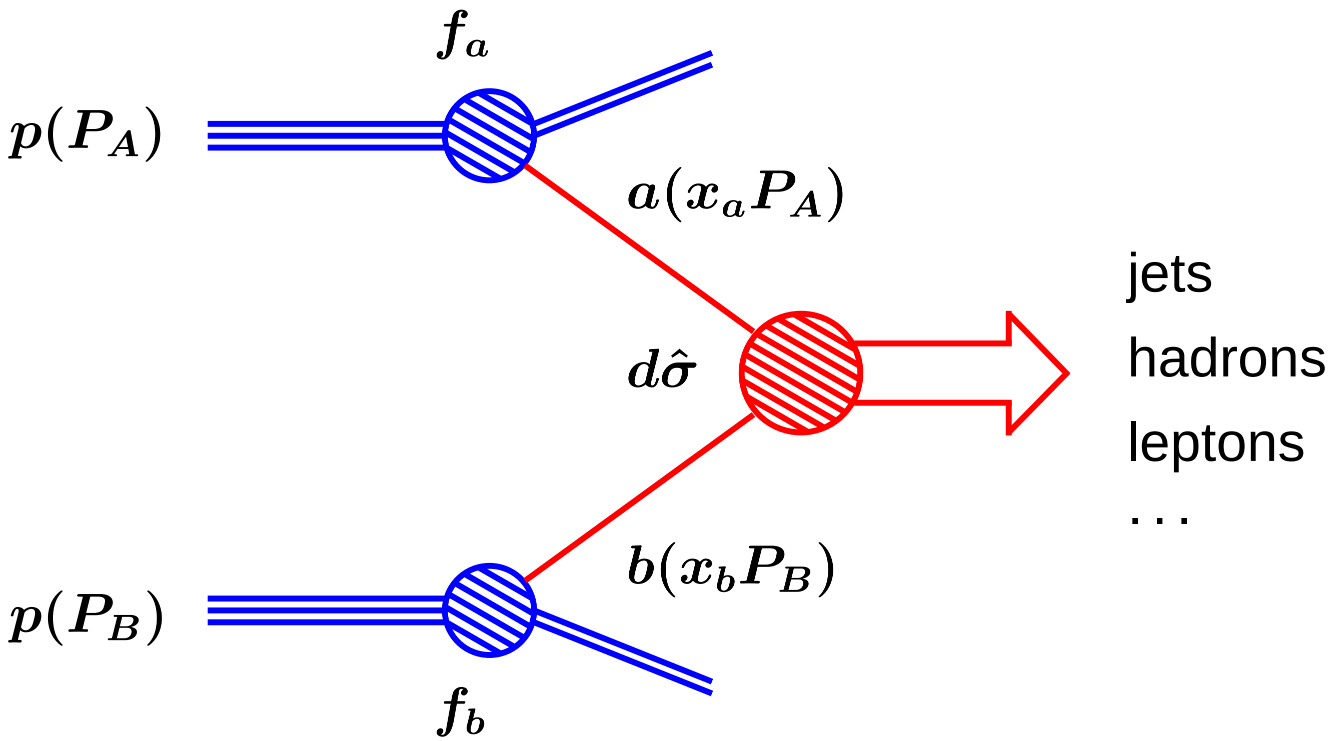
N_f ... # of flavors

increasing energy scale: coupling decreases = "asymptotic freedom"

☞ at high scales: $\alpha_s < 1 \rightarrow$ perturbation theory applicable

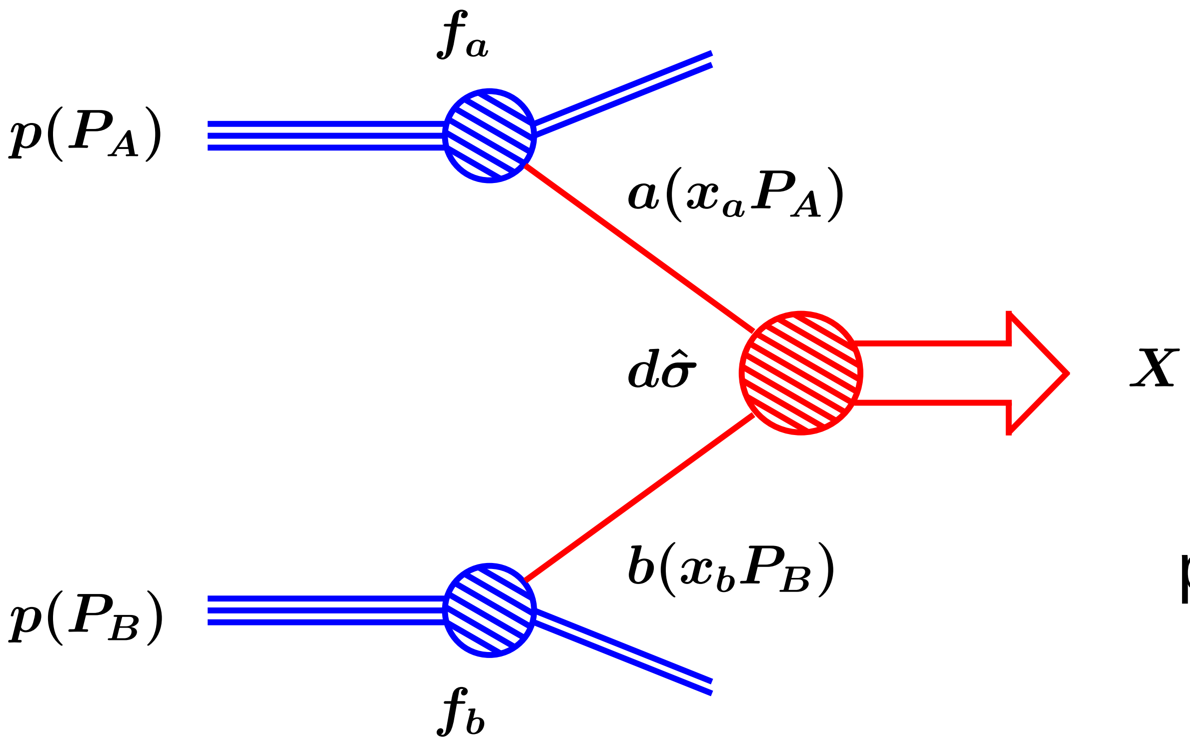
[consequence of non-Abelian interaction, contrary to U(1) of QED]

hadron-hadron collision



$$d\sigma^{pp\to X} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b\, f_a(x_a,\mu_F) f_b(x_b,\mu_F) \times d\hat{\sigma}^{ab\to X}(x_a P_A, x_b P_B, \mu_F, \mu_R)$$

hadron-hadron collision

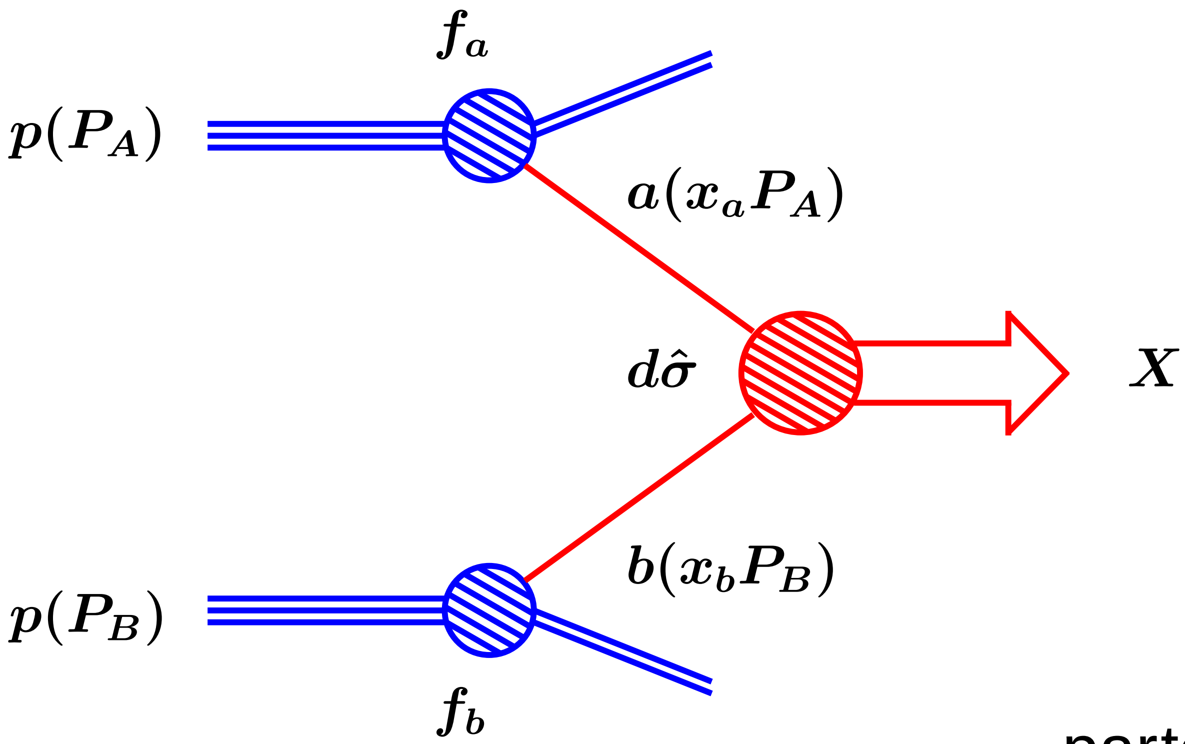


parton distribution functions

$$d\sigma^{pp\to X} = \sum_{a,b}\int_0^1 dx_a \int_0^1 dx_b\, f_a(x_a,\mu_F) f_b(x_b,\mu_F)$$

$$\times d\hat{\sigma}^{ab\to X}(x_a P_A, x_b P_B, \mu_F, \mu_R)$$

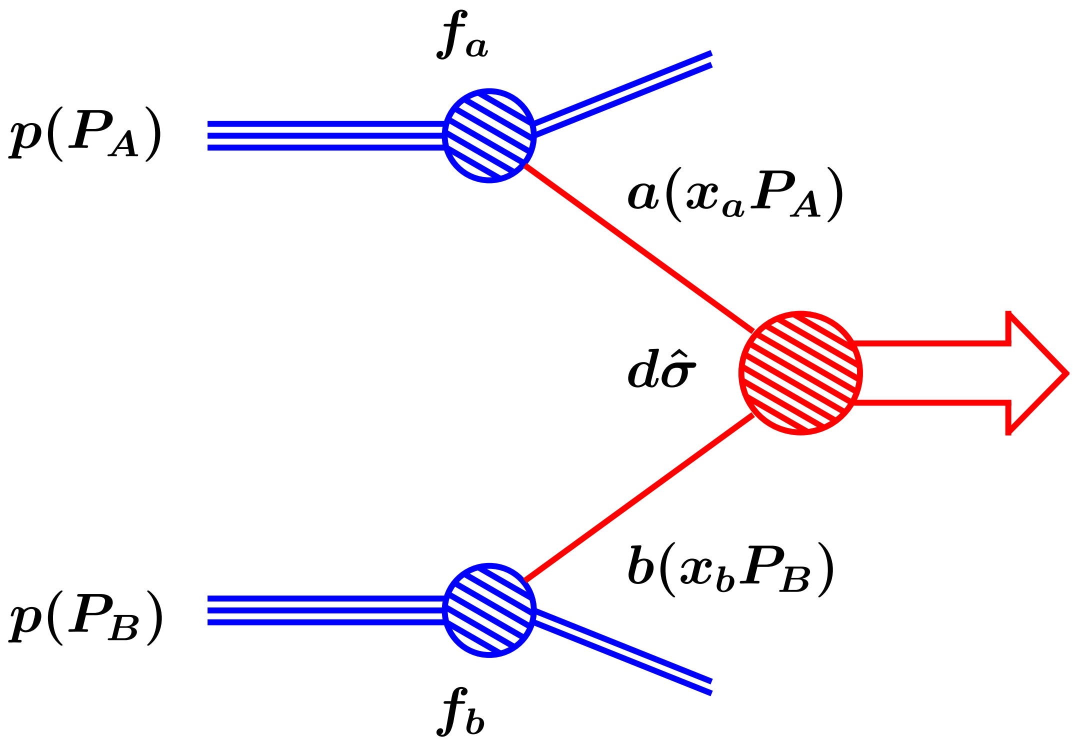
hadron-hadron collision



partonic cross section

$$d\sigma^{pp\to X} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b\, f_a(x_a,\mu_F) f_b(x_b,\mu_F) \times d\hat{\sigma}^{ab\to X}(x_a P_A, x_b P_B, \mu_F, \mu_R)$$

hadron-hadron collision

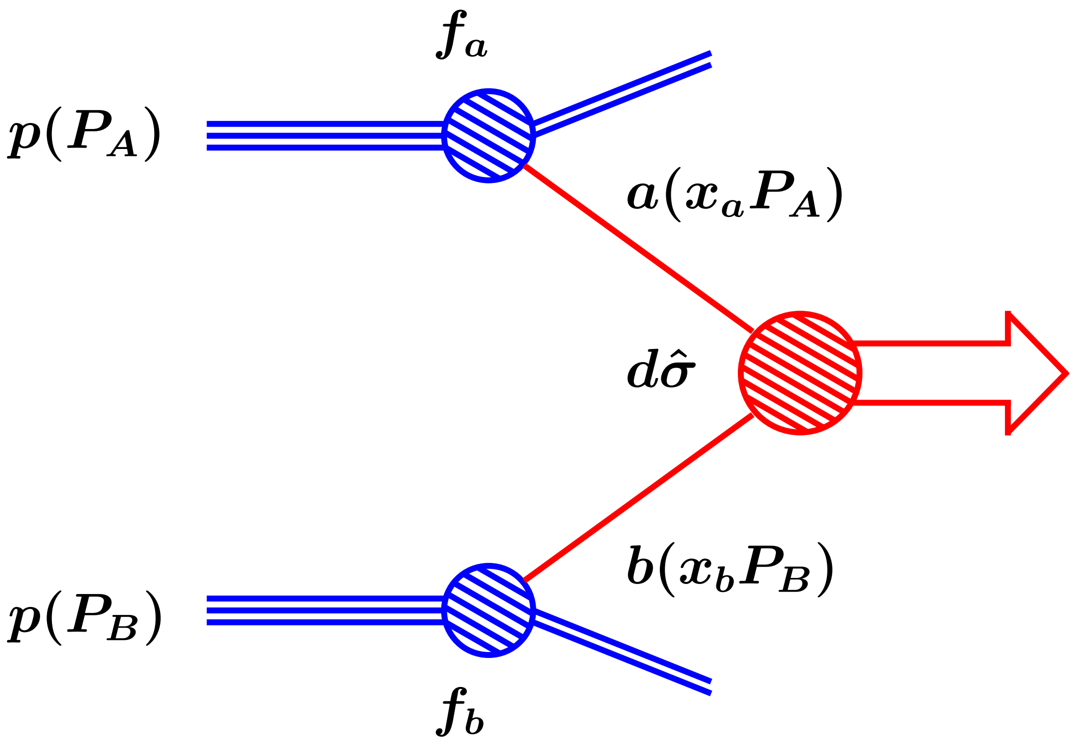


energy available for hard scattering:

$$\sqrt{\hat{s}} = \sqrt{x_a x_b S}$$

$$d\sigma^{pp\to X} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b\, f_a(x_a,\mu_F) f_b(x_b,\mu_F) \times d\hat{\sigma}^{ab\to X}(x_a P_A, x_b P_B, \mu_F, \mu_R)$$

factorization



foundation for predictive power of pQCD:

long-distance structure of hadrons
can be separated from
hard parton scattering
at specific scale μ_F

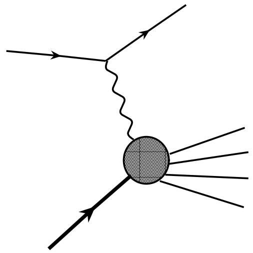
$$d\sigma^{pp\to X} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_a(x_a, \mu_F) f_b(x_b, \mu_F)$$
$$\times d\hat{\sigma}^{ab\to X}(x_a P_A, x_b P_B, \mu_F, \mu_R)$$

parton distribution functions

- extracted from experiment at a scale μ_0, e.g.:

  $f_q(x, \mu_0)$ ... DIS: e$^-$ p $\to$ e$^-$ X

  (CTEQ, MSTW, NNPDF ...)



- further constraints provided by lattice QCD

- universal: PDFs do not depend on reaction / experiment

- μ dependence predicted by perturbative QCD:

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} f_q(x,\mu) \\ f_g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s(\mu))} \cdot \begin{pmatrix} f_q \\ f_g \end{pmatrix} \left(\frac{x}{z}, \mu\right)$$

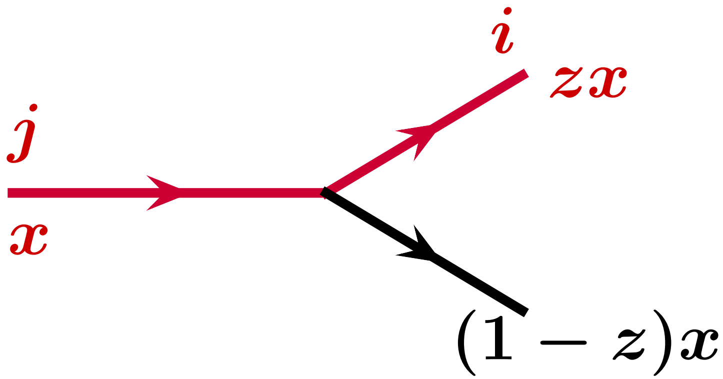
DGLAP equations

$$\mu^2 \frac{\partial f_i(x,\mu)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(\frac{x}{z},\mu\right)$$

[Altarelli, Parisi; Gribov, Lipatov, Dokshitzer (1977)]

- system of coupled integro-differential equations
- splitting functions can be computed perturbatively:

$$P_{ij}(z) = \tfrac{\alpha_s}{2\pi} P_{ij}^{(0)} + \left(\tfrac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(1)} + \ldots$$



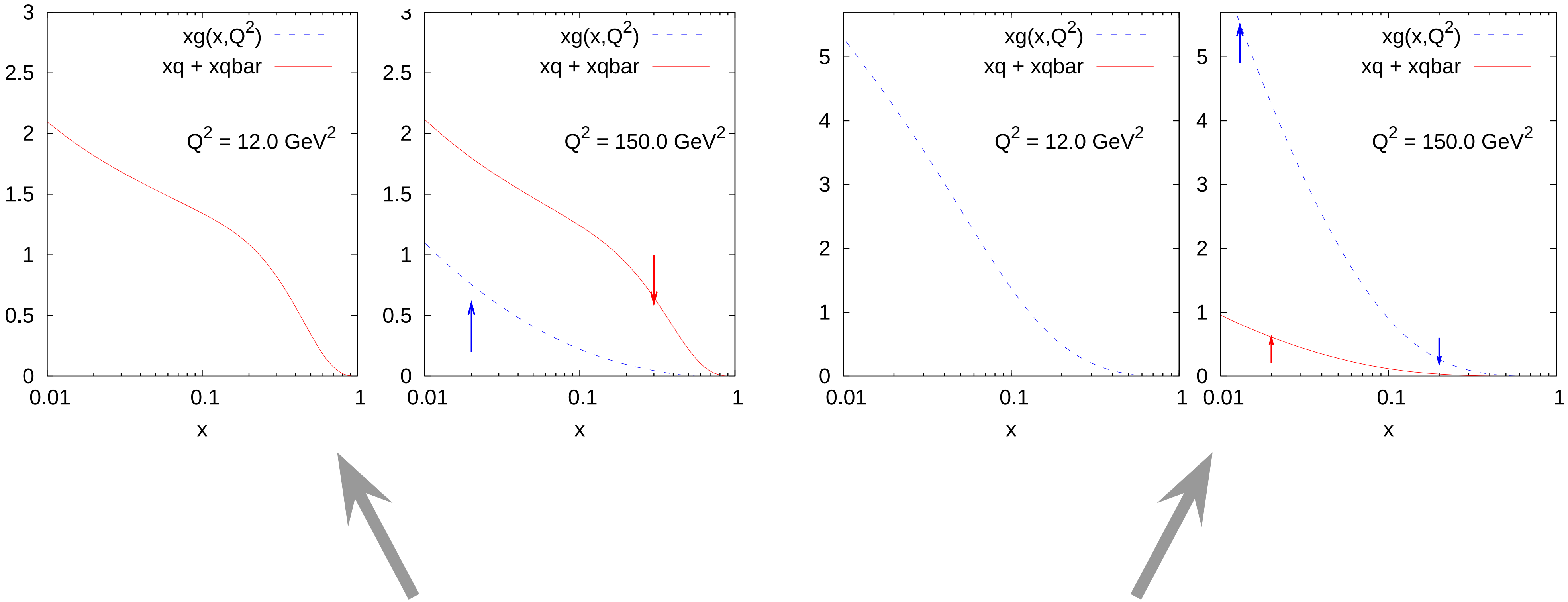
at leading order:

$$P_{qg}^{(0)} = \frac{1}{2}\left[z^2 + (1-z)^2\right] ,$$

$$P_{gq}^{(0)} = C_F \frac{1+(1-z)^2}{z} ;$$ $P_{qq}^{(0)}$ and $P_{gg}^{(0)}$ ... more complicated

DGLAP evolution

[Salam (2011)]

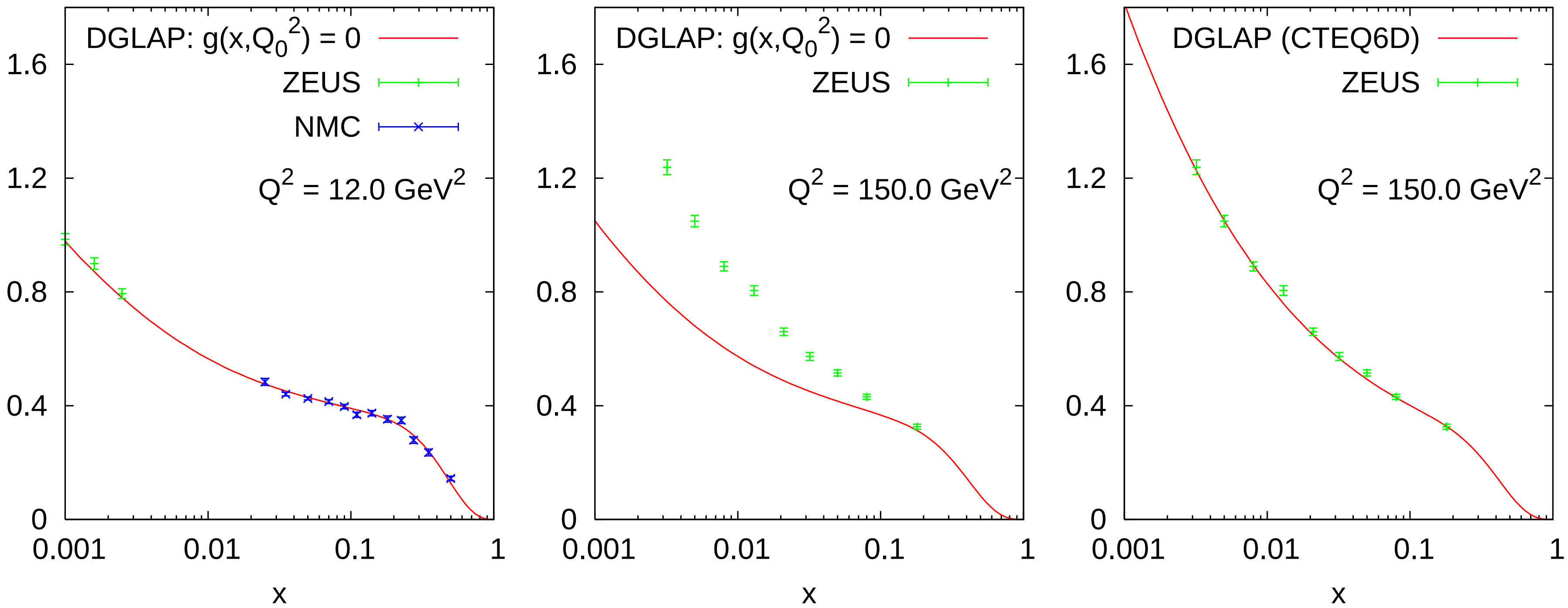


start from pure quark input
$\longrightarrow$ evolution generates gluon

start from pure gluon input
$\longrightarrow$ evolution generates
quarks / anti-quarks

DGLAP evolution confronted with data

[Salam (2011)]



pure quark input does not describe high-Q^2 data on $F_2^p(x, Q^2)$ structure function well

(CTEQ includes significant low-scale gluon component)

parton luminosities

total hadronic cross section σ can be expressed as

$$\sigma(s) = \sum_{a,b} \int_{\tau_0}^{1} \frac{d\tau}{\tau} \left[\frac{\tau}{\hat{s}} \frac{d\mathcal{L}_{ab}}{d\tau} \right] \hat{s}\hat{\sigma}_{ab}(\hat{s})\,, \quad \text{with } \tau = x_a x_b = \hat{s}/s$$

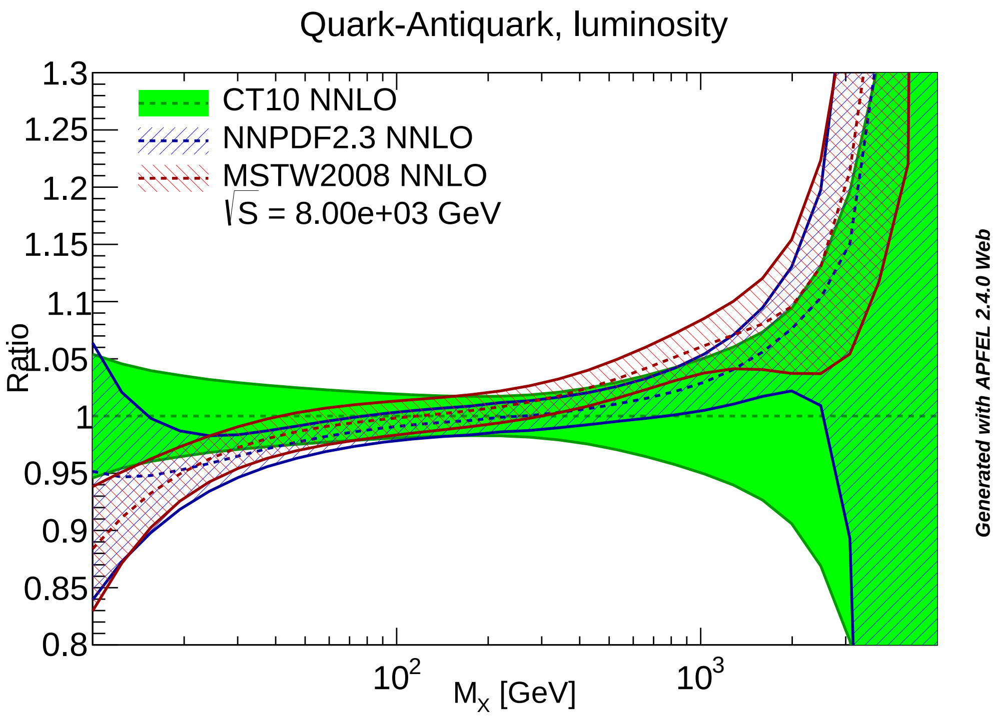
using the differential parton luminosity

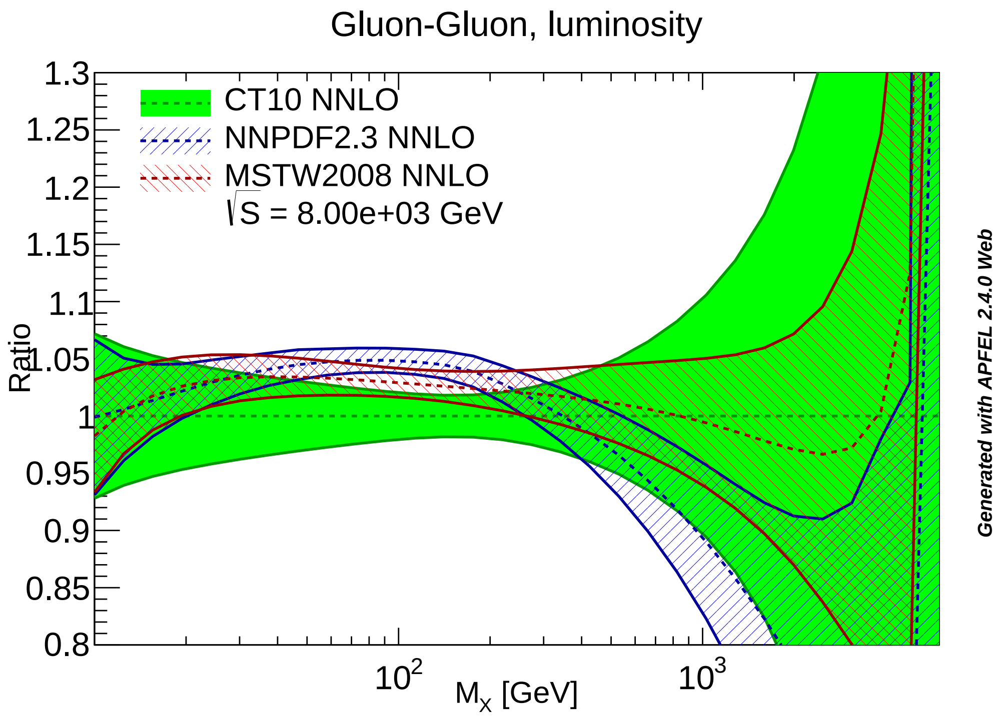
$$\tau \frac{d\mathcal{L}_{ab}}{d\tau} = \int_0^1 dx dy \left[x f_a(x,\mu_F)\, y f_b(y,\mu_F) + (x \leftrightarrow y) \right] \delta(\tau - xy)$$

$\rightarrow$ helpful to estimate production rate due to specific partonic channels at hadron collider

PDF uncertainties

[PDF4LHC 2015]



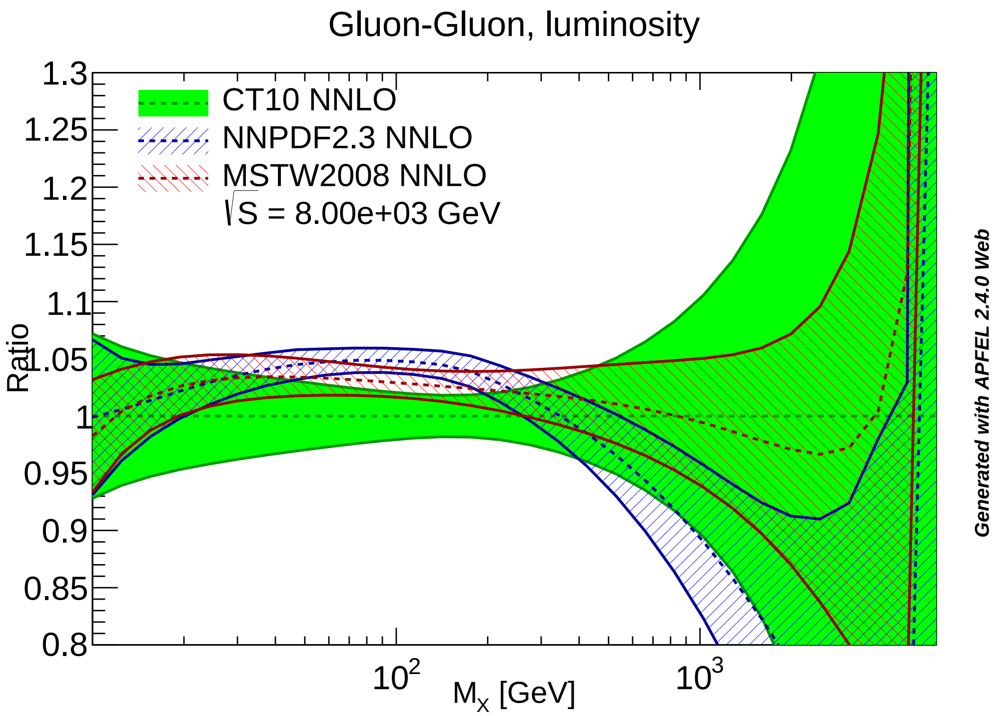


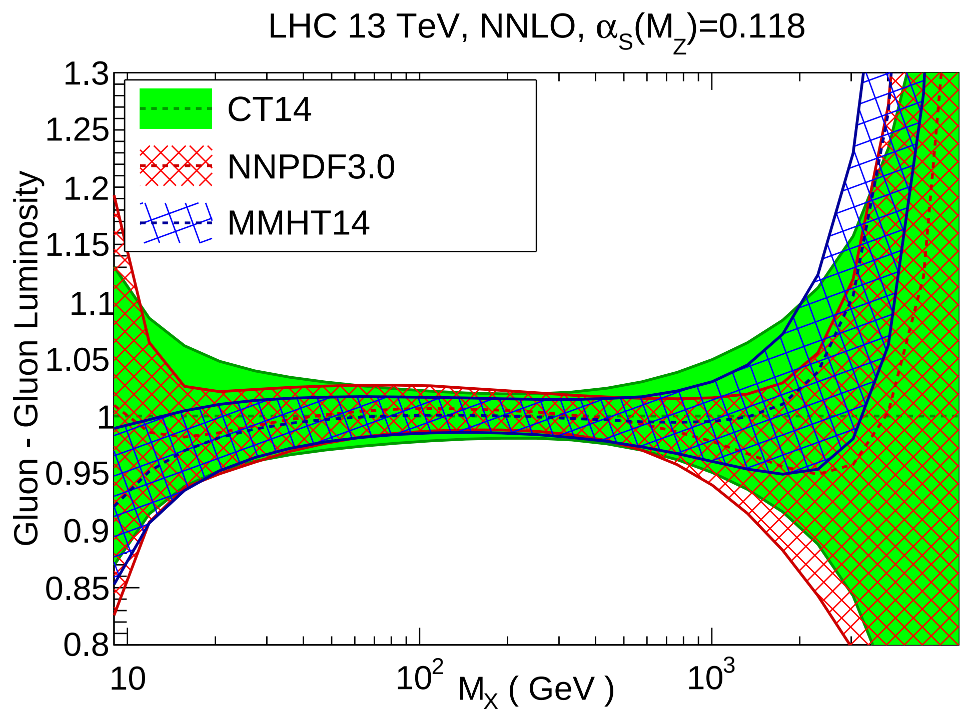
✔ each PDF set is associated with intrinsic uncertainty

✘ in some regions no overlap of CT10, MSTW2008, NNPDF2.3 uncertainty bands

PDF uncertainties

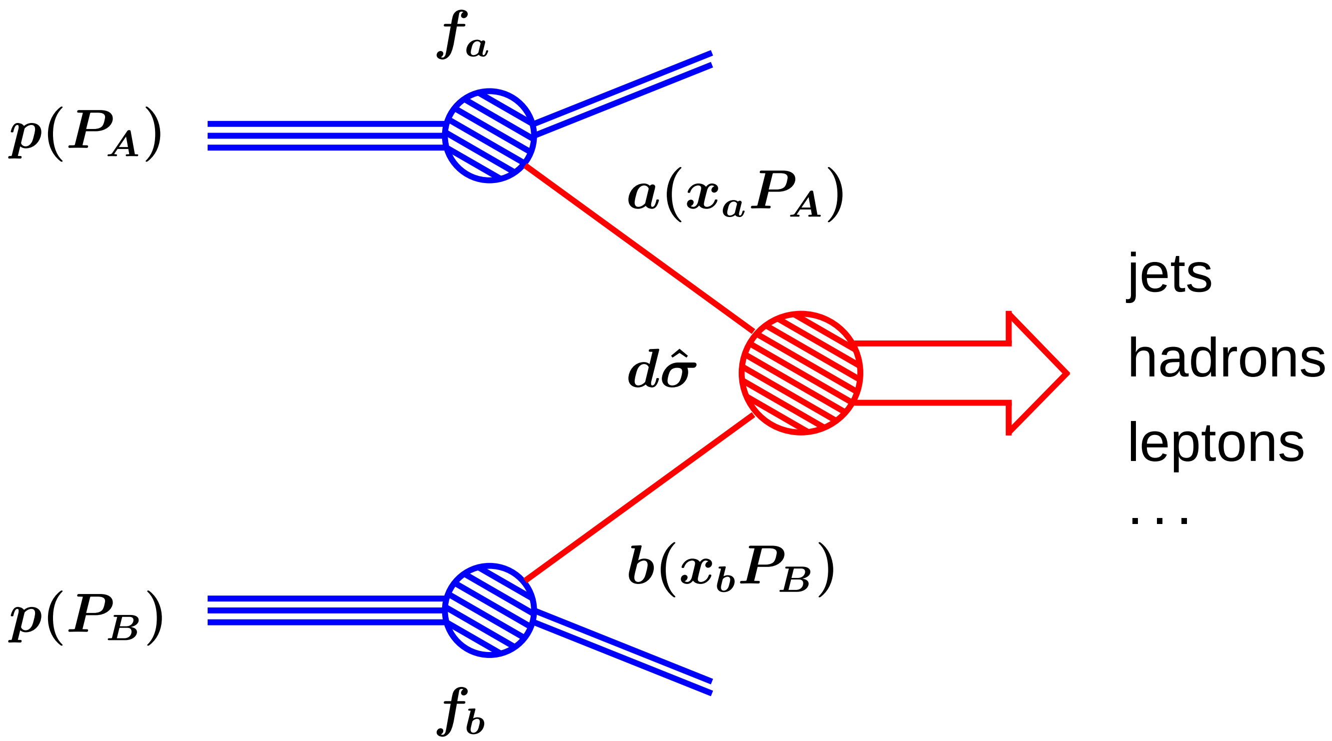
[PDF4LHC 2015]





✔ newer PDF sets CT14, NNPDF3.0, MMHT14 exhibit better consistency

hadron-hadron collision



$$d\sigma^{pp\to X} = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_a(x_a, \mu_F) f_b(x_b, \mu_F) \times d\hat{\sigma}^{ab\to X}(x_a P_A, x_b P_B, \mu_F, \mu_R)$$

recipe: calculation of partonic cross sections

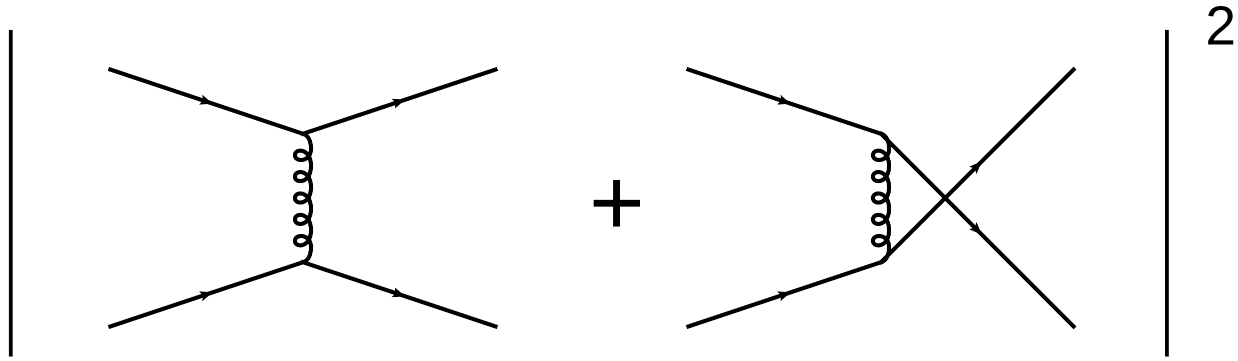
$$d\hat{\sigma}_{ab\to\dots} \sim \overline{\sum} |\mathcal{M}|^2_{ab\to cd\dots} \, \mathcal{F}_{\text{cuts}}(p_f) \, dPS$$

- calculation of scattering amplitude squared $|\mathcal{M}|^2$ at desired perturbative order (in α_s or α)
- proper treatment of ultraviolet and infrared divergences:
  - regularization
  - renormalization
  - subtraction of infrared singularities
- phase space integration and convolution with PDFs

the leading order

need to compute scattering amplitude squared, e.g.:

$$|\mathcal{M}_{\mathrm{LO}}|^2 =$$



(here: only two tree-level Feynman diagrams occur for $qq \to qq$)

matrix elements can be computed numerically
using helicity amplitude techniques

evaluation of Feynman diagrams

need to evaluate

$$\sum_{\text{helicities}} |\mathcal{M}|^2 = \sum_{\text{helicities}} (\mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_3+\ldots)\cdot(\mathcal{M}_1+\mathcal{M}_2+\mathcal{M}_3+\ldots)^\star$$

amplitude techniques:

evaluate $\mathcal{M} = (\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \ldots)$ first numerically for specific helicities of external particles, then square it!

fast numerical programs and many implementations available, e.g.

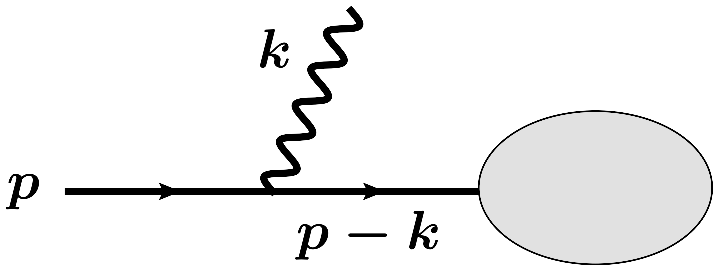
approach proposed by *Hagiwara, Zeppenfeld (1986,1989)*:

implemented in `HELAS` *(Murayama et al., 1992)*

employed by `MadGraph` *(Stelzer et al., 1994ff)*

amplitude techniques

basic approach of `HELAS/MadGraph`:



- at each phase space point

  $\longrightarrow$ take numerical values of external 4-momenta p_i^μ, k_i^μ

- polarization vectors $\varepsilon^\mu(k,\lambda)$ and spinors $u(p,\sigma)$

  $\Longleftrightarrow$ complex 4-arrays

- products like

$$\frac{1}{\not{p} - \not{k} - m}\not{\varepsilon}(k,\lambda)u(p,\sigma)$$

of momenta, polarization vectors, spinors, and γ^μ-matrices are computed via numerical 4×4 matrix multiplication

☞ perfect for LO amplitudes (all building blocks and results are completely finite)

the leading order

several public programs on the market for
automated generation of hard scattering matrix elements at
tree level in the Standard Model:

`Alpgen, CompHep, Helac, MadGraph, Sherpa, ...`

extra features:

- physics beyond the Standard Model
- facilities for phase-space integration
- analysis tools
- interfaces to parton-shower generators
- ...

need for higher-order corrections

- more reliable information:
  - higher order corrections often large
  - closer to experiment (more realistic final state)
  - test of methods and underlying theory
- search for physics beyond the Standard Model:

  since deviations of nature from SM small:
  - need very precise predictions for signal
    to spot effects of new physics
  - requires thorough understanding of SM backgrounds

. . . more precision . . .

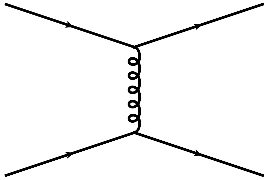
the next-to-leading order:

- real emission
- virtual corrections

next-to-leading-order (NLO) calculation: ingredients

example process: $qq \to qq$:

$$\mathcal{M}_{\mathrm{LO}} =$$



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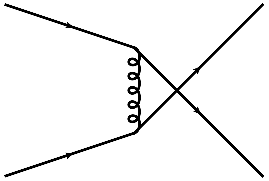
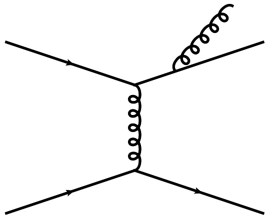


the leading order:

$$d\hat{\sigma}_{\mathrm{LO}} \sim |\mathcal{M}_{\mathrm{LO}}|^2 \sim \mathcal{O}(\alpha_s^2)$$

real-emission contributions:

$$\mathcal{M}_{\mathrm{real}} =$$



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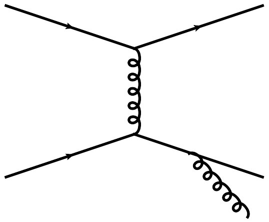


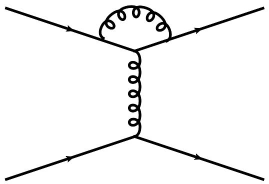
$+ \cdots$

diagrams with emission of one extra parton

$$d\hat{\sigma}_{\mathrm{R}} \sim |\mathcal{M}_{\mathrm{real}}|^2 \sim \mathcal{O}(\alpha_s^3)$$

virtual corrections:

$$\mathcal{M}_{\mathrm{virt}} =$$



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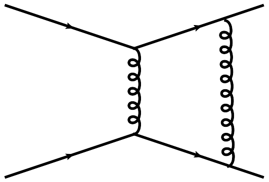


$+ \cdots$

loop diagrams yield interference contribution of wanted order

$$d\hat{\sigma}_{\mathrm{V}} \sim 2\mathrm{Re}\left[\mathcal{M}_{\mathrm{virt}}\mathcal{M}^{\star}_{\mathrm{LO}}\right] \sim \mathcal{O}(\alpha_s^3)$$

some complications at NLO

obvious: meaningful observables

$\downarrow$

theoretical prediction: finite result

but: how is finite result obtained in practice?

generally: perturbative calculation beyond LO
$\longrightarrow$ singularities encountered in intermediate steps

even though they will eventually cancel,
divergencies need to be treated properly
throughout!

regularization

☞ regularization needed to manifest singularities in intermediate steps of a calculation

various prescriptions on the market:

- cut-off regularization
- mass regularization
- dimensional regularization
- . . .

result for a meaningful observable:

independent of regulator and regularization prescription

regularization schemes

- momentum cut-off:

  can be used to regulate UV and / or

  IR divergent loop integrals, schematically:

$$\int_0^\infty \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2)^n} \to \int_{\Lambda_0}^{\Lambda_\infty} \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2)^n}$$

✔ simple to implement

✘ violates translation and gauge invariance

regularization schemes

- mass regularization:
  introduce auxiliary mass m for massless gauge bosons

  e.g., photon: propagator $$\frac{1}{q^2 + i\delta} \rightarrow \frac{1}{q^2 - m^2 + i\delta}$$

✘ calculations more complicated due to additional mass scale

✘ problems with gauge invariance in Non-Abelian case (QCD)

✔ frequently used for QED calculations

regularization schemes

- many other schemes are on the market, e.g.:
  - Pauli Villars regularization
  - analytical regularization
  - lattice regularization
  - ...

✘ can be problematic if Lorentz invariance or gauge symmetries are to be preserved

✔ can be useful for specific applications

regularization

✔ dimensional regularization:

dimension of space-time $d = 4 \to d = 4 - 2\varepsilon$

$$\int_0^\infty \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2)^n} \to \int_0^\infty \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2)^n}$$

$\varepsilon > 0 \ldots$ UV regulator, $\varepsilon < 0 \ldots$ IR regulator

divergencies $\to$ poles in ε

- preserves Lorentz and gauge invariance
- problem: have to perform Dirac algebra in d dimensions; $\varepsilon^{\mu\nu\rho\sigma}$ and γ^5 a priori undefined in $d \neq 4$

still: THE method of choice in QCD

dimensional regularization

different (but finally equivalent) implementations:

· “genuine” dimensional regularization:
polarization vectors/spinors of external particles and
internal loop momenta d-dimensional

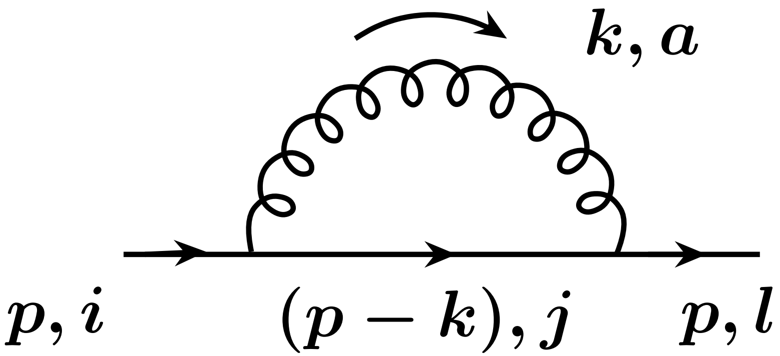
· dimensional reduction:
polarization vectors/spinors of external particles
4-dimensional,
internal loop momenta d-dimensional

well-defined transformation rules between different schemes

our method of choice: dimensional reduction

dimensional regularization: an example

let's calculate the quark selfenergy in d dim ($\overline{\mathrm{MS}}$ scheme):



$$= \quad \Sigma^b_{il}(p)$$

(un-renormalized)

compute color factor $\sum_{a,j} T^a_{ij} T^a_{jl} = C_F \delta_{il}$ and

replace coupling by dimensional one $g_s^2 \to \left(\frac{e^\gamma}{4\pi}\mu^2\right)^\varepsilon g_s^2$

$\Downarrow$

$$\Sigma^b_{il}(p) = -g_s^2 \mu^{2\varepsilon} C_F \, \delta_{il} \int \frac{d^d k}{(2\pi)^d} \frac{\gamma_\mu (\not{p} - \not{k}) \gamma^\mu}{k^2 (k-p)^2} = -i \not{p} C_F \delta_{il} \Sigma^b(p^2)$$

quark selfenergy

for evaluation of Σ^b we need scalar integral

$$\tilde{B}_0 = \frac{1}{i}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k-p)^2} = \frac{1}{16\pi^2}\left(\frac{-p^2}{4\pi}\right)^{-\varepsilon}\Gamma(1+\varepsilon)\left(2+\frac{1}{\varepsilon}\right)$$

and find after some algebra
(details on computation of loop integrals: see below)

$$\Sigma^b(p^2) = -\frac{\alpha_s}{4\pi}\left(\frac{\mu^2}{-p^2}\right)^{\varepsilon}\left(1+\frac{1}{\varepsilon}\right)$$

UV pole! remove by renormalization

quark selfenergy

☞ renormalized selfenergy for off-shell quarks:

$$\Sigma(p^2 \neq 0) = -\frac{\alpha_s}{4\pi}\left[\left(\frac{\mu^2}{-p^2}\right)^{\varepsilon}\left(1+\frac{1}{\varepsilon}\right) - \frac{1}{\varepsilon}\right]$$
$$= -\frac{\alpha_s}{4\pi}\left[1 + \ln\left(\frac{\mu^2}{-p^2}\right) + \mathcal{O}(\varepsilon)\right]$$

note:

- result finite as $\varepsilon \to 0$
- introduced arbitrary mass scale μ

cancelation of divergencies at NLO

UV divergencies

$\updownarrow$

renormalization

at scale μ_r

collinear singularities

$\updownarrow$

factorization

at scale μ_f

soft singularities

$\updownarrow$

cancel in sum of virtual and real emission contributions

sum of all real and virtual contributions to well-defined observable:

finite

cancelation of divergencies at NLO

intermediate
steps: regularize
all divergencies by
$d \to 4 - 2\varepsilon$

collinear singularities
$\updownarrow$
factorization
at scale μ_f

soft singularities
$\updownarrow$
cancel in sum of
virtual and real
emission contributions

sum of all real and
virtual contributions to
well-defined
observable:

finite for $\varepsilon \to 0$

cancelation of divergencies at NLO

cancelation of ε poles can be performed explicitly in analytical calculation, but how can divergencies be handled in numerical calculation?

collinear singularities

$\updownarrow$

factorization

at scale μ_f

soft singularities

$\updownarrow$

cancel in sum of virtual and real emission contributions

sum of all real and virtual contributions to well-defined observable:

finite for $\varepsilon \to 0$

cancelation of divergencies at NLO

typical NLO QCD calculation up to 1990ies:

- compute $|\mathcal{M}_{\text{real}}|^2$ and $2\text{Re}[\mathcal{M}_V \mathcal{M}_B^\star]$
  analytically in d dimensions
- perform phase-space integration analytically in d dim
  (considering acceptance cuts etc.)
- cancel matching poles in real emission and virtual contributions
- set $\varepsilon \to 0$ and convolute $d\hat{\sigma}$ with PDFs numerically for $d = 4$

cancelation of divergencies at NLO

procedure perfect for processes with only a few particles and minimal set of cuts (e.g., total cross sections):

- poles cancelled analytically
  $\rightarrow$ no delicate numerical cancelations needed

- resulting code fast and efficient
- procedure still used, e.g., for global PDF analyses

but:

- complete calculation has to be performed analytically in d dim (Dirac algebra can become very complicated; γ^5 problem . . .)
- PS integration can be done explicitly for “simple” reactions only
- implementation of cuts for realistic distributions hard

cancelation of divergencies at NLO

basic idea of modern approaches:

- treat only minimal part of full calculation analytically

  (utilize universality of pieces containing divergencies)

- finite contributions are treated numerically

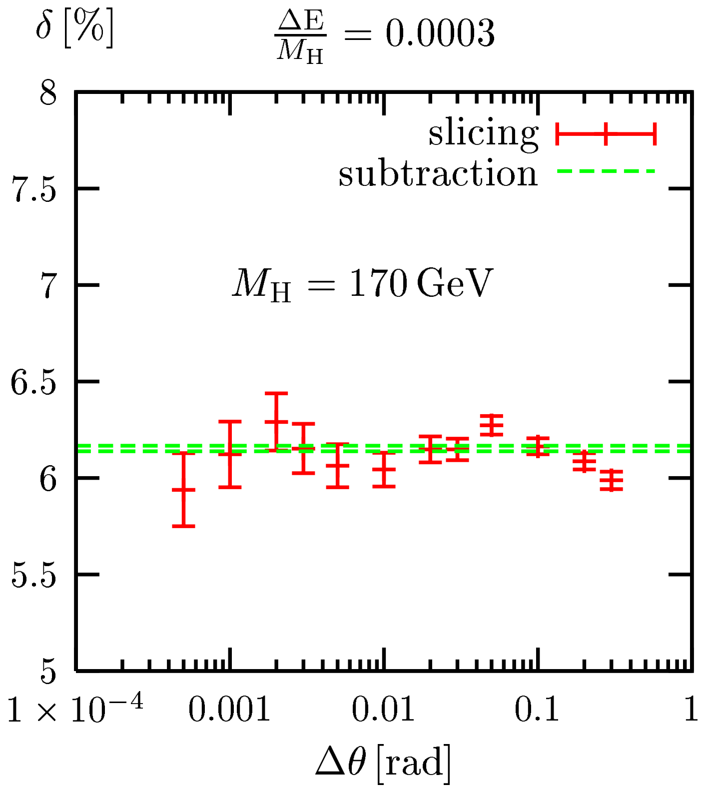
two types of algorithm to handle divergencies numerically:

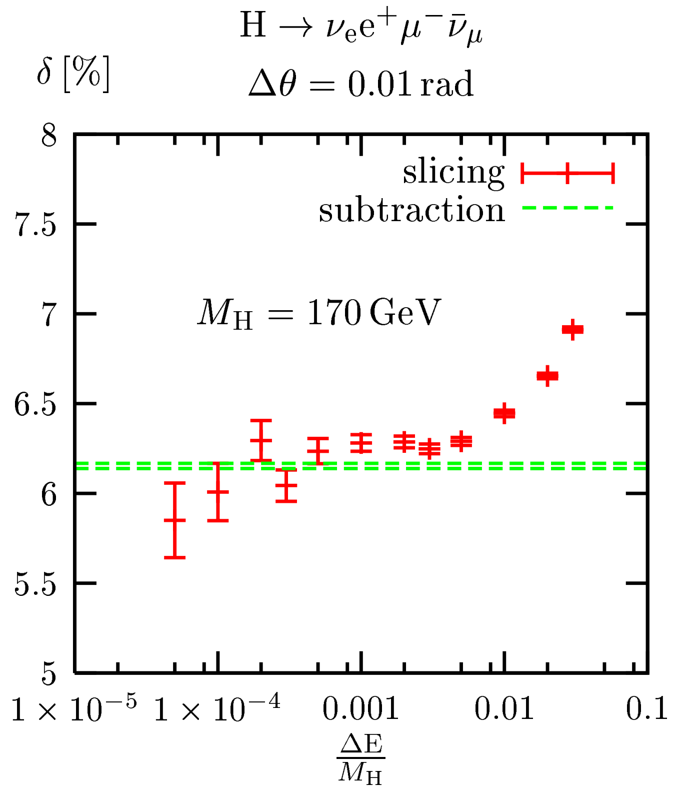
- phase space slicing
- subtraction method

actual details vary depending on specific implementation/variant, but basic concepts are general

Monte Carlo methods: a comparison

phase space slicing and subtraction techniques are in priciple equivalent, but are they in practice?





taken from *Bredenstein, Denner, Dittmaier, Weber,*
“ Precise predictions for the Higgs-boson decay
$H \to WW/ZZ \to 4$ *leptons”, hep-ph/0604011*

phase space slicing

- introduce cut parameter δ_S to split phase space into soft and hard regions that are evaluated separately
- after phase-space integration: $\ln \delta_S$ dependence in virtual and real emission contributions cancels numerically
- disadvantage: perform integration over potentially large terms first, cancel large contributions afterwards

$\longrightarrow$ procedure can cause numerical problems

see, e.g., *Harris, Owens, hep-ph/0102128*

subtraction methods

introduce local counterterm which

cancels divergencies before integration

↓

numerically stable

- first applied in $e^+e^- \to \mathbf{3}$ jets

  in process-specific manner by

  Ellis, Ross, Terrano (1981)

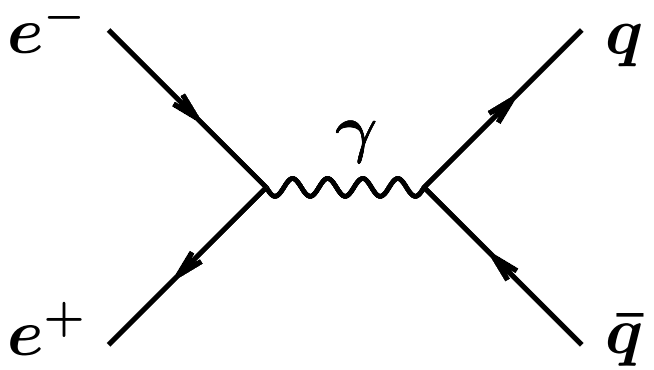
- extended to the general case by

  – *Frixione, Kunszt, Signer (1995)*

  – *Catani, Seymour (1996)*

  (later extensions/refinements exist)

dipole subtraction: a simple example



the most transparent case:

no identified hadrons in process, e.g. $e^+e^- \to 2$ jets:

$m \ldots \#$ of final state partons

$$\sigma^{LO} = \int_m d\sigma^B$$

finite!

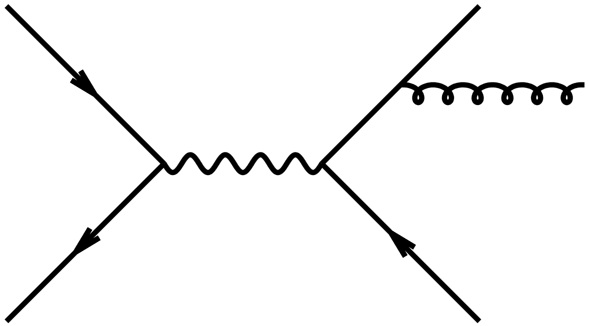
no regularization needed

calculate in $d = 4$ dimensions

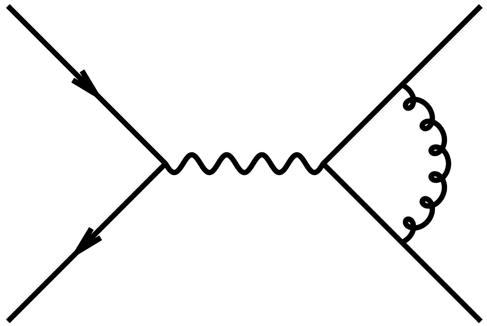
m-parton phase space integral

Born x-sec for $e^+e^- \to q\bar{q}$ $(m = 2)$

dipole subtraction: NLO ingredients



real emission contributions
$m+1$ parton kinematics



virtual corrections
m parton kinematics

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

IR divergent
☞regularize in $d = 4 - 2\varepsilon$ dim

dipole subtraction: counterterms

introduce local counterterm $\boldsymbol{d\sigma^A}$ with same singularity structure as $\boldsymbol{d\sigma^R}$:

$$\boldsymbol{\sigma^{NLO} = \underbrace{\int_{m+1} \left[d\sigma^R - d\sigma^A\right]}_{\text{finite}} + \int_{m+1} d\sigma^A + \int_m d\sigma^V}$$

↓

can safely set $\varepsilon \to \mathbf{0}$

perform integral numerically in four dimension

singularity structure

$$|\mathcal{M}_{m+1}(Q\,;\,p_1,\ldots,p_i,\ldots,{\color{red}p_j},\ldots,p_{m+1})|^2$$

soft region:

$$p_j = \lambda q,\ \lambda \to 0$$

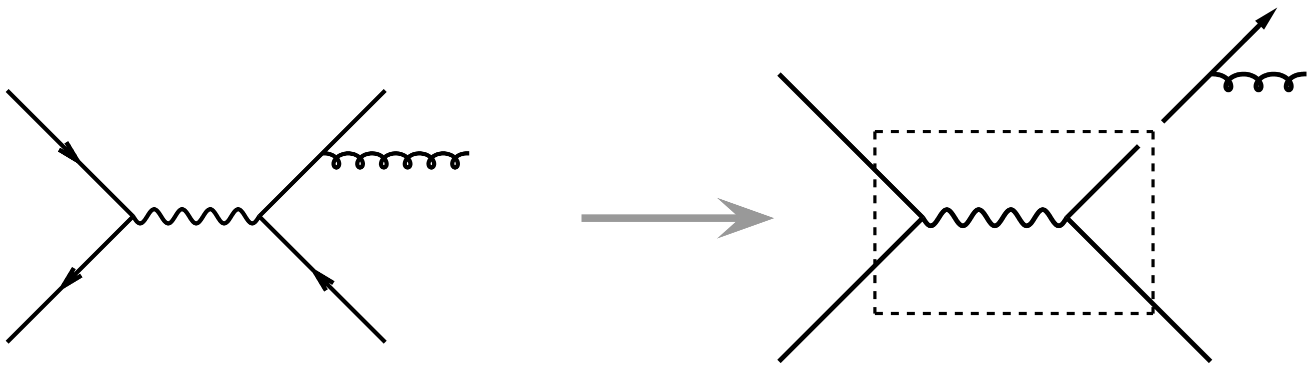
$$|\mathcal{M}_{m+1}|^2 \sim \frac{1}{\lambda^2}$$

collinear region:

$$p_j = \frac{(1-z)}{z}\,p_i$$

$$|\mathcal{M}_{m+1}|^2 \sim \frac{1}{p_i p_j}$$

e. g.:



universal structure: for each singular configuration

$$|\mathcal{M}_{m+1}|^2 \to |\mathcal{M}_m|^2 \otimes \mathrm{V}_{ij,k}$$

dipole subtraction:counterterms

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \left[d\boldsymbol{\sigma}^R - d\boldsymbol{\sigma}^A \right] \Big|_{\varepsilon=0} + \int_m d\boldsymbol{\sigma}^V + \int_{m+1} d\boldsymbol{\sigma}^A$$

$$\downarrow$$

integrate over one-parton PS analytically

explicitly cancel poles & then set $\varepsilon \to 0$

$$\downarrow$$

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \left[d\boldsymbol{\sigma}^R_{\varepsilon=0} - d\boldsymbol{\sigma}^A_{\varepsilon=0} \right] + \int_m \left[d\boldsymbol{\sigma}^V + \int_1 d\boldsymbol{\sigma}^A \right]_{\varepsilon=0}$$

dipole subtraction: the counterterm

wish list:

- matches singular behavior of $d\sigma^R$ exactly in d dim
- convenient for Monte Carlo integration
- exactly integrable analytically over one-parton PS in d dim
- for given process: independent of specific observable
- extra feature: universal structure

dipole subtraction: the counterterm

wish list:

- matches singular behavior of $d\sigma^R$ exactly in d dim
- convenient for Monte Carlo integration
- exactly integrable analytically over one-parton PS in d dim
- for given process: independent of specific observable
- extra feature: universal structure

a solution: dipole subtraction method
`[Catani and Seymour, hep-ph/9605323]`

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

(other approaches: Ellis et al.; Kunszt and Soper; Dittmaier, . . .)

dipole subtraction: the counterterm

color/spin
projection of
Born x-sec

universal dipole
factors

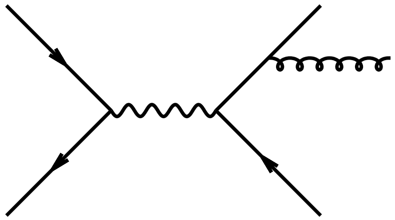
$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

dipoles for all $(m+1)$
configurations
corresponding to given
m-parton state

PS convolution &
color/spin summation

real emission contributions

for the computation of $d\sigma^R$ we need numerical value for

$|\mathcal{M}_R|^2 = \Big|$  $+ \quad \cdots \Big|^2$

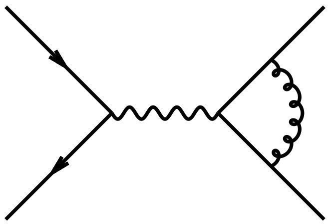
at each generated phase space point in 4 dimensions

can apply same (numerical) amplitude techniques as at LO

keep in mind: kinematics different from LO
($2 \to 3$ instead of $2 \to 2$ particles)

virtual corrections

…interference of LO diagrams with one-loop graphs

$\mathcal{M}_V =$  $+ \;\ldots$

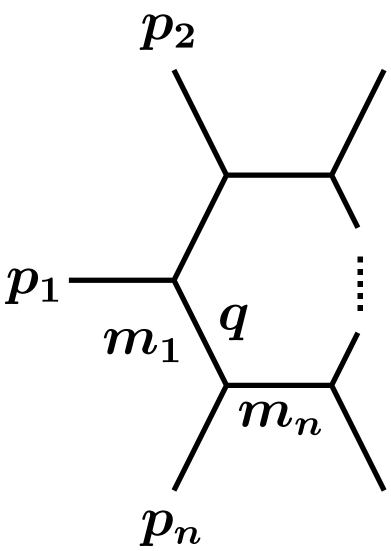
note: Born-type parton kinematics

recall: poles are needed explicitly, finite remainder
can be computed in 4 dimensions

requires computation of one-loop scalar and tensor integrals
(increasing complexity the more propagators are involved)

loop integrals

in any loop calculation we encounter tensor integrals of type



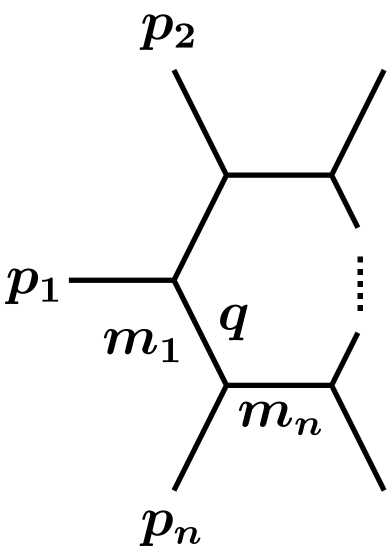
$$T_{\mu_1\ldots\mu_m}(p_1,\ldots,p_n;m_1,\ldots,m_n) = \int \frac{d^dq}{i\pi^2}\,\frac{q_{\mu_1}\cdots q_{\mu_m}}{D_1D_2\ldots D_n}$$

with

$$\begin{aligned} D_1 &= q^2 - m_1^2 + i\epsilon \\ D_2 &= (q+p_1)^2 - m_2^2 + i\epsilon \\ &\cdots \\ D_n &= (q+\ldots+p_{n-1})^2 - m_n^2 + i\epsilon \end{aligned}$$

loop integrals

in any loop calculation we encounter tensor integrals of type



$$T_{\mu_1 \dots \mu_m}(p_1, \dots, p_n; m_1, \dots, m_n) = \int_0^\infty \frac{d^d q}{i\pi^2} \frac{q_{\mu_1} \dots q_{\mu_m}}{D_1 D_2 \dots D_n}$$

nomenclature:

scalar integrals with

$$n = 1, 2, 3, 4, 5, \dots$$

$\downarrow$

$$A_0, B_0, C_0, D_0, E_0, \dots$$

and analogous for tensor integrals:

$$A_\mu, B_\mu, B_{\mu\nu}, \dots$$

tensor integrals

…calculable from scalar integrals by Passarino-Veltman reduction

$$T^{\{0,\mu,\mu\nu,\dots\}}(p_1,\dots) = \int \frac{d^dq}{i\pi^2} \frac{\{1, q^\mu, q^\mu q^\nu, \dots\}}{D_1 \dots D_n}$$

bubbles :

$$\begin{aligned} B^\mu &= p_1^\mu B_1 \\ B^{\mu\nu} &= p_1^\mu p_1^\nu B_{21} + g^{\mu\nu} B_{22} \end{aligned}$$

triangles :

$$\begin{aligned} C^\mu &= p_1^\mu C_{11} + p_2^\mu C_{12} \\ C^{\mu\nu} &= p_1^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + \{p_1 p_2\}^{\mu\nu} C_{23} + g^{\mu\nu} C_{24} \\ C^{\mu\nu\rho} &= p_1^\mu p_1^\nu p_1^\rho C_{31} + p_2^\mu p_2^\nu p_2^\rho C_{32} + \{p_1 p_1 p_2\}^{\mu\nu\rho} C_{33} \\ &\quad + \{p_1 p_2 p_2\}^{\mu\nu\rho} C_{34} + \{p_1 g\}^{\mu\nu\rho} C_{35} + \{p_2 g\}^{\mu\nu\rho} C_{36} \end{aligned}$$

tensor integrals

boxes:

$$D^{\mu} = p_1^{\mu} D_{11} + p_2^{\mu} D_{12} + p_3^{\mu} D_{13}$$

$$\begin{aligned} D^{\mu\nu} &= p_1^{\mu} p_1^{\nu} D_{21} + p_2^{\mu} p_2^{\nu} D_{22} + p_3^{\mu} p_3^{\nu} D_{23} + \{p_1 p_2\}^{\mu\nu} D_{24} \\ &\quad + \{p_1 p_3\}^{\mu\nu} D_{25} + \{p_2 p_3\}^{\mu\nu} D_{26} + g^{\mu\nu} D_{27} \end{aligned}$$

$$\begin{aligned} D^{\mu\nu\rho} &= p_1^{\mu} p_1^{\nu} p_1^{\rho} D_{31} + p_2^{\mu} p_2^{\nu} p_2^{\rho} D_{32} + p_3^{\mu} p_3^{\nu} p_3^{\rho} D_{33} + \{p_1 p_1 p_2\}^{\mu\nu\rho} D_{34} \\ &\quad + \{p_1 p_1 p_3\}^{\mu\nu\rho} D_{35} + \{p_1 p_2 p_2\}^{\mu\nu\rho} D_{36} + \{p_1 p_3 p_3\}^{\mu\nu\rho} D_{37} \\ &\quad + \{p_2 p_2 p_3\}^{\mu\nu\rho} D_{38} + \{p_2 p_3 p_3\}^{\mu\nu\rho} D_{39} + \{p_1 p_2 p_3\}^{\mu\nu\rho} D_{310} \\ &\quad + \{p_1 g\}^{\mu\nu\rho} D_{311} + \{p_2 g\}^{\mu\nu\rho} D_{312} + \{p_3 g\}^{\mu\nu\rho} D_{313} \end{aligned}$$

scalar coefficients D_{ij} depend on B_0, C_0, D_0

tensor integrals

example:

$$B_\mu(p) = p_\mu B_1(p) = \int \frac{d^d q}{i\pi^2} \frac{q_\mu}{q^2(q+p)^2}$$

compute B_1 by suitable contractions:

$$\begin{aligned} p^\mu B_\mu(p) = p^2 B_1(p) &= \int \frac{d^d q}{i\pi^2} \frac{p \cdot q}{q^2(q+p)^2} \\ &= \int \frac{d^d q}{i\pi^2} \frac{1}{2} \frac{\left[(p+q)^2 - p^2 - q^2\right]}{q^2(q+p)^2} \\ &= \frac{1}{2}\left[A(0) - A(0) - p^2 B_0\right] \end{aligned}$$

$$\longrightarrow \quad B_1 = -\frac{1}{2} B_0$$

tensor reduction methods

newer approaches:

refinements of Passarino-Veltman tensor reduction, e.g.:

- Binoth, Guillet, Heinrich et al. (1999, 2005)
- Denner, Dittmaier: (2002,2005)
- Ellis, Giele, Zanderighi (2005)

alternative: reduction of one-loop amplitudes

to scalar integrals at the integrand level

Ossola, Papadopolous, Pittau (2006)

verification

checks

to ensure reliability of calculation: perform some checks!

✔ comparison of LO and real emission amplitudes with alternative code, e.g. `MadGraph`:

- compare numerical value of $\mathcal{M}_B$ and $\mathcal{M}_R$ at every generated phase space point

  keep in mind: real-emission corrections to $ab \to X$ correspond to

  Born amplitudes for $ab \to X$ + parton

  $\to$ generation with tree-level amplitude generators possible

- expect agreement at 10^{-10} level

checks

✔ check infrared subtraction procedure:

- in soft / collinear limits subtraction terms approach real-emission contributions (non-singular contributions become sub-dominant)

- generate events in singular regions:

  expect $d\sigma^A/d\sigma^R \to 1$ as

  two partons become collinear ($p_i \cdot p_j \to 0$) or

  gluon becomes soft ($E_g \to 0$)

checks

✔ QCD gauge invariance:
easy to check for processes with external gluon, as

$$\mathcal{M} = \varepsilon_\mu(p_g)\,\mathcal{M}^\mu = [\varepsilon_\mu(p_g) + \beta\, p_{g\,\mu}]\,\mathcal{M}^\mu$$

☞ expect $p_{g\,\mu}\mathcal{M}^\mu = 0$

✧ practically: in code for computation of $\mathcal{M}$
replace $\varepsilon_\mu(p_g)$ throughout with $p_{g\mu} \to \mathcal{M}'$

$\to$ expected relation ($\mathcal{M}' = 0$) fulfilled within
numerical accuracy of the program

checks

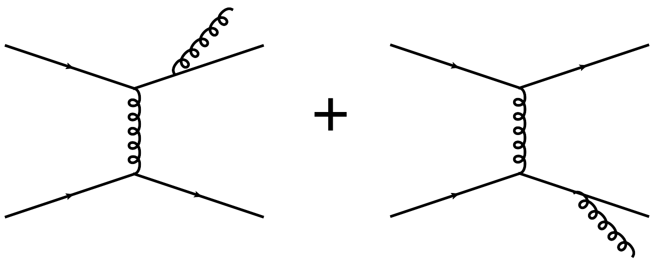
✔ produce two independent codes

↓

require agreement within
numerical accuracy of the two programs

recap: ingredients of an NLO calculation

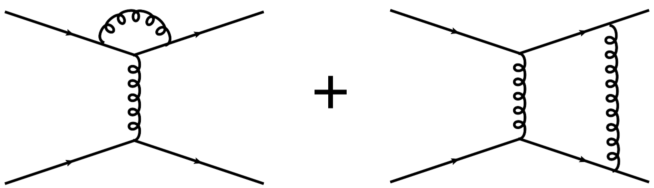
real-emission contributions:

$\mathcal{M}_{\text{real}} =$  $+ \cdots$

diagrams with emission of one extra parton

$$d\hat{\sigma}_{\text{R}} \sim |\mathcal{M}_{\text{real}}|^2 \sim \mathcal{O}(\alpha_s^3)$$

virtual corrections:

$\mathcal{M}_{\text{virt}} =$  $+ \cdots$

loop diagrams yield interference contribution of wanted order

$$d\hat{\sigma}_{\text{V}} \sim 2\text{Re}\left[\mathcal{M}_{\text{virt}}\mathcal{M}^{\star}_{\text{LO}}\right] \sim \mathcal{O}(\alpha_s^3)$$

extra ingredients for handling of divergences:

- subtraction procedure for infrared divergences
- renormalization of UV divergences

tools for the next-to-leading order in QCD

development of new techniques over last 15 years:

OPP algorithm, generalized unitarity, loops from trees, recursion relations, . . .

☞ starting point of automated approaches to loop calculations

multi-purpose tools for (more or less) automated computation of NLO QCD amplitudes

`MadGraph5_aMC@NLO, OpenLoops, GoSam, ...`

dedicated tools for efficient calculation of specific processes

`HAWK, MCFM, VBFNLO, ...`

public loop integral libraries

Carazza, Ellis, Zanderighi (2007, 2016)

QCDLoop

Object-oriented one-loop scalar Feynman integrals framework

DOWNLOAD LIVE DEMO

Denner, Dittmaier, Hofer (2016)

COLLIER

A Complex One-Loop LIbrary with Extended Regularizations

frontiers of NLO QCD

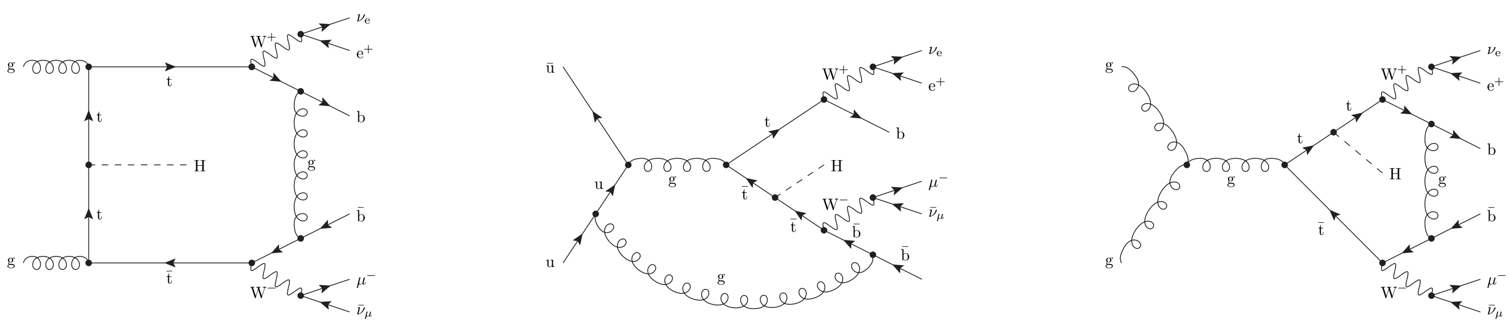
exact NLO calculation of multi-leg processes possible

☞ accurate treatment of off-shell configurations
(narrow-width approximation no longer necessary)

example: $t\bar{t}H$ (with $t \to Wb \to \ell\nu b$)

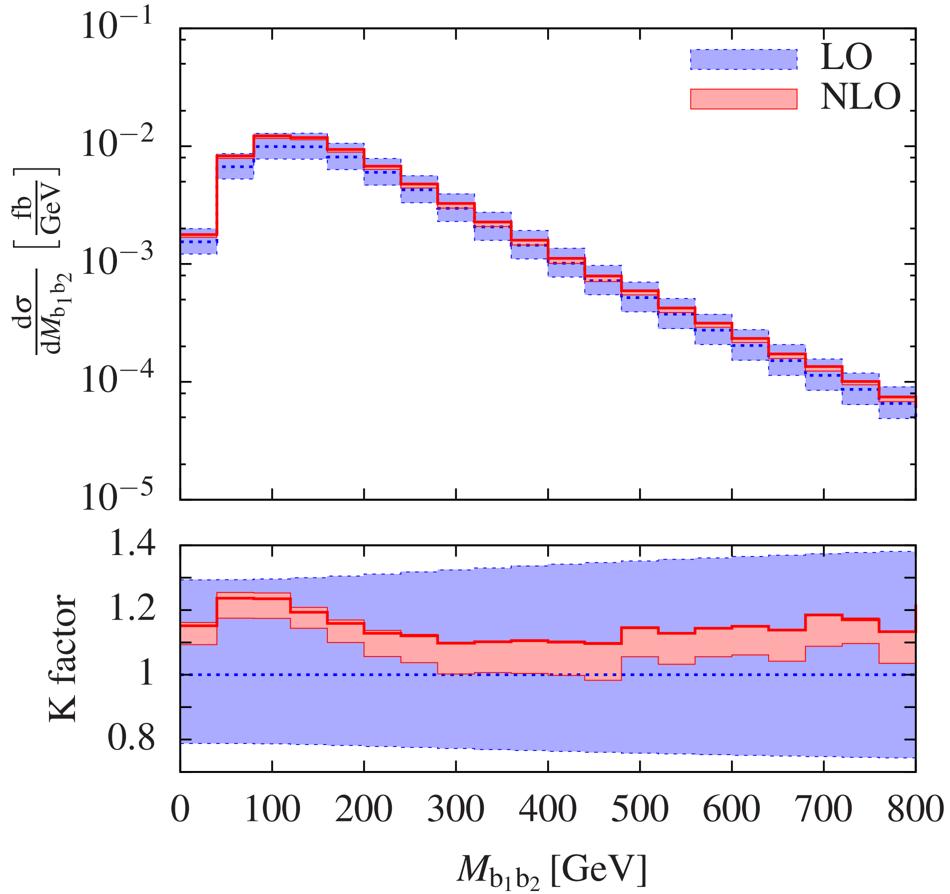
[Beenakker et al.; Dawson et al. (2001-03)]

$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}H$ *[Denner, Feger (2015)]*



$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}H$ at NLO QCD

Denner, Feger (2015)



tremendous complexity:

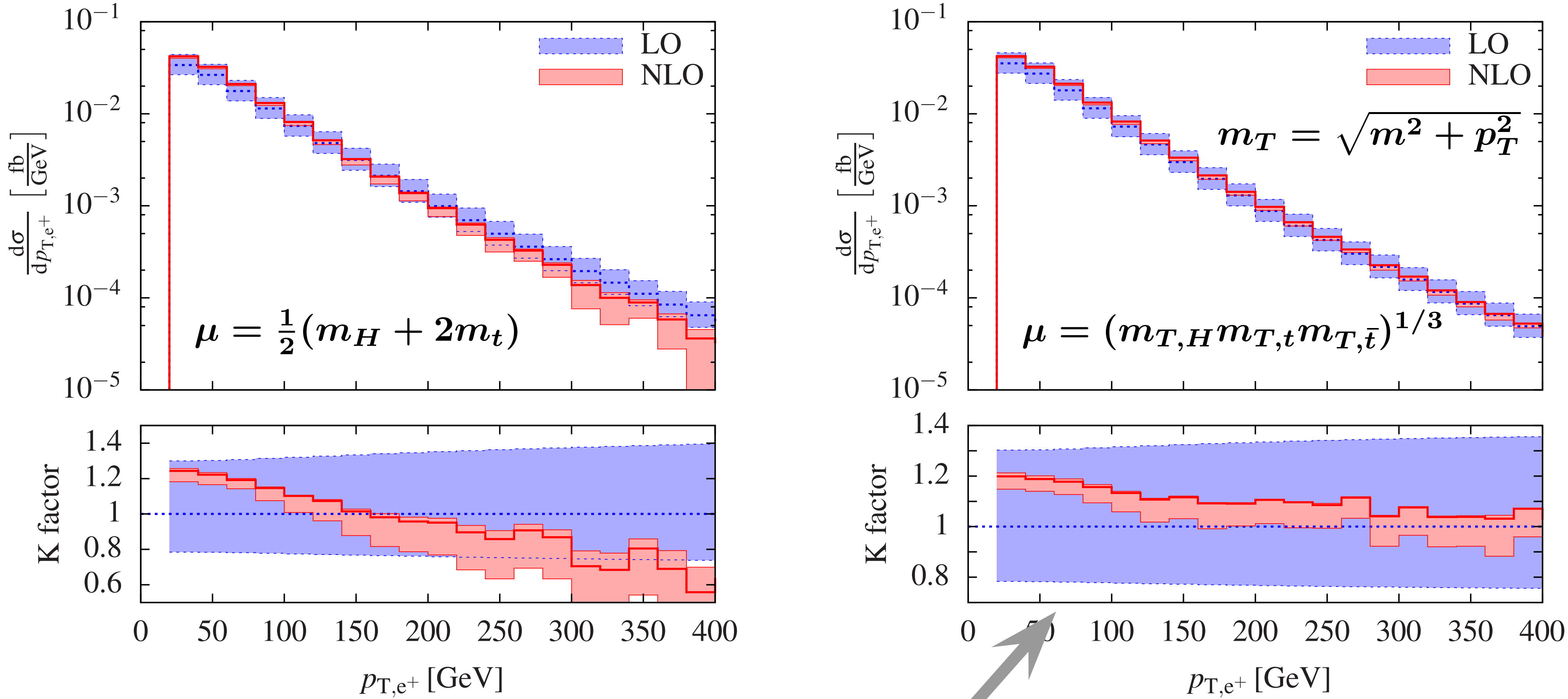
- amplitudes generated with the help of automated tool `RECOLA`
- loop integrals are evaluated with the `COLLIER` library
- bottle neck: efficient phase-space integration

gain: full control on final-state particles

(realistic cuts on leptons and b-jets, access to decay correlations, . . .)

$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}H$ at NLO QCD

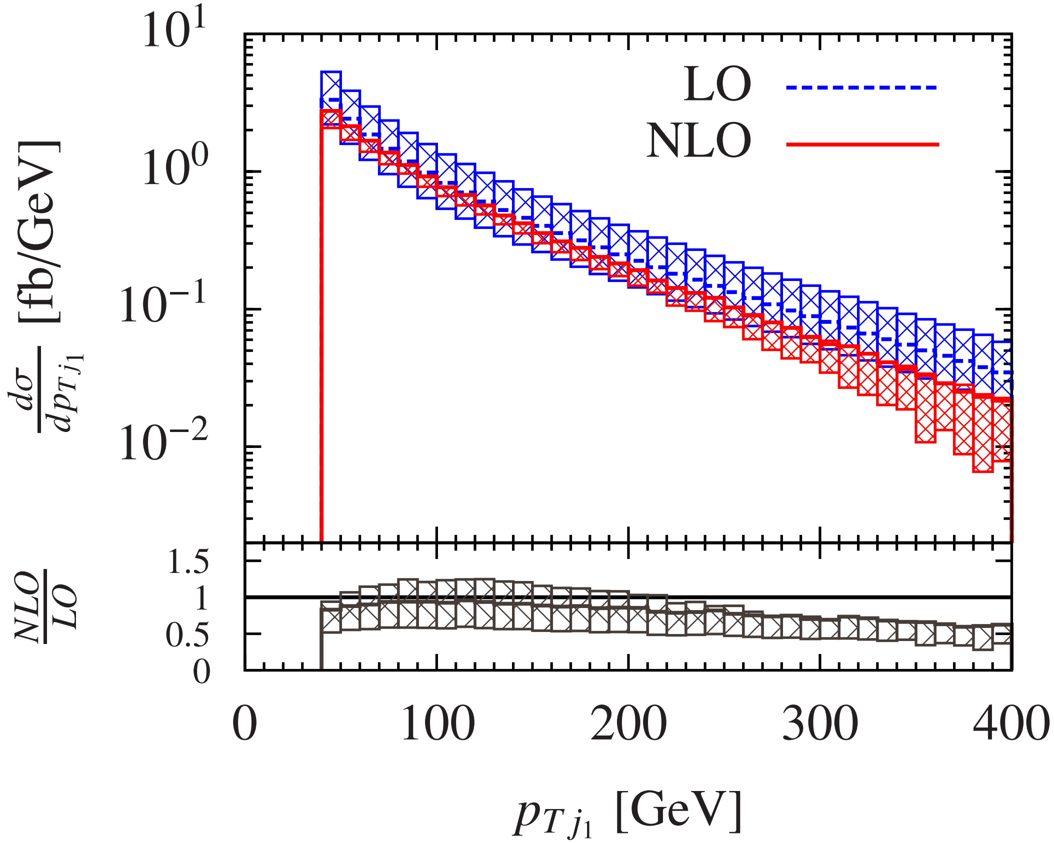
Denner, Feger (2015)

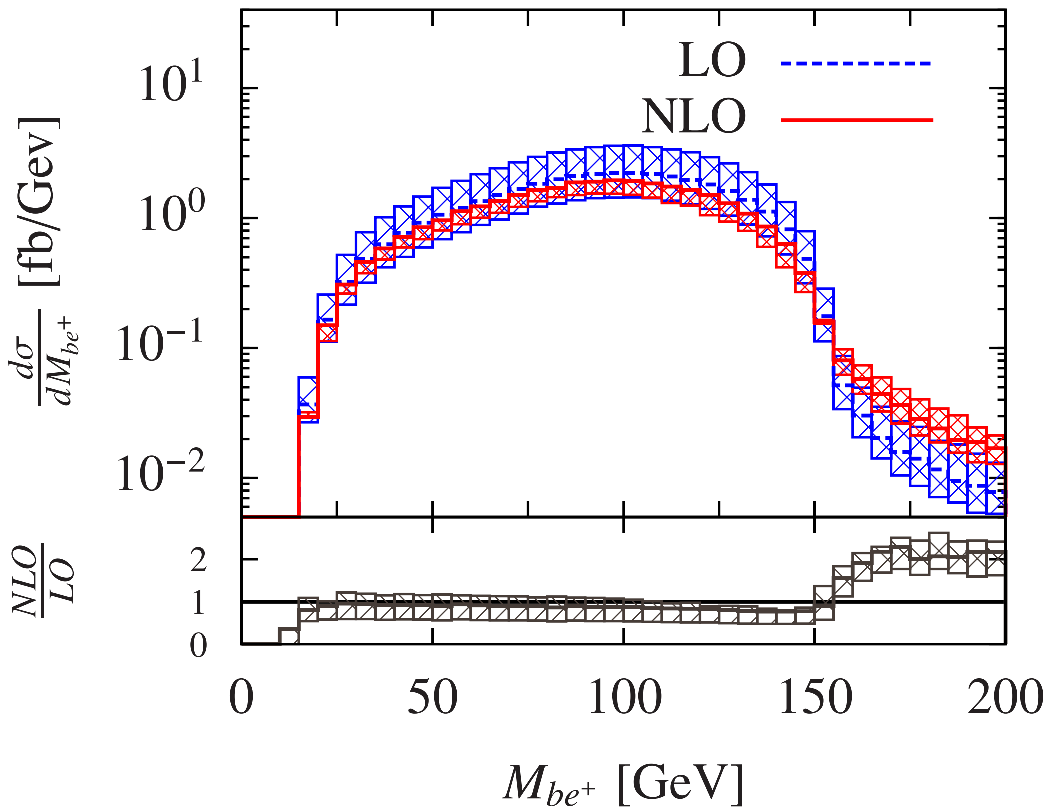


dynamical scale improves perturbative stability

from $pp \to t\bar{t}j$ to $pp \to e^+\nu_e\mu^-\bar{\nu}_\mu b\bar{b}j$

Bevilaqua et al. (2015)





full off-shell effects for $pp \to t\bar{t}j$ using the programs `Helac-1Loop`, `OneLoop`, `CutTools`

. . . even more precision . . .

- the next-to-next-leading order (NNLO) in QCD
- NLO electroweak (EW) corrections
- mixed QCD-EW effects

more types of perturbative corrections

- fixed order QCD corrections: LO, NLO, NNLO, . . .
- QCD resummations:
  - with analytical methods (LL, NLL, NNLL, . . .)
  - via parton shower Monte Carlo tools
- NLO EW corrections:

  generically $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$, but systematic enhancements by

  - Sudakov logarithms $\sim \ln^n(M_W/Q)$ at high scales Q
  - kinematic effects from photon radiation off leptons
- consistent combination of various types of corrections

QCD: the next-to-next-to leading order

amazing progress in computation of total cross sections and differential distributions for benchmark processes at NNLO QCD

requiring: two-loop amplitudes for a process X, one-loop amplitudes for the processes $X+1$ parton, tree-level amplitudes for the processes $X+2$ partons

prerequisites:

✔ availability of 2-loop master integrals

✔ efficient subtraction techniques for infrared divergences

(q_T subtraction, N-jettiness, antenna subtraction, sector decomposition, projection to Born)

✔ powerful Monte-Carlo programs of high numerical stability

$pp \to X$ beyond one loop

process	motivation
dijets	PDFs, strong coupling, BSM
H	Higgs couplings
H+jet	Higgs couplings
$t\bar{t}$	top properties, PDFs, BSM
single top	top properties, PDFs
VBF	Higgs couplings
V+jet	PDFs
VH	Higgs couplings
VV	gauge couplings, BSM
HH	Higgs potential

NNLO QCD: new public Monte Carlo programs

brand-new: implementation of several NNLO QCD processes with color-singlet final states in the public Monte Carlo program `MCFM`

$\boldsymbol{pp \to H, Z, W, HZ, HW, \gamma\gamma}$ (including decays)

performance: very CPU efficient
(1% statistical accuracy within a few hours on 8 cores)

Boughezal et al. (05/2016)

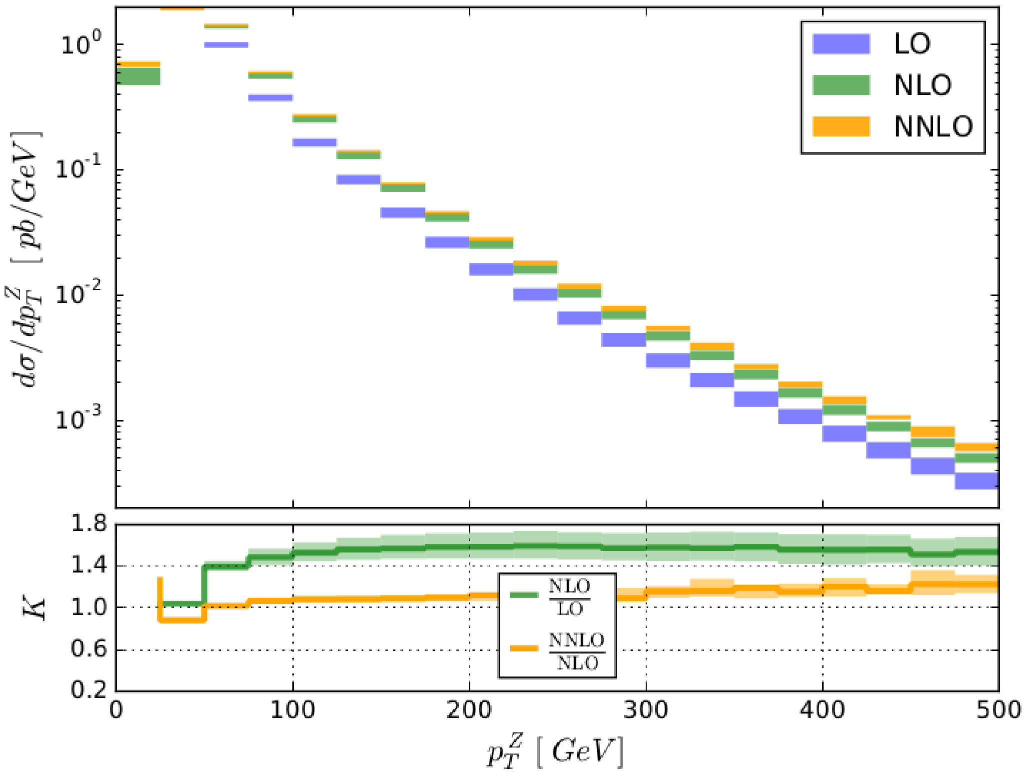
in preparation: fully differential NNLO process library `MATRIX`

$\boldsymbol{pp \to Z, W, H, \gamma\gamma, ZZ, WW, WZ}$ (partly including decays)

Grazzini et al. (release planned for this year)

$pp \to Zj$ at NNLO QCD

Boughezal et al. (2015)



2015: two completely independent calculations

[Gehrmann-De Ridder et al. & Boughezal et al.]

using different techniques

(antenna vs. N-jettiness subtraction)

✔ scale uncertainties reduced

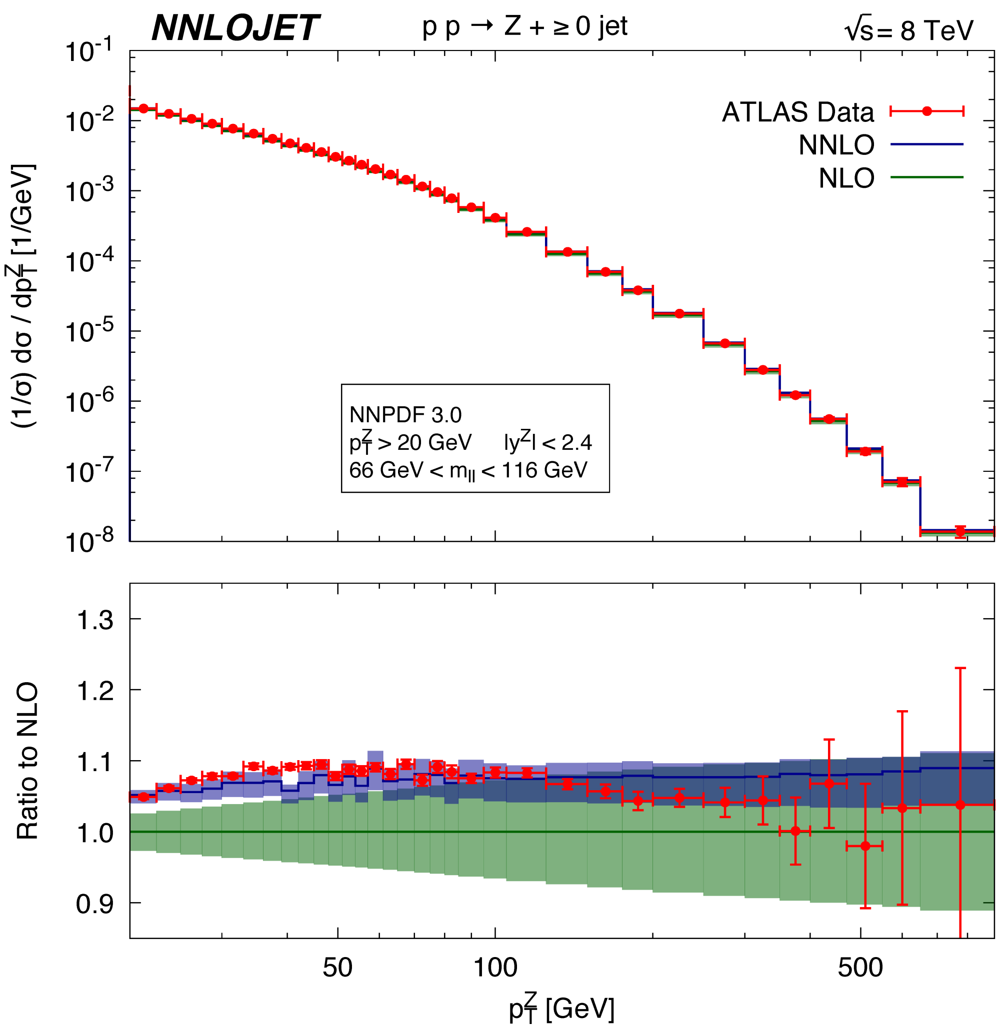
✔ perturbative expansion stable

NNLO QCD corrections are at percent level for inclusive xsec,

up to 10% in tails of distributions

$pp \to \ell^+\ell^- j$ at NNLO QCD

Gehrmann-De Ridder et al. (2016)



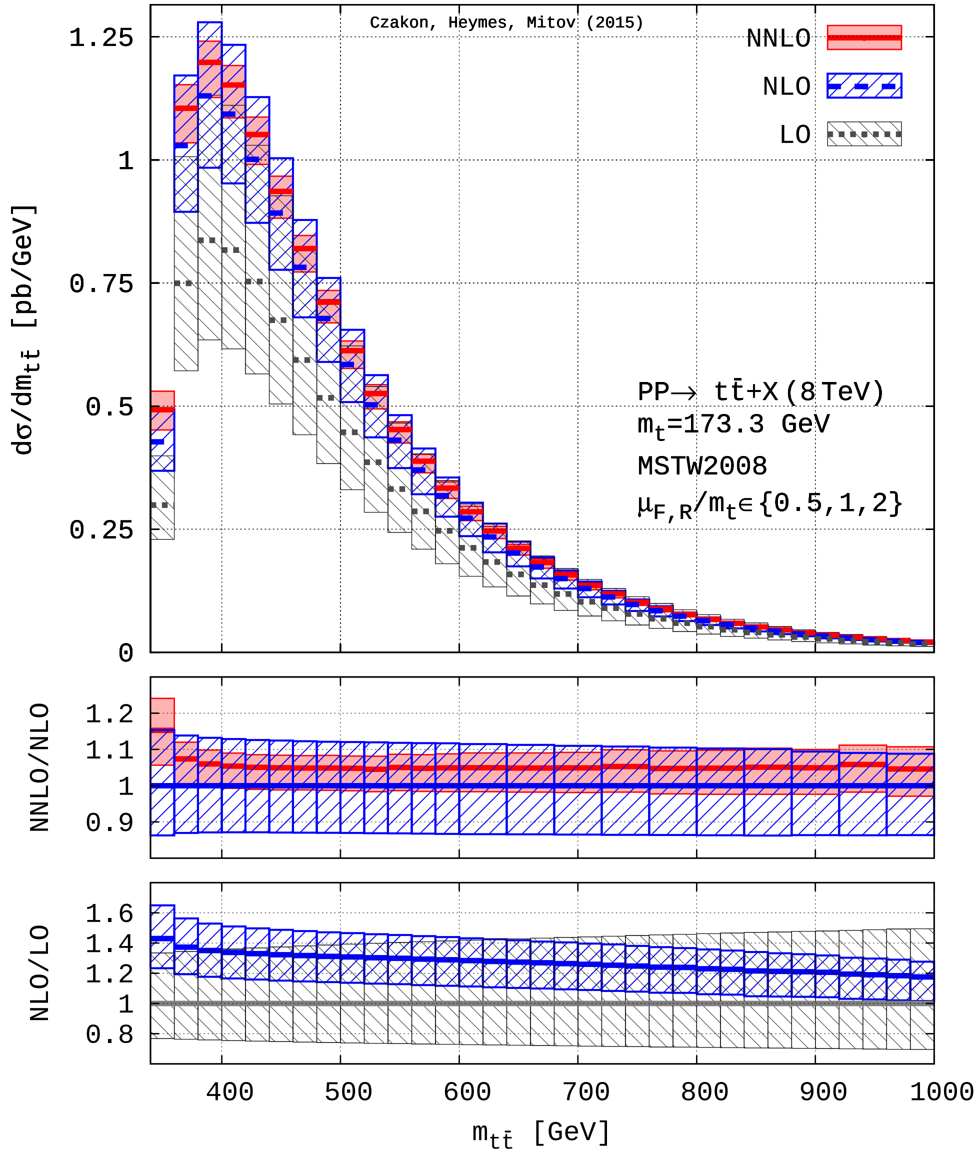
differential predictions at NNLO accuracy soften tension between theory and experiment

optimal: normalize to inclusive Drell-Yan xsec

($\to$ minimize impact of experimental uncertainties)

$pp \to t\bar{t}$: going differential at NNLO QCD

Czakon, Heymes, Mitov (2015)

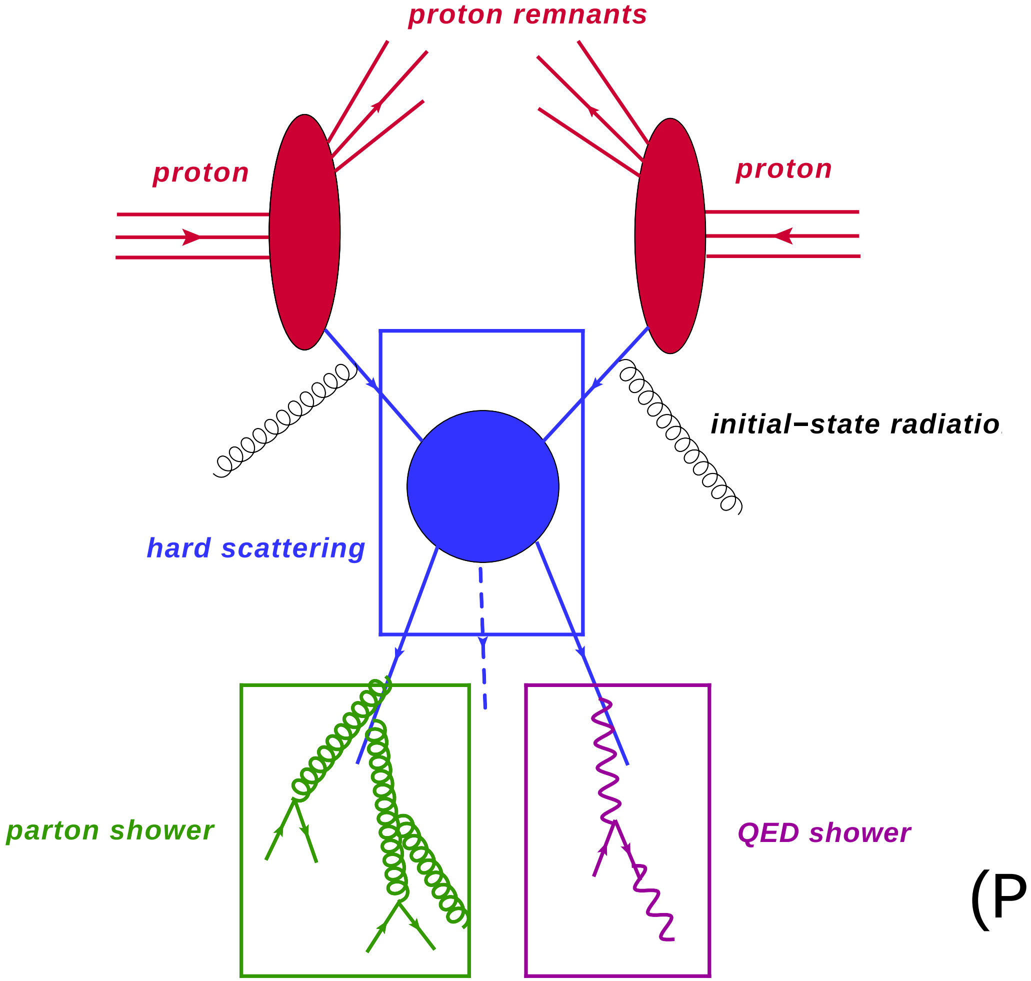


- perturbative result stabilized
- scale dependence reduced
- improved agreement with data from Tevatron and LHC

future applications:

PDF fits, precision measurements of the top mass, α_s extraction

more realistic simulations



for realistic description of scattering processes at hadron colliders:

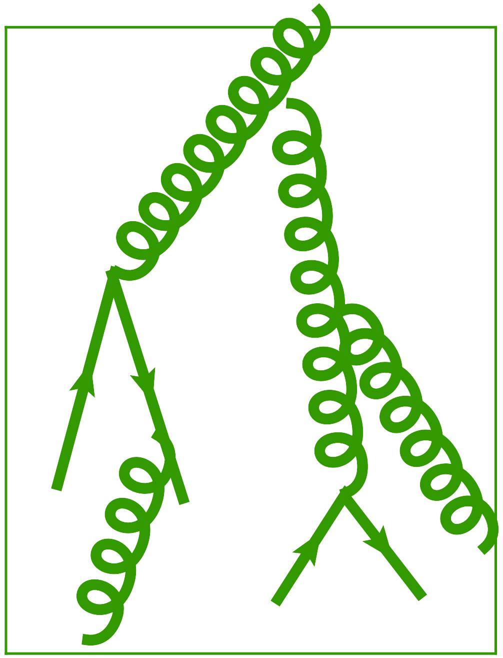
- combine matrix elements for hard scattering with programs for simulation of

underlying event, parton shower, and hadronization

(PYTHIA, HERWIG, SHERPA,...)

parton-shower event generators

parton shower



= computer programs for simulation of collider events down to the level of stable particles:

start from hard scattering process

↓

energetic partons radiate soft/collinear daughter partons $\longrightarrow$ energy scale decreases

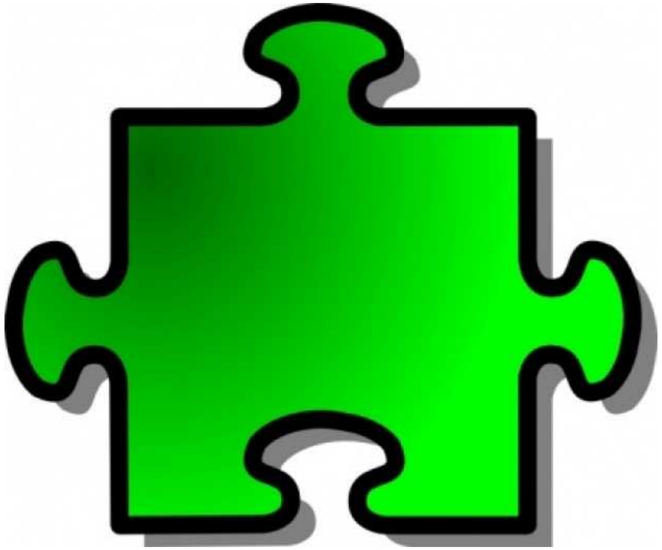
↓

at low scales partons hadronize

most common generators: `HERWIG`, `PYTHIA`, `SHERPA`

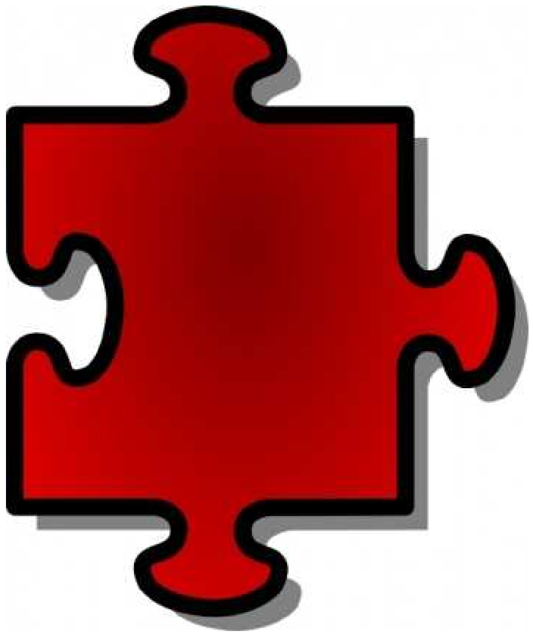
include many other useful features, e.g.: hadronization models, simulation of underlying event, multi-parton interactions, generators for hard scattering amplitudes

realistic & precise predictions

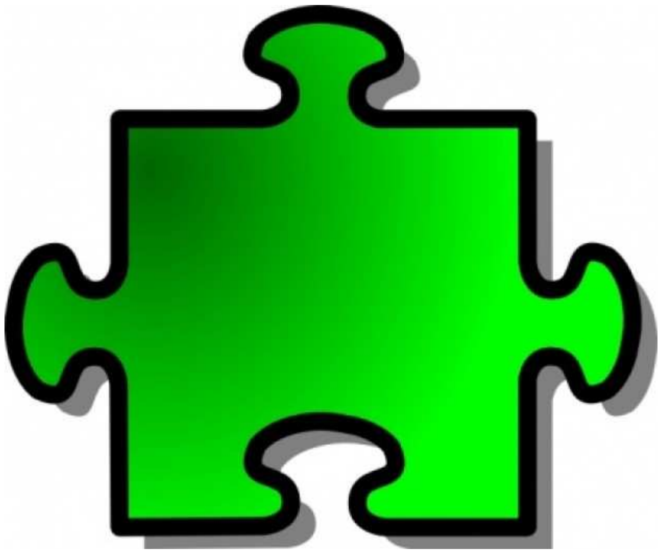


exploit merits of flexible
Monte Carlo tools

retain NLO accuracy
for hard scattering

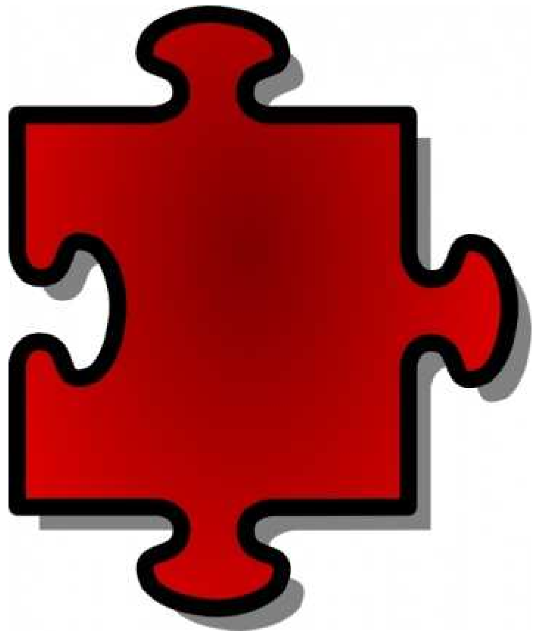


realistic & precise predictions



<u>shower Monte Carlo:</u>

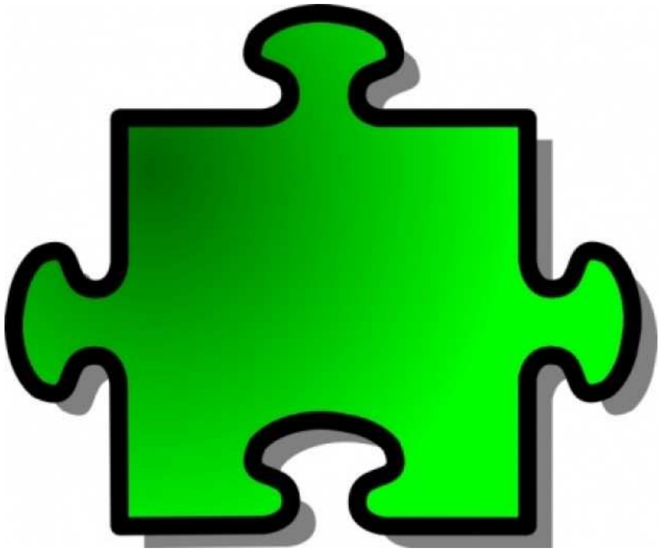
- good description at low transverse momenta (p_T)
- events at hadron level



<u>NLO-QCD calculation:</u>

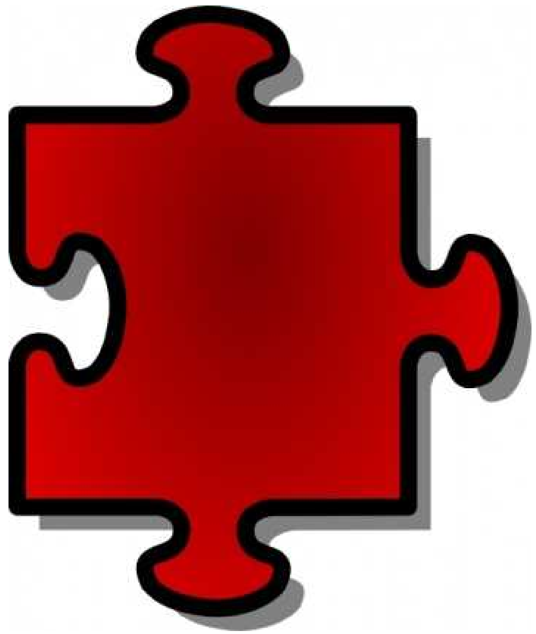
- accurate shapes at high p_T
- normalization accurate at NLO
- reduced scale dependence

realistic & precise predictions



POWHEG

MC@NLO

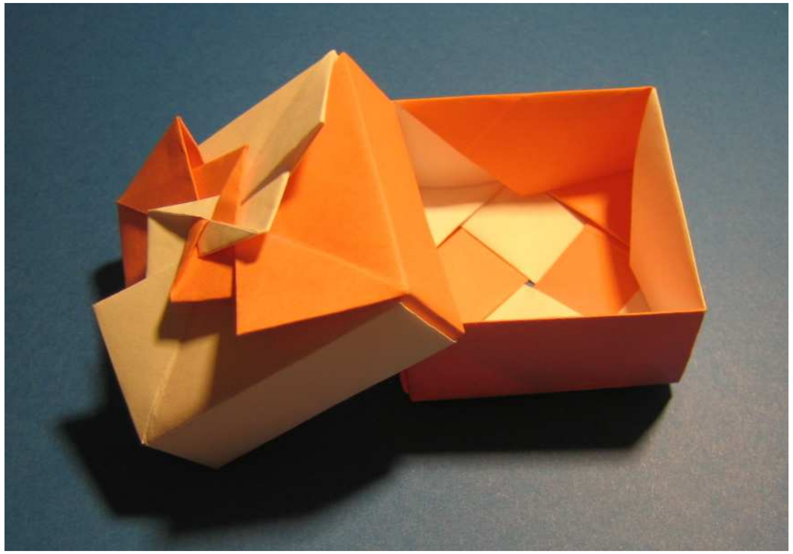


realistic & precise predictions

general presciption for matching
parton-level NLO-QCD calculation
with parton-shower programs

[Frixione, Nason, Oleari]

POWHEG



a public multi-purpose tool
for “do-it-yourself” implementations:

the `POWHEG-BOX`

`http://powhegbox.mib.infn.it/`

[Alioli, Nason, Oleari, Re]

parton showers & NLO-QCD: the `POWHEG` method

POsitive Weight Hardest Emission Generator

general prescription for matching parton-level NLO-QCD
calculations with parton shower programs

[Frixione, Nason, Oleari]

- generate partonic event with single emission at NLO-QCD
- all subsequent radiation must be softer than the first one
- event is written on a file in standard Les Houches format

  $\longrightarrow$ can be processed by default
  parton shower program
  (`HERWIG`, `PYTHIA`, ...)

parton showers & NLO-QCD: the POWHEG method

POsitive Weight Hardest Emission Generator

general prescription for matching parton-level NLO-QCD calculations with parton shower programs

[Frixione, Nason, Oleari]

- applicable to any p_T-ordered parton shower program
- no double counting of real-emission contributions
- produces events with positive weights
- tools for “do-it-yourself” implementation publicly available (the `POWHEG-BOX`)

[Alioli, Nason, Oleari, Re]

NLO cross sections

reminder: differential NLO cross section

Born

real emission and counter-terms

$$d\sigma_{\mathrm{NLO}} = d\Phi_n \left\{ B(\Phi_n) + V(\Phi_n) + \Big[R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \Big] d\Phi_r \right\}$$

finite virtuals:

$$V_b(\Phi_n) + \int d\phi_r \, C(\Phi_n, \Phi_r)$$

radiation phase space:

$$d\Phi_r = dt dz d\phi$$

shower Monte Carlo cross sections

leading order shower Monte Carlo cross section

Born

first emission (governed by splitting function P)

$$d\sigma_{\mathrm{LO-SMC}} = d\Phi_n B(\Phi_n) \left\{ \Delta_{t_0} + \Delta_t \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} d\Phi_r \right\}$$

Sudakov factor:

$$\Delta_t = \exp\left[- \int d\Phi'_r \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \theta(t' - t) \right]$$

…probability for no emission at scale $t' > t$

POWHEG cross sections

$$\overline{B} \;=\; \left\{ B(\Phi_n) + V(\Phi_n) + \int d\Phi_r \Big[R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \Big] \right\}$$

$$d\sigma_{\mathrm{POWHEG}} \;=\; d\Phi_n \overline{B}(\Phi_n) \left\{ \Delta(\Phi_n, p_T^{\min}) + \Delta(\Phi_n, p_T) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n, \Phi_r)} d\Phi_r \right\}$$

POWHEG "Sudakov" factor:

$$\Delta(\Phi_n, p_T) \;=\; \exp\left[-\int d\Phi'_r \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta\left(k_T(\Phi_n, \Phi'_r) - p_T \right) \right]$$

the POWHEG cross section

$$d\sigma_{\mathrm{NLO}} = d\Phi_n \left\{ B(\Phi_n) + V(\Phi_n) + \Big[R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \Big] d\Phi_r \right\}$$

$$d\sigma_{\mathrm{LO-SMC}} = d\Phi_n B(\Phi_n) \left\{ \Delta_{t_0} + \Delta_t \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \, d\Phi_r \right\}$$

$$d\sigma_{\mathrm{POWHEG}} = d\Phi_n \overline{B}(\Phi_n) \left\{ \Delta(\Phi_n, p_T^{\mathrm{min}}) + \Delta(\Phi_n, p_T) \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n, \Phi_r)} \, d\Phi_r \right\}$$

parton showers & NLO-QCD: the POWHEG-BOX

up-to-date info on the POWHEG-BOX and code download:

`http://powhegbox.mib.infn.it/`

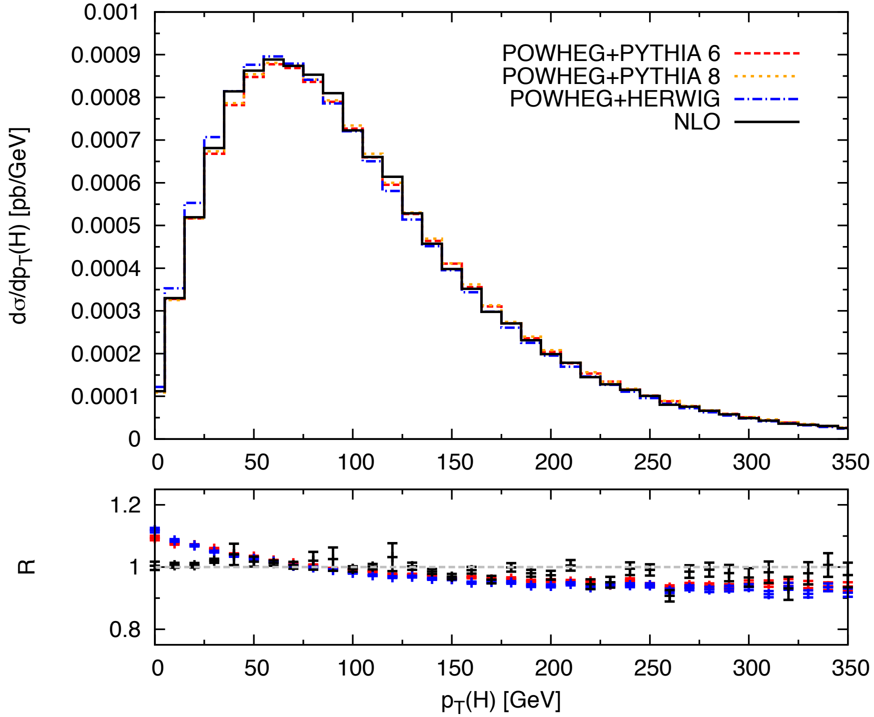
✘ user has to supply process-specific quantities:

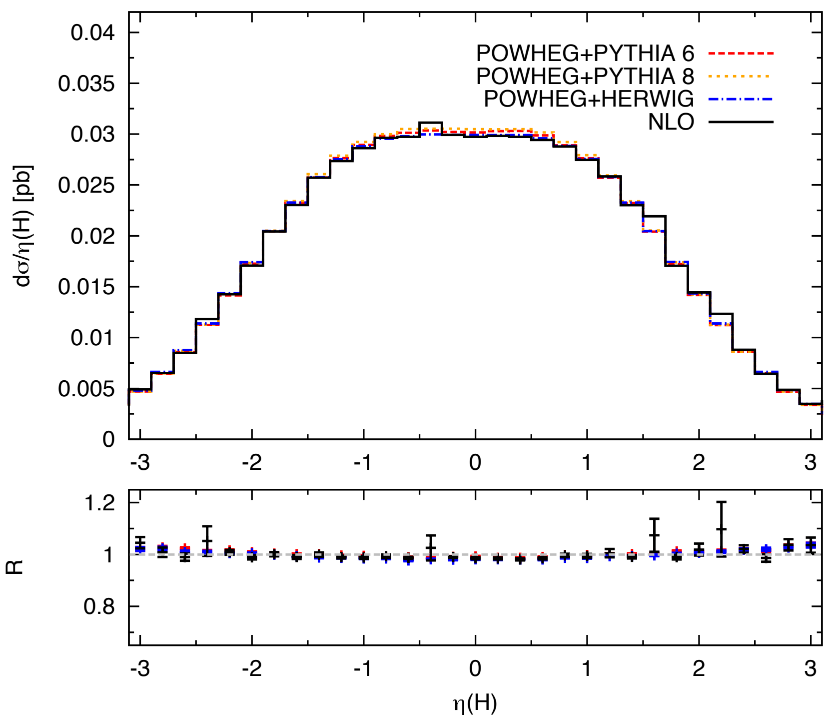
- lists of flavor structures for Born and real emission processes
- Born phase space
- Born amplitudes squared, color-and spin-correlated amplitudes
- real-emission amplitudes squared
- finite part of the virtual corrections
- Born color structure in the limit of a large number of colors

✔ all general, process-independent aspects of the matching are provided by the POWHEG-BOX

$pp \to t\bar{t}H$: NLO-QCD and parton-shower effects

Hartanto et al. (2015)





transverse-momentum distributions shifted to slightly smaller values

little impact on rapidity distributions

NNLO QCD and parton showers

first steps toward matching of NNLO QCD calculations
with parton shower programs:

- ✔ realistic exclusive description of specific final state
- ✔ multi-parton interactions, hadronization, underlying event
- ✔ best possible perturbative accuracy of hard interaction
- ✔ proper modeling of jets (e.g. sub-structure)

☞ immediate impact on LHC physics program

(Higgs, EW precision measurements, . . .)

NNLO QCD and parton showers

first steps toward matching of NNLO QCD calculations with parton shower programs:

- POWHEG+MINLO

  $pp \to H$, HW, Drell-Yan *[Zanderighi et al. (2013-16)]*

- UNNLOPS

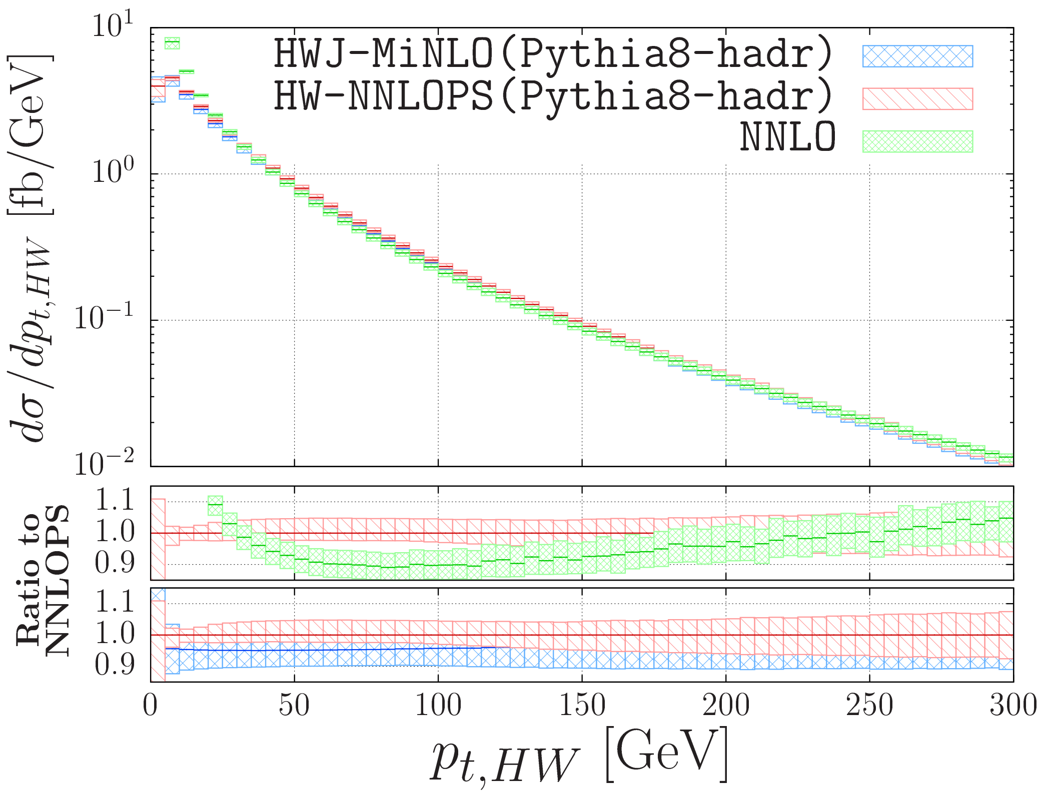
  $pp \to H$, Drell-Yan *[Höche, Li, Prestel (2014)]*

- GENEVA

  Drell-Yan *[Alioli et al. (2014)]*

NNLO QCD and parton showers

Astill et al. (2016)



- scale uncertainties reduced from about 10% to 2%
- agreement with NNLO results for inclusive lepton observables
- jet distributions sensitive to parton-shower effects
- NNLO+PS tool more flexible than pure NNLO calculation

NNLO+PS accurate description of $pp \to HW$ using the `POWHEG+MINLO` approach

EW corrections: why worry?

- LHC-2 is operating at 13 TeV

  $\longrightarrow$ reach energy range (more) sensitive to EW effects;
  EW corrections ($\delta_{\mathbf{EW}}$) can reach some 10%

- integrated LHC luminosity will reach several 100 fb^{-1}

  $\longrightarrow$ many measurements at few-percent level
  (= typical size of EW corrections)

- planned high-precision measurements:

  EW parameters, (anomalous) couplings,...
  $\longrightarrow$ $\delta_{\mathbf{EW}}$ is crucial ingredient

EW corrections: generic features

naive expectation:

$\alpha \sim \alpha_s^2 \rightarrow$ NLO EW $\sim$ NNLO QCD ?

but: systematic enhancements possible, e.g.:

- kinematic effects
- photon emission $\longrightarrow$ mass-singular logs, e.g. $\frac{\alpha}{\pi} \ln\left(\frac{Q}{m_\mu}\right)$
- high energies $\longrightarrow$ EW Sudakov logs, e.g. $\frac{\alpha}{\pi} \ln^2\left(\frac{Q}{M_W}\right)$

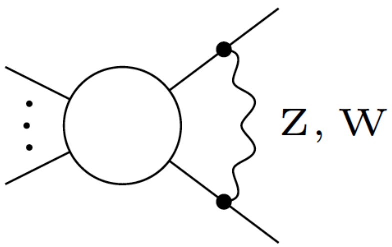
EW corrections: Sudakov logarithms

typical $\mathbf{2} \to \mathbf{2}$ process: at high energy
EW corrections enhanced by large logs

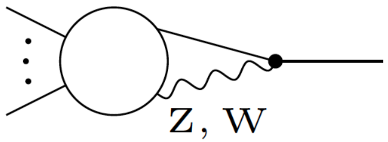
$$\ln^2\left(\frac{Q^2}{M_W^2}\right) \sim \mathbf{25}$$ @ energy scale of 1 TeV

universal origin of leading EW logs:

mass singularities in virtual corrections related to external lines



soft and collinear virtual gauge bosons: $\to$ double logs



soft or collinear virtual gauge bosons: $\to$ single logs

EW corrections: Sudakov logarithms

compare to QED / QCD:
IR singularities of virtuals canceled
by real-emission contributions

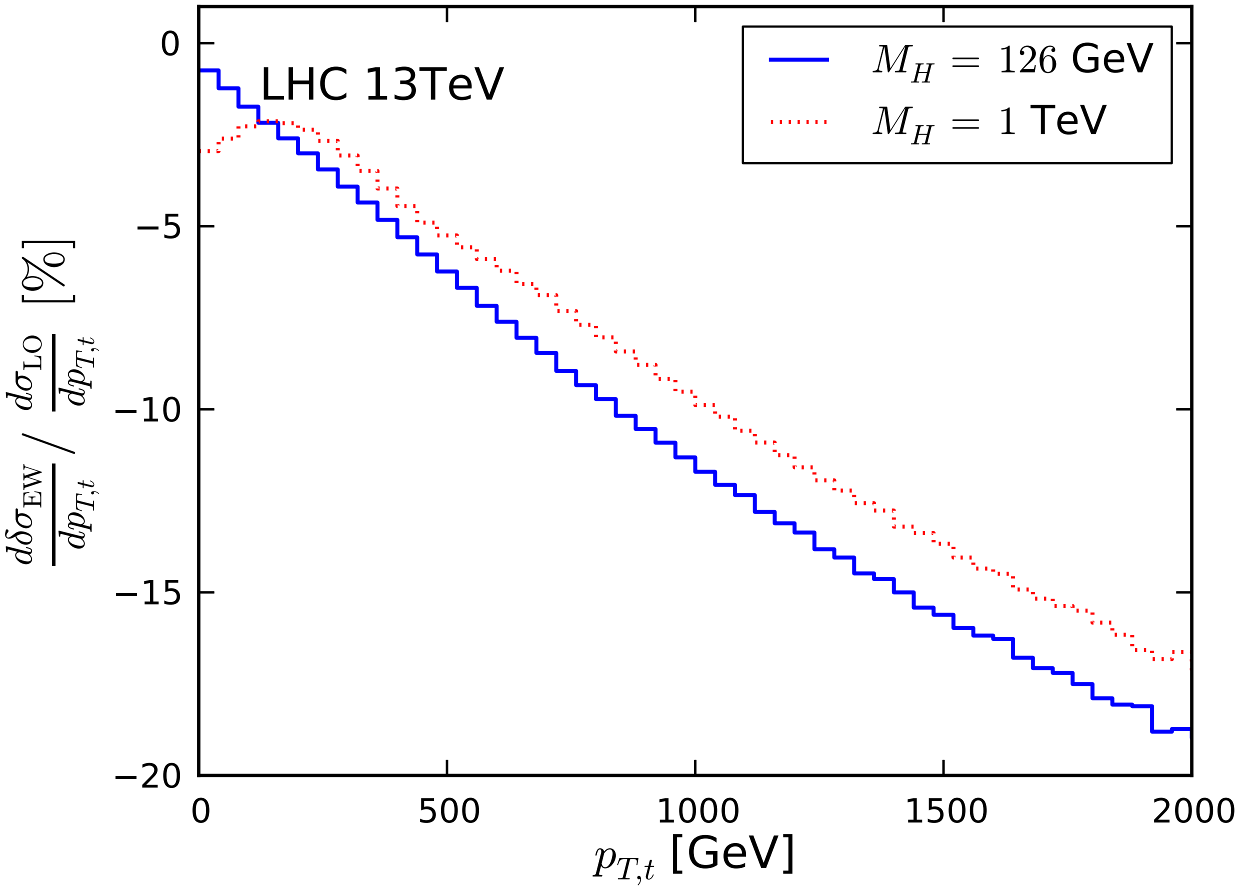
electroweak bosons massive
$\longrightarrow$ real radiation experimentally distinguishable

non-Abelian charges of W, Z are open
$\longrightarrow$ Bloch-Nordsieck theorem not applicable

M. Ciafaloni, P. Ciafaloni, Comelli; Beenakker, Werthenbach; Denner, Pozzorini; Kühn et al., Baur; . . .

impact of EW Sudakov logarithms

Kühr, Scharf, Uwer (2013)



$pp \to t\bar{t}$ at 13 TeV:

tails of distributions receive large corrections!

EW effects in PDFs

consistent calculation at NLO EW requires PDFs including
$\mathcal{O}(\alpha)$ corrections and new photon PDF

MRST2004QED: first PDF set with $\mathcal{O}(\alpha)$ corrections

NNPDF2.3QED (2013): NNPDF set with $\mathcal{O}(\alpha)$ corrections

- 2013: best PDF prediction at (N)NLO QCD + NLO QED
- PDF samples for error estimate provided
- photon PDF fitted to DIS and Drell-Yan data $(10^{-5} \lesssim x \lesssim 10^{-1})$
  (note lack of experimental information for large x)
- being updated; currently: NNPDF3.0QED

progress in NLO EW calculations

- NLO EW often more demanding than NLO QCD calculations

  (richer resonance structure, more mass scales, . . .)

- most NLO EW results available based on dedicated calculations

  ($pp \to V$, Vj, HV, VV, 4 leptons, dijets, VBF, . . .)

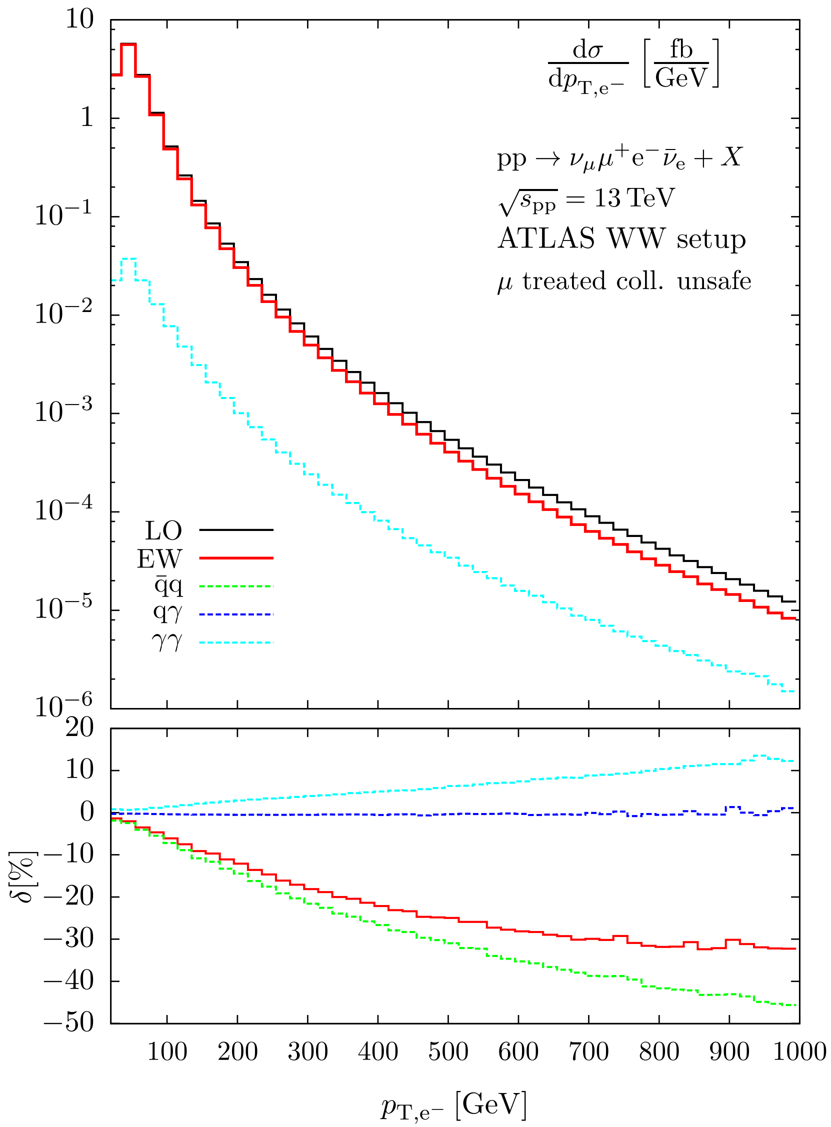
- automated tools start to play a more important role:

  `Recola, OpenLoops, MadGraph5_aMC@NLO`

  ($pp \to Vjj$, 4 leptons, $t\bar{t}V$, . . .)

$pp \to WW \to 4f$: full NLO EW calculation

Biedermann et al. (05/2016)



flexible Monte-Carlo approach gives full control on lepton distributions and correlations with realistic selection cuts:

EW corrections small for total XS, but large and negative at high scales

note: based on two independent calculations (`Recola` vs. dedicated standalone calculation)

combination of QCD and EW corrections

current experimental precision requires combination of NLO EW corrections with best QCD prediction

how to combine?
factorized or additive approach?

$$\left(\mathbf{1}+\boldsymbol{\delta}^{\mathbf{QCD}}\right)\times\left(\mathbf{1}+\boldsymbol{\delta}^{\mathbf{EW}}\right)$$

versus

$$\left(\mathbf{1}+\boldsymbol{\delta}^{\mathbf{QCD}}+\boldsymbol{\delta}^{\mathbf{EW}}\right)$$

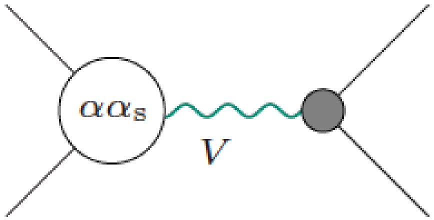
can only be resolved by computing

mixed QCD-EW corrections $\mathcal{O}(\boldsymbol{\delta}^{\mathbf{QCD}}\boldsymbol{\delta}^{\mathbf{EW}})$

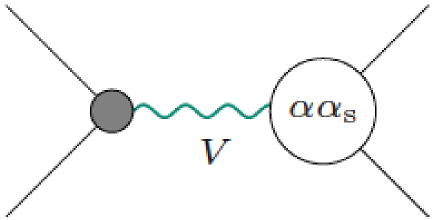
Drell-Yan: mixed QCD$\times$EW corrections

Dittmaier, Huss, Schwinn (2014-16):

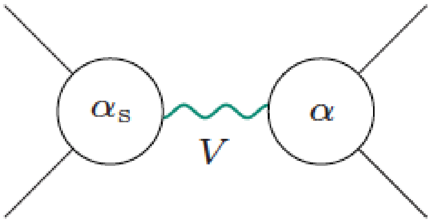
Factorizable contributions: (only virtual contributions indicated)



- no significant resonance distortion expected
- no PDFs with $\mathcal{O}(\alpha\alpha_s)$ corrections

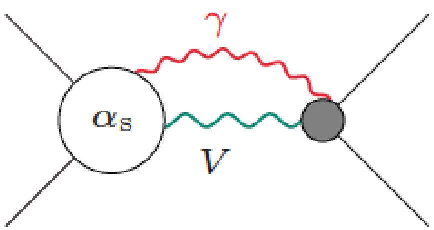


- only $Vl\bar{l}'$ counterterm contributions
  $\hookrightarrow$ uniform rescaling, no distortions



- significant resonance distortions from FSR
- calculated, preliminary results

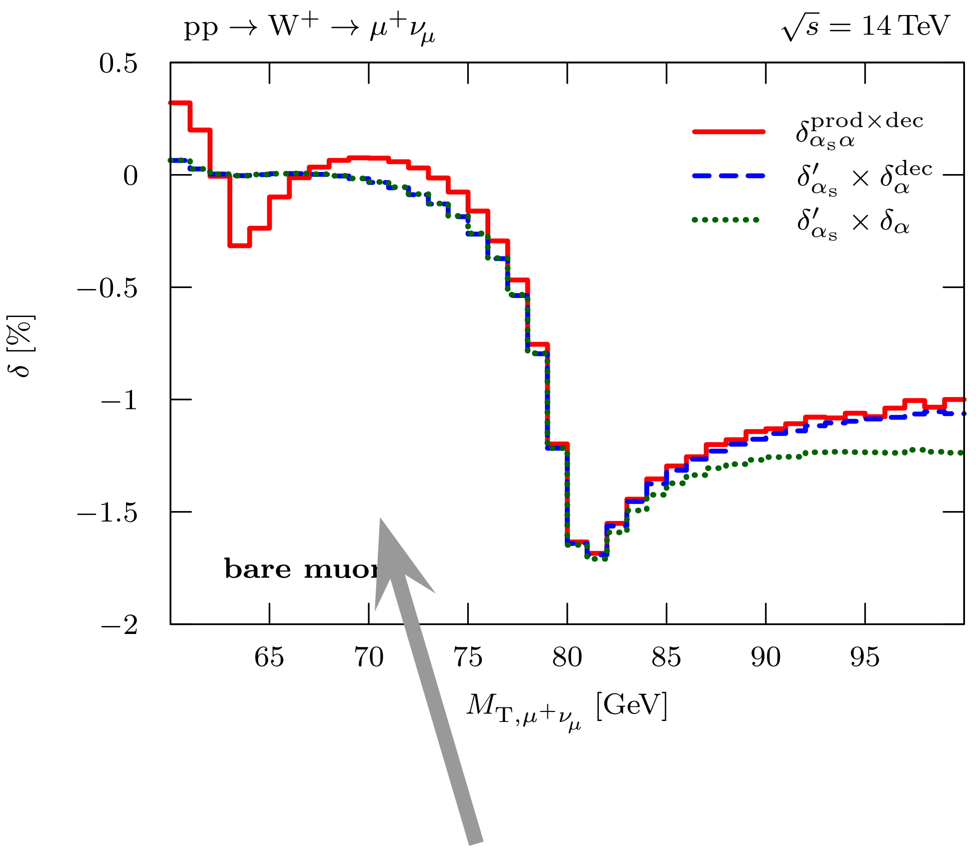
Non-factorizable contributions: (only virtual contributions indicated)

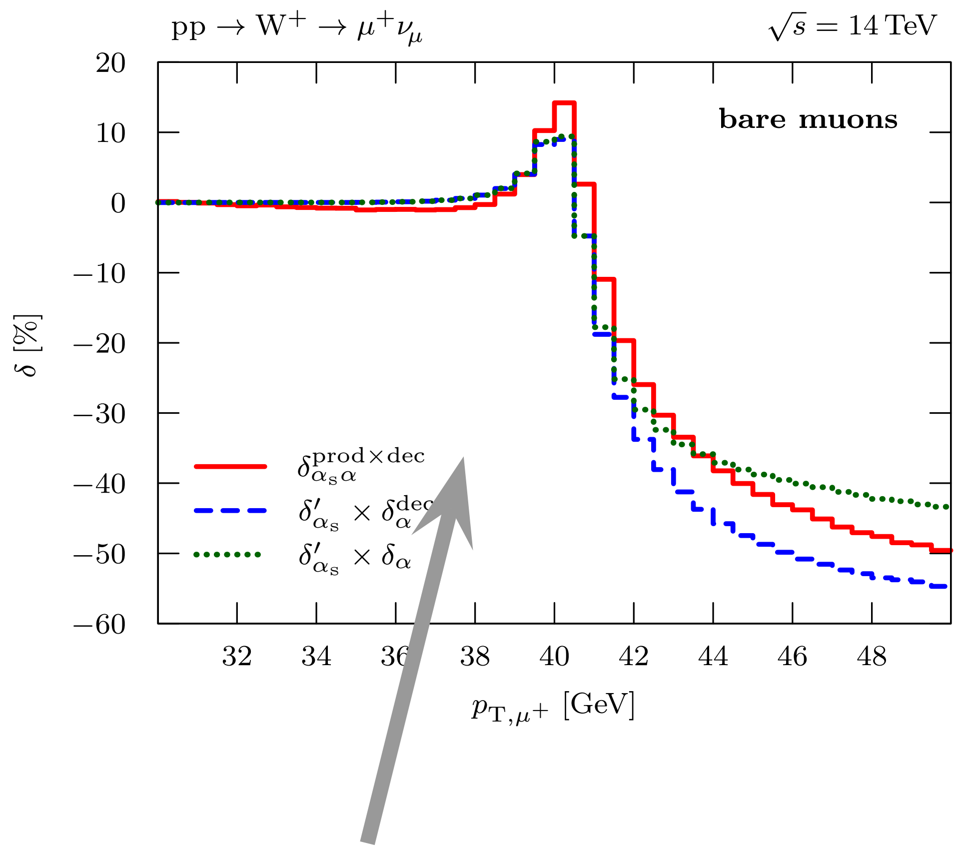


- could induce shape distortions
- calculated, turn out to be small

Drell-Yan: mixed QCD×EW corrections

Dittmaier, Huss, Schwinn (2014-16):



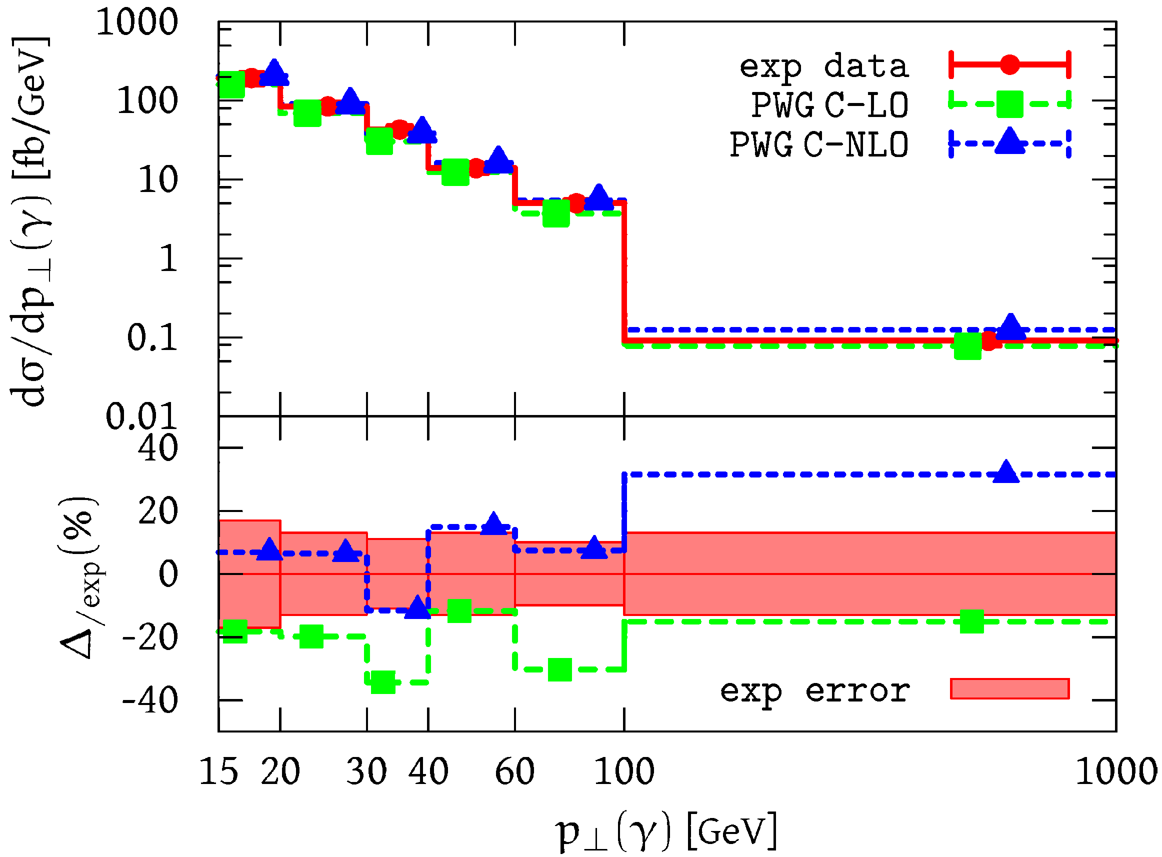


naive factorization QCD×EW works

naive factorization poor for $p_{T,\mu} > M_W/2$

NLO QED and NLO QCD with parton showers

[Barzè et al. (2014)]



QED and QCD corrections can be combined and matched consistently with parton shower using the POWHEG framework

first implementation: $pp \to W\gamma$

the SM and precision calculations: summary

- guiding principle of modern particle physics: local gauge theories
- cornerstone of our understanding:
  electroweak symmetry breaking $\leftrightarrow$ Higgs mechanism
- tool of choice for better understanding: (hadron) colliders
- interpretation of experimental results requires precise theoretical predictions beyond LO in perturbation theory:
  - consider (N)NLO QCD and NLO EW corrections
  - match precision calculations to parton-shower programs
- status of theory predictions advanced, several public tools available