BFKL and CCFM evolutions with absorptive boundary

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Outline

- How to approach non-linear QCD evolution

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- Results for BFKL, comparison to BK
- Explaining CCFM with cartoons
- Applying saturation philosophy to CCFM

Experimental (LHC) Motivation

- Considerably enlarged phase space at LHC, so important to account for non-linear effects.
- May expect many effects in the UE, but complicated both for perturbative and non-perturbative component.
- Deviations from linear evolution expected also to influence “hard” observables, ex: Jet production at forward η.
- LHC jets: $Q \gtrsim 10$ GeV. Not necessarily DGLAP physics. BFKL type physics important when $Y = \ln s \gtrsim \ln Q^2$.

Experimental (LHC) Motivation

- Saturation effects can be felt also for $Q > Q_s$. $\mathcal{A}(x, k_\perp)$ above Q_s modified by saturation below Q_s.
- At LHC expect $Q_s \sim 2, 3$ GeV. Much higher Q_s in rare events if one focus on “hot spots” inside the proton.
- Thus important to make realistic and quantitative predictions on the effects of saturation when $Q^2 > Q_s^2$.

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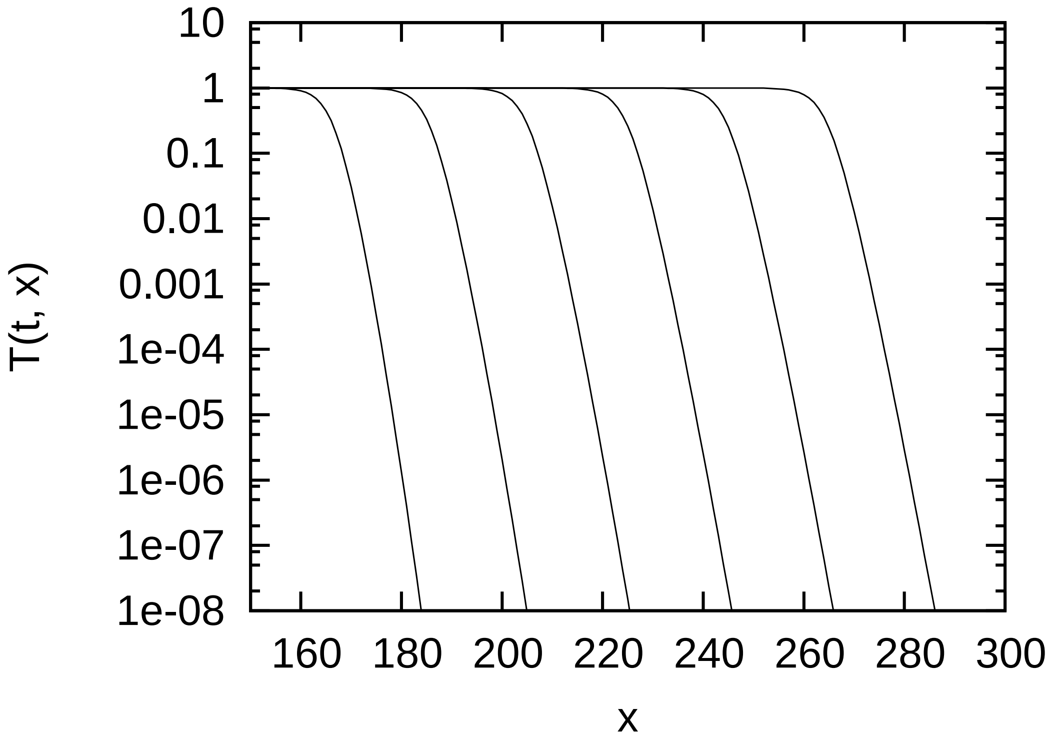
  - No detailed knowledge of saturation mechanism necessary. Q_s determined fully by linear evolution, *if* the linear evolution is endowed by an absorptive boundary restoring unitarity.
- Opens possibility to study effects of saturation on formalism whose non-linear generalization not known yet, e.g. CCFM or BFKL beyond LL approx.

Theoretical Motivation and Philosophy

- For fixed $\bar{\alpha}_s$ and $Y \to \infty$ we know this is case from analogy to statistical physics.
- *A priori* not clear if this strategy works for running $\bar{\alpha}_s$ or for realistic Y.
- For running $\bar{\alpha}_s$ linear evolution IR unstable, and resembles more of a “pushed” type evolution rather than “pulled” type.
- Yet, our analysis demonstrate the strategy works also in this case: for running $\bar{\alpha}_s$ and for all Y.

Traveling waves and fronts

$$\partial_t T(t,x) = T(t,x) + \partial_x^2 T(t,x) - T^2(t,x)$$



Application: Absorptive boundary

- Define first line of constant amplitude: $T(Y, \rho = \rho_c(Y)) = c < 1$, $\rho \equiv \ln(r_0^2/r^2)$ or $\ln(k_\perp^2/k_0^2)$.

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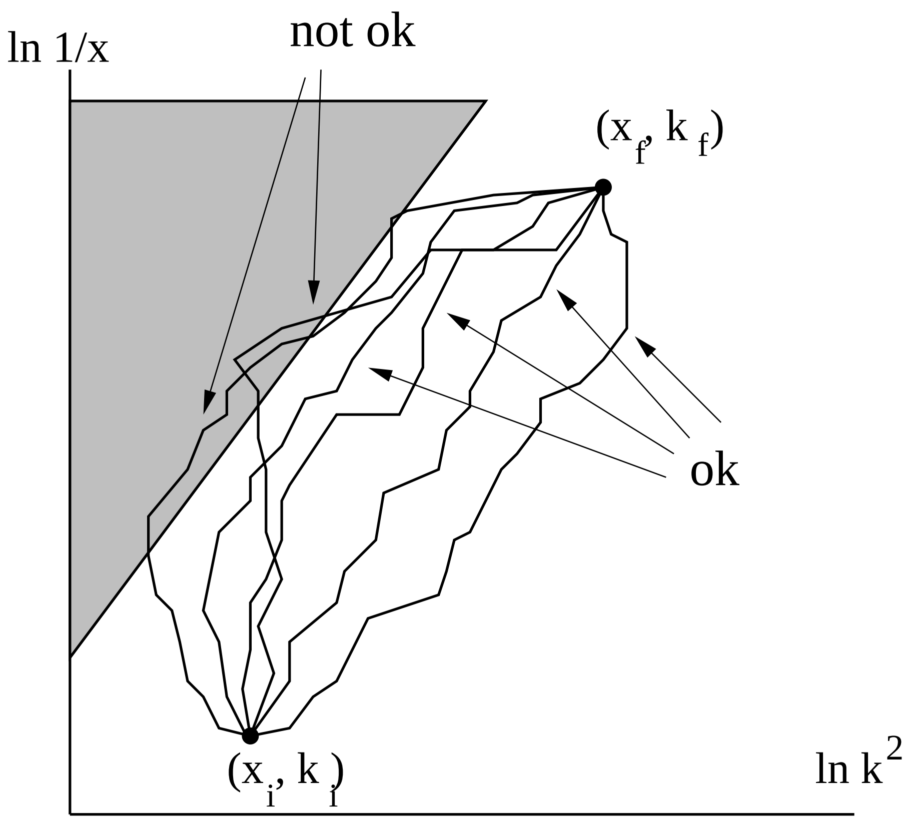
- A distance Δ behind ρ_c, apply boundary condition.
- Most natural choice: $T(Y, \rho) = 0$ for $\rho \leq \rho_c - \Delta$.
- Δ and c as free parameters. However, correlated as $\Delta \sim \ln(1/c)$

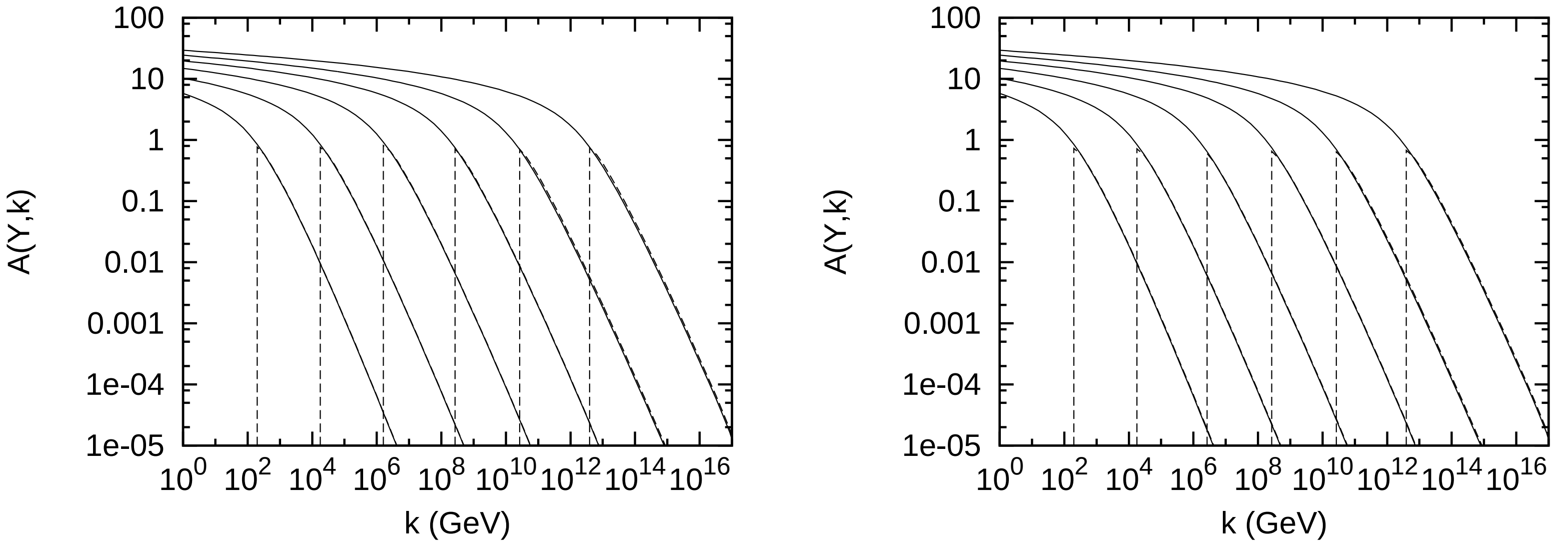
Meaning of absorptive boundary



A few comments

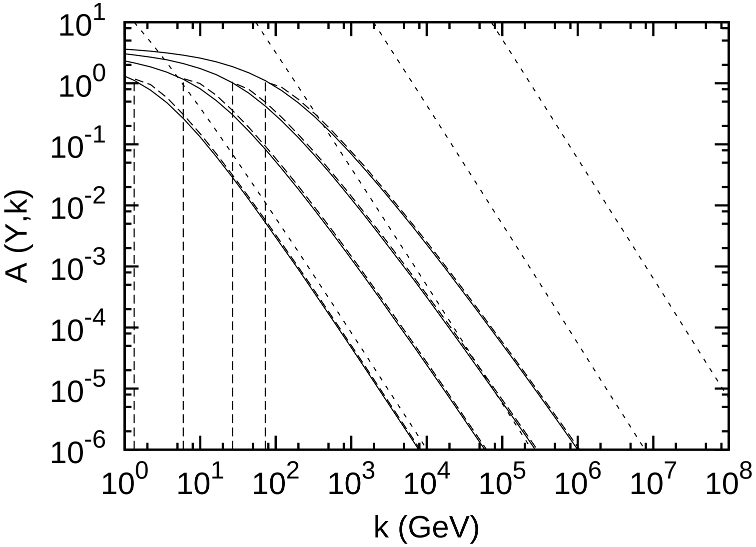
- ρ_s can be identified with maximum of T, or any point of constant T between c and 1.
- Cannot use this procedure to study any problem sensitive to $\rho \lesssim \rho_s$. For $\rho > \rho_c \approx \rho_s$ works very good.
- We are interested in $\mathcal{A}(Y, k_\perp)$ and not $T(Y, r)$. However, same conditions apply on $\mathcal{A}(Y, k_\perp)$, $\mathcal{A} \sim \mathcal{O}(1)$ at saturation

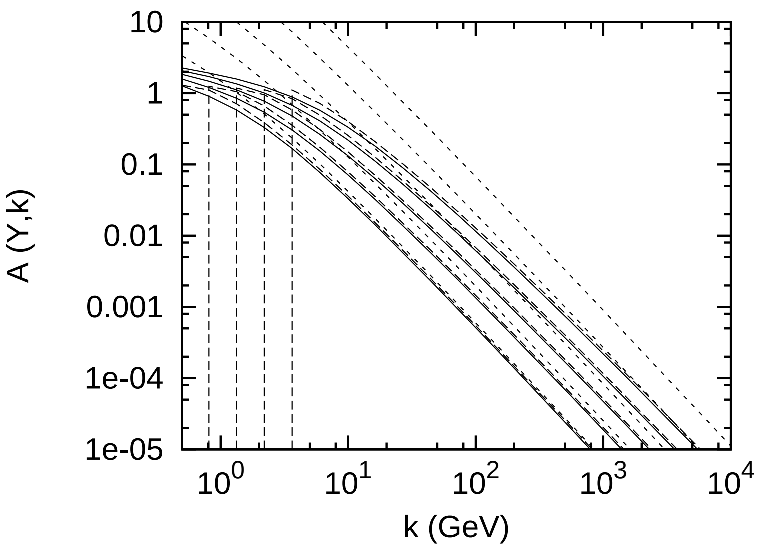
Results: BFKL with ab vs BK



- Left: $c = 0.1, \Delta = 5.0$ Right: $c = 0.3, \Delta = 3.0$.
  $Y = 20 + 10 * i$, $\bar{\alpha}_s = 0.2$.

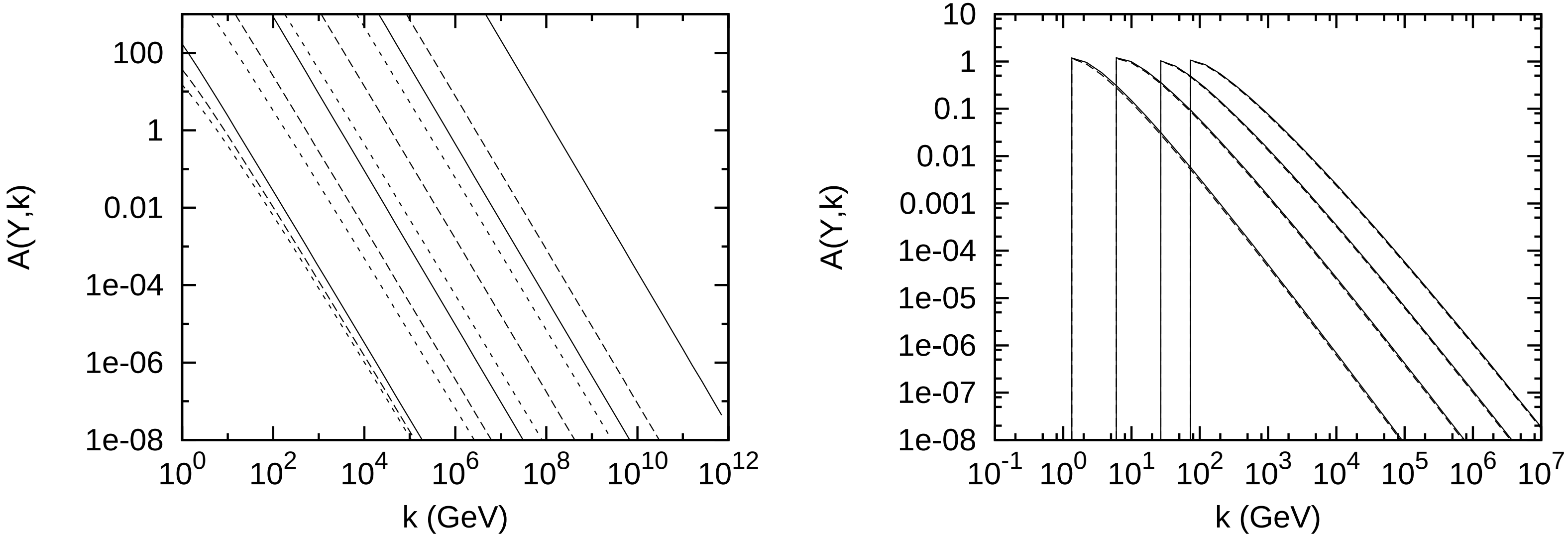
Results: Running $\bar{\alpha}_s$





- Left: $Y = 10, 20, 30, 40$. Right: $Y = 6, 8, 10, 12, 14$. $c = 0.1, \Delta = 5.0$.

Results: IR sensitivity



- $\bar{\alpha}_s(Q^2) \to \bar{\alpha}_s(Q^2 + \mu^2)$. Checking sensitivity to μ.

CCFM

- Real emission density in CCFM: $\bar{\alpha}_s \frac{dy_k}{y_k} \frac{d\xi_k}{\xi_k}$, y_k energy fraction, ξ_k squared angle: $\xi_k = q_k^2/(y_k^2 E^2)$
- Virtual form factors S_{eik} and S_{ne}:

$$\begin{aligned} S_{eik}^2 &= \exp\left(-\bar{\alpha}_s \int_{y_{k+1}}^{y_k} \frac{dy}{y} \int^{\bar{\xi}} \frac{d\xi}{\xi}\right), \\ S_{ne}^2 &= \exp\left(+\bar{\alpha}_s \int_{y_{k+1}}^{y_k} \frac{dy}{y} \int_{\xi(Q_k)}^{\bar{\xi}} \frac{d\xi}{\xi}\right), \end{aligned}$$

- $\bar{\xi}$: Maximal angle allowed by coherence

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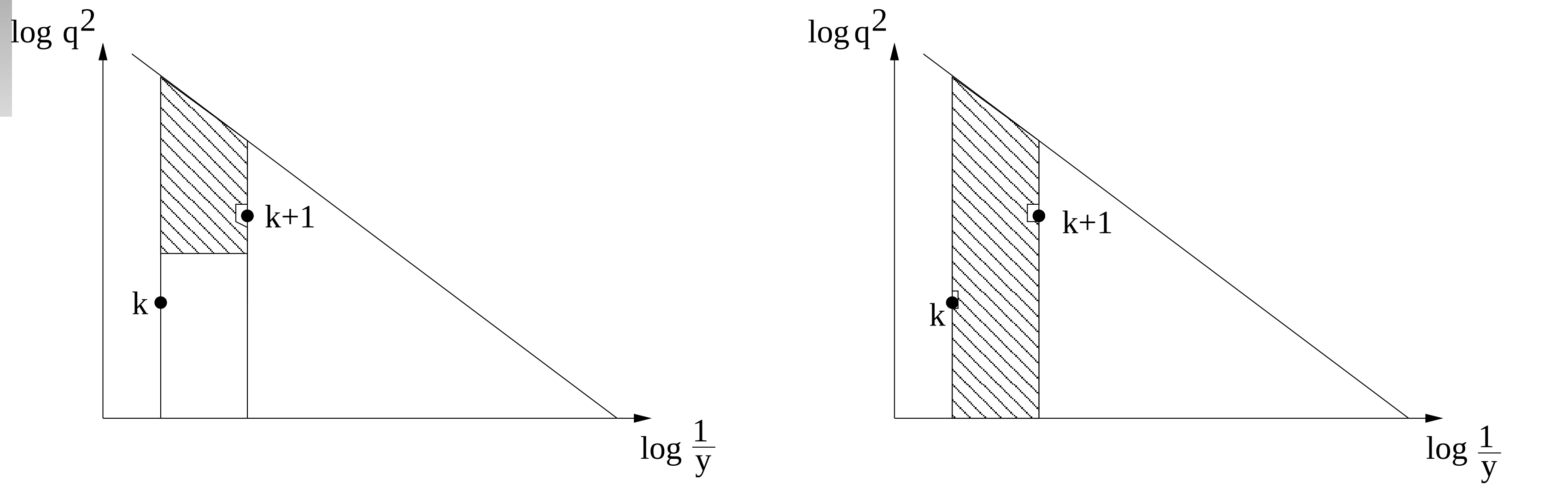
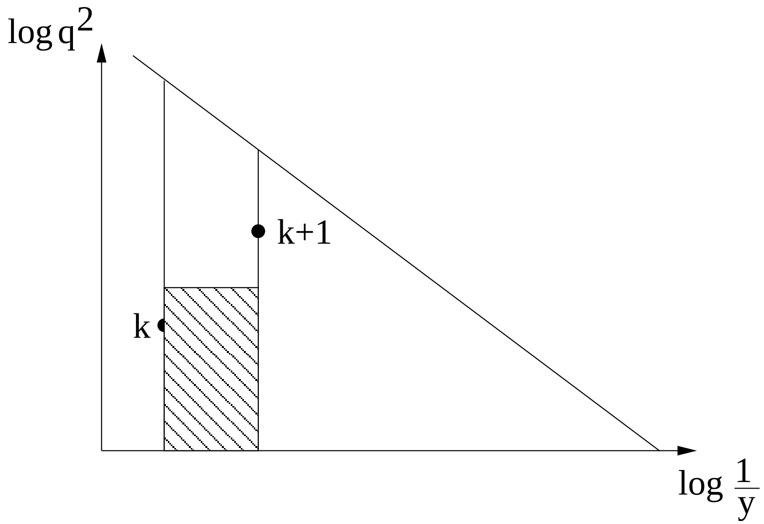
- Hard: $1/z_k$, Soft: $1/(1-z_k)$, where $y_k = (1-z_k)x_{k-1}$.

Virtual form factors



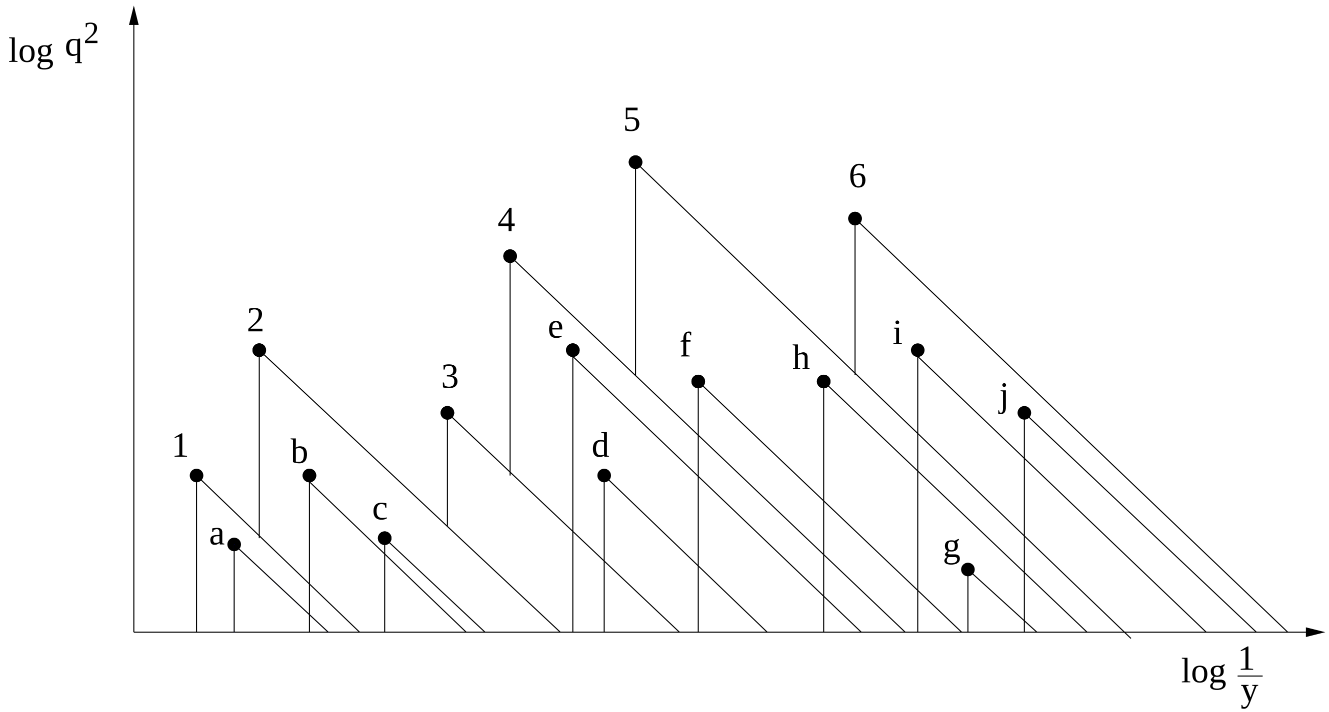
- Left: $S^2_{ne}(k)$, Right: $S^2_{eik}(k)$, $Q_k = -\sum_{i=1}^{k} q_i$.

Relation to BFKL



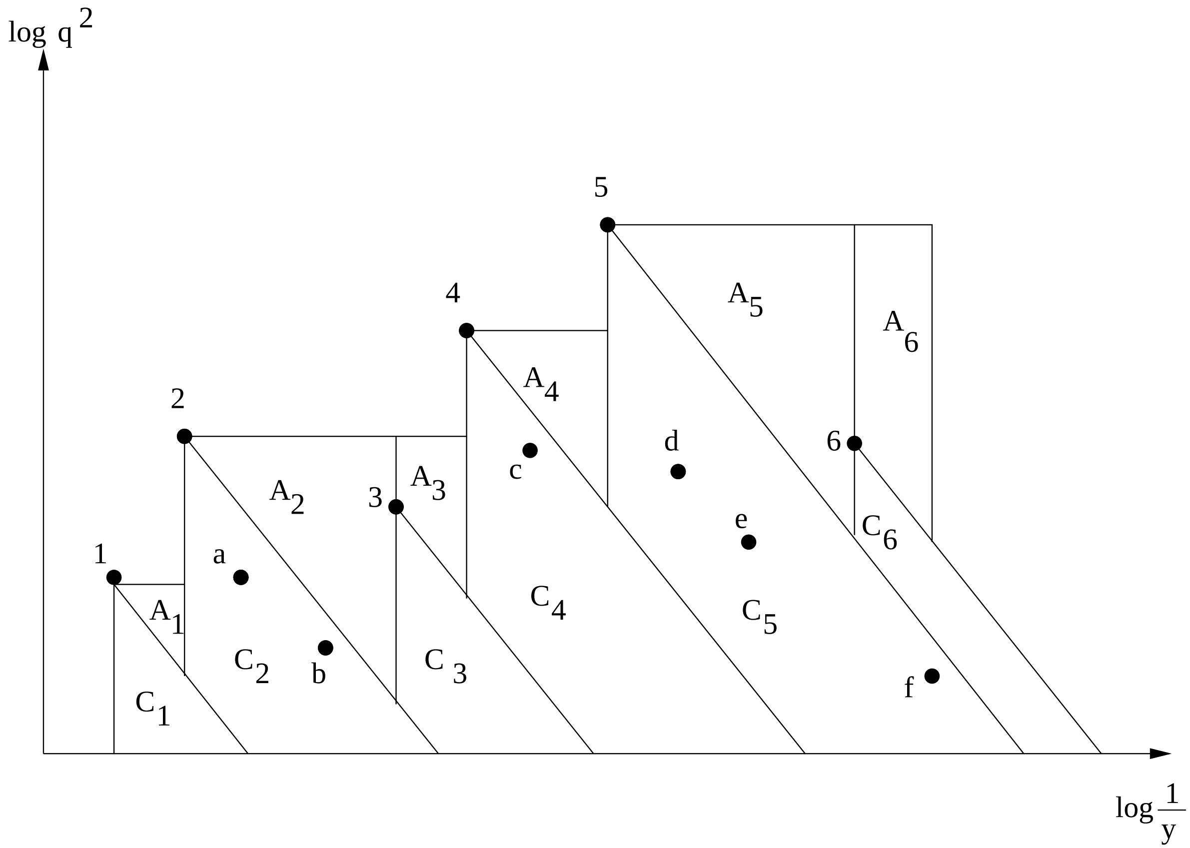
$$S_{ne}^2(k) * S_{eik}^2(k) \;=\; \exp\left(-\bar{\alpha}_s \int_{y_{k+1}}^{y_k} \frac{dy}{y} \int^{Q_k^2} \frac{dq^2}{q^2}\right)$$
$$\;=\; \Delta_{ne}^{(BFKL)}(k)$$

Hard and Soft emissions



- $1, 2, \ldots$ hard emissions, $a, b, \ldots$ soft emissions.

The “Sudakov” and the “non-Sudakov”



- Define the “Sudakov” form factors by $\Delta_s(k) = e^{-\bar{\alpha}_s C_k}$, and “non-Sudakov” by $\Delta_{ns}(k) = e^{-\bar{\alpha}_s A_k}$.

Extracting out the Soft Emissions

- Sum exclusively over soft emissions in each C_k: $e^{\bar{\alpha}_s C_k}$.
- $\Delta_s(k) \cdot e^{\bar{\alpha}_s C_k} = 1$, *i.e.* soft emissions cancelled by Sudakov (Probability conservation).
- Left with only $1/z$ pole and Δ_{ns}, simpler gluon distribution.
- Important that soft emissions do not change t-channel $k_\perp$.
- Actually also holds for certain subset of hard emissions

Simplifying more

- Sum over real hard emissions in A_k. $\Rightarrow$ Cancellation of Δ_{ns} and simpler formula.
- Most importantly, the gluon distrb. $\mathcal{A}(Y,k,\bar{q}) \Rightarrow \mathcal{A}(Y,k)$ for $\bar{q} \geq k$.
- Much easier and faster numerical solution to integral eq.
- However, eq. to be derived not exactly unique, and different eq. $\Rightarrow$ different intercepts.

The Equations

- First equation:

$$\partial_Y \mathcal{A}(Y,k) = \bar{\alpha}_s \int \frac{dk'^2}{|k^2 - k'^2|}\, h(\kappa) \Bigg(\theta(k^2 - k'^2)\mathcal{A}(Y,k')$$

$$+\theta(k'^2 - k^2)\theta(Y - \ln(k'^2/k^2))\mathcal{A}(Y - \ln(k'^2/k^2), k') \Bigg).$$

where $\kappa \equiv \min(k^2, k'^2)/\max(k^2, k'^2)$ and

$$h(\kappa) = 1 - \frac{2}{\pi} \arctan\left(\frac{1+\sqrt{\kappa}}{1-\sqrt{\kappa}} \sqrt{\frac{2\sqrt{\kappa}-1}{2\sqrt{\kappa}+1}} \right) \theta(\kappa - 1/4).$$

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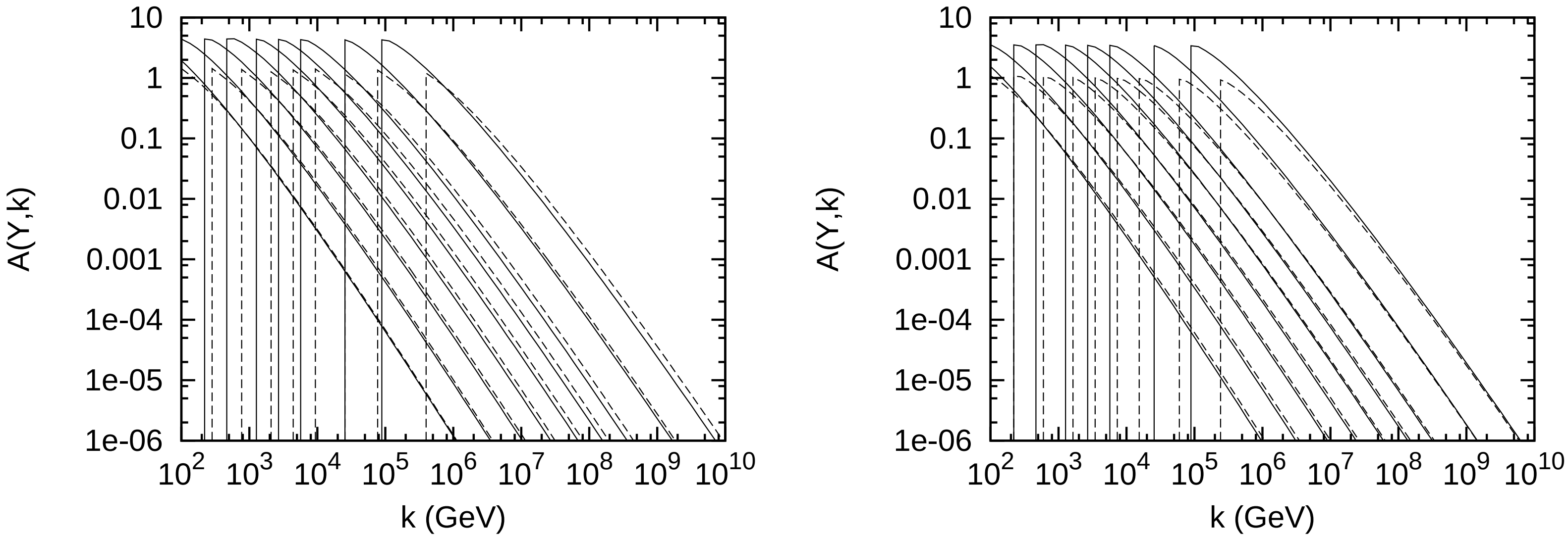
$$+\theta(k'^2 - k^2)\theta(Y - \ln(k'^2/k^2))\mathcal{A}(Y - \ln(k'^2/k^2), k')\Bigg).$$

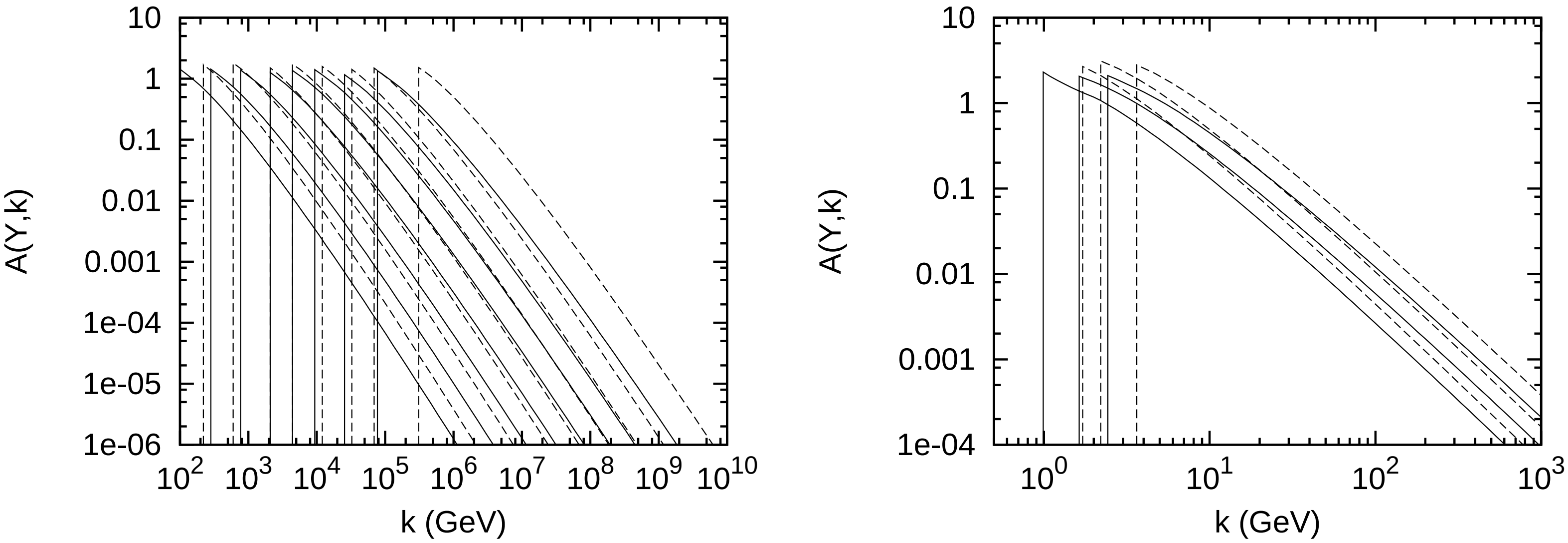
- Solve these equations in the presence of the saturation boundary.

Results



- Left: 1st eq. vs BFKL with kin. cons. Right: BFKL without kin. cons. vs BFKL with kin. cons. $Y = 40 \rightarrow 140$, running $\bar{\alpha}_s$.

Results



- 1st equation vs 2nd. Left: $Y = 40 \rightarrow 120$, Right: $Y = 10, 12, 14$.

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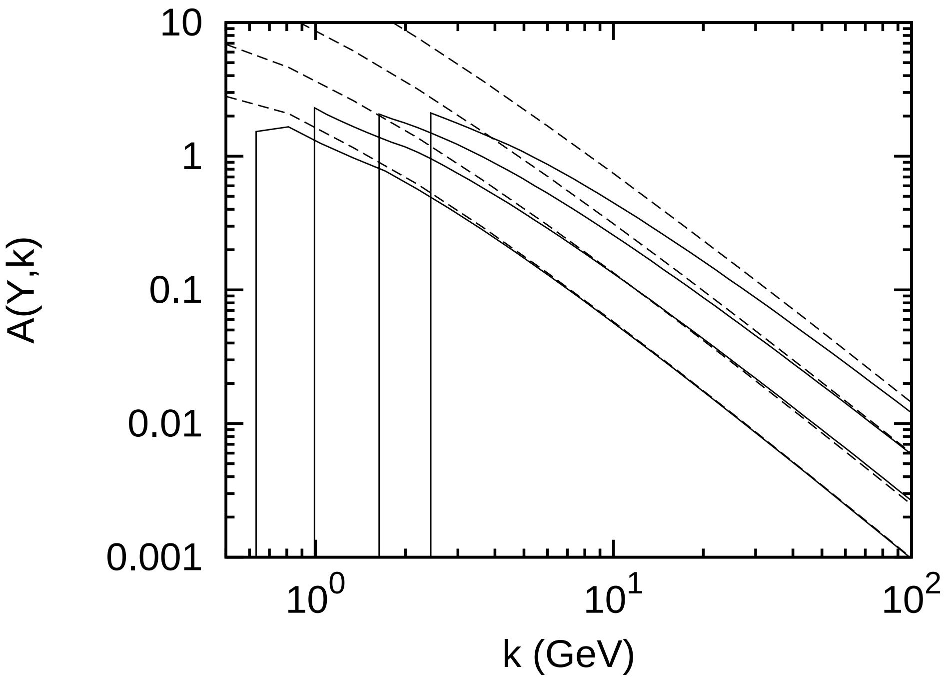

- Solution to 1st equation with and without saturation boundary for $Y = 8, 10, 12, 14$.

Saturation momentum

- For running $\bar{\alpha}_s$, the saturation momentum Q_s can be parametrized as $Q_s = Q_0 \exp(\lambda_r \sqrt{Y})$
- For the 1st eq, and BFKL (with and without kin. cons.) we find $\lambda_r \approx 3.0$
- For 2nd equation we find $\lambda_r \approx 3.2$
- For fixed $\bar{\alpha}_s$ all results consistent with $Q_s = Q_0 \exp(\lambda_f(\bar{\alpha}_s)Y)$.

Some final comments

- Method described can be implemented immediately in present MC's.
- Collaboration with H. Jung at Desy for implementing idea on CASCADE.
- Valuable as one can check effects of saturation on exclusive final states.