Frequency comb spectrum of periodic patterned signals

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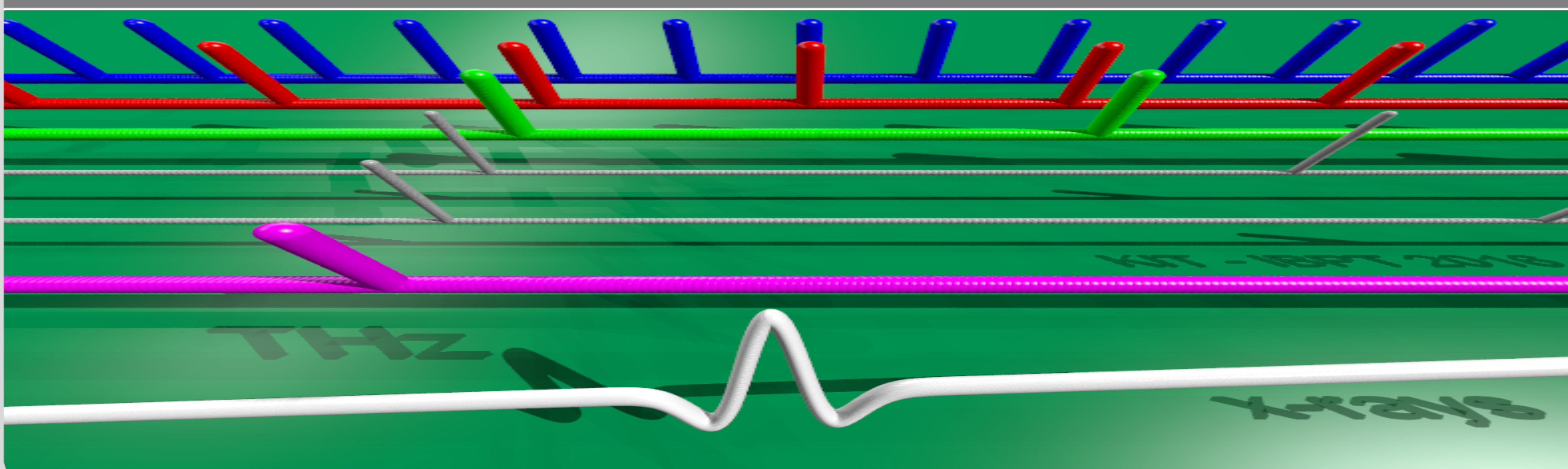
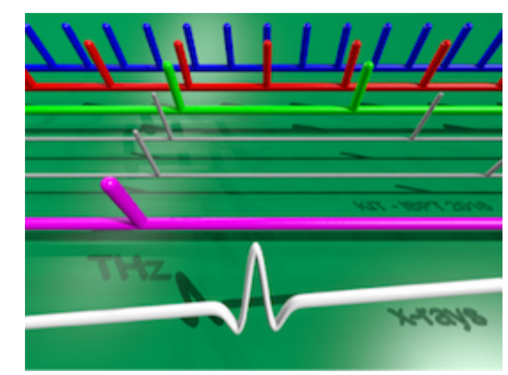
3rd annual meeting of the programme "Matter and Technologies"

Laboratorium für Applikationen der Synchrotronstrahlung (LAS)



Motivation: Frequency Combs (FC)

- Nobel price (Physics 2005)
- Used in
  - atomic clocks
  - frequency metrology
  - high-resolution spectroscopy
  - …
- Generated by repeated emission of identical pulses
- Laser-based FC commercially available in VIS and NIR range
- Less attention paid to accelerator based FC (FEL/synchrotron)
  - High power
  - High stability
  - Broadband spectrum
  - Demonstrated in THz and FIR range



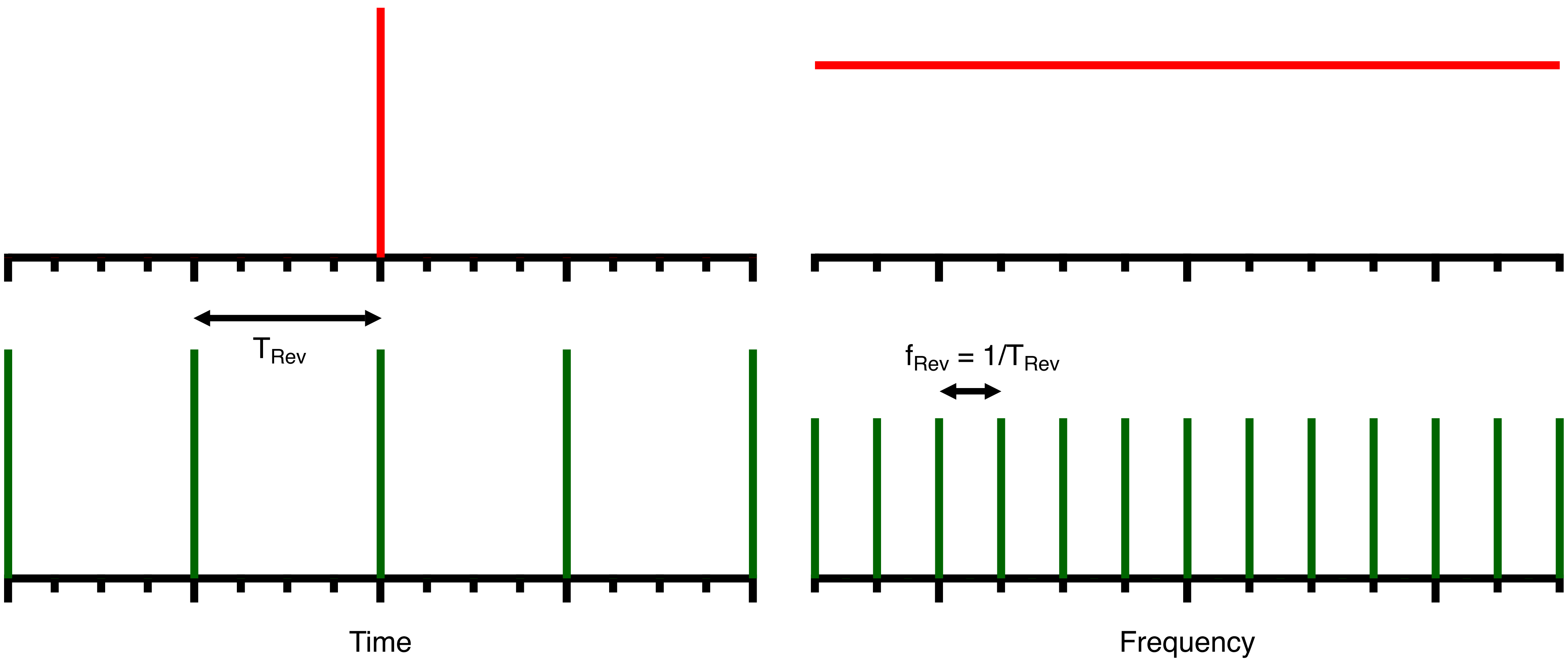
See:

S. Tammaro, et.al., *High density terahertz frequency comb produced by coherent synchrotron radiation* Nat. Commun. **6**, 7733 (2015).

J.L. Steinmann, et.al., *Frequency-Comb Spectrum of Periodic-Patterned Signals* Phys Rev Lett 117 (**17**), 174802 (2016).

Frequency comb generation

- Repeated emission => discrete spectrum

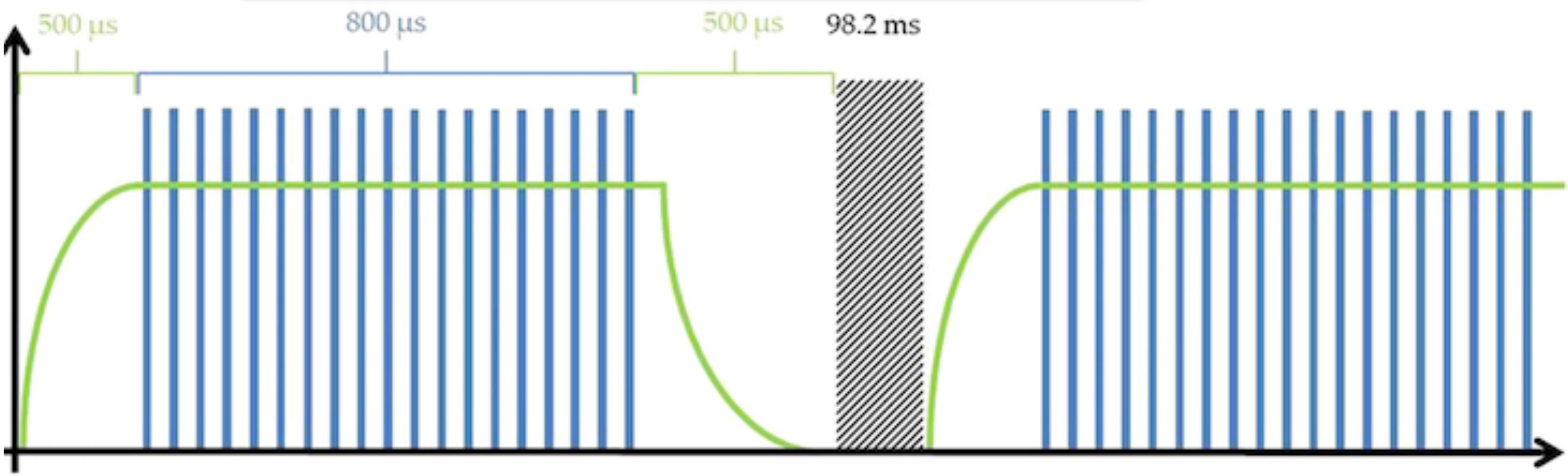


- Accelerators **always** emit periodically.
- How does the spectrum look like?
- What can it be used for?

Frequency comb generation

- Repeated emission => discrete spectrum

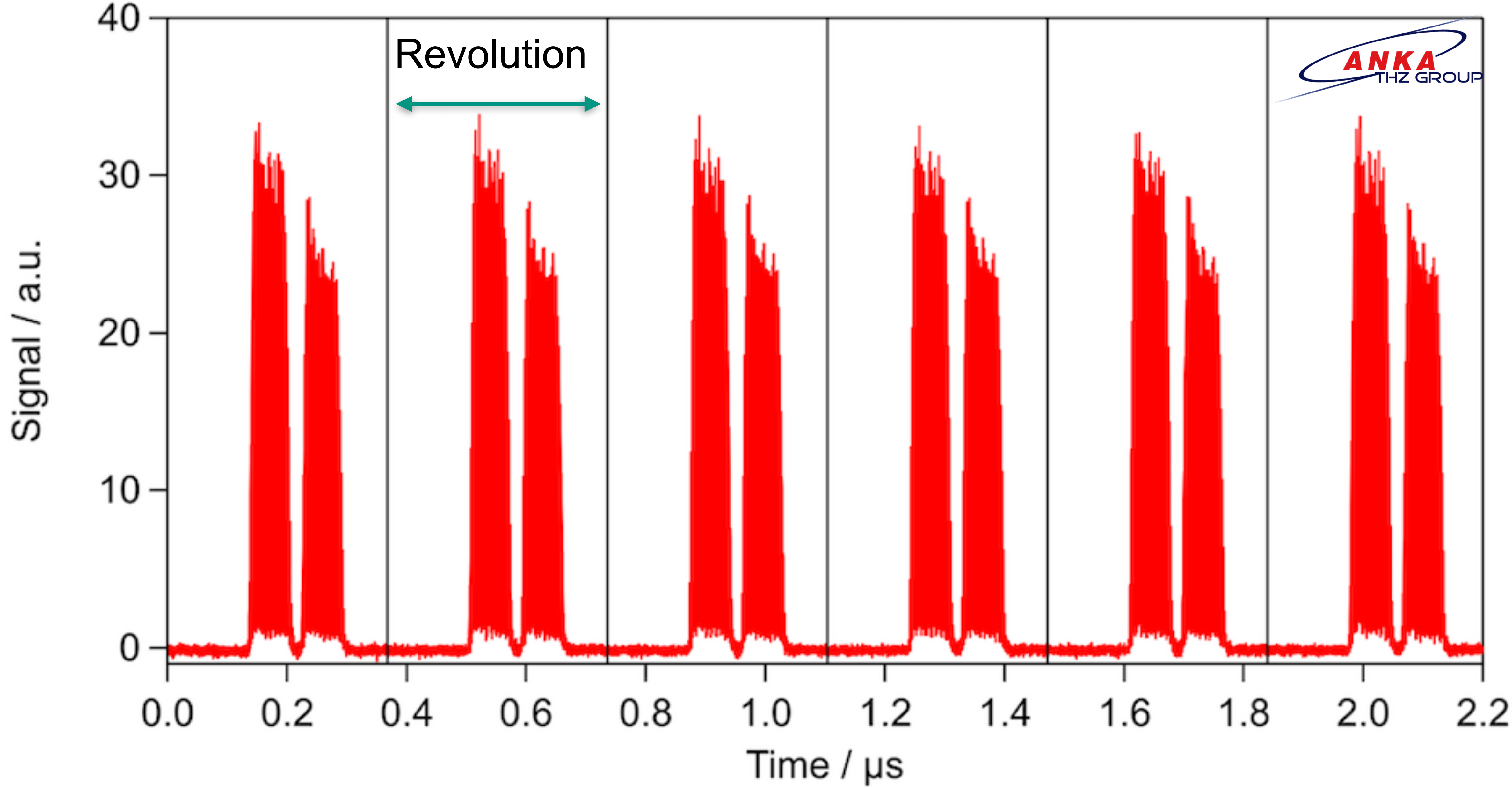
Also linacs emit repeatedly! Example: FLASH



B Faatz et al 2016 New J. Phys. 18 062002

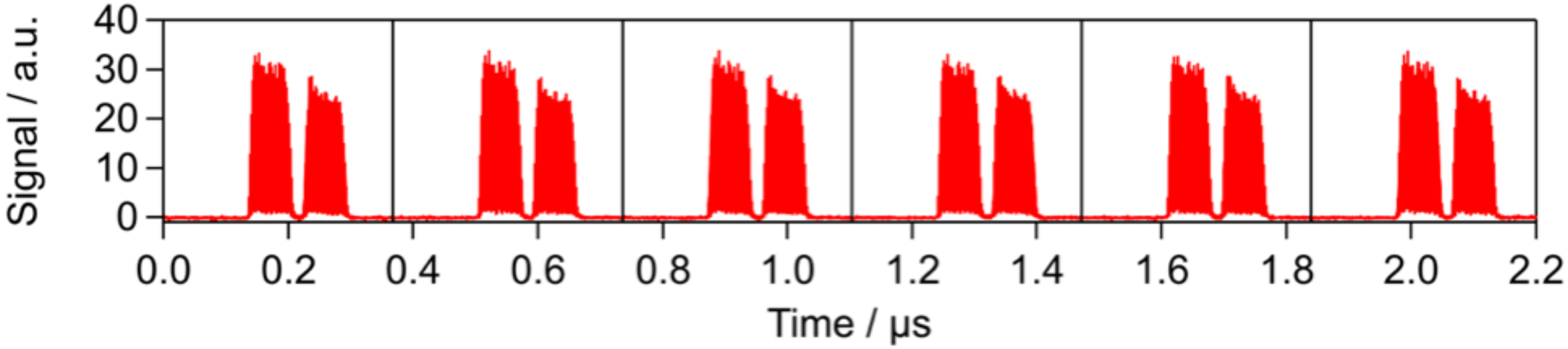
- Accelerators **always** emit periodically.
- How does the spectrum look like?
- What can it be used for?

Example: Signal observed in a synchrotron



Task: find mathematical description of signal

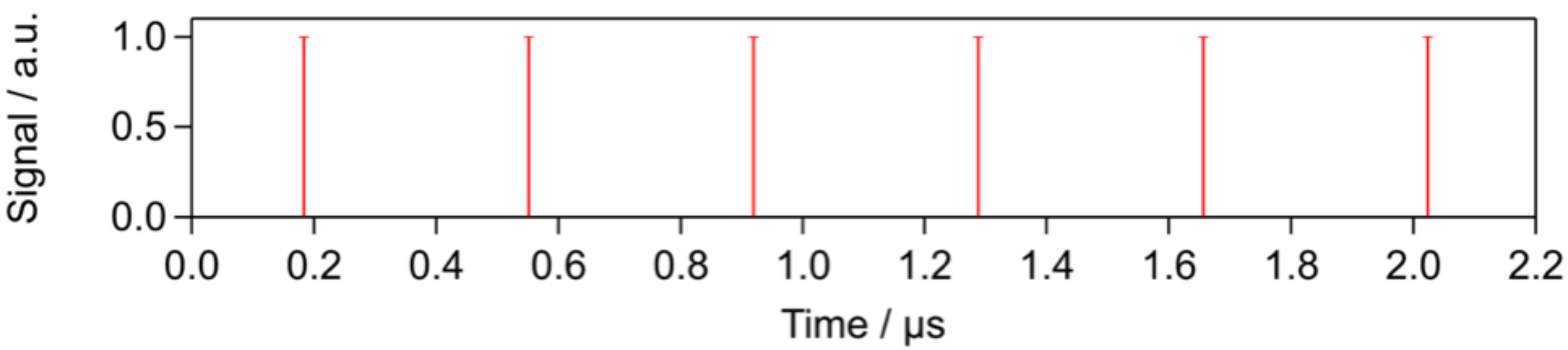
Convolution with infinite series



$$\sum_{n=-\infty}^{\infty} s_0(t-nT_0)$$

$$=$$

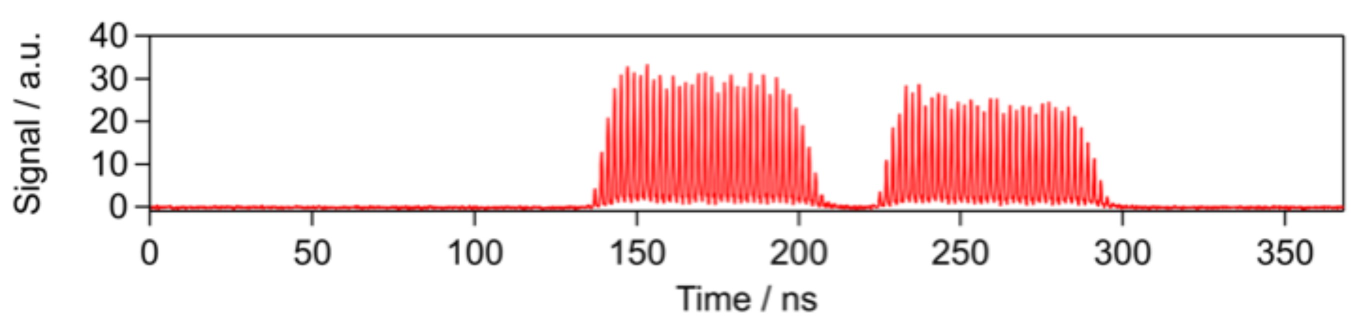
"Shah"-distribution $Ш_{T_0}(t)$ $=$



$$\sum_{n=-\infty}^{\infty} \delta(t-nT_0)$$

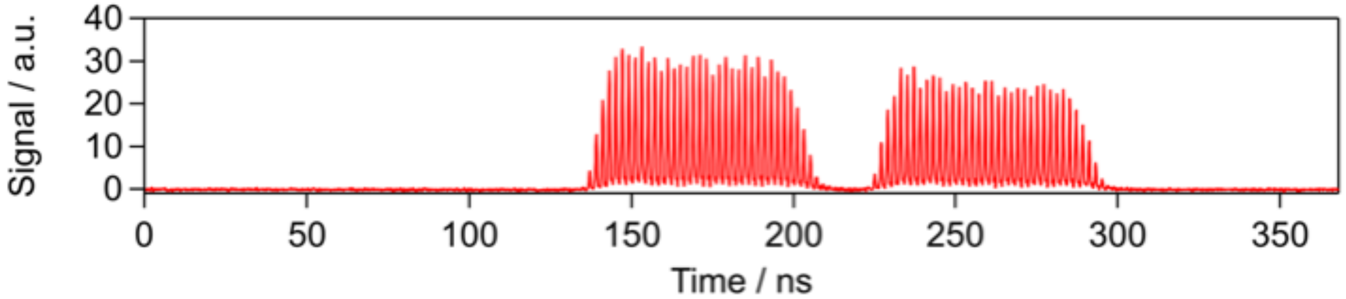
$$*$$

One-turn signal $*$



$$s_0(t)$$

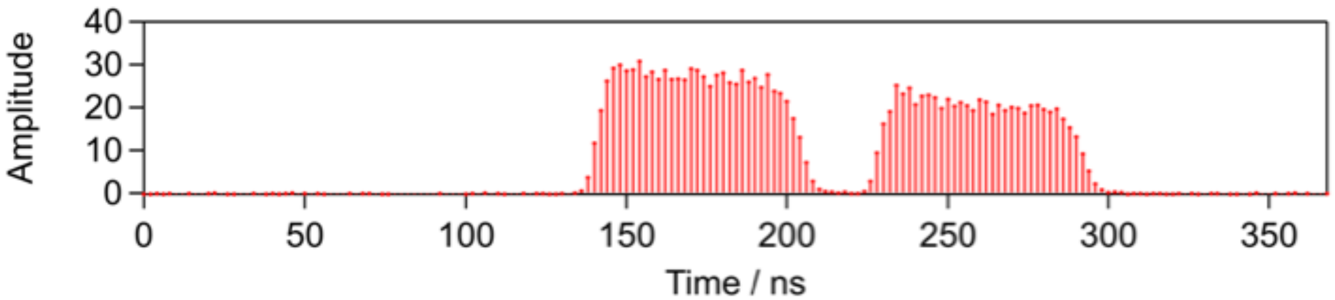
Revolution Signal



$$s_0(t) = \sum_{k=1}^{h} V_K \, s_P(t - kT_P)$$

$=$

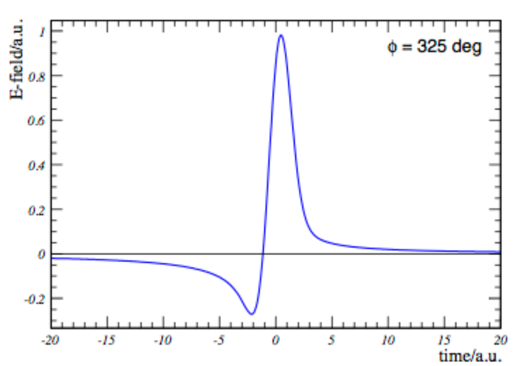
$=$



$$s_F(t) = \sum_{k=1}^{h} V_K \delta(t - kT_P)$$

$*$

$*$



$$s_P(t)$$

From Time to Frequency Domain

$$s(t) = \text{Ш}_{T_0}(t) * s_F(t) * s_P(t)$$

From Time to Frequency Domain

$$s(t) = \text{Ш}_{T_0}(t) * s_F(t) * s_P(t)$$

Fourier Transformation

$$S(f) = \frac{1}{T_0} \text{Ш}_{\frac{1}{T_0}}(f)$$

From Time to Frequency Domain

$$s(t) = Ш_{T_0}(t) * s_F(t) * s_P(t)$$

Fourier Transformation

$$S(f) = \frac{1}{T_0} Ш_{\frac{1}{T_0}}(f) \times \mathrm{DFT}\left\{s_F(t)\right\}$$

From Time to Frequency Domain

$$s(t) = Ш_{T_0}(t) * s_F(t) * s_P(t)$$

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Fourier Transformation

$$S(f) = \frac{1}{T_0}\text{Ш}_{\frac{1}{T_0}}(f) \times \text{DFT}\{s_F(t)\} \times S_P(f)$$

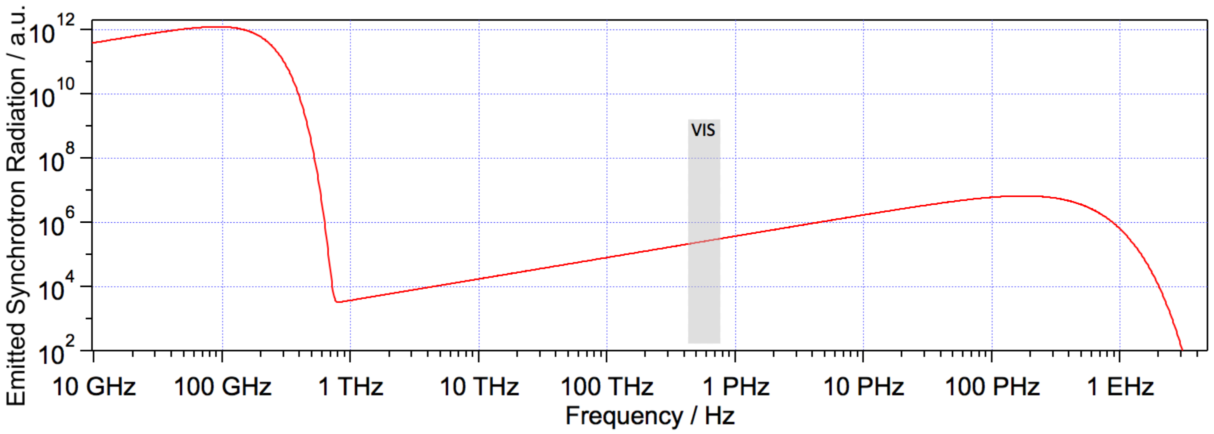
$$\text{DFT}\{s_F(t)\} \equiv \sum_{k=1}^{h} V_k e^{-j2\pi f k T_P}$$

Spectrum: Multiplication of Three Parts

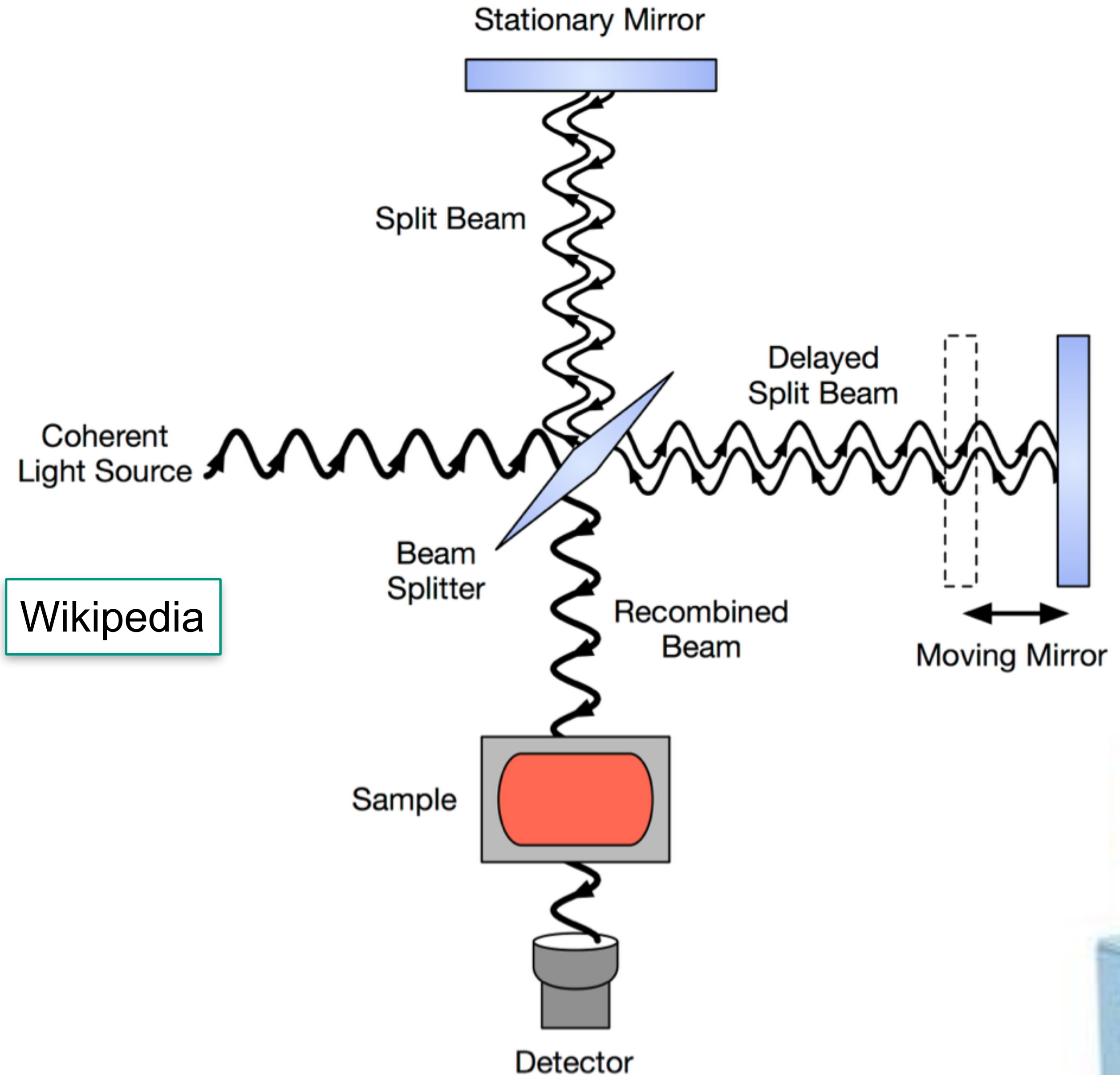
$$S(f) = \frac{1}{T_0} \text{Ш}_{\frac{1}{T_0}}(f) \times \text{DFT}\{s_F(t)\} \times S_P(f)$$

- Shah distribution
  - Discretization, samples spectrum at multiples of revolution frequency
- Discrete Fourier Transform of the filling pattern
  - Sum of every pulse according to its position
  - Periodic with period $f_p = 1/T_p$
  - Repeats up to infinity
  - Continuous
- Spectrum of a single pulse
  - Determines overall spectral shape

$$\text{DFT}\{s_F(t)\} \equiv \sum_{k=1}^{h} V_k e^{-j2\pi f k T_P}$$



How to measure? Interferometer?

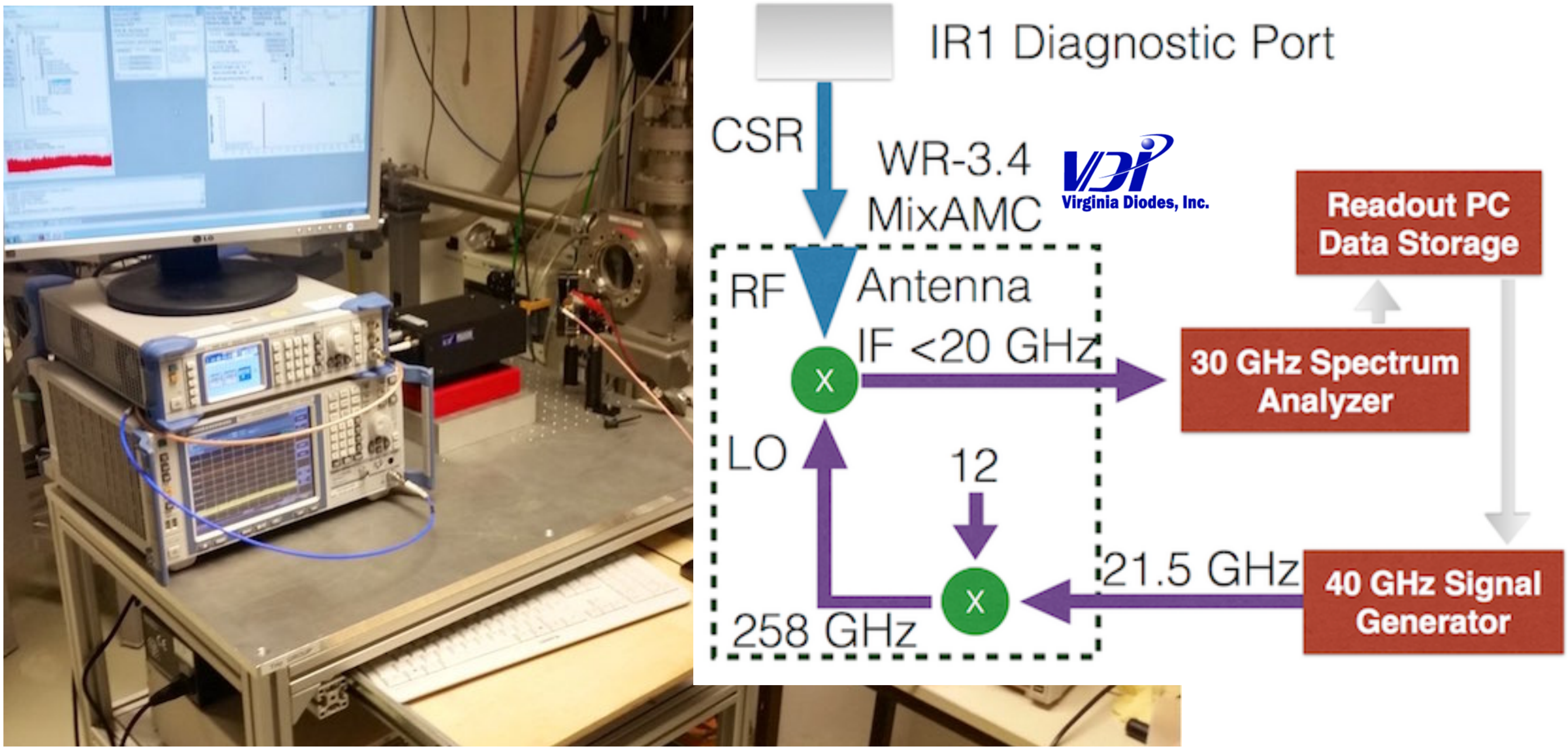


Wikipedia

Resolution	*Path length*
3 GHz	*10 cm*
300 MHz	*1 m*
30 MHz	*10 m*
3 MHz	*100 m*

IFS125HR, © Bruker Optics GmbH (up to 6m arm)

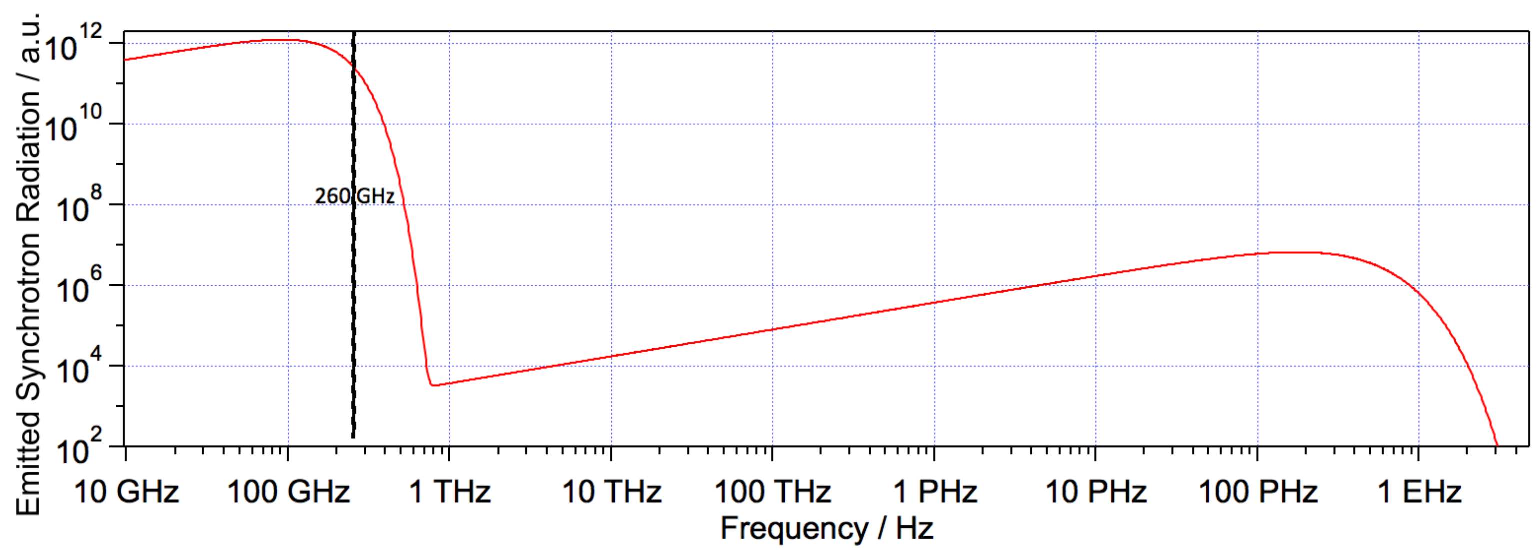
Heterodyne Measurements

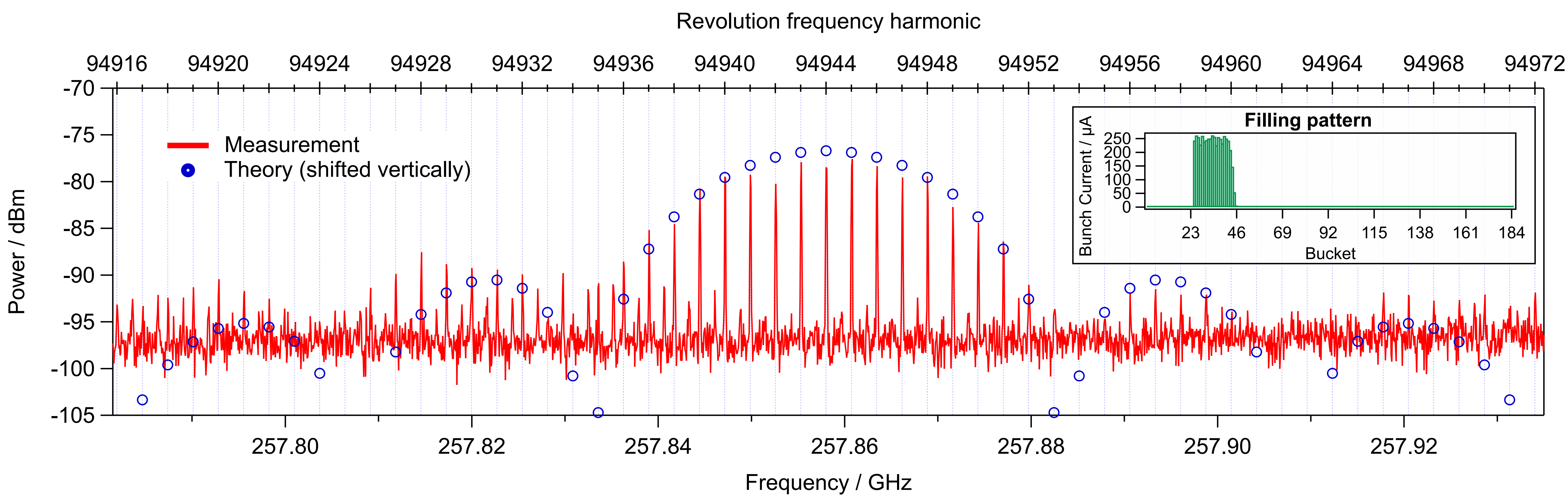


- Resolution limited by spectrum analyzer: 1 Hz
- Bandwidth limited by mixer: 40 GHz (LO ± 20 GHz)

One Train

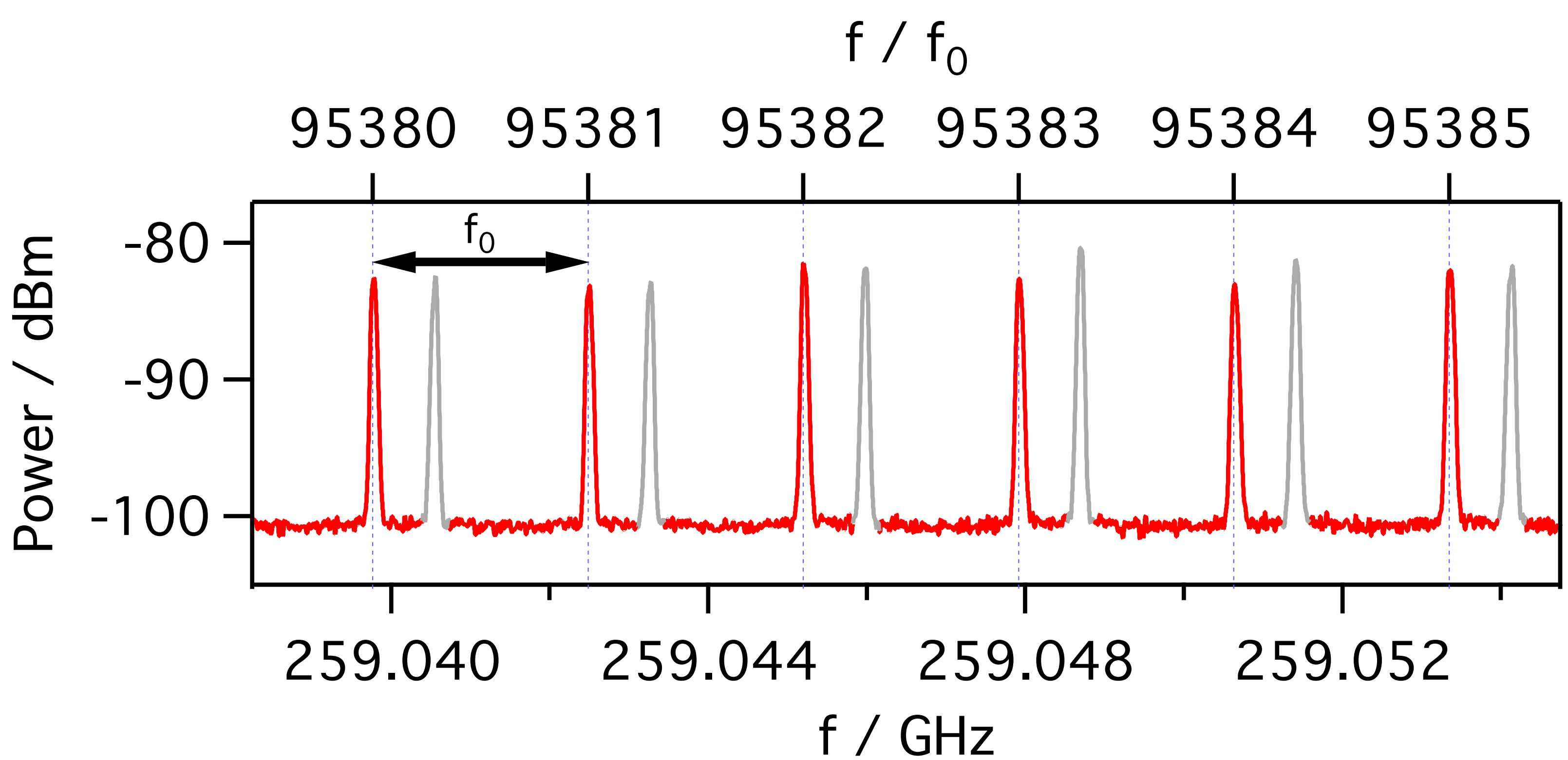
- A single train
- Almost identical currents
- DFT mimics a sinc function

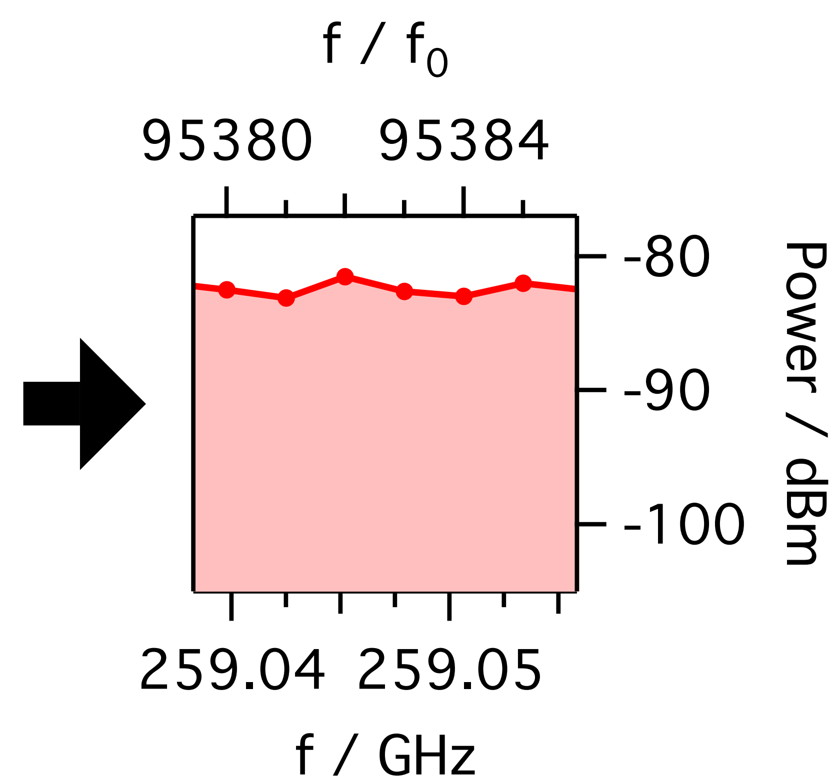




Observed discrete nature of SR at THz frequencies

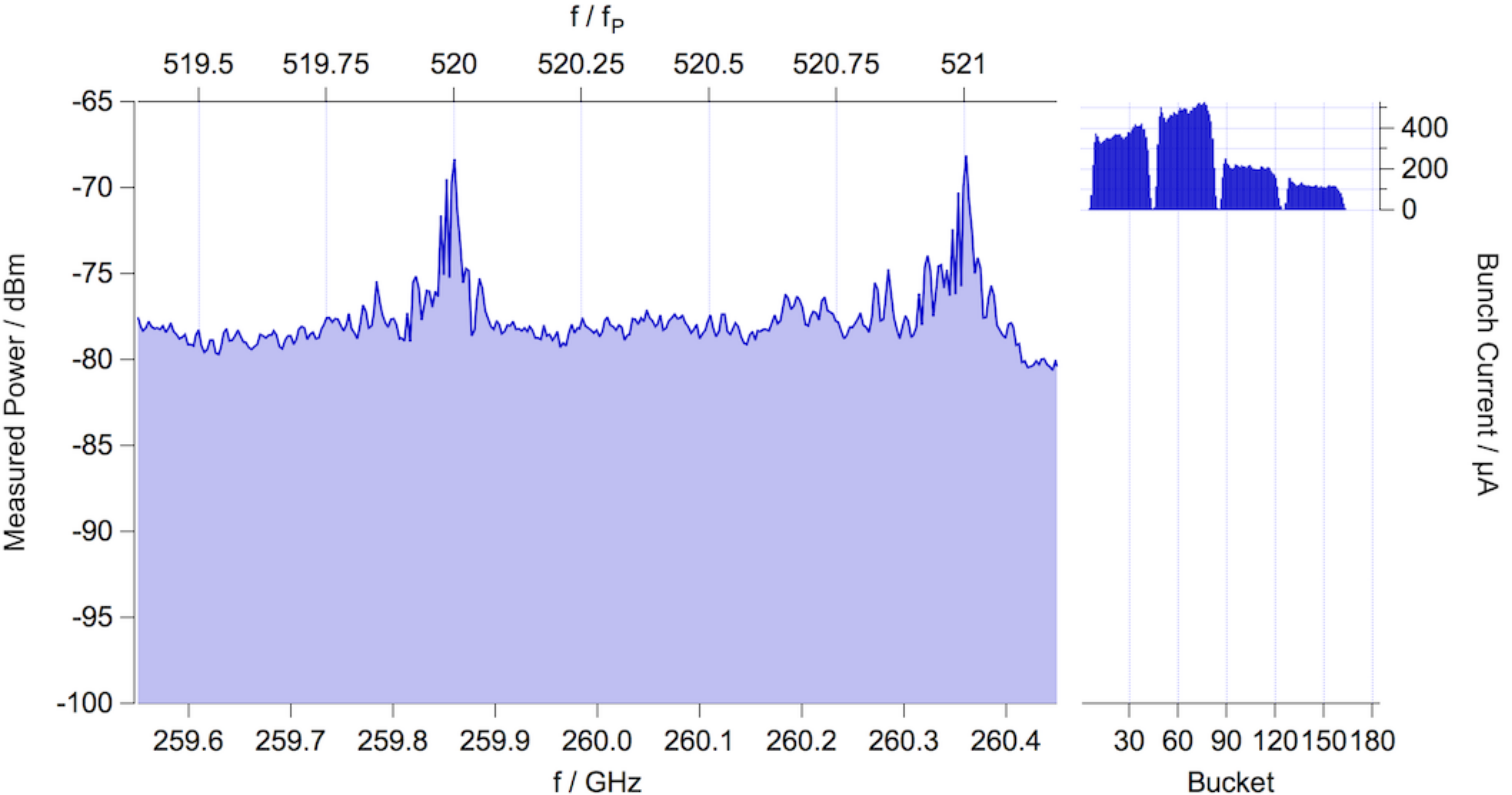
- Position of frequency peaks can be adjusted with RF => revolution frequency



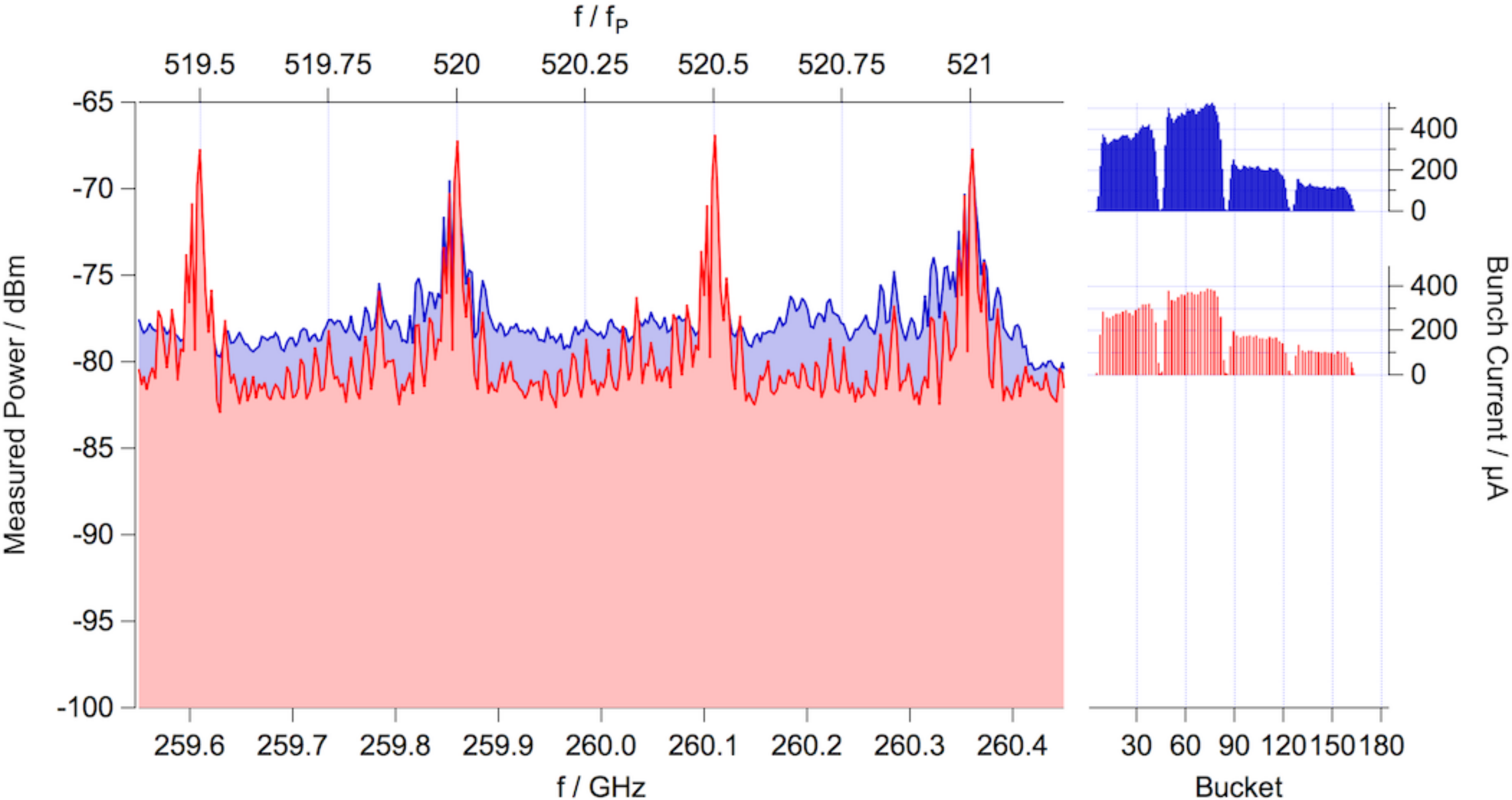


- Data reduction: Save only power of every revolution harmonic

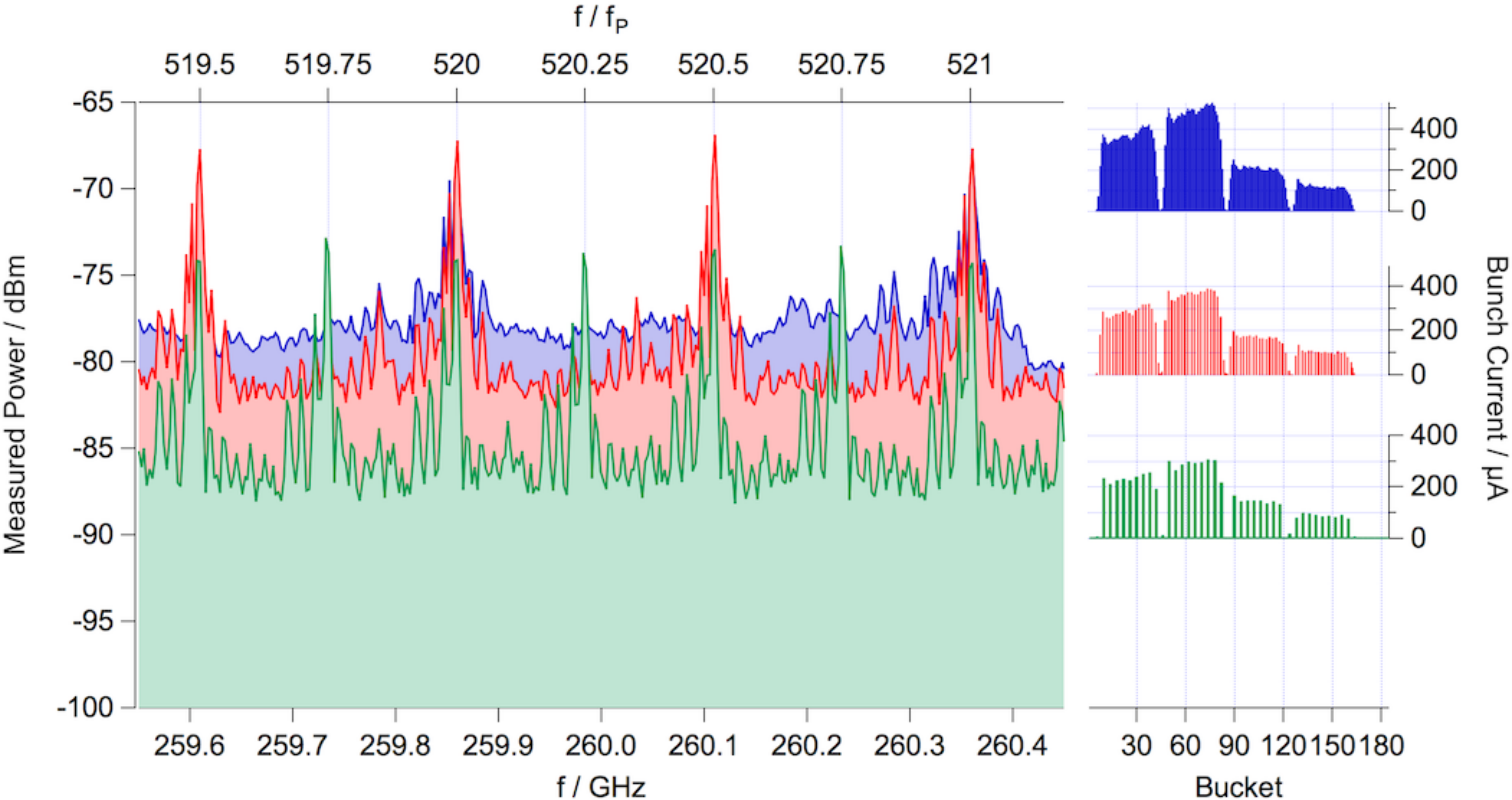
Spectra of different filling patterns



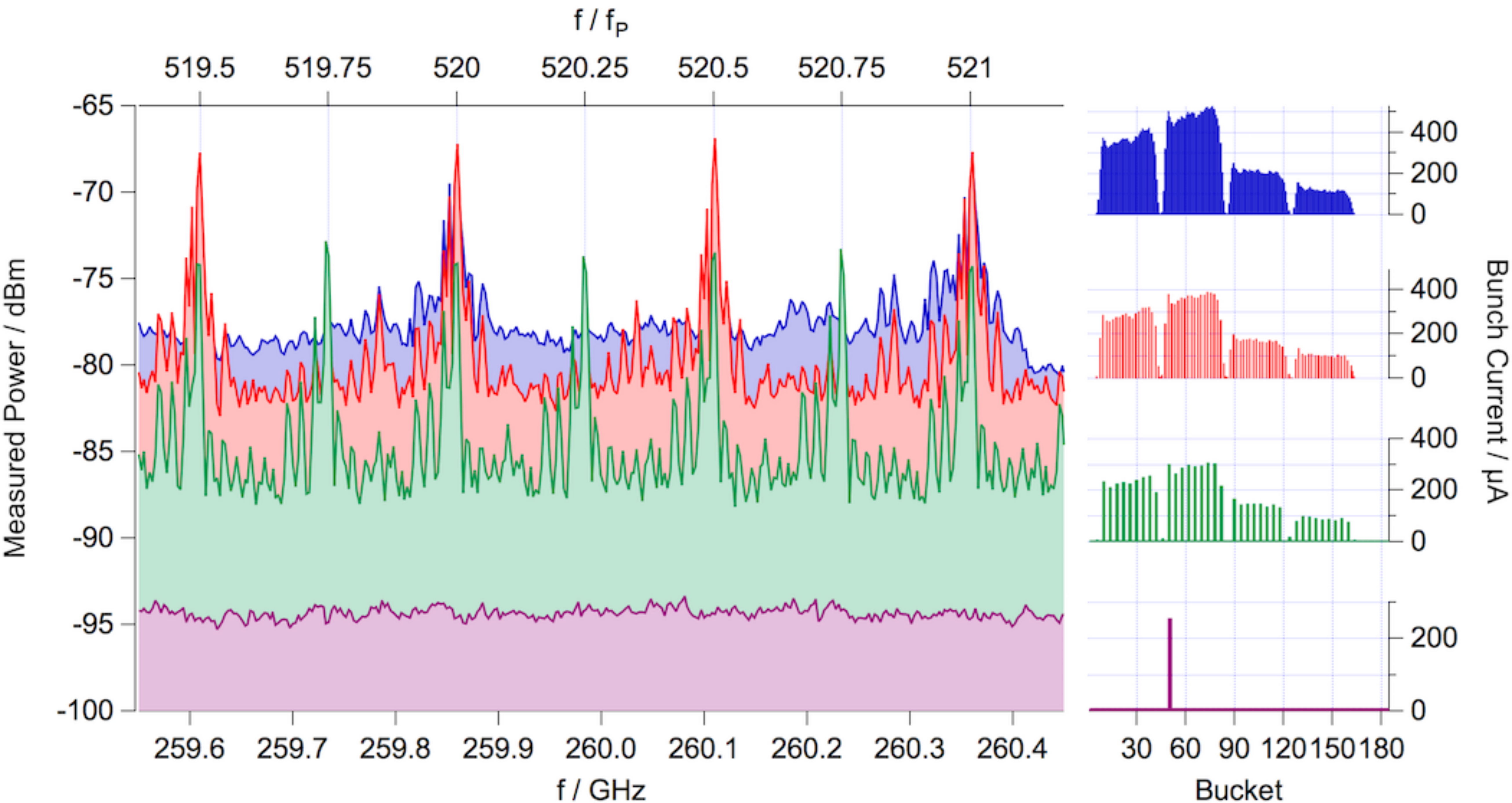
Spectra of different filling patterns



Spectra of different filling patterns

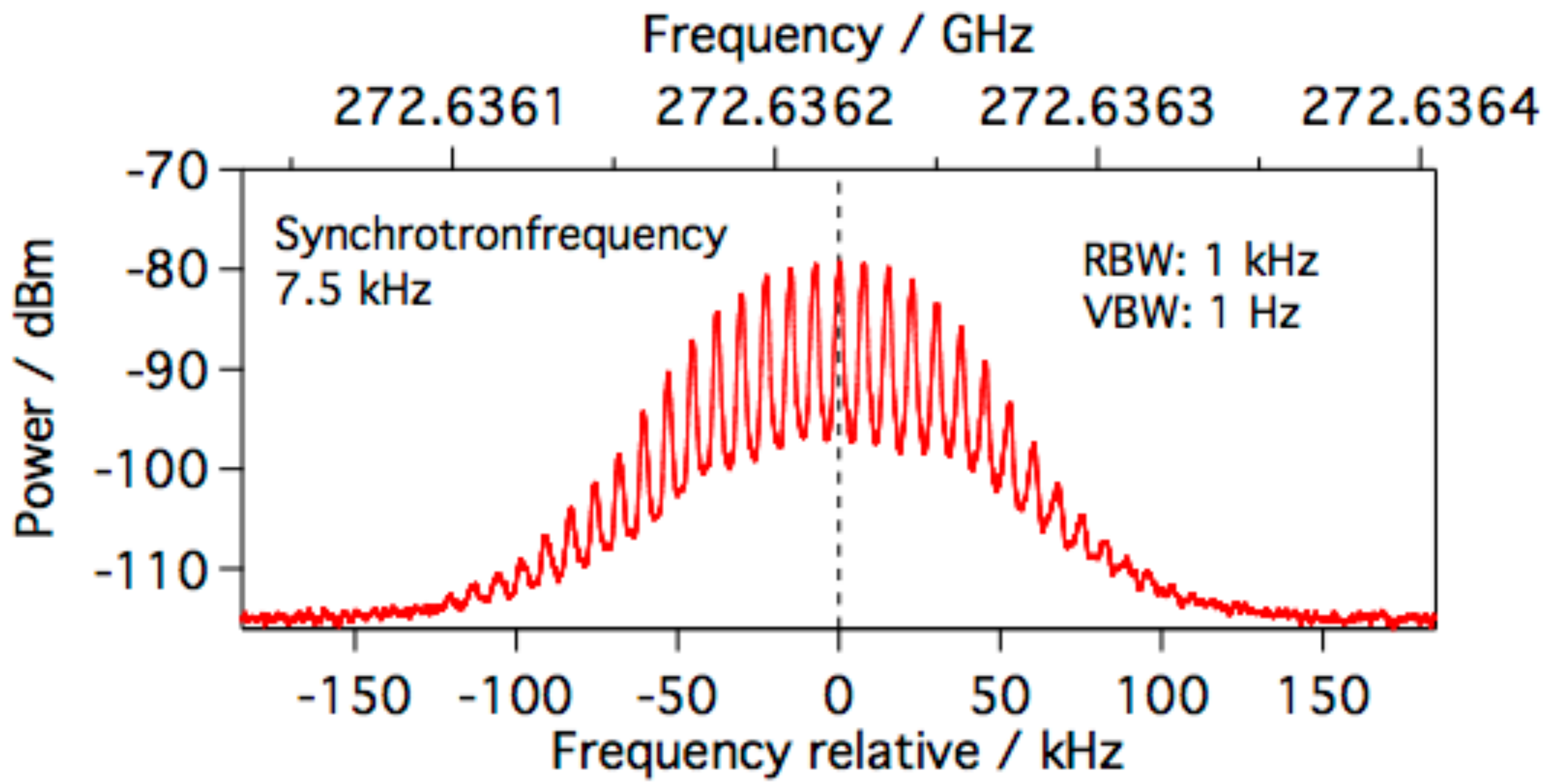


Spectra of different filling patterns



Zoom into single frequency peak

- Synchrotron motion leads to sidebands
- Intensity of sideband *m* dependent on Bessel function of order *m*
- Effect scales with number of harmonic and amplitude of oscillation
- Diagnostic tool (?)



Take-Home-Message

- Repeated radiation leads to frequency comb
  - Synchrotron radiation from storage ring / FEL is discrete
  - Can be resolved by high resolution spectroscopy

- Spectrum consists of
  - Revolution harmonics
  - Single pulse spectrum
  - Discrete Fourier Transformation of filling pattern

- Filling pattern “adjustment” can create “super-radiant” frequencies
- Spectrum can be scanned by changing the revolution frequency

Thanks to all colleagues who made this possible

- **Fruitful collaboration of**
  - Accelerator Research and Development
  - Detector Technologies and Systems
  - Industry Partner

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Frequency-Comb Spectrum of Periodic-Patterned Signals

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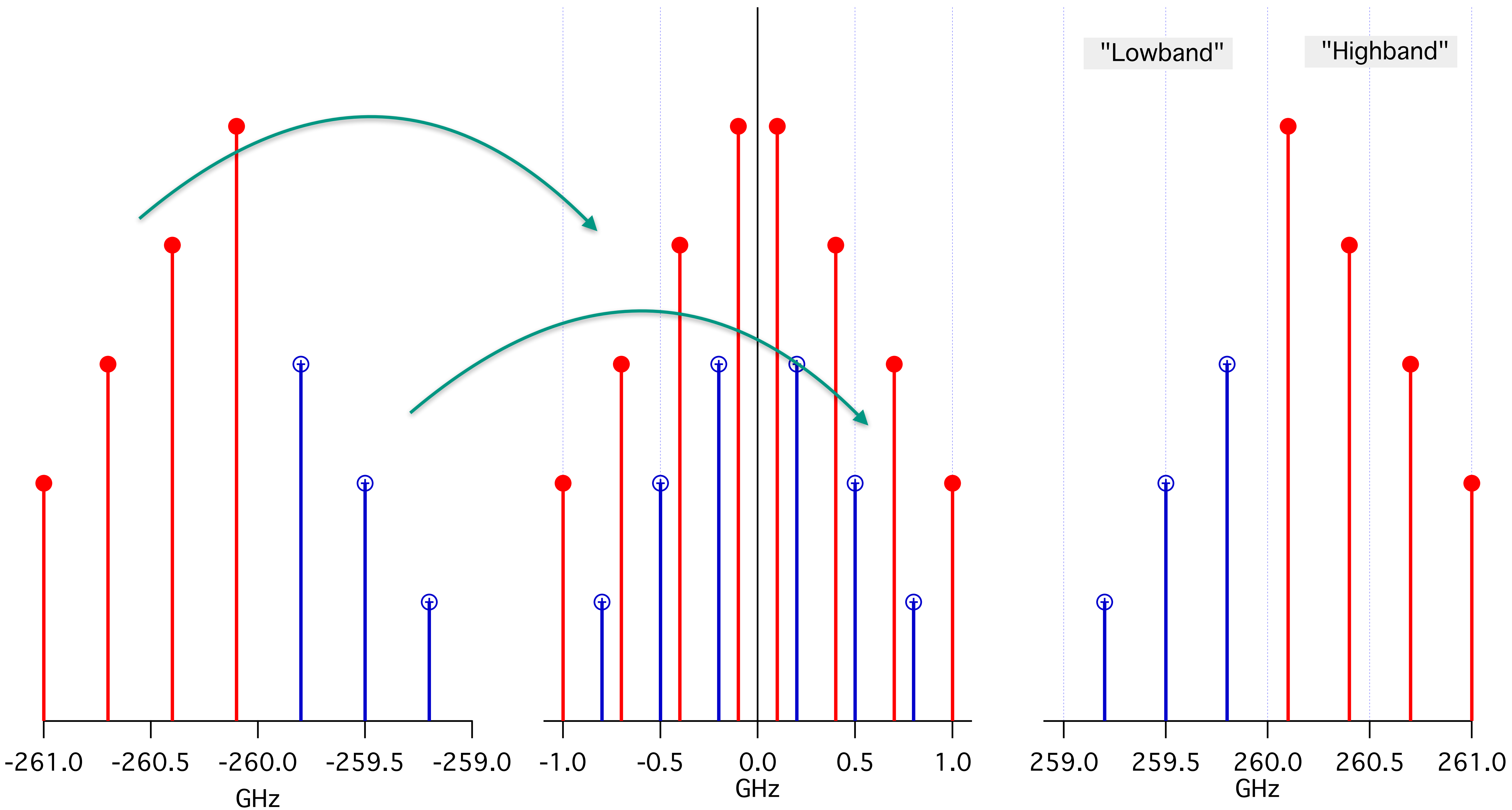
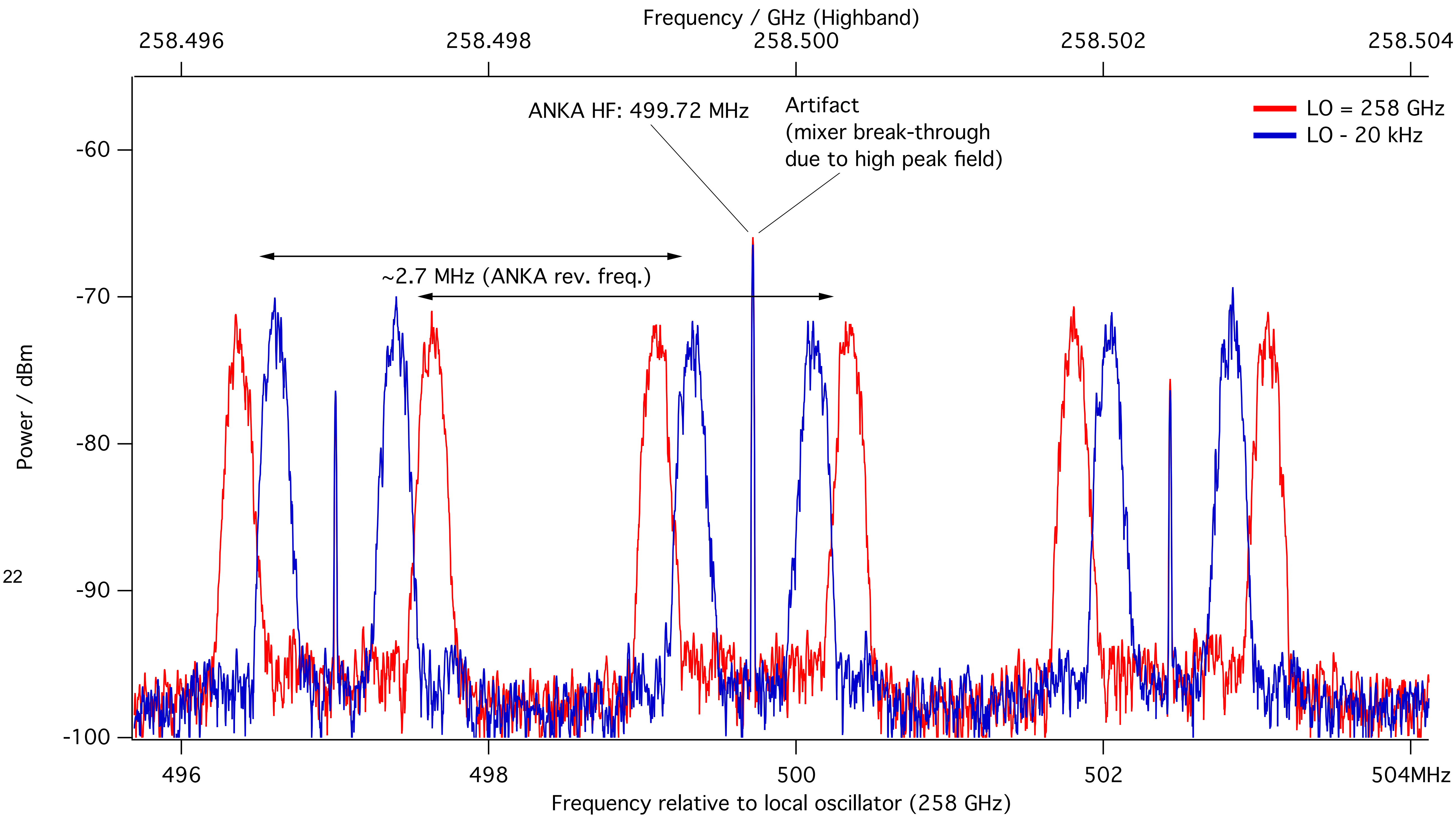
Down mixing

“Double-Side-Band” (DSB) mixing



Measurements: Highband and Lowband



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Measurements: Full IF spectrum