Flavor Physics Theory

Lecture at the DESY Summer Student Programme 2016

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Motivation

In general Flavor is the multiplicity of the same gauge representation

In the Standard Model we observe three generations = 3 flavors in both quark and lepton sectors, which leads to all the rich structure in particles that we see in our colliders: most parameters of the SM are from flavor sector

Therefore flavor is an essential ingredient of the SM, and is crucial to understand its success

Motivation

Besides, Flavor serves as a good motivation for New Physics beyond the SM, but also strongly constraints it

This was used already when the “New Physics” was the nowadays SM: in 1970 Glashow, Iliopoulos and Maiani used flavor physics to predict not only the existence of the charm quark, but also its mass

Motivation

Unfortunately in these days we do not have such strong hints for New Physics from flavor, but rather general ones, related to e.g. the strong hierarchies in the flavor parameters that we observe ("The SM Flavor Puzzle"), which can be addressed in SM extensions

Such SM extensions, and in general all extensions with a generic flavor structure, are strongly constrained by precision observables, up to 10^5 TeV. This implies that all NP models around the TeV scale must have a highly non-generic flavor structure: "The NP Flavor Problem"

Outline

Lecture I: The flavor stucture of the Standard Model

- The SM in quite some detail
- Counting parameters
- CP Violation
- The CKM Matrix

Outline

Lecture II: Testing the flavor stucture of the SM

- Experimental Status of the CKM fit
- Meson Mixing
- CP Violating Observables

Part I

The flavour structure of the Standard Model

The Standard Model

Most general Lagrangian
invariant under gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

and field content

$$\text{Quarks}\begin{cases} Q_{Li} = (3,2)_{1/6} \\ U_{Ri} = (3,1)_{2/3} \\ D_{Ri} = (3,1)_{-1/3} \end{cases}$$

$$\text{Leptons}\begin{cases} L_{Li} = (1,2)_{-1/2} \\ E_{Ri} = (1,1)_{-1} \end{cases}$$

$$\text{Higgs}\ \{\ \phi = (1,2)_{1/2}$$

Chirality: **Left-handed** (LH) Dirac Spinor

Gauge representation: SU(3) triplet, SU(2) doublet, hypercharge 1/6

$$Q_{Li} = \begin{pmatrix} U_{Li} \\ D_{Li} \end{pmatrix} = (3,2)_{1/6}$$

Generation index i=1,2,3: three copies of field with same gauge quantum numbers = FLAVOR index

Note that SM is **chiral** theory: LH and RH fields in different gauge representations: forbids explicit mass terms

$$\boxed{\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}}$$

kinetic terms (includes gauge boson couplings)

Higgs self-couplings

fermion-fermion-Higgs couplings

$$\mathcal{L}_{\text{kinetic}} = \sum_{F_i} i\overline{F}_i \gamma_\mu D^\mu F_i + \left(D_\mu \phi\right)^\dagger D^\mu \phi + \mathcal{L}_{\text{gauge-kinetic}}$$

$$-\mathcal{L}_{\text{Higgs}} = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$$-\mathcal{L}_{\text{Yukawa}} = Y^e_{ij} \overline{L}_{Li} E_{Rj} \phi + Y^d_{ij} \overline{Q}_{Li} D_{Rj} \phi + Y^u_{ij} \overline{Q}_{Li} U_{Rj} \phi^* + \text{h.c.}$$

$$\mathcal{L}_{\text{kinetic,ferm}} = \sum i\overline{F}_i \gamma_\mu D^\mu F_i = i\overline{Q}_{Li}\gamma_\mu D^\mu Q_{Li} + \ldots$$

$$F_i = Q_{Li}, U_{Ri}, D_{Ri}, L_{Li}, E_{Ri}$$

- Have to use appropriate covariant derivative depending on fermion gauge representation

$$D^\mu = \partial^\mu + ig_s G^\mu_A \underbrace{T_A(F)}_{T_A(\mathbf{1})=0\ \ T_A(\mathbf{3})=\lambda_A/2} + igW^\mu_a \underbrace{T_a(F)}_{T_a(\mathbf{1})=0\ \ T_a(\mathbf{2})=\sigma_a/2} + ig' B^\mu Y(F)$$

[3 parameters]

- Can always choose to have **flavor-universal** form

$$a_{ij}\left[i\overline{Q}_{Li}\gamma_\mu D^\mu Q_{Lj}\right] \xrightarrow[\text{re-definitions}]{\text{field}} \delta_{ij}\left[i\overline{Q}_{Li}\gamma_\mu D^\mu Q_{Lj}\right]$$

kinetic terms have large **global flavor symmetry**:

invariant under $F \to V_F F,\ V_F \in U(3)$ ➡ $U(3)^5$

[kinetic terms do not distinguish different flavors]

$$-\mathcal{L}_{\text{Higgs}} = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$$

[2 parameters]

- When $\mu^2 > 0$ (and $\lambda > 0$) Higgs acquires vacuum expectation value

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$$

that leads to electroweak symmetry breaking (EWSB)

$$SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$$

- 3 Goldstone Bosons are eaten by W,Z, which get masses from Higgs kinetic terms; one real scalar is left over (Higgs boson h), which couples according to replacement

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h\end{pmatrix}$$

$$-\mathcal{L}_{\mathrm{Yukawa}} = Y^e_{ij}\overline{L}_{Li}E_{Rj}\phi + Y^d_{ij}\overline{Q}_{Li}D_{Rj}\phi + Y^u_{ij}\overline{Q}_{Li}U_{Rj}\phi^* + \mathrm{h.c.}$$

- SU(2) contractions are defined as

$$\overline{L}_{Li}\phi \equiv \left(\overline{L}_{Li}\right)_a \phi_a \qquad \overline{Q}_{Li}\phi \equiv \left(\overline{Q}_{Li}\right)_a \phi_a \qquad \overline{Q}_{Li}\phi^* \equiv \epsilon_{ab}\left(\overline{Q}_{Li}\right)_a \phi^*_b$$

$$Q_{Li} = \begin{pmatrix} U_{Li} \\ D_{Li} \end{pmatrix} \qquad L_{Li} = \begin{pmatrix} \nu_{Li} \\ E_{Li} \end{pmatrix}$$

- Yukawa couplings are 3x3 complex matrices that break global symmetries of fermion kinetic terms:

$$V^\dagger_{Q_L} Y^u V_{U_R} \neq Y^u$$

Yukawas allow distinguish different flavors:

Flavor physics is physics of U(3)^5 breaking

Counting Parameters

- Can use $U(3)^5$ rotations to eliminate parameters in Yukawa matrices: many parameters are not physical, because they can be absorbed by field re-definitions; <u>how do we count number of physical parameters?</u>

- Analogous to H-atom in ext. magnetic field: when B=0 have SO(3) symmetry, but with non-zero B SO(3) broken; B described by 3 parameters, but can use SO(3) rotations to eliminate parameters: rotate B in z-direction

$$B = (B_x, B_y, B_Z) \xrightarrow[\text{rotations}]{\text{SO(3)}} B = (0, 0, B_Z)$$

Although SO(3) has 3 parameters, cannot use to eliminate 3 parameters in B! Reason is that SO(2) subgroup is unbroken (rotations in orthogonal plane leave B invariant)

# of parameters that can be absorbed is $N_{\text{global}} - N_{\text{unbroken}}$

$$Y^u \to V_{Q_L}^\dagger Y^u V_{U_R} \quad Y^d \to V_{Q_L}^\dagger Y^d V_{D_R} \quad Y^e \to V_{L_L}^\dagger Y^e V_{E_R}$$

- Can choose convenient basis in flavor space with minimal number of physical parameters; count carefully, because some subgroup of $U(3)^5$ rotations leaves Yukawas invariant: this subgroup is global symmetry of full $\mathcal{L}_{\mathrm{SM}}$

Show below: unbroken symmetries are
Baryon number and individual Lepton number

$$U(3)^5 \supset G_{\text{unbroken}} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$

- Need number of parameters in $U(3)^5$: For U(3) have 9 real parameters + 9 phases minus unitarity conditions, 6 for real and 3 for phases = 3 real + 6 phases

[U(N): n (n-1)/2 real + n (n+1)/2 phases]

N_{global} = 5 (3 real + 6 phases) = 15 real + 30 phases

N_{unbroken} = 4 phases

field re-definitions can absorb $N_{\text{global}} - N_{\text{unbroken}}$ = 15 real + 26 phases

Yukawas are 3 complex 3x3 matrices = 27 real + 27 phases

# of physical parameters in Yukawa sector = 12 real + 1 phase

- lepton sector alone: [9-6] real + [9- (12-3)] phases = 3 real
- can choose to parametrize by e.g.

$$Y^e \to \underbrace{Y^e_{\text{diag}}}_{\text{3 real}} \qquad Y^u \to \underbrace{Y^u_{\text{diag}}}_{\text{3 real}} \qquad Y^d \to \overbrace{V_{\text{CKM}}}^{\text{unitary matrix with 3 real + 1 phase}} \underbrace{Y^d_{\text{diag}}}_{\text{3 real}}$$

- one physical phase in quark sector: implies **CP violation**

CP-Violation

- On field level C and P flip chirality: “maximally” violated in SM since LH and RH fields in different gauge representation

e.g. $$\underset{\text{SU(2) doublet}}{E_L} \stackrel{\mathrm{P}}{\rightarrow} \eta\gamma^0 \underset{\text{SU(2) singlet}}{E_R}$$

- Instead combined CP-transformation compatible with gauge symmetry: always symmetry of $\mathcal{L}_{\text{kinetic}}$

- CP is symmetry of full Lagrangian if all couplings real

e.g. $$Y^u_{ij}\overline{Q}_{Li}U_{Rj}\phi^* + \text{h.c.} \stackrel{\mathrm{CP}}{\rightarrow} Y^u_{ij}\overline{U}_{Rj}Q_{Li}\phi + \text{h.c.} = \left(Y^u_{ij}\right)^* \overline{Q}_{Li}U_{Rj}\phi^* + \text{h.c.}$$

- To prove CP violation (CPV) in general more subtle: have to show that no flavor basis exists where all couplings real; but we have just demonstrated that in quark sector there is always one physical phase: There is CPV in quark sector

- Note that it's the simultaneous presence of up and down sector that gives CPV: if just up or down, same as lepton sector: no CPV

- Also crucial is fact that we have 3 generations: if we would have only 2 generations no CPV. Led to prediction of 3rd generation in 1973 by Kobayashi and Maskawa (Nobel Prize 2008)

  Ex.: show this

- In SM CPV related to Yukawa couplings -> flavor sector; Higgs sector respects CP since there is only one scalar doublet; in models with more Higgses can have CPV also in Higgs sector, e.g. in SUSY

Flavor after EWSB

- After EWSB have to rewrite fermion kinetic terms in gauge boson mass eigenstates W^+, W^-, Z, A: get for gauge currents

$$-\mathcal{L}_{\text{current}} = \frac{g}{c_W} Z_\mu j^\mu_{\text{N}} + e A_\mu j^\mu_{\text{EM}} + \frac{g}{\sqrt{2}} \left(W^+_\mu j^\mu_+ + \text{h.c.} \right)$$

$$\left.\begin{aligned} j^\mu_{\text{N}} &= \sum_{F_i} \overline{F}_i \gamma^\mu \left(T^3_F - s^2_W Q_F \right) F_i \\ j^\mu_{\text{EM}} &= \sum_{F_i} \overline{F}_i \gamma^\mu Q_F F_i \end{aligned}\right\} \text{respect global } U(3)^7$$

$$\left. j^\mu_+ = \overline{U}_{Li} \gamma^\mu D_{Li} + \overline{\nu}_{Li} \gamma^\mu E_{Li} \right\} \text{break global } U(3)^7$$

$$F_i = \{U_L, D_L, U_R, D_R, E_L, E_R, \nu_L\} \qquad T^3_{\mathbf{2}} = \pm\frac{1}{2}, T^3_{\mathbf{1}} = 0, Q_F = T^3_F + Y_F$$

- Also have to decompose SU(2) structure in Yukawas

$$-\mathcal{L}_{\text{Yukawa}} = \left(Y^e_{ij}\overline{E}_{Li}E_{Rj} + Y^d_{ij}\overline{D}_{Li}D_{Rj} + Y^u_{ij}\overline{U}_{Li}U_{Rj}\right)\frac{v+h}{\sqrt{2}} + \text{h.c.}$$

- Use broken $U(3)^7$ rotations to absorb unphysical parameters in Yukawa couplings: can diagonalize all three matrices by bi-unitary transformations ("Singular Value Decomposition")

$$E_R \to V_R^E E_R \equiv e_R \qquad E_L \to V_L^E E_L \equiv e_L$$

$$V_L^E\, Y^E \left(V_R^E\right)^\dagger = Y^E_{\text{diag}} \qquad \text{etc...}$$

- Defines mass eigenstates ("mass basis")

$$-\mathcal{L}_{\text{mass}} = m_i^e \overline{e}_{Li}e_{Ri} + m_i^d \overline{d}_{Li}d_{Ri} + m_i^u \overline{u}_{Li}u_{Ri} + \text{h.c.}$$

$$m_i^e = v/\sqrt{2}\left(Y^E_{\text{diag}}\right)_i > 0 \qquad i = \{1,2,3\} \to \{e,\mu,\tau\} \qquad \text{etc...}$$

- Note that still have freedom to rotate without changing masses:

$$e_R \to P_e e_R, \quad e_L \to P_e e_L \qquad P_e = \begin{pmatrix} e^{i\phi_e} & & \\ & e^{i\phi_\mu} & \\ & & e^{i\phi_\tau} \end{pmatrix}$$

$+\ P_u, P_d$

and also full U(3) rotations for massless neutrinos $\nu_L \to V_L^\nu \nu_L$

- In mass basis also Higgs couplings are flavor-diagonal and neutral and electromagnetic currents remain flavor-universal

$$\mathcal{L}_h = h\frac{m_i^e}{v}\bar{e}_{Li}e_{Ri} + \ldots \qquad j^\mu_{\rm N}, j^\mu_{\rm EM} \sim \delta_{ij}\overline{f}_i\gamma^\mu f_j$$

Higgs couplings aligned to masses

Identity matrix in flavor space

same field: uu,dd,..

- Note true for charged currents: identity matrix in flavor space, but connect different SU(2) components

$$
\begin{aligned}
j_+^\mu &= \overline{U}_L\, \mathbb{1}_{3\times 3}\, \gamma^\mu D_L + \overline{\nu}_L\, \mathbb{1}_{3\times 3}\, \gamma^\mu E_L \\
&= \overline{u}_L \left(V_L^U\right)^\dagger V_L^D \gamma^\mu d_L + \overline{\nu}_L V_L^E \gamma^\mu e_L
\end{aligned}
$$

- Can use remaining neutrino rotations to make lepton current universal, but quark currents are non-diagonal: CKM matrix

$$
\nu_L \to V_L^E \nu_L \qquad\qquad V_{\mathrm{CKM}} \equiv \left(V_L^U\right)^\dagger V_L^D
$$

Final form charged currents:

$$
-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[\left(\overline{u}_{Li} V_{ij} \gamma^\mu d_{Lj} + \overline{\nu}_{Li}\gamma^\mu e_{Li}\right) W_\mu^+ + \left(\overline{d}_{Li} V_{ji}^* \gamma^\mu e_{Lj} + \overline{e}_{Li}\gamma^\mu \nu_{Li}\right) W_\mu^-\right]
$$

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[\overline{u}_L V_{\mathrm{CKM}} \gamma^\mu d_L + \overline{\nu}_L \gamma^\mu e_L \right] W_\mu^+ + \mathrm{h.c.}$$

- This is only term that partially breaks left-over re-phasing; still invariant if common universal phase in u and d (Baryon number) and common diagonal phase in e, ν (Lepton number)

$$e_R \to P_e e_R, \quad e_L \to P_e e_L \qquad \nu_L \to P_e \nu_L \qquad P_e = \begin{pmatrix} e^{i\phi_e} & & \\ & e^{i\phi_\mu} & \\ & & e^{i\phi_\tau} \end{pmatrix}$$

$$\boxed{U(1)_e \times U(1)_\mu \times U(1)_\tau}$$

$$u_R \to P_B u_R, \quad u_L \to P_B u_L \qquad d_R \to P_B d_R, \quad d_L \to P_B d_L \qquad P_B = e^{i\phi_B} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\boxed{U(1)_B}$$

The CKM Matrix

$$V_{\rm CKM} \equiv \left(V_L^U\right)^\dagger V_L^D \qquad\qquad \frac{g}{\sqrt{2}}\overline{u}_L V_{\rm CKM}\gamma^\mu d_L + \text{h.c.}$$

- Unitary matrix: in general 3 real + 6 phases; can use broken phase transformations to absorb 5 phases (6 - unbroken B)

$$\begin{array}{lll} u_R \to P_u u_R, & u_L \to P_u u_L & P_u = \text{diag}\left(e^{i\phi_u}, e^{i\phi_c}, e^{i\phi_t}\right) \\ d_R \to P_d d_R, & d_L \to P_d d_L & P_d = \text{diag}\left(e^{i\phi_d}, e^{i\phi_s}, e^{i\phi_b}\right) \end{array}$$

$$V_{\rm CKM} \to P_u^\dagger V_{\rm CKM} P_d$$

Physical observables must be independent of this re-phasing!

e.g. Jarlskog invariant: $J = \text{Im}\left(V_{ud}V_{cs}V_{us}^*V_{cd}^*\right)$

CKM Parametrizations

$$(V_{\mathrm{CKM}})_{ij} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

PDG parametrization $(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$

CPV

$$V_{\mathrm{CKM}} = R_{23} P_{33} R_{13} P_{33}^{\dagger} R_{12}$$

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad P_{33} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}$$

$$c_{12} \equiv \cos\theta_{12} \qquad s_{12} \equiv \sin\theta_{12}$$

Wolfenstein parametrization (λ, A, ρ, η)

CPV

$$V_{\mathrm{CKM}} \approx \begin{pmatrix} 1 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda = |V_{us}| \approx 0.2$

Cabibbo angle

Unitarity Triangles

Can obtain geometric interpretation of
CKM elements from unitarity relations

$$\sum_i V_{ia} V_{ib}^* = 0 \qquad a \neq b$$

e.g. $$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

sum of three C-numbers = 0 $\leftrightarrow$ closed vector sum in complex plane $\leftrightarrow$ triangle in complex plane

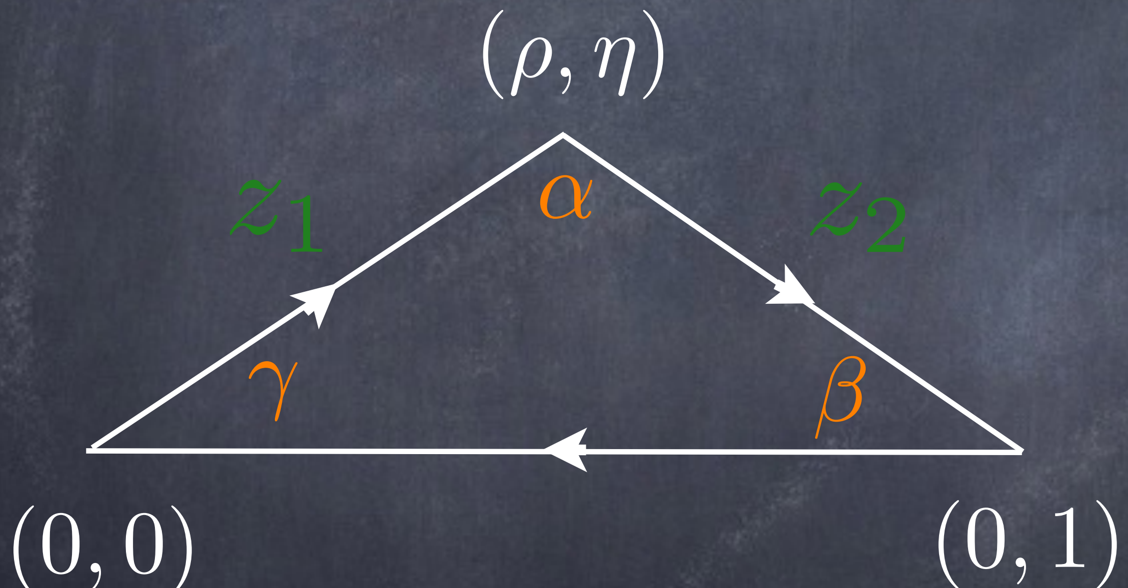
only one triangle with sides of comparable length:
"The Unitarity Triangle"

The Unitarity Triangle

$$1 + \underbrace{\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}}_{-z_1} + \underbrace{\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}}_{-z_2} = 0$$

Wolfenstein:

$$z_1 = \rho + i\eta$$



$$\alpha = \arg\left(-\frac{z_2}{z_1}\right) = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta = \arg z_2^* = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \arg z_1 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$R_u \equiv |z_1| = \sqrt{\rho^2 + \eta^2} \qquad R_t \equiv |z_2| = \sqrt{(1-\rho)^2 + \eta^2}$$

Note that if CP conserved have $\eta = 0$ and triangle becomes line

The Flavor Structure of the SM

1. **No Lepton Flavor Violation** (e.g. $\mu \to e\gamma$)

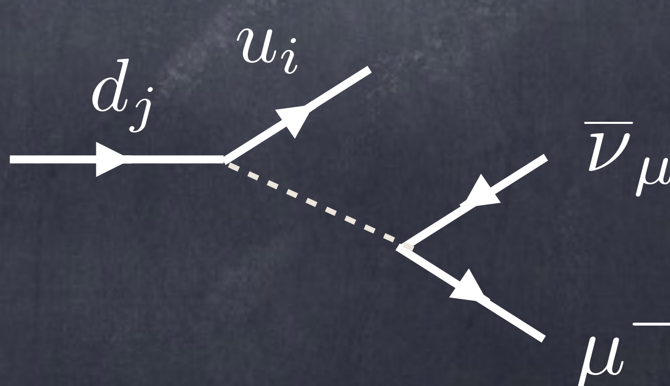
due to conserved $U(1)_e \times U(1)_\mu \times U(1)_\tau$

[still tiny when include neutrino masses]

2. **Quark Flavor Violation only in (LH) charged currents and controlled by hierarchical CKM matrix, parametrised by 3 angles + 1 phase**

$$\mathcal{L}_{\rm FV} = -\frac{g}{\sqrt{2}} V_{ij}\, \overline{u}_{Li} \gamma^\mu d_{Lj} W^+_\mu + \text{h.c.}$$

$\sim V_{ij} V^*_{lk}$



$\sim V_{ij}$

Flavour Changing Charged Currents (FCCC) at tree-level

The Flavor Structure of the SM

3. **In particular no flavor violation in gluon, photon, Higgs, Z interactions (at tree-level!)**

Gluon and photon couplings are universal, since aligned with kinetic terms (due to unbroken gauge invariance)

Higgs couplings are diagonal, since aligned with mass terms (due to Higgs as only source of mass terms)

Z couplings are universal, since all up and down quarks have same $T^3 = \pm 1/2$ (despite gauge invariance is broken)

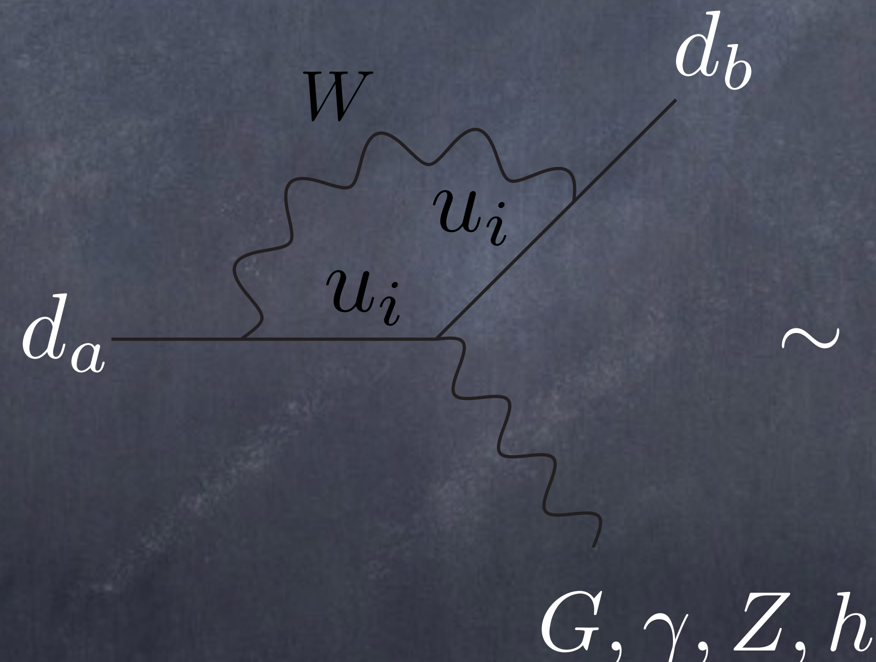
No Flavour Changing Neutral Currents (FCNC) at tree-level!

[general FCNC process: external fermions have same T^3]

The Flavor Structure of the SM

4. **Flavor violation in gluon, photon, Higgs, Z interactions can arise at one-loop, but suppressed by GIM-mechanism: loop factor, small CKM factors and** m^2_{quark}/M_W^2

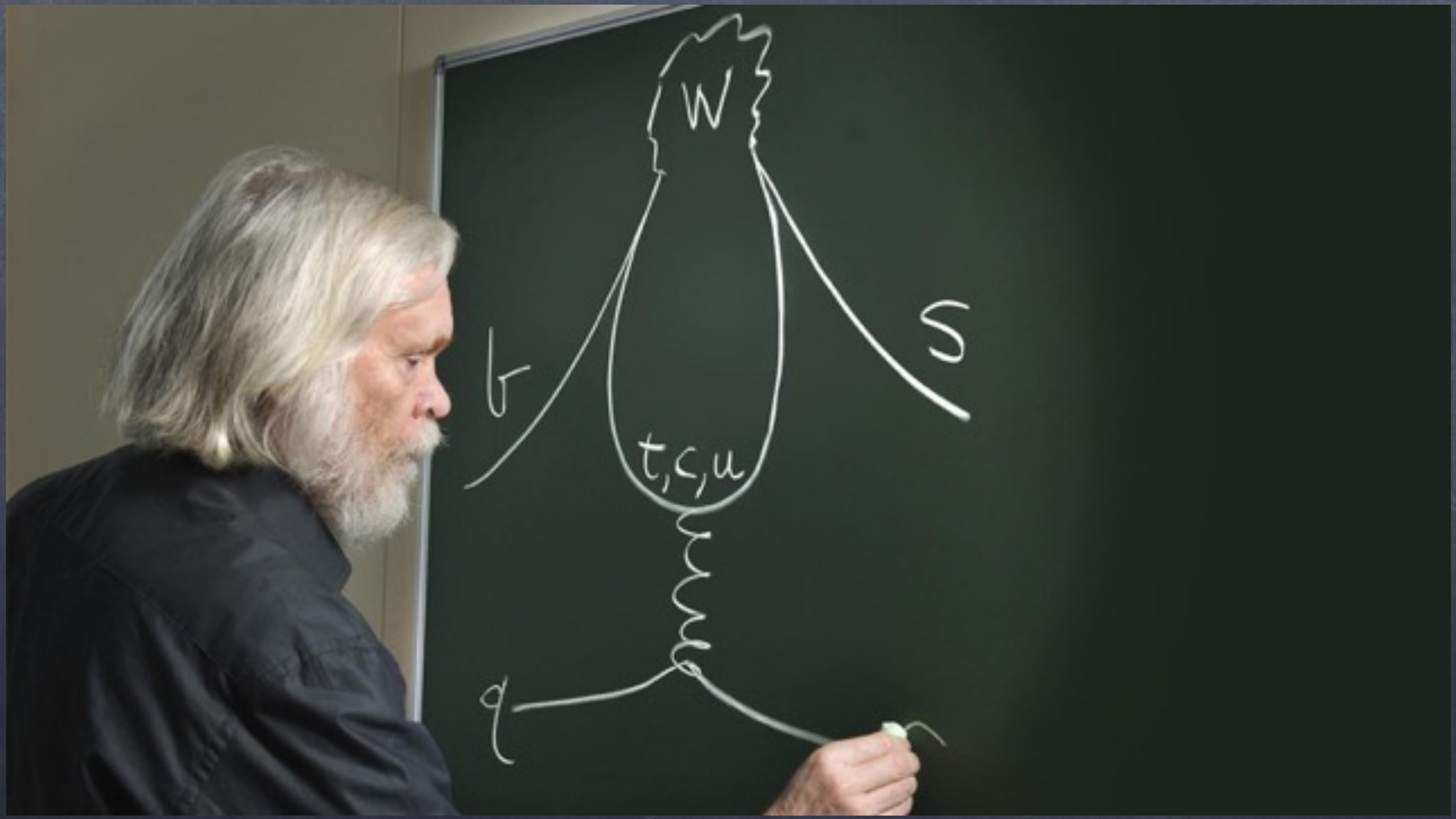
Penguin diagram:



$$\sim \frac{g^2}{16\pi^2}\sum_i V_{ia}V_{ib}^*\, f\underbrace{\left(\frac{m_i^2}{M_W^2}\right)}_{\frac{m_i^2}{M_W^2}+\dots}$$

FCNC are strongly suppressed by GIM mechanism

...in case you wonder about the name:
the inventor can draw it better



The Flavor Structure of the SM

Example of FCNC/FCCC suppression: estimate decay rates

$K_L(\bar{s}d/\bar{d}s) \to \mu^+\mu^-$ all T^3 = −1/2: FCNC

$K^+(\bar{s}u) \to \mu^+\nu$ both T^3 = ±1/2: FCCC

$$\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K^+ \to \mu^+\nu)}\Big|_{\exp} \approx 3 \cdot 10^{-9} \qquad \left(\frac{g^2}{16\pi^2}\frac{m_t^2}{M_W^2}\frac{V_{ts}V_{td}}{V_{us}}\right)^2 \sim 0.01^2\lambda^8 \approx 8 \cdot 10^{-10}$$

5. **Only source of CPV is single phase in CKM matrix, therefore accompanied by small CKM elements: CPV in SM is strongly suppressed**

Part II

Testing the flavour structure of the SM

The General Philosophy

- SM flavor violation described by 4 CKM parameters: measure these 4 and then make predictions

- In practice limited by experimental and theoretical precision:

  Experimentally have to deal with fact that flavor and CP violating effects are tiny effects; need very good resolution

  Theoretically have to deal with fact that we observe hadrons, not quarks, and therefore predictions are polluted by non-perturbative QCD effects: need smart ways to either measure or eliminate these hadronic uncertainties

  For light quarks can often use approximate symmetries of QCD like isospin ($m_u=m_d$) or $SU(3)_F$ ($m_s=m_u=m_d$), for heavy quarks can use heavy quark effective theory ($m_b \gg \Lambda_{\mathrm{QCD}}$)

The General Philosophy

CKM measurements can be divided into two classes

"Direct" measurements	**"Indirect" measurements**
tree-level meson decays, in particular semi-leptonic ones	loop processes with internal W exchange (penguin or box diagrams)
directly probe magnitude of CKM elements $\lvert V_{ij} \rvert$ (sides of unitarity triangle)	can probe combinations of CKM elements V_{ij} and phases (angles of unitarity triangle)
likely not affected by possible New Physics beyond SM	might be affected by possible New Physics beyond SM

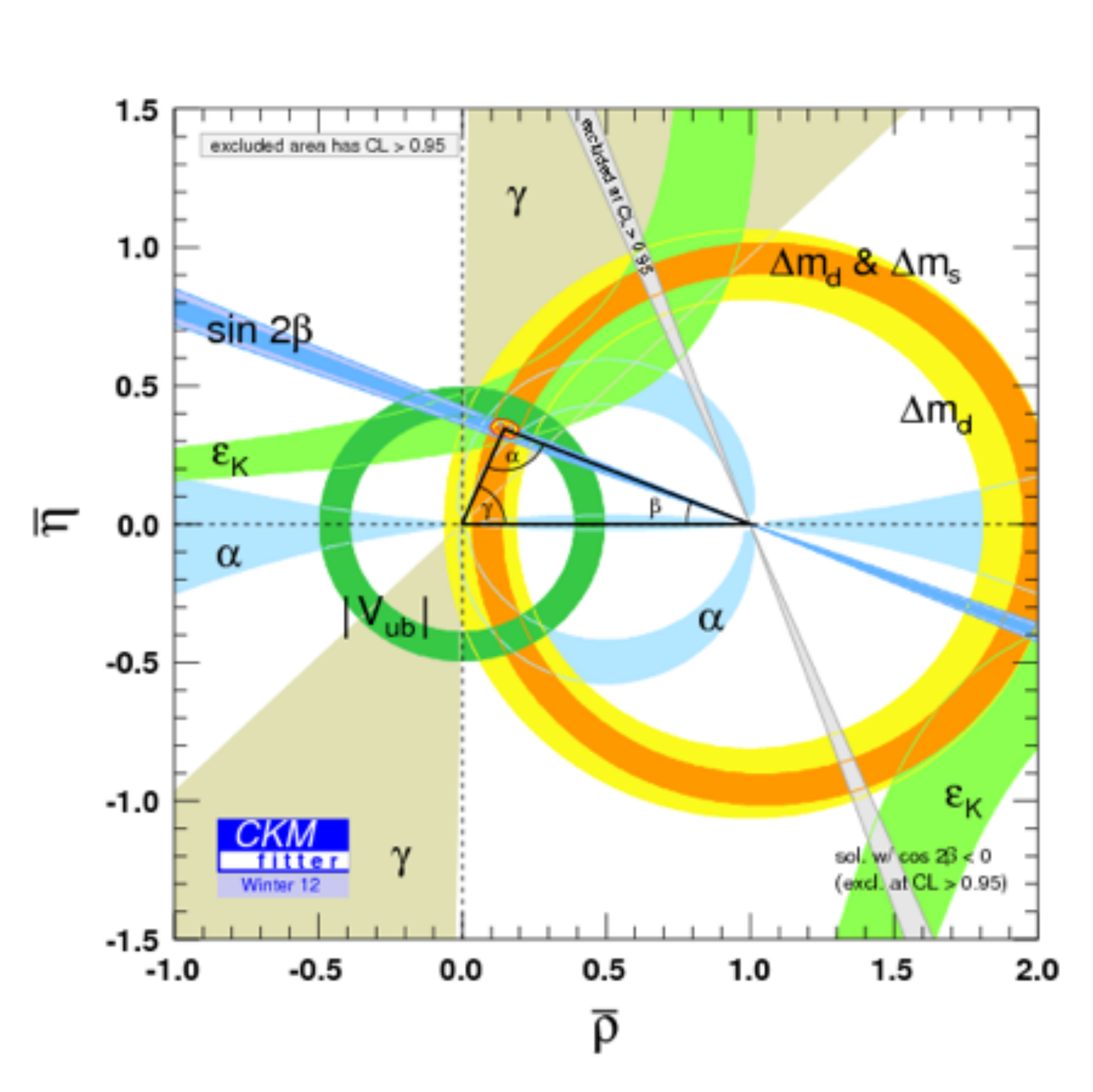
Experimental Results

Have pretty good direct measurements for $|V_{us}|$ [0.4%] and $|V_{cb}|$ [3%] from semi-leptonic K and B meson decays

→ determine A and λ in Wolfenstein: left are ρ and η

Triangle sides:

$R_u = |V_{ub}|$: semi-leptonic B decays



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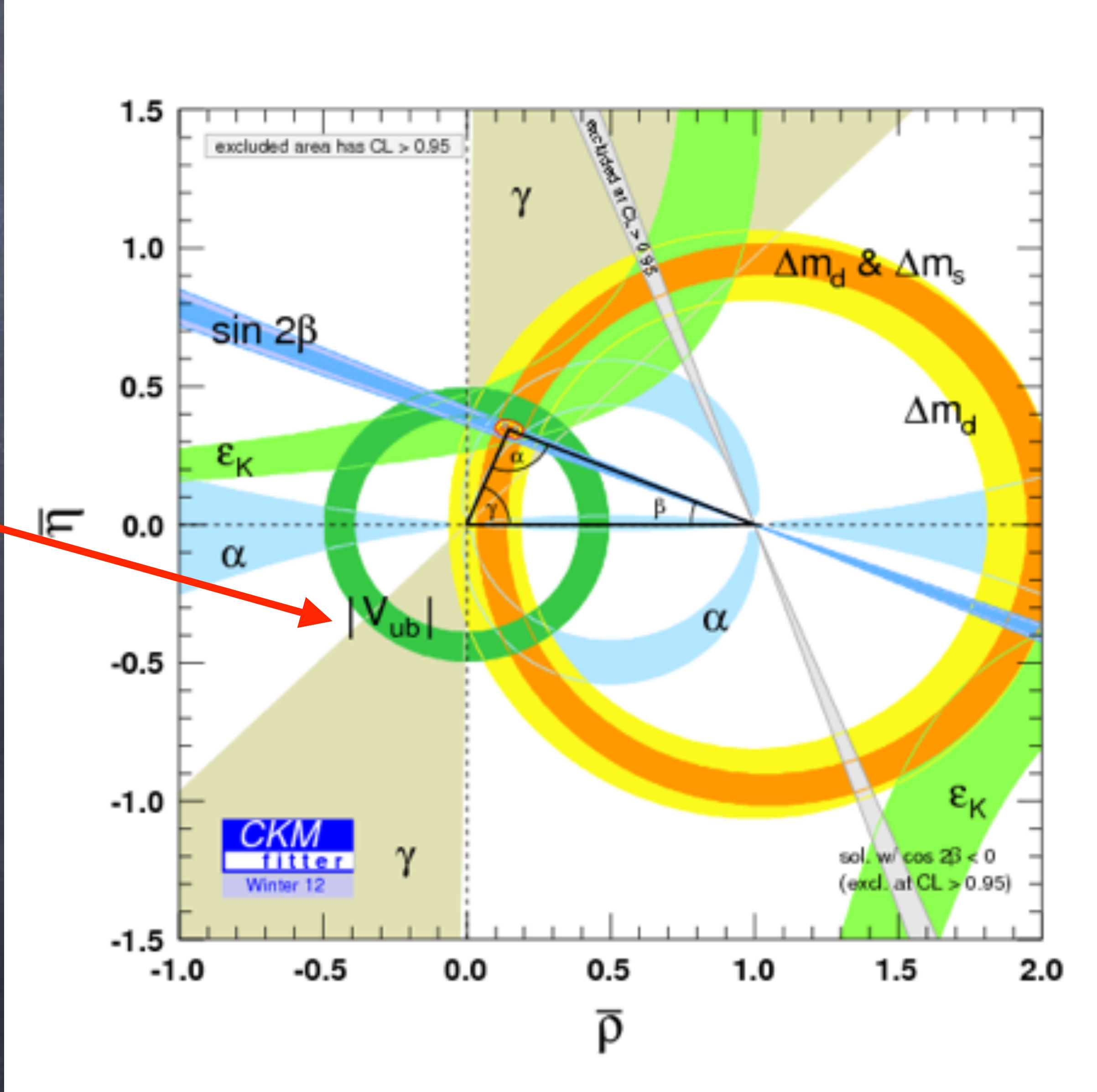

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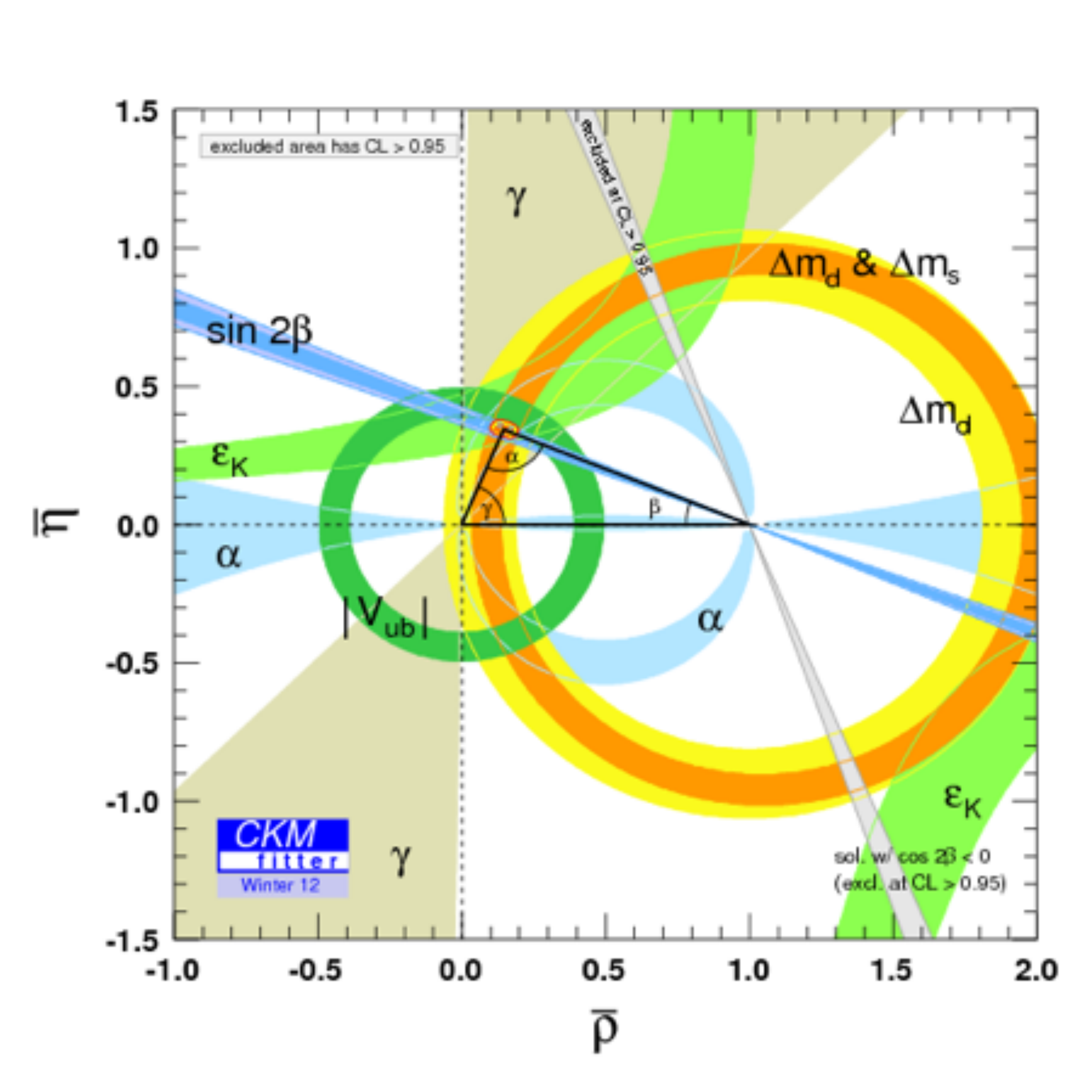
$R_u = |V_{ub}|$: semi-leptonic B decays

$R_t = |V_{td}|$: B_d mixing



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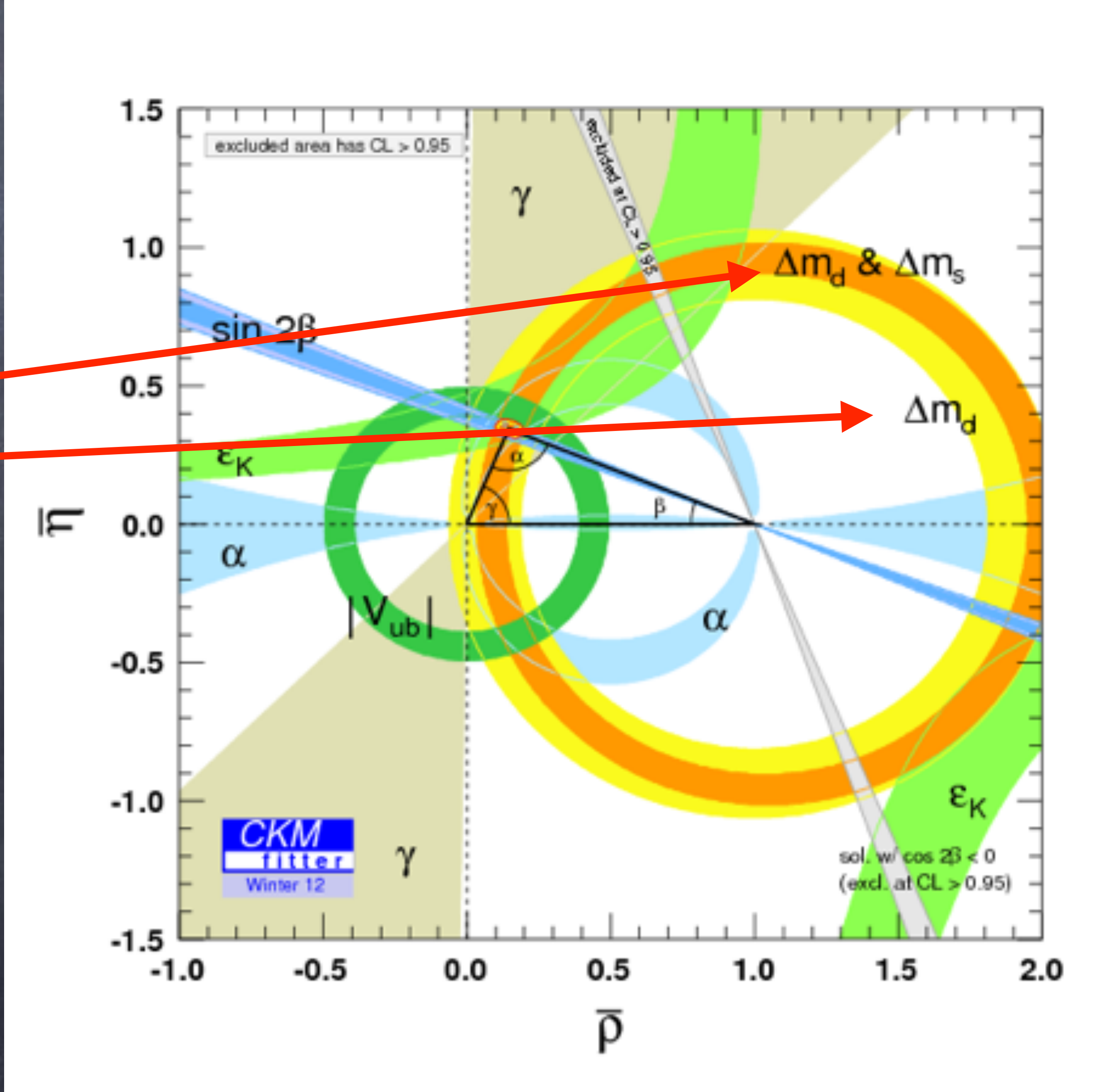

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⇨ determine A and λ in Wolfenstein: left are ρ and η

Triangle sides:

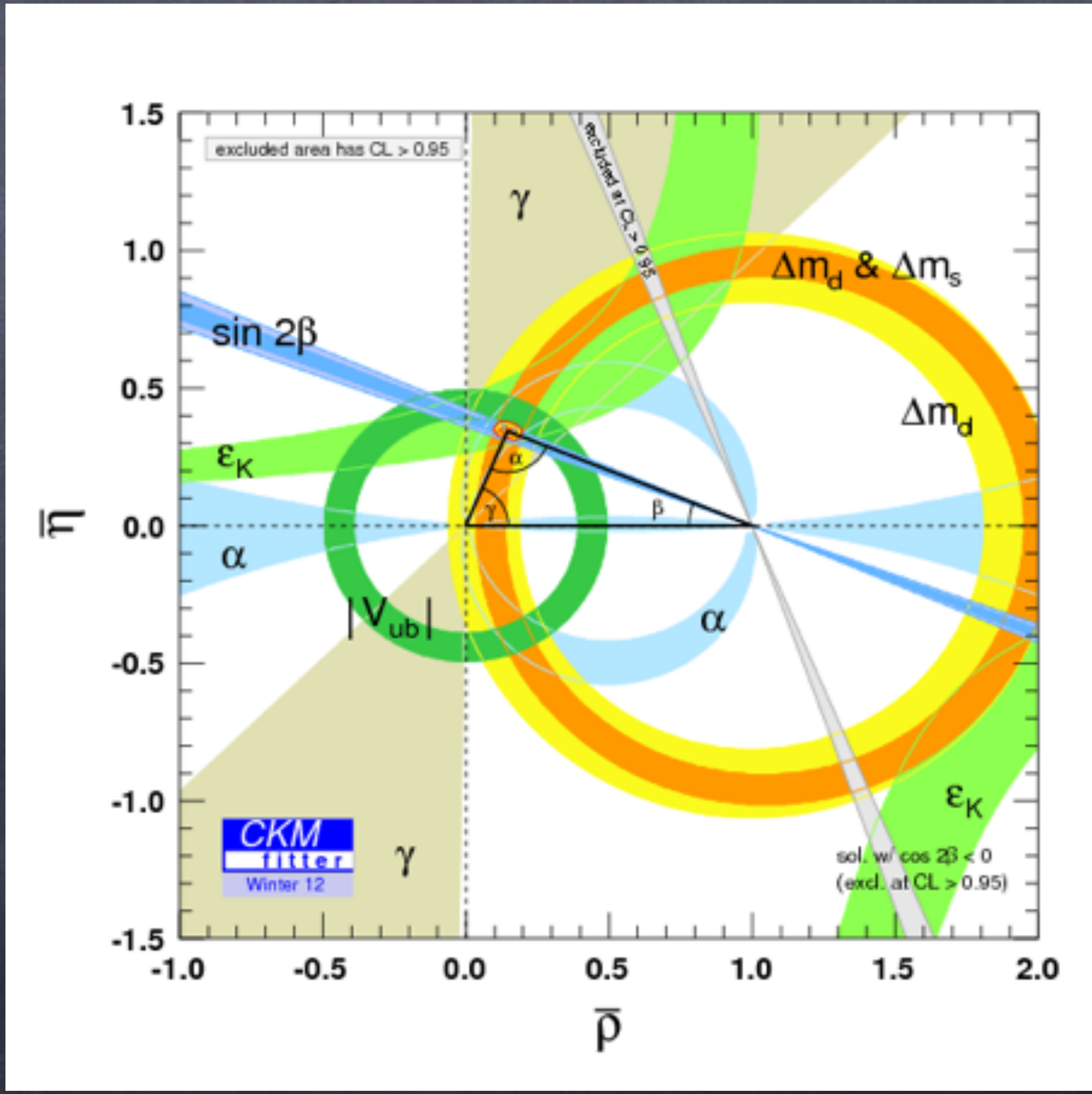
$R_u = |V_{ub}|$: semi-leptonic B decays

$R_t = |V_{td}|$: B_d mixing

note: cam measure CPV without measuring phase

Triangle angles:

from CPV, very precise determination of $\sin(2\beta)$ from $B \to J/\psi K_S$



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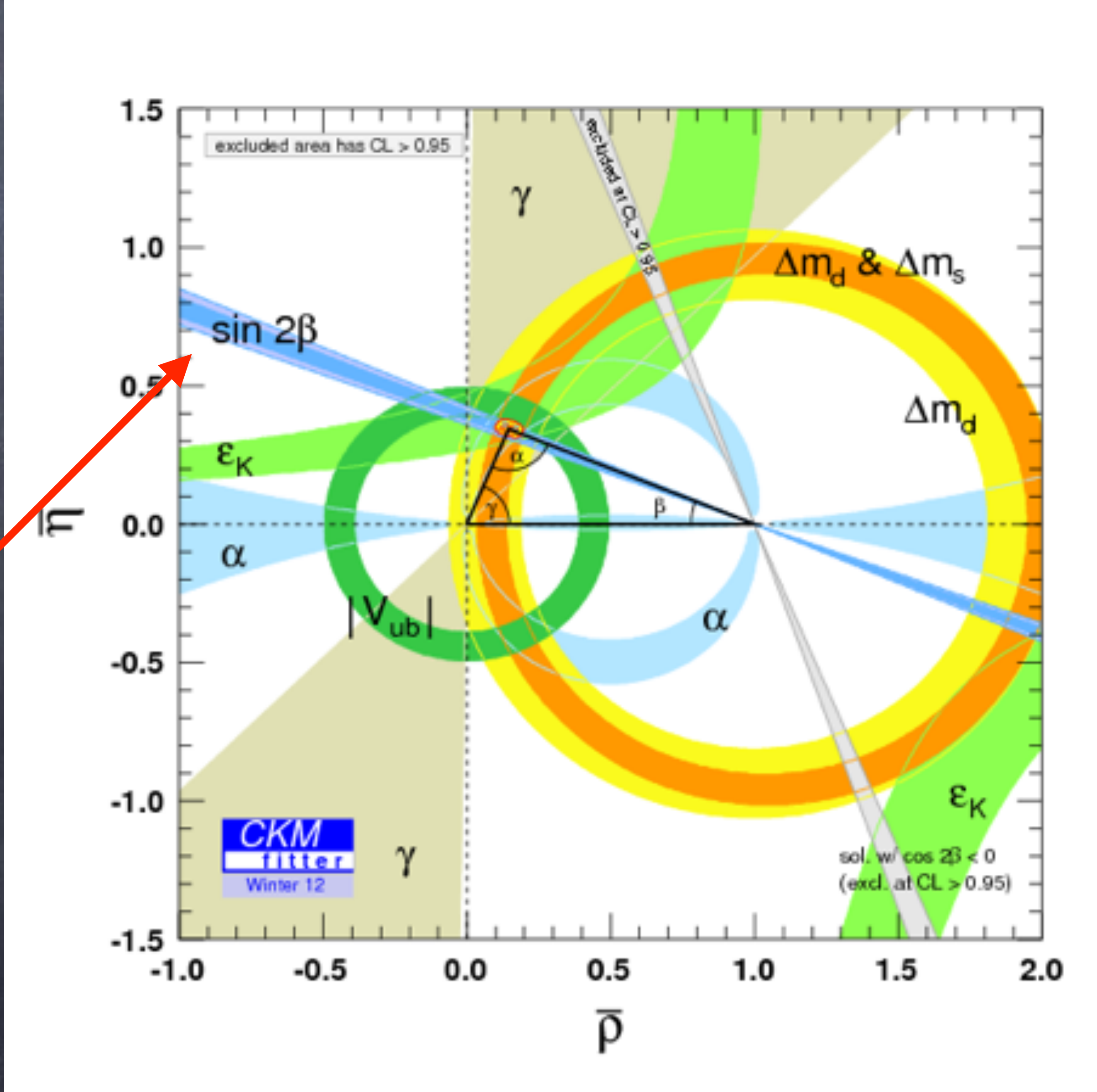

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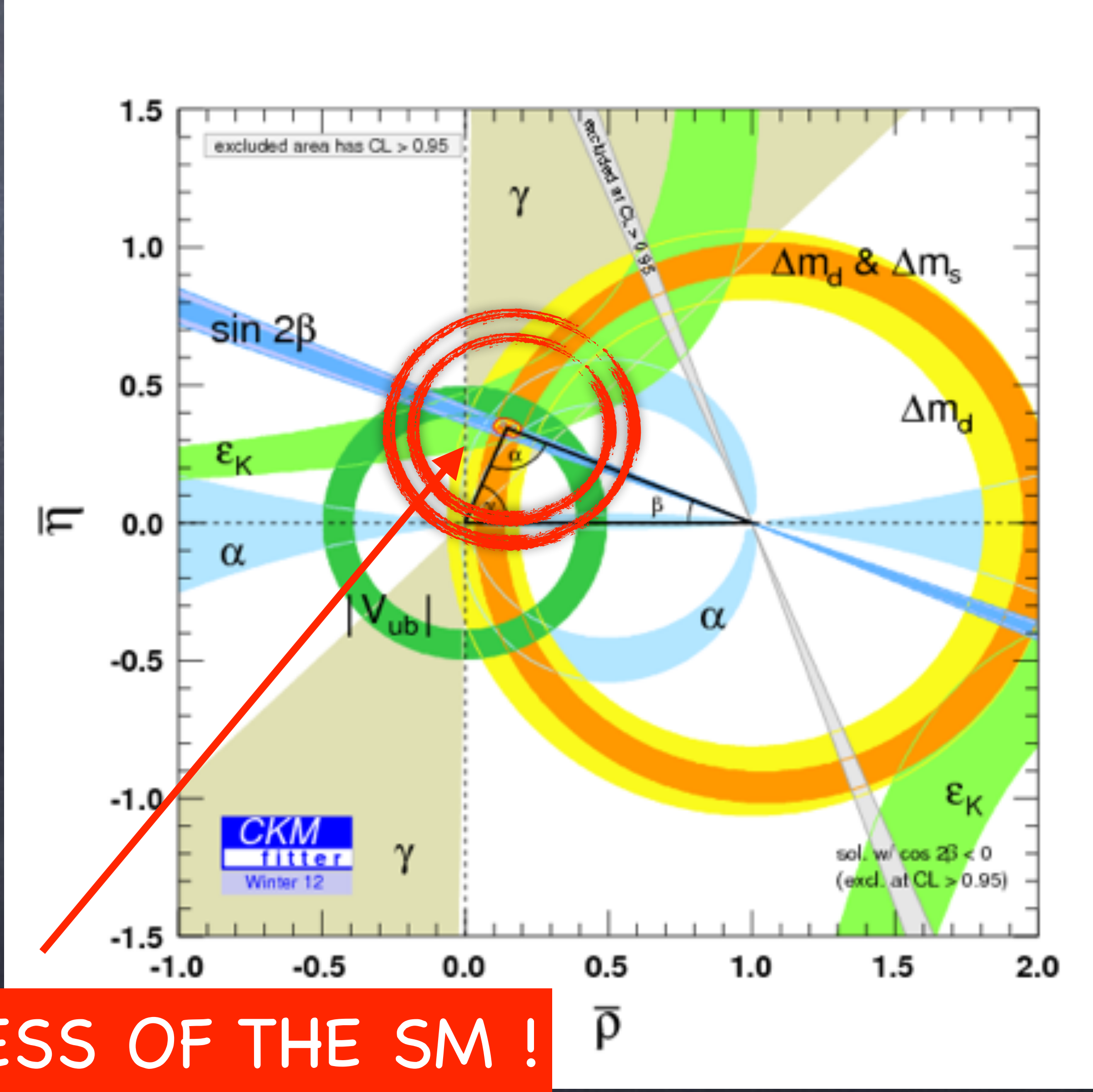


A GREAT SUCCESS OF THE SM !

Meson Mixing

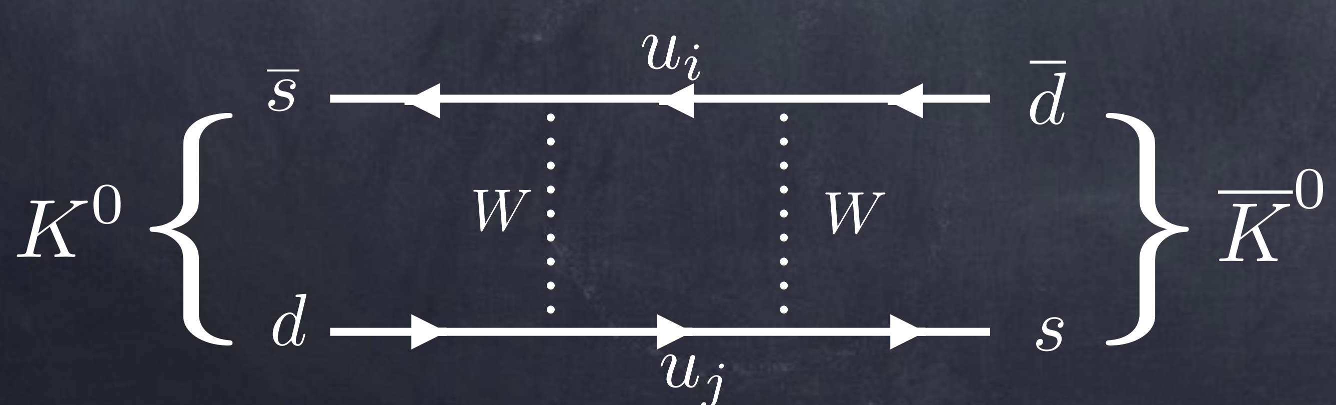
Have neutral mesons and anti-mesons with S, C, B

$$K^0 \sim \bar{s}d, \qquad D^0 \sim c\bar{u}, \qquad B_d^0 \sim \bar{b}d, \qquad B_s^0 \sim \bar{b}s$$

$$\overline{K}^0 \sim s\bar{d}, \qquad \overline{D}^0 \sim \bar{c}u, \qquad \overline{B}_d^0 \sim b\bar{d}, \qquad \overline{B}_s^0 \sim b\bar{s}$$

S, C, B are good quantum numbers for QCD and without weak interactions mesons would be stable; with weak interactions they can decay and mix with their anti-particles

e.g. $K^0 \longleftrightarrow \overline{K}^0 \ : \ \Delta S = 2$ FCNC process through box diagram



highly suppressed, but leads to measurable effect in meson oscillations

Mixing Formalism

[do for B_d system, but valid also for other neutral mesons]

When B^0 produced from initial states with B=0 from strong interactions, always produced with anti-B^0

In general meson described by coherent superposition

$$|\psi(t=0)\rangle = a(t=0)\,|B^0\rangle + b(t=0)\,|\overline{B}^0\rangle$$

When at t=0 we see other state X decaying (semi-leptonically), we know its flavor (= "tag" its flavor), so that our meson must be a pure state of anti-X at t=0

We call $|B^0(t)\rangle$ the state that at t=0 was pure B^0, $|B^0(t=0)\rangle = |B^0\rangle$

[and similar for the anti-B]

Mixing Formalism

At later times meson is described by

$$|\psi(t)\rangle = \underbrace{a(t)\,|B^0\rangle + b(t)\,|\overline{B}^0\rangle}_{\text{mixing}} + \underbrace{c_1(t)\,|f_1\rangle + c_2(t)\,|f_2\rangle + \ldots}_{\text{decays}}$$

We are interested just in a(t) and b(t), whose time evolution is governed by Schrödinger equation with 2x2 non-hermitian Hamiltonian

$$H_{ij} = \begin{pmatrix} \langle B^0|\,\hat{H}\,|B^0\rangle & \langle B^0|\,\hat{H}\,|\overline{B}^0\rangle \\ \langle \overline{B}^0|\,\hat{H}\,|B^0\rangle & \langle \overline{B}^0|\,\hat{H}\,|\overline{B}^0\rangle \end{pmatrix}$$

that we can write as sum of two hermitian matrices

$$H = M - \frac{i}{2}\Gamma$$

Mixing Formalism

Therefore time evolution of pure states at t=0 is given by

$$i\frac{d}{dt}\begin{pmatrix}|B^0(t)\rangle \\ |\overline{B}^0(t)\rangle\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}|B^0(t)\rangle \\ |\overline{B}^0(t)\rangle\end{pmatrix}$$

CPT invariance requires that diagonal entries are equal

$$M_{22} = M_{11}, \Gamma_{11} = \Gamma_{22}$$

The off-diagonal entries are related to dispersive (off-shell intermediate states) and absorptive (on-shell intermediate states) part of mixing amplitude; presence implies that $|B^0(t)\rangle$ has not well-defined mass and width

Need to diagonalize Hamiltonian

Mixing Formalism

Hamiltonian can be diagonalized by

$$PHP^{-1} = H_{\text{diag}} = \begin{pmatrix} M_L - \frac{i}{2}\Gamma_L & \\ & M_H - \frac{i}{2}\Gamma_H \end{pmatrix}$$

with eigenstates and diagonalizing matrix

$$\begin{pmatrix} |B_L\rangle \\ |B_H\rangle \end{pmatrix} = P \begin{pmatrix} |B^0\rangle \\ |\overline{B}^0\rangle \end{pmatrix} \qquad P = \begin{pmatrix} p & q \\ p & -q \end{pmatrix}$$

Instead of eigenvalues use sum and differences

$$M \equiv \frac{M_L + M_H}{2}, \quad \Delta m \equiv M_H - M_L \,(> 0)$$

$$\Gamma \equiv \frac{\Gamma_L + \Gamma_H}{2}, \quad \Delta\Gamma \equiv \Gamma_L - \Gamma_H$$

Mixing Formalism

Solution in terms of original Hamiltonian parameters is

$$M = M_{11} \qquad \Delta m^2 - \frac{1}{4}\Delta\Gamma^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$$

$$\Gamma = \Gamma_{11} \qquad \Delta m \Delta\Gamma = -4\mathrm{Re}\left(M_{12}\Gamma_{12}^*\right)$$

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12} - i/2\Gamma_{12}}{M_{12}^* - i/2\Gamma_{12}^*} \qquad \frac{q}{p} = -\frac{\Delta m + i/2\Delta\Gamma}{-2M_{12}^* + i\Gamma_{12}^*}$$

with normalization condition $|q|^2 + |p^2| = 1$

Ex.: derive these equations

CP Violation

When CP is conserved one has $H_{12} = H_{21}$ (up to a phase)
Then have same phase for M_{12} and Γ_{12}, so that q/p is pure phase (that we choose to be 0). Therefore we have:

$$\boxed{\text{CPV in mixing} \iff |q/p| \neq 1}$$

if CP conserved can choose $q = p = \dfrac{1}{\sqrt{2}}$ and equations simplify: $\Delta m = 2|M_{12}|, \quad \Delta\Gamma = 2|\Gamma_{12}|$

Note that CP approximately conserved when large hierarchy between M_{12} and Γ_{12}

Time Evolution

Now we can study what happens with state that is pure at t=0

Time evolution of mass eigenstates determined by diagonal Hamiltonian

$$|B_{L,H}(t)\rangle = e^{-(iM_{L,H}+\Gamma_{L,H}/2)}\,|B_{L,H}\rangle$$

Can use to obtain time evolution of $|B^0(t)\rangle\,, |\overline{B}^0(t)\rangle$

Meson Oscillations

For simplicity assume CP conservation (|q/p|=1) and neglect width difference (good approx for B_d system)

$$|B^0(t)\rangle = e^{-\Gamma t/2} e^{-iMt} \left[\cos\frac{\Delta m\, t}{2} |B^0\rangle + i \sin\frac{\Delta m\, t}{2} |\overline{B}^0\rangle \right]$$

Note that:

- Indeed is at t=0 pure B^0 state
- Exponential decay governed by width
- At later times acquires also anti-B^0 component: Probabilities to find B^0/anti-B^0 at time t:

$$P_{B^0\to B^0,t} = |\langle B^0|B^0(t)\rangle|^2 = e^{-\Gamma t}\frac{1+\cos\Delta m\, t}{2} \qquad P_{B^0\to \overline{B}^0,t} = e^{-\Gamma t}\frac{1-\cos\Delta m\, t}{2}$$

Neutral mesons oscillate with frequency Δm

Time Scales

If we can observe oscillations, we can determine $\Delta m \stackrel{\text{CP}}{=} 2|M_{12}|$, which is tiny (induced by FCNCs)

In order to observe oscillations, have to measure probability of being B or anti-B after time t

Measurement is done by observing decay, which happens after typical time scale $\tau = 1/\Gamma$

Have to compare to oscillation frequency Δm so that relevant parameter is $x \equiv \Delta m/\Gamma$

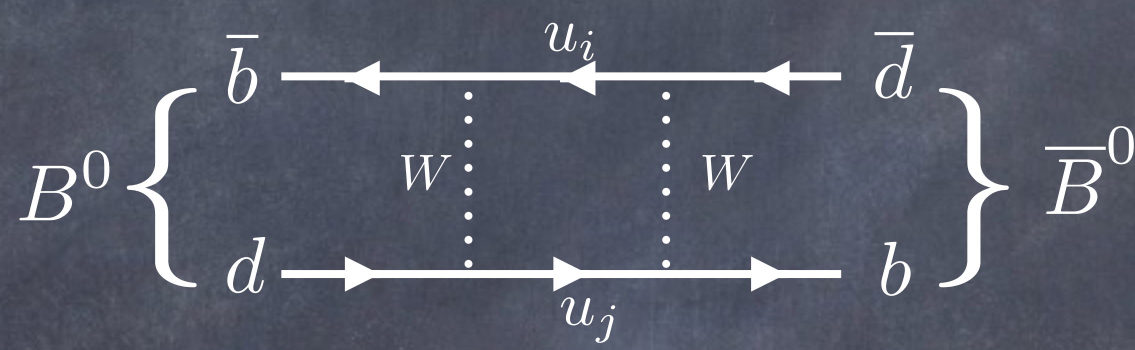
$x \ll 1$: no time to oscillate $x \gg 1$: fast oscillations, average out

$x \sim 1$: can measure oscillation well! Is case for K and B_d

What predicts the SM?

In B sector have approximately CP conservation and therefore $\Delta m = 2|M_{12}|$

M_{12} is given by FCNC box diagram



$\sim (V_{tb}V_{td}^*)^2$

Calculation needs hadronic matrix elements that must be taken from lattice and dominate error; use result and compare to data to determine $|V_{td}|$ -> CKM triangle

Note that measured effects are very tiny:

$$\Delta m_{B_d}|_{\mathrm{exp}} = 3 \cdot 10^{-13}\,\mathrm{GeV} = 0.5\,\mathrm{ps}^{-1} \sim \Gamma \ll M_{B^0} = 5.2\,\mathrm{GeV}$$

CP Violation

CP transforms particles into anti-particles, up to phases

$$CP\,|f\rangle = e^{i\omega_f}\,|\overline{f}\rangle$$
$$CP\,|\overline{f}\rangle = e^{-i\omega_f}\,|f\rangle$$

Define CP-conjugated decay amplitudes

$$A_f = \mathcal{A}(P \to f) = \langle f|\,\mathcal{H}\,|P\rangle \qquad \overline{A}_{\overline{f}} = \mathcal{A}\left(\overline{P} \to \overline{f}\right) = \langle \overline{f}|\,\mathcal{H}\,|\overline{P}\rangle$$

CP is violated when $\Gamma\,(P \to f) \neq \Gamma\left(\overline{P} \to \overline{f}\right)$

This requires that we have at least two different contributions to amplitudes with different CP-odd (“weak”) phases and CP-even (“strong”) phases

Ex.: show this

$$A_f = |a_1|e^{i(\delta_1+\phi_1)} + |a_2|e^{i(\delta_2+\phi_2)}$$
$$\overline{A}_{\overline{f}} = |a_1|e^{i(\delta_1-\phi_1)} + |a_2|e^{i(\delta_2-\phi_2)}$$

strong phase

weak phase

CP Violation in Meson Decays

When we talk about neutral meson decays, in general we have to include meson mixing

Have 3 Types of CPV:

A) CPV in decays ("direct CPV"): $|\overline{A}_{\overline{f}}/A_f| \neq 1$

Interference of 2 decay amplitudes, strong phase from intermediate on-shell states

B) CPV in mixing ("indirect CPV"): $|q/p| \neq 1$

Interference of absorptive (Γ_{12}) and dispersive (M_{12}) mixing amplitudes, strong phase from time evolution

C) CPV in decays w/wo mixing: $\mathrm{Im}\left(q/p\,\overline{A}_{f_{CP}}/A_{f_{CP}}\right) \neq 0$

Interference of mixing and decay amplitude (with no CPV in mixing and decays), strong phase from time evolution

CP Violation in Meson Decays

A) CPV in decays ("direct CPV"): $|\overline{A}_{\overline{f}}/A_f| \neq 1$

Interference of 2 decay amplitudes, strong phase from intermediate on-shell states

Can be measured by CP asymmetry

$$a_{\mathrm{CP}} = \frac{\Gamma\left(\overline{P} \to \overline{f}\right) - \Gamma\left(P \to f\right)}{\Gamma\left(\overline{P} \to \overline{f}\right) + \Gamma\left(P \to f\right)} = \frac{|\overline{A}_{\overline{f}}/A_f|^2 - 1}{|\overline{A}_{\overline{f}}/A_f|^2 + 1}$$

$$= \frac{|a_2|}{|a_1|} \sin\left(\phi_2 - \phi_1\right) \sin\left(\delta_2 - \delta_1\right)$$

[see previous amplitude parametrization]

Relevant for charged mesons (no mixing) and systems with small CPV in mixing, but strong phase is difficult to calculate; nice example are $B^\pm \to (D^0, \overline{D}^0) K^\pm \to f_D K^\pm$ decays that measure CKM phase γ

CP Violation in Meson Decays

B) CPV in mixing ("indirect CPV"): $|q/p| \neq 1$

Interference of absorptive (Γ_{12}) and dispersive (M_{12})
mixing amplitudes, strong phase from time evolution

Back to mixing formalism: in general have

$$|P^0(t)\rangle = g_+(t)\,|P^0\rangle - \frac{q}{p} g_-(t)\,|\overline{P}^0\rangle$$

$$|\overline{P}^0(t)\rangle = g_+(t)\,|\overline{P}^0\rangle - \frac{p}{q} g_-(t)\,|P^0\rangle$$

$$g_\pm(t) = \frac{1}{2}\left(e^{-iM_H t - \Gamma_H/2\, t} \pm e^{-iM_L t - \Gamma_L/2\, t}\right)$$

from which one calculate decay rates

$$\Gamma\left(P^0(t) \to f\right), \Gamma\left(\overline{P}^0(t) \to \overline{f}\right)$$

that depend in general also on $A_{\overline{f}}$ and $\overline{A}_f$

CP Violation in Meson Decays

B) CPV in mixing ("indirect CPV"): $|q/p| \neq 1$

Interference of absorptive (Γ_{12}) and dispersive (M_{12})
mixing amplitudes, strong phase from time evolution

Now look for simple decay such that no CPV from decays, i.e. $|\overline{A}_{\overline{f}}/A_f| = 1$, and $A_f = A_{\overline{f}} = 0$

➡ semi-leptonic decays of neutral mesons:

decays of P to l and anti-P to anti-l are tree-level, while
decays of P to anti-l and anti-P to l are strongly suppressed

Measure in semi-leptonic CP asymmetries

$$a_{\mathrm{SL}} = \frac{\Gamma\left(\overline{P}(t) \to l^+ X\right) - \Gamma\left(P(t) \to l^- \overline{X}\right)}{\Gamma\left(\overline{P}(t) \to l^+ X\right) + \Gamma\left(P(t) \to l^- \overline{X}\right)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \sim \mathrm{Im}\frac{\Gamma_{12}}{M_{12}}$$

CP Violation in Meson Decays

C) CPV in decays w/wo mixing: $\mathrm{Im}\left(q/p\,\overline{A}_{f_{CP}}/A_{f_{CP}}\right) \neq 0$

Interference of mixing and decay amplitude (with no CPV in mixing and decays), strong phase from time evolution

Need system where no CPV in mixing or decays

$$|q/p| = 1 \quad |\overline{A}_f/A_f| = 1$$

P decays directly to CP eigenstate f, or first oscillate into anti-P and then decays to f

Measure in time-dependent CP asymmetries

$$a_{f_{CP}}(t) = \frac{\Gamma\left(\overline{P}(t) \to f\right) - \Gamma\left(P(t) \to f\right)}{\Gamma\left(\overline{P}(t) \to f\right) + \Gamma\left(P(t) \to f\right)}$$

CP Violation in Meson Decays

C) CPV in decays w/wo mixing: $\mathrm{Im}\left(q/p\,\overline{A}_{f_{CP}}/A_{f_{CP}}\right) \neq 0$

Interference of mixing and decay amplitude (with no CPV in mixing and decays), strong phase from time evolution

Use mixing formalism for this case; in simplifying case with small width difference (good in B_d system) find

$$a_{f_{CP}}(t) = -\sin(\Delta m\, t)\,\mathrm{Im}\,\lambda_{f_{CP}} \qquad \lambda_{f_{CP}} \equiv \frac{p}{q}\frac{\bar{A}_f}{A_f}$$

If know mass difference can measure, very clean since small hadronic uncertainties

nice example is $B \to J/\psi K_S$ that measures CKM angle β

$$\lambda_{J/\psi K_S} = -\sin 2\beta$$

References

There are many good reviews on flavor physics on the arxiv with much more material; to name just a few:

- Y. Grossman, arXiv:1006.3534
- U. Nierste, arXiv:0904.1869
- Y. Nir, arXiv:1605.00433, hep-ph/9911321
- A. Buras, hep-ph/980647