Covariant diagrams for one-loop EFT matching

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Based on 1610.00710

The EFT matching problem

- Given $\mathcal{L}_{\rm UV}[\Phi, \phi]$ with $M_\Phi \gg m_\phi$, at low energy $E \ll M_\Phi$,

$$\mathcal{L}_{\rm EFT}[\phi] = ?$$

- Why interesting?
  - Interpretation of model-independent EFT results in UV models.
  - Precise calculation of low-energy observables.
  - Theoretical curiosity — matching without Feynman diagrams!

The EFT matching problem

$$\mathcal{L}_{\text{EFT}}[\phi] = ?$$

- Solution #1: Feynman diagram matching (familiar).
  - Calculate scattering amplitudes/correlation functions with Feynman diagrams in both UV theory and EFT.
  - Equate the results and solve for EFT operator coefficients.
- Solution #2: functional matching (more elegant, often simpler).
  - Manipulate path integral to directly derive EFT operators.
  - Can preserve gauge covariance in intermediate steps via CDE.
  - Can obtain universal results (master formulas).

Literature overview

- Earlier developments
  - M.K. Gaillard [Nucl.Phys.B268,669 (1986)]
  - L.H. Chan [Phys.Rev.Lett.57,1199 (1986)]
  - O. Cheyette [Nucl.Phys.B297,183 (1988)]
- Recent revival, developments and applications
  - B. Henning, X. Lu, H. Murayama [1412.1837, 1604.01019].
  - C.-W. Chiang, R. Huo [1505.06334].
  - R. Huo [1506.00840, 1509.05942].
  - A. Drozd, J. Ellis, J. Quevillon, T. You [1512.03003].
  - F. del Aguila, Z. Kunszt, J. Santiago [1602.00126].
  - M. Boggia, R. Gomez-Ambrosio, G. Passarino [1603.03660].
  - S. Ellis, J. Quevillon, T. You, ZZ [1604.02445].
  - J. Fuentes-Martin, J. Portoles, P. Ruiz-Femenia [1607.02142].
  - ZZ [1610.00710].

This talk

- Basic ideas of gauge-covariant functional matching.
- Functional matching at one loop with covariant diagrams.
  - Diagrammatic representation of CDE.
    - Compare Feynman diagrams --- diagrammatic representation of (non-gauge-covariant) expansion of correlation functions.
  - Builds upon previous literature on functional matching, but…
    - No complicated functional manipulations; rules easy to use.
    - Applicable beyond minimal case.

Basic idea (minimal technical details)

background field

- Matching condition: $\Gamma_{\mathrm{L,UV}}[\phi_{\mathrm{b}}] = \Gamma_{\mathrm{EFT}}[\phi_{\mathrm{b}}]$.
  - 1LPI effective action calculated in the UV theory:

source

$$\Gamma_{\mathrm{L,UV}}[\phi_{\mathrm{b}}] = -i \log Z_{\mathrm{UV}}[J_\Phi = 0, J_\phi] - \int d^d x \, J_\phi \phi_{\mathrm{b}}$$

  - 1PI effective action calculated in the EFT:

$$\Gamma_{\mathrm{EFT}}[\phi_{\mathrm{b}}] = -i \log Z_{\mathrm{EFT}}[J_\phi] - \int d^d x \, J_\phi \phi_{\mathrm{b}}$$

where

$$Z_{\mathrm{UV}}[J_\Phi, J_\phi] = \int [D\Phi][D\phi] \, e^{\, i \int d^d x (\mathcal{L}_{\mathrm{UV}}[\Phi,\phi] + J_\Phi \Phi + J_\phi \phi)}$$

$$Z_{\mathrm{EFT}}[J_\phi] = \int [D\phi] \, e^{\, i \int d^d x (\mathcal{L}_{\mathrm{EFT}}[\phi] + J_\phi \phi)}$$

Background field method

- To calculate 1(L)PI effective actions (real scalar example):

$$\Phi = \Phi_{\rm b} + \Phi' \,, \quad \phi = \phi_{\rm b} + \phi' \quad \Rightarrow$$

"quadratic matrix"

$$\begin{aligned}
&\mathcal{L}_{\rm UV}[\Phi, \phi] + J_\Phi \Phi + J_\phi \phi \\
&= \mathcal{L}_{\rm UV}[\Phi_{\rm b}, \phi_{\rm b}] + J_\Phi \Phi_{\rm b} + J_\phi \phi_{\rm b} \\
&\quad - \frac{1}{2} \begin{pmatrix} \Phi'^T & \phi'^T \end{pmatrix} \begin{pmatrix} \Delta_H[\Phi_{\rm b}, \phi_{\rm b}] & X_{HL}[\Phi_{\rm b}, \phi_{\rm b}] \\ X_{LH}[\Phi_{\rm b}, \phi_{\rm b}] & \Delta_L[\Phi_{\rm b}, \phi_{\rm b}] \end{pmatrix} \begin{pmatrix} \Phi' \\ \phi' \end{pmatrix} \\
&\quad + \ldots
\end{aligned}$$

➡ tree (classical)

➡ 1-loop

➡ higher orders

  - Recall $0 = \frac{\delta \mathcal{L}_{\rm UV}}{\delta \Phi}[\Phi_{\rm b}, \phi_{\rm b}] + J_\Phi = \frac{\delta \mathcal{L}_{\rm UV}}{\delta \phi}[\Phi_{\rm b}, \phi_{\rm b}] + J_\phi \Rightarrow$ linear terms vanish.

- Similar expansion on the EFT side.

Matching results

- Assuming $X_{HL} = X_{LH} = 0$ for now (will generalize later),

$$\boxed{\begin{gathered} \mathcal{L}_{\text{EFT}}^{\text{tree}}[\phi] = \mathcal{L}_{\text{UV}}\big[\Phi_{\text{c}}[\phi], \phi\big] \\ \int d^d x\, \mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\phi] = \frac{i}{2}\log\det \Delta_H\big[\Phi_{\text{c}}[\phi], \phi\big] = \frac{i}{2}\text{Tr}\,\log \Delta_H\big[\Phi_{\text{c}}[\phi], \phi\big] \end{gathered}}$$

where $\Phi_{\text{c}}[\phi]$ solves the classical EoM: $\frac{\delta \mathcal{L}_{\text{UV}}}{\delta \Phi}\big[\Phi_{\text{c}}[\phi], \phi\big] = 0$.

Evaluating functional trace
(familiar from quantum mechanics)

$$\begin{aligned}
& \mathrm{Tr}\,\mathcal{O}(\hat{x},\hat{p}) \\
= \quad & \int \frac{d^dq}{(2\pi)^d}\,\langle q|\,\mathrm{tr}\,\mathcal{O}(\hat{x},\hat{p})\,|q\rangle = \int d^dx \int \frac{d^dq}{(2\pi)^d}\,\langle q|\,x\rangle\langle x|\,\mathrm{tr}\,\mathcal{O}(\hat{x},\hat{p})\,|q\rangle \\
= \quad & \int d^dx \int \frac{d^dq}{(2\pi)^d}\,e^{iq\cdot x}\,\mathrm{tr}\,\mathcal{O}(x,i\partial_x)\,e^{-iq\cdot x} = \int d^dx \int \frac{d^dq}{(2\pi)^d}\,\mathrm{tr}\,\mathcal{O}(x,i\partial_x+q) \\
= \quad & \int d^dx \int \frac{d^dq}{(2\pi)^d}\,\mathrm{tr}\,\mathcal{O}(x,i\partial_x-q)
\end{aligned}$$

- Recall $\int d^dx\,\mathcal{L}_{\mathrm{EFT}}^{\text{1-loop}}[\phi] = \frac{i}{2}\mathrm{Tr}\,\log\Delta_H$

$P_\mu \equiv iD_\mu$ (hermitian)

$$\Rightarrow \quad \mathcal{L}_{\mathrm{EFT}}^{\text{1-loop}}[\phi] = \frac{i}{2}\int \frac{d^dq}{(2\pi)^d}\,\mathrm{tr}\log\Delta_H|_{P\to P-q}$$

Covariant derivative expansion (CDE)

Fuentes-Martin-Portoles-Ruiz-Femenia version (1607.02142)

- Recall $\mathcal{L}_{\mathrm{UV}}[\Phi = \Phi_{\mathrm{b}} + \Phi',\, \phi = \phi_{\mathrm{b}} + \phi'] \supset -\frac{1}{2}\,\Phi'^{T}\,\Delta_H[\Phi_{\mathrm{b}}, \phi_{\mathrm{b}}]\,\Phi'$
- General form of quadratic terms for bosonic fields:

$$\Delta_H = -P^2 + M^2 + X_H$$

where

$$P_\mu = iD_\mu\,, \quad M = \mathrm{diag}\,(M_1, M_2, \dots)\,,$$

$$\begin{aligned} X_H[\Phi, \phi, P_\mu] = {} & U_H[\Phi, \phi] + P_\mu Z_H^\mu[\Phi, \phi] + Z_H^{\dagger\mu}[\Phi, \phi] P_\mu \\ & + P_\mu P_\nu Z_H^{\mu\nu}[\Phi, \phi] + Z_H^{\dagger\mu\nu}[\Phi, \phi] P_\nu P^\mu + \dots \end{aligned}$$

Covariant derivative expansion (CDE)

Fuentes-Martin-Portoles-Ruiz-Femenia version (1607.02142)

$$\Delta_H = -P^2 + M^2 + X_H$$

- Expand for $M \to \infty$ while keeping P_μ's intact (as opposed to separating into $i\partial_\mu$ and $gA_\mu^a T^a$).

$$\begin{aligned}\Rightarrow \quad \mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\phi] &= \frac{i}{2}\int \frac{d^d q}{(2\pi)^d}\, \text{tr} \log \Delta_H|_{P\to P-q} \\ &= \frac{i}{2}\int \frac{d^d q}{(2\pi)^d}\, \text{tr} \log\big(-q^2 + M^2 + 2q\cdot P - P^2 + X_H|_{P\to P-q}\big) \\ &= \text{const.} - \frac{i}{2}\, \text{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d q}{(2\pi)^d} \Big[\big(q^2 - M^2\big)^{-1}\big(2q\cdot P - P^2 + X_H|_{P\to P-q}\big)\Big]^n\end{aligned}$$

To recap...

- Matching condition: $\Gamma_{\mathrm{L,UV}}[\phi_{\mathrm{b}}] = \Gamma_{\mathrm{EFT}}[\phi_{\mathrm{b}}]\,.$
- Background field method $\Phi = \Phi_{\mathrm{b}} + \Phi'\,,\ \phi = \phi_{\mathrm{b}} + \phi'\ \Rightarrow$ (details skipped)

$$\int d^dx\,\mathcal{L}^{\text{1-loop}}_{\mathrm{EFT}}[\phi] = \frac{i}{2}\log\det\Delta_H\left[\Phi_{\mathrm{c}}[\phi],\phi\right] = \frac{i}{2}\mathrm{Tr}\,\log\Delta_H\left[\Phi_{\mathrm{c}}[\phi],\phi\right]$$

- Evaluate trace $\Rightarrow\ \mathcal{L}^{\text{1-loop}}_{\mathrm{EFT}}[\phi] = \frac{i}{2}\int\frac{d^dq}{(2\pi)^d}\,\mathrm{tr}\log\Delta_H|_{P\to P-q}$
- Plug in general form $\Delta_H = -P^2 + M^2 + X_H$ and perform CDE

$$\mathcal{L}^{\text{1-loop}}_{\mathrm{EFT}}[\phi] = -\frac{i}{2}\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\left[\left(q^2 - M^2\right)^{-1}\left(2q\cdot P - P^2 + X_H|_{P\to P-q}\right)\right]^n$$

Covariant diagrams

$$\mathcal{L}_{\mathrm{EFT}}^{\text{1-loop}}[\phi] = -\frac{i}{2}\,\mathrm{tr}\sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^dq}{(2\pi)^d}\Big[\underbrace{\left(q^2-M^2\right)^{-1}}_{\text{propagator}}\underbrace{\left(2q\cdot P-P^2+X_H|_{P\to P-q}\right)}_{\text{vertex insertions}}\Big]^n$$

- This is a sum of one-loop diagrams!
- Loop integrals factor out, with the form

$$\int\frac{d^dq}{(2\pi)^d}\frac{q^{\mu_1}\cdots q^{\mu_{2n_c}}}{(q^2-M_i^2)^{n_i}(q^2-M_j^2)^{n_j}\cdots}\equiv g^{\mu_1\cdots\mu_{2n_c}}\,\mathcal{I}[q^{2n_c}]_{ij\ldots}^{n_in_j\ldots}$$

master integral

completely symmetric tensor, e.g. $g^{\mu\nu\rho\sigma}=g^{\mu\nu}g^{\rho\sigma}+g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}$

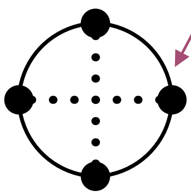
  - All terms Lorentz-contract P_μ's from $2q\cdot P$ insertions in all possible ways.

Example:

threshold matching of gauge coupling (scalar case)

- Integrate out a complex scalar of mass M, and extract $P^4 \sim D^4 \sim G^2$ terms in the EFT.

represents one term in the CDE



$$= -2 \cdot \frac{i}{2} \cdot \frac{1}{4} \cdot \mathcal{I}[q^4]_i^4 \,\mathrm{tr}(2P^\mu \cdot 2P^\nu \cdot 2P_\mu \cdot 2P_\nu) \subset -2i\,\mathcal{I}[q^4]_i^4 \,\mathrm{tr}\big([P^\mu, P^\nu][P_\mu, P_\nu]\big)$$

$$= 2ig^2\,\mathcal{I}[q^4]_i^4 \,\mathrm{tr}(G^{\mu\nu}G_{\mu\nu}) = -\frac{g^2}{16\pi^2}\frac{1}{3}\log\frac{M^2}{\mu^2}\left[-\frac{1}{4}\,\mathrm{tr}(G^{\mu\nu}G_{\mu\nu})\right]$$

Dynkin index of representation R

$$\Rightarrow \qquad \frac{g^2_{\text{eff}}(\mu)}{g^2(\mu)} = 1 + \frac{g^2}{16\pi^2}T(R)\cdot\frac{1}{3}\log\frac{M^2}{\mu^2}$$

- 1 complex scalar = 2 real scalars.
- Symmetry factor due to Z_4 symmetry under rotation.
- The diagram calculated on the LHS is sufficient to fix the coefficient of the operator on the RHS.

Universal master formula
(EFT matching reduced to matrix algebra!)

- Assuming $X_{HL} = X_{LH} = 0$ (as I did above) and $X_H[\Phi, \phi, P_\mu] = U_H[\Phi, \phi]$
  - A. Drozd, J. Ellis, J. Quevillon, T. You (1512.03003) derived a master formula for one-loop matching up to dimension-six level, dubbed the Universal One-Loop Effective Action (UOLEA).
  - The degenerate limit of the UOLEA was reported earlier by B. Henning, X. Lu, H. Murayama (1412.1837).

- In 1610.00710, I re-derived the UOLEA using covariant diagrams, which greatly simplify the calculation.

Universal master formula
(EFT matching reduced to matrix algebra!)

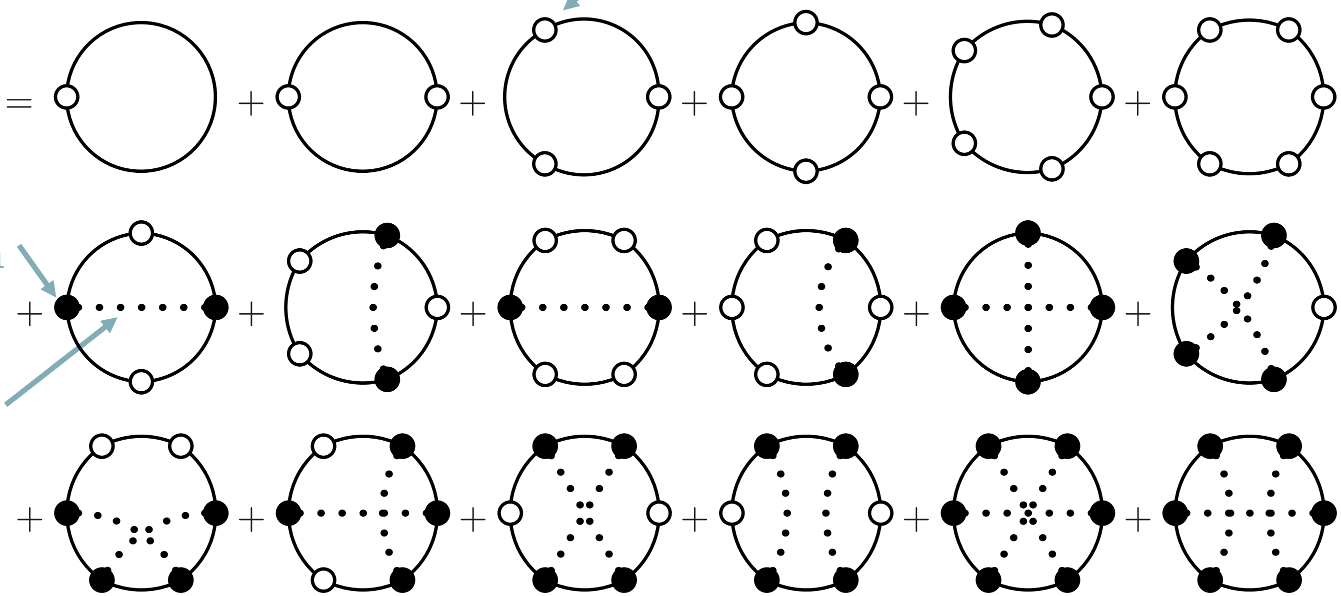
Up to dimension-6:

empty circle = U insertion

filled circle = P insertion

dotted line = Lorentz contraction

$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}} =$$



$$= \sum \left(\text{universal coefficient} \times \text{tr}(\text{operator})\right)$$

=> see next slide

18 covariant diagrams => 18 terms in the CDE => 18 independent operator traces

Universal master formula

(EFT matching reduced to matrix algebra!)

$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}} = \sum \left(\text{universal coefficient} \times \text{tr}(\text{operator})\right)$$

- How to use the master formula:
  Given a UV model,
  - Extract the U matrix
  - Plug in and calculate traces
  - Add up all terms
  - Done!
- For examples, see
  - Henning, Lu, Murayama (1412.1837)
  - Chiang, Huo (1505.06334)
  - Huo (1506.00840, 1509.05942)
  - Drozd, Ellis, Quevillon, You (1512.03003)

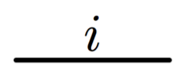
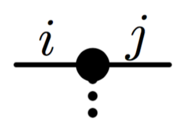
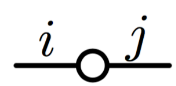
Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\mathcal{I}[q^4]_i^4$	$G_i^{\prime\mu\nu} G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i^{\prime\rho\nu}]$
$f_6^i = \frac{32}{3}\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i} G'^{\nu}{}_{\rho,i} G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\mathcal{I}[q^4]_i^5$	$U_{ii}G_i^{\prime\mu\nu}G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\right)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu,[P_\mu,U_{ij}]]\,[P^\nu,[P_\nu,U_{ji}]]$
$f_{13}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{42} + 2\mathcal{I}[q^4]_{ij}^{51}\right)$	$U_{ij}U_{ji}G_i^{\prime\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = \left(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\right)$	$\left(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji}\right)[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\left(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\right)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

But that is not all!

- The UOLEA master formula does not account for
  - $X_{HL}, X_{LH} \neq 0$
  - Z terms in $X_H[\Phi, \phi, P_\mu] = U_H[\Phi, \phi] + P_\mu Z_H^\mu[\Phi, \phi] + Z_H^{\dagger\mu}[\Phi, \phi] P_\mu$
    $+ P_\mu P_\nu Z_H^{\mu\nu}[\Phi, \phi] + Z_H^{\dagger\mu\nu}[\Phi, \phi] P_\nu P^\mu + \dots$
  - Mixed statistics (bosonic+fermionic)
- Covariant diagrams are capable of dealing with all these additional structures, and so can be used to
  - derive extended master formulas;
  - do matching calculations for specific UV models.

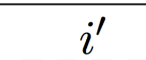
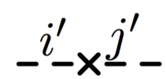
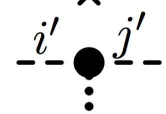
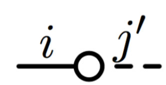
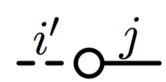
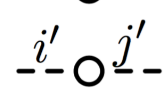
The rules (1/4)

- The derivations involve technical functional manipulations, so here I only tell you the rules (which are very simple and intuitive and you can immediately use!).
- We already encountered the following rules for heavy loops:

Element of diagram	Symbol	Expression
heavy propagator (bosonic)		1
P insertion (bosonic, heavy)		$2P_\mu\delta_{ij}$
U insertion (heavy-heavy)		$U_{H\,ij}$

The rules (2/4)

- When $X_{HL}, X_{LH} \neq 0$, we should also draw mixed heavy-light loop diagrams, with the following additional ingredients:

Element of diagram	Symbol	Expression
light propagator (bosonic)		1
light mass insertion (bosonic)		$m_{i'}^2\, \delta_{i'j'}$
P insertion (bosonic, light)		$2P_\mu\, \delta_{i'j'}$
U insertion (heavy-light)		$U_{HL\, ij'}$
U insertion (light-heavy)		$U_{LH\, i'j}$
U insertion (light-light)		$U_{L\, i'j'}$

The rules (3/4)

- When $\mathbf{X}[\Phi,\phi,P_\mu] = \mathbf{U}[\Phi,\phi] + P_\mu \mathbf{Z}^\mu[\Phi,\phi] + \mathbf{Z}^{\dagger\mu}[\Phi,\phi] P_\mu$ (boldface for full matrix containing H, HL, LH, L blocks), draw additional diagrams with *Z* insertions, with the following rules:

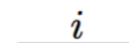
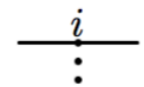
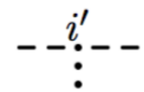
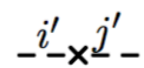
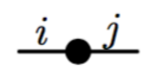
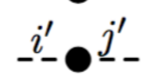
Element of diagram	Symbol	Expression
Z insertion (uncontracted, heavy-heavy)	$i \; j$	$P_\mu Z^\mu_{H\,ij}$
Z insertion (uncontracted, heavy-light)	$i \; j'$	$P_\mu Z^\mu_{HL\,ij'}$
Z insertion (uncontracted, light-heavy)	$i' \; j$	$P_\mu Z^\mu_{LH\,i'j}$
Z insertion (uncontracted, light-light)	$i' \; j'$	$P_\mu Z^\mu_{L\,i'j'}$
Z insertion (contracted, heavy-heavy)	$i \; j$	$-Z^\mu_{H\,ij}$
Z insertion (contracted, heavy-light)	$i \; j'$	$-Z^\mu_{HL\,ij'}$
Z insertion (contracted, light-heavy)	$i' \; j$	$-Z^\mu_{LH\,i'j}$
Z insertion (contracted, light-light)	$i' \; j'$	$-Z^\mu_{L\,i'j'}$

Element of diagram	Symbol	Expression
$Z^\dagger$ insertion (uncontracted, heavy-heavy)	$i \; j$	$Z^{\dagger\mu}_{H\,ij} P_\mu$
$Z^\dagger$ insertion (uncontracted, heavy-light)	$i \; j'$	$Z^{\dagger\mu}_{LH\,ij'} P_\mu$
$Z^\dagger$ insertion (uncontracted, light-heavy)	$i' \; j$	$Z^{\dagger\mu}_{HL\,i'j} P_\mu$
$Z^\dagger$ insertion (uncontracted, light-light)	$i' \; j'$	$Z^{\dagger\mu}_{L\,i'j'} P_\mu$
$Z^\dagger$ insertion (contracted, heavy-heavy)	$i \; j$	$-Z^{\dagger\mu}_{H\,ij}$
$Z^\dagger$ insertion (contracted, heavy-light)	$i \; j'$	$-Z^{\dagger\mu}_{LH\,ij'}$
$Z^\dagger$ insertion (contracted, light-heavy)	$i' \; j$	$-Z^{\dagger\mu}_{HL\,i'j}$
$Z^\dagger$ insertion (contracted, light-light)	$i' \; j'$	$-Z^{\dagger\mu}_{L\,i'j'}$

  - Terms with more P_μ's can be similarly dealt with.

The rules (4/4)

- For matching calculations involving Dirac fermions, use the following rules:

Element of diagram	Symbol	Expression
heavy propagator (fermionic, uncontracted)		M_i
heavy propagator (fermionic, contracted)		$-\gamma^\mu$
light propagator (fermionic)		$-\gamma^\mu$
light mass insertion (fermionic)		$m_{i'}\delta_{i'j'}$
P insertion (fermionic, heavy)		$-\not{P}\delta_{ij}$
P insertion (fermionic, light)		$-\not{P}\delta_{i'j'}$

  - Rules for Weyl fermions can be derived similarly.

The rules (further comments)

- Master integrals are generalized for mixed heavy-light loops:

$$\int \frac{d^dq}{(2\pi)^d} \frac{q^{\mu_1} \cdots q^{\mu_{2n_c}}}{(q^2 - M_i^2)^{n_i} (q^2 - M_j^2)^{n_j} \cdots (q^2)^{n_L}} \equiv g^{\mu_1 \cdots \mu_{2n_c}} \mathcal{I}[q^{2n_c}]_{ij\ldots 0}^{n_i n_j \ldots n_L}$$

  - n_i, n_j, … = number of heavy propagators of type i, j, …
  - n_L = number of light propagators
  - n_c = number of Lorentz contractions (dotted lines)
- Prefactor has opposite signs for bosonic vs. fermionic loops.
  - For mixed bosonic-fermionic loops, prefactor is determined by the propagator from which one starts reading the diagram.
- Diagrams giving rise to $\mathrm{tr}(\ldots P^2 \ldots)$ can be omitted as other diagrams are sufficient to determine all independent operator coefficients.

Example:
Integrating out a scalar triplet

- Test case for mixed heavy-light matching.

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \frac{1}{2}(D_\mu \Phi^a)^2 - \frac{1}{2}M^2\Phi^a\Phi^a - \frac{1}{4}\lambda_\Phi(\Phi^a\Phi^a)^2 + \kappa\phi^\dagger\sigma^a\phi\Phi^a - \eta|\phi|^2\Phi^a\Phi^a$$

- We shall focus on mixed heavy-light contributions to

$$\mathcal{O}_T = \frac{1}{2}\left(\phi^\dagger \overleftrightarrow{D}_\mu \phi\right)^2, \quad \mathcal{O}_H = \frac{1}{2}\left(\partial_\mu|\phi|^2\right)^2, \quad \mathcal{O}_R = |\phi|^2|D_\mu\phi|^2$$

  - Scalar sector: contributions independent of *g, g'*.
  - Gauge sector: contributions dependent of *g, g'* (involve Z terms).

Example:
Integrating out a scalar triplet (scalar sector)

- First, extract quadratic pieces:

$$\mathcal{L}_{\text{UV, quad.}} \supset -\frac{1}{2}\left(\Phi'^a \;\phi'^\dagger \;\tilde{\phi}'^\dagger\right)\left(-P^2 + \mathbf{M}^2 + \mathbf{U}[\Phi_\text{b}, \phi_\text{b}, \tilde{\phi}_\text{b}]\right)\begin{pmatrix}\Phi'^b \\ \phi' \\ \tilde{\phi}'\end{pmatrix} \qquad \tilde{\phi} \equiv i\sigma^2\phi^*$$

where

$$\mathbf{M}^2 = \text{diag}(M^2\delta^{ab}, m^2, m^2) \qquad \mathbf{U} = \begin{pmatrix} U_H & (U_{HL})_{1\times 2} \\ (U_{LH})_{2\times 1} & (U_L)_{2\times 2}\end{pmatrix} = \begin{pmatrix} U_\Phi^{ab} & (U_{\phi\Phi}^{\dagger a})_{1\times 2} \\ (U_{\phi\Phi}^b)_{2\times 1} & (U_\phi)_{2\times 2}\end{pmatrix}$$

$$\Phi_\text{c}^a[\phi] = \frac{\kappa}{M^2}\phi^\dagger\sigma^a\phi - \frac{\kappa}{M^4}\left[2\eta|\phi|^2\left(\phi^\dagger\sigma^a\phi\right) + D^2\left(\phi^\dagger\sigma^a\phi\right)\right] + \mathcal{O}(M^{-5})$$

$$\begin{aligned}
U_\Phi^{ab} &= 2\eta\,|\phi|^2\delta^{ab} + \lambda_\Phi\left(\Phi_\text{c}^d\Phi_\text{c}^d\,\delta^{ab} + 2\,\Phi_\text{c}^a\Phi_\text{c}^b\right) \sim \mathcal{O}(\phi^2,\, \phi^4,\, P^2\phi^4, \dots) \\
U_{\phi\Phi}^b &= \begin{pmatrix} -\kappa\,\sigma^b\phi + 2\eta\,\phi\,\Phi_\text{c}^b \\ \kappa\,\sigma^b\tilde{\phi} + 2\eta\,\tilde{\phi}\,\Phi_\text{c}^b \end{pmatrix} \sim \mathcal{O}(\phi,\, \phi^3,\, P^2\phi^3, \dots) \\
U_\phi &= \begin{pmatrix} 2\lambda\left(|\phi|^2\,\mathbb{1}_2 + \phi\,\phi^\dagger\right) - \kappa\,\Phi_\text{c}^d\sigma^d + \eta\,\Phi_\text{c}^d\Phi_\text{c}^d\,\mathbb{1}_2 & 2\lambda\,\phi\,\tilde{\phi}^\dagger \\ 2\lambda\,\tilde{\phi}\,\phi^\dagger & 2\lambda\left(|\phi|^2\,\mathbb{1}_2 + \tilde{\phi}\,\tilde{\phi}^\dagger\right) + \kappa\,\Phi_\text{c}^d\sigma^d + \eta\,\Phi_\text{c}^d\Phi_\text{c}^d\,\mathbb{1}_2 \end{pmatrix} \\
&\sim \mathcal{O}(\phi^2,\, \phi^4,\, P^2\phi^2,\, P^2\phi^4, \dots)
\end{aligned}$$

Example:
Integrating out a scalar triplet (scalar sector)

- Second, identify terms to be computed

$$\mathcal{O}_T = \frac{1}{2}\left(\phi^\dagger \overleftrightarrow{D}_\mu \phi\right)^2, \quad \mathcal{O}_H = \frac{1}{2}\left(\partial_\mu |\phi|^2\right)^2, \quad \mathcal{O}_R = |\phi|^2 |D_\mu \phi|^2$$

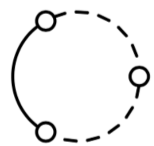
  - All these operators are $\mathcal{O}(P^2\phi^4)$, so need to compute covariant diagrams proportional to

$$U_{HL}U_{LH},\ U_{HL}U_LU_{LH},\ P^2U_{HL}U_{LH},\ P^2U_{HL}U_{LH}U_H,\ P^2U_{HL}U_LU_{LH},\ P^2(U_{HL}U_{LH})^2.$$

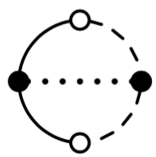
- Then, draw diagrams and calculate!

Example:
Integrating out a scalar triplet (scalar sector)

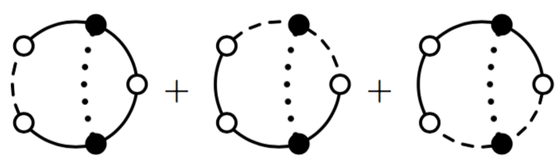
$$= -ic_s\,\mathcal{I}_{i0}^{11}\,\mathrm{tr}(U_{HL}U_{LH}), \tag{4.29a}$$



$$= -ic_s\,\mathcal{I}_{i0}^{12}\,\mathrm{tr}(U_{HL}U_LU_{LH}), \tag{4.29b}$$



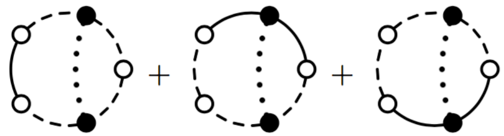
$$= -ic_s\,2^2\,\mathcal{I}[q^2]_{i0}^{22}\,\mathrm{tr}(P^\mu U_{HL}P_\mu U_{LH}) \subset -ic_s\,2\,\mathcal{I}[q^2]_{i0}^{22}\,\mathrm{tr}\big([P^\mu,U_{HL}][P_\mu,U_{LH}]\big)\,, \tag{4.29c}$$



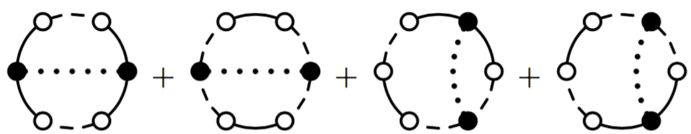
$$\begin{aligned}
&= -ic_s 2^2\big\{\mathcal{I}[q^2]_{i0}^{41}\mathrm{tr}(P_\mu U_{HL}U_{LH}P^\mu U_H)\\
&\qquad +\mathcal{I}[q^2]_{i0}^{32}\,\mathrm{tr}(P^\mu U_H U_{HL}P_\mu U_{LH}+P^\mu U_{HL}P_\mu U_{LH}U_H)\big\}\\
&\subset -ic_s\big\{4\,\mathcal{I}[q^2]_{i0}^{32}\,\mathrm{tr}\big([P^\mu,U_{HL}][P_\mu,U_{LH}]U_H\big)\\
&\qquad +2\,\big(\mathcal{I}[q^2]_{i0}^{41}+\mathcal{I}[q^2]_{i0}^{32}\big)\,\mathrm{tr}\big([P^\mu,U_{HL}U_{LH}][P_\mu,U_H]\big)\big\},
\end{aligned} \tag{4.29d}$$

Example:

Integrating out a scalar triplet (scalar sector)



$$
\begin{aligned}
&= -ic_s\, 2^2 \big\{ \mathcal{I}[q^2]^{14}_{i0}\, \mathrm{tr}(P_\mu U_{LH} U_{HL} P^\mu U_L) \\
&\qquad + \mathcal{I}[q^2]^{23}_{i0}\, \mathrm{tr}(P^\mu U_L U_{LH} P_\mu U_{HL} + P^\mu U_{LH} P_\mu U_{HL} U_L) \big\} \\
&\subset -ic_s \big\{ 4\,\mathcal{I}[q^2]^{23}_{i0}\, \mathrm{tr}\big([P^\mu, U_{LH}][P_\mu, U_{HL}] U_L\big) \\
&\qquad + 2\,(\mathcal{I}[q^2]^{14}_{i0} + \mathcal{I}[q^2]^{23}_{i0})\, \mathrm{tr}\big([P^\mu, U_{LH} U_{HL}][P_\mu, U_L]\big) \big\},
\end{aligned} \tag{4.29e}
$$



$$
\begin{aligned}
&= -ic_s\, 2^2 \Big\{ \frac{1}{2}\,\mathcal{I}[q^2]^{42}_{i0}\, \mathrm{tr}(P^\mu U_{HL} U_{LH} P_\mu U_{HL} U_{LH}) + \frac{1}{2}\,\mathcal{I}[q^2]^{24}_{i0}\, \mathrm{tr}(P^\mu U_{LH} U_{HL} P_\mu U_{LH} U_{HL}) \\
&\qquad + \mathcal{I}[q^2]^{33}_{i0}\, \mathrm{tr}(P^\mu U_{HL} P_\mu U_{LH} U_{HL} U_{LH} + P^\mu U_{LH} P_\mu U_{HL} U_{LH} U_{HL}) \Big\} \\
&\subset -ic_s \big\{ (2\,\mathcal{I}[q^2]^{24}_{i0} + 4\,\mathcal{I}[q^2]^{33}_{i0})\, \mathrm{tr}\big([P^\mu, U_{HL}][P_\mu, U_{LH}] U_{HL} U_{LH}\big) \\
&\qquad + (2\,\mathcal{I}[q^2]^{42}_{i0} + 4\,\mathcal{I}[q^2]^{33}_{i0})\, \mathrm{tr}\big([P^\mu, U_{LH}][P_\mu, U_{HL}] U_{LH} U_{HL}\big) \\
&\qquad + (\mathcal{I}[q^2]^{42}_{i0} + \mathcal{I}[q^2]^{24}_{i0} + 2\,\mathcal{I}[q^2]^{33}_{i0}) \\
&\qquad\quad \mathrm{tr}\,\big([P^\mu, U_{HL}] U_{LH} [P_\mu, U_{HL}] U_{LH} + U_{HL} [P^\mu, U_{LH}] U_{HL} [P_\mu, U_{LH}]\big) \big\}.
\end{aligned} \tag{4.29f}
$$

Coefficient	Operator
$-ic_s\,\mathcal{I}_{i0}^{11} = \frac{c_s}{16\pi^2}\big(1-\log\frac{M^2}{\mu^2}\big)$	$\mathrm{tr}(U_{HL}U_{LH})$ $\to U^{\dagger a}_{\phi\Phi}U^{a}_{\phi\Phi} \supset -\frac{16\kappa^2\eta}{M^4}(\mathcal{O}_T+2\mathcal{O}_R)$
$-ic_s\,\mathcal{I}_{i0}^{12} = \frac{c_s}{16\pi^2}\frac{1}{M^2}\big(1-\log\frac{M^2}{\mu^2}\big)$	$\mathrm{tr}(U_{HL}U_{L}U_{LH})$ $\to U^{\dagger a}_{\phi\Phi}U_\phi U^{a}_{\phi\Phi} \supset \frac{4\kappa^4}{M^4}(\mathcal{O}_T+2\mathcal{O}_R)$
$-ic_s\,2\,\mathcal{I}[q^2]_{i0}^{22} = \frac{c_s}{16\pi^2}\big(-\frac{1}{2M^2}\big)$	$\mathrm{tr}\big([P^\mu,U_{HL}][P_\mu,U_{LH}]\big)$ $\to [P^\mu,U^{\dagger a}_{\phi\Phi}][P_\mu,U^{a}_{\phi\Phi}]$ $\supset -6\kappa^2\lvert D_\mu\phi\rvert^2+\frac{8\kappa^2\eta}{M^2}(\mathcal{O}_H+\mathcal{O}_R)$
$-ic_s\,4\,\mathcal{I}[q^2]_{i0}^{32} = \frac{c_s}{16\pi^2}\frac{1}{2M^4}$	$\mathrm{tr}\big([P^\mu,U_{HL}][P_\mu,U_{LH}]U_H\big)$ $\to [P^\mu,U^{\dagger a}_{\phi\Phi}][P_\mu,U^{b}_{\phi\Phi}]U^{ba}_{\Phi} \supset -12\kappa^2\eta\mathcal{O}_R$
$-ic_s\,2\,(\mathcal{I}[q^2]_{i0}^{41}+\mathcal{I}[q^2]_{i0}^{32}) = \frac{c_s}{16\pi^2}\frac{1}{3M^4}$	$\mathrm{tr}\big([P^\mu,U_{HL}U_{LH}][P_\mu,U_H]\big)$ $\to [P^\mu,U^{\dagger a}_{\phi\Phi}U^{b}_{\phi\Phi}][P_\mu,U^{ba}_{\Phi}] \supset -24\kappa^2\eta\mathcal{O}_H$
$-ic_s\,4\,\mathcal{I}[q^2]_{i0}^{23} = \frac{c_s}{16\pi^2}\frac{1}{M^4}\big(-\frac{5}{2}+\log\frac{M^2}{\mu^2}\big)$	$\mathrm{tr}\big([P^\mu,U_{LH}][P_\mu,U_{HL}]U_L\big)$ $\to [P^\mu,U^{a}_{\phi\Phi}][P_\mu,U^{\dagger a}_{\phi\Phi}]U_\phi$ $\supset 2\kappa^2\big[\big(\frac{\kappa^2}{M^2}-2\lambda\big)\mathcal{O}_T-\frac{\kappa^2}{M^2}\mathcal{O}_H$ $+\big(\frac{\kappa^2}{M^2}-10\lambda\big)\mathcal{O}_R\big]$
$-ic_s\,2\,(\mathcal{I}[q^2]_{i0}^{14}+\mathcal{I}[q^2]_{i0}^{23}) = \frac{c_s}{16\pi^2}\big(-\frac{1}{2M^4}\big)$	$\mathrm{tr}\big([P^\mu,U_{LH}U_{HL}][P_\mu,U_L]\big)$ $\to [P^\mu,U^{a}_{\phi\Phi}U^{\dagger a}_{\phi\Phi}][P_\mu,U_\phi]$ $\supset 4\kappa^2\big[\big(-\frac{\kappa^2}{M^2}+2\lambda\big)\mathcal{O}_T$ $-10\lambda\mathcal{O}_H-\frac{2\kappa^2}{M^2}\mathcal{O}_R\big]$
$-ic_s(2\,\mathcal{I}[q^2]_{i0}^{24}+4\,\mathcal{I}[q^2]_{i0}^{33}) = \frac{c_s}{16\pi^2}\frac{1}{M^6}$	$\mathrm{tr}\big([P^\mu,U_{HL}][P_\mu,U_{LH}]U_{HL}U_{LH}\big)$ $\to [P^\mu,U^{\dagger a}_{\phi\Phi}][P_\mu,U^{b}_{\phi\Phi}]U^{\dagger b}_{\phi\Phi}U^{a}_{\phi\Phi} \supset -12\kappa^4\mathcal{O}_R$
$-ic_s(2\,\mathcal{I}[q^2]_{i0}^{42}+4\,\mathcal{I}[q^2]_{i0}^{33}) = \frac{c_s}{16\pi^2}\frac{1}{M^6}\big(\frac{17}{6}-\log\frac{M^2}{\mu^2}\big)$	$\mathrm{tr}\big([P^\mu,U_{LH}][P_\mu,U_{HL}]U_{LH}U_{HL}\big)$ $\to [P^\mu,U^{a}_{\phi\Phi}][P_\mu,U^{\dagger a}_{\phi\Phi}]U^{b}_{\phi\Phi}U^{\dagger b}_{\phi\Phi}$ $\supset -2\kappa^4(\mathcal{O}_H+4\mathcal{O}_R)$
$-ic_s(\mathcal{I}[q^2]_{i0}^{42}+\mathcal{I}[q^2]_{i0}^{24}+2\,\mathcal{I}[q^2]_{i0}^{33}) = \frac{c_s}{16\pi^2}\frac{5}{12M^6}$	$\mathrm{tr}\big([P^\mu,U_{HL}]U_{LH}[P_\mu,U_{HL}]U_{LH}$ $+U_{HL}[P^\mu,U_{LH}]U_{HL}[P_\mu,U_{LH}]\big)$ $\to [P^\mu,U^{\dagger a}_{\phi\Phi}]U^{b}_{\phi\Phi}[P_\mu,U^{\dagger b}_{\phi\Phi}]U^{a}_{\phi\Phi}$ $+U^{\dagger a}_{\phi\Phi}[P^\mu,U^{b}_{\phi\Phi}]U^{\dagger b}_{\phi\Phi}[P_\mu,U^{a}_{\phi\Phi}]$ $\supset 4\kappa^4(-5\mathcal{O}_H+4\mathcal{O}_R)$

Example:
Integrating out a scalar triplet (scalar sector)

- Final result (add up all terms):

$$\mathcal{L}_{\text{EFT}}^{\text{1-loop}}[\phi] \supset \frac{1}{16\pi^2}\frac{3\kappa^2}{2M^2}|D_\mu\phi|^2 + \frac{1}{16\pi^2}\frac{\kappa^2}{M^4}\left[\left(\frac{\kappa^2}{2M^2} - 8\eta + 3\lambda\right)\mathcal{O}_T + \left(-\frac{9\kappa^2}{2M^2} - 6\eta + 10\lambda\right)\mathcal{O}_H + \left(-\frac{21\kappa^2}{2M^2} - 21\eta + 25\lambda\right)\mathcal{O}_R\right]$$

  - in agreement with earlier calculations with Feynman diagrams (F. del Aguila, Z. Kunszt, J. Santiago, 1602.00126) and different functional methods (B. Henning, X. Lu, H. Murayama, 1604.01019; S. Ellis, J. Quevillon, T. You, ZZ, 1604.02445).

Example:
Integrating out a scalar triplet (gauge sector)

- Extended quadratic pieces:

$$\mathcal{L}_{\text{UV, quad.}} \supset -\frac{1}{2}\left(\Phi'^{a}\ \phi'^{\dagger}\ \tilde{\phi}'^{\dagger}\ W'^{a}_{\alpha}\ B'_{\alpha}\right)\left(-P^2+\mathbf{M}^2+\mathbf{U}+P_\mu\mathbf{Z}^\mu+\mathbf{Z}^{\dagger\mu}P_\mu\right)\begin{pmatrix}\Phi'^{b}\\ \phi'\\ \tilde{\phi}'\\ W'^{b}_{\beta}\\ B'_{\beta}\end{pmatrix}$$

where

$$\mathbf{M}^2 = \operatorname{diag}(M^2, m^2, m^2, 0, 0)\,,$$

$$\mathbf{U} = \begin{pmatrix} U_H & (U_{HL})_{1\times 4} \\ (U_{LH})_{4\times 1} & (U_L)_{4\times 4}\end{pmatrix} = \begin{pmatrix} U_{\Phi}^{ab} & (U_{\phi\Phi}^{\dagger a})_{1\times 2} & U_{\Phi W}^{ab\beta} & 0 \\ (U_{\phi\Phi}^{b})_{2\times 1} & (U_{\phi})_{2\times 2} & (U_{\phi W}^{b\beta})_{2\times 1} & (U_{\phi B}^{\beta})_{2\times 1} \\ U_{\Phi W}^{\dagger ab\alpha} & (U_{\phi W}^{\dagger a\alpha})_{1\times 2} & U_{W}^{ab\alpha\beta} & U_{BW}^{a\alpha\beta} \\ 0 & (U_{\phi B}^{\dagger\alpha})_{1\times 2} & U_{BW}^{b\alpha\beta} & U_{B}^{\alpha\beta}\end{pmatrix},$$

$$\mathbf{Z}^\mu = \begin{pmatrix} Z_H^\mu & (Z_{HL}^\mu)_{1\times 4} \\ (Z_{LH}^\mu)_{4\times 1} & (Z_L^\mu)_{4\times 4}\end{pmatrix} = \begin{pmatrix} 0 & \mathbf{0}_{1\times 2} & Z_{\Phi W}^{\mu\, ab\beta} & 0 \\ \mathbf{0}_{2\times 1} & \mathbf{0}_{2\times 2} & (Z_{\phi W}^{\mu\, b\beta})_{2\times 1} & (Z_{\phi B}^{\mu\,\beta})_{2\times 1} \\ 0 & \mathbf{0}_{1\times 2} & 0 & 0 \\ 0 & \mathbf{0}_{1\times 2} & 0 & 0\end{pmatrix}$$

Example:

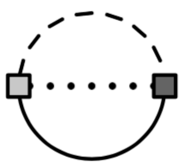
Integrating out a scalar triplet (gauge sector)

$$Z^{\mu\,ab\beta}_{\Phi W} = g^{\mu\beta} i g \epsilon^{adb} \Phi^d_c \sim \mathcal{O}(g\phi^2,\, gP^2\phi^2,\, g\phi^4, \dots)\,, \quad U_{\Phi W} = [P_\mu, Z^\mu_{\Phi W}]\,,$$

$$Z^{\mu\,b\beta}_{\phi W} = -g^{\mu\beta} \frac{g}{2} \begin{pmatrix} \sigma^b \phi \\ \sigma^b \tilde{\phi} \end{pmatrix} \sim \mathcal{O}(g\phi)\,, \quad U_{\phi W} = [P_\mu, Z^\mu_{\phi W}]\,,$$

$$Z^{\mu\,\beta}_{\phi B} = -g^{\mu\beta} \frac{g'}{2} \begin{pmatrix} \phi \\ -\tilde{\phi} \end{pmatrix} \sim \mathcal{O}(g'\phi)\,, \quad U_{\phi B} = [P_\mu, Z^\mu_{\phi B}]\,.$$

- Calculate diagrams like

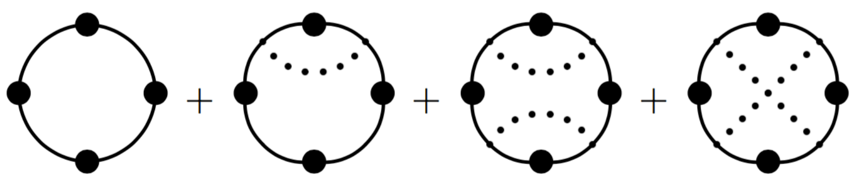


$$= -i c_s \mathcal{I}[q^2]^{11}_{i0} \operatorname{tr}(Z^\mu_{HL} Z^\dagger_{HL\,\mu}) = \frac{c_s}{16\pi^2}\left(\frac{3}{8} - \frac{1}{4}\log\frac{M^2}{\mu^2}\right) M^2 \operatorname{tr}(Z^\mu_{HL} Z^\dagger_{HL\,\mu})$$

- Final result: $\mathcal{L}^{\text{1-loop}}_{\text{EFT}}[\phi] \supset \frac{1}{16\pi^2}\frac{5\kappa^2}{8M^4}\left[\, g^2\,\mathcal{O}_T + g'^2\,\mathcal{O}_H - (4g^2 + 2g'^2)\,\mathcal{O}_R \,\right].$
  - First time this has been calculated with functional methods!
  - In agreement with earlier calculation with Feynman diagrams (F. del Aguila, Z. Kunszt, J. Santiago, 1602.00126).

Example:
threshold matching of gauge coupling (fermion case)

- Integrate out a vector-like fermion of mass M, and extract $P^4 \sim D^4 \sim G^2$ terms in the EFT.



Heavy fermionic propagators can be contracted or uncontracted; Fermionic P insertions are never contracted.

$$= -ic_s\Big\{\frac{1}{4}M^4\mathcal{I}_i^4\,\mathrm{tr}(\slashed{P}^4) + M^2\mathcal{I}[q^2]_i^4\,\mathrm{tr}(\gamma^\alpha\slashed{P}\gamma_\alpha\slashed{P}^3)$$
$$+\mathcal{I}[q^4]_i^4\Big(\frac{1}{2}\mathrm{tr}(\gamma^\alpha\slashed{P}\gamma_\alpha\slashed{P}\gamma^\beta\slashed{P}\gamma_\beta\slashed{P}) + \frac{1}{4}\mathrm{tr}(\gamma^\alpha\slashed{P}\gamma^\beta\slashed{P}\gamma_\alpha\slashed{P}\gamma_\beta\slashed{P})\Big)\Big\}$$

- Gamma matrix algebra + plug in master integrals =>

$$\frac{c_s}{16\pi^2}\frac{2}{3}\log\frac{M^2}{\mu^2}\,\mathrm{tr}(P^\mu P^\nu P_\mu P_\nu) \subset -\frac{1}{16\pi^2}\frac{1}{3}\log\frac{M^2}{\mu^2}\,\mathrm{tr}\big([P^\mu,P^\nu][P_\mu,P_\nu]\big)$$
$$= -\frac{g^2}{16\pi^2}\frac{4}{3}\log\frac{M^2}{\mu^2}\Big[-\frac{1}{4}\,\mathrm{tr}(G^{\mu\nu}G_{\mu\nu})\Big] \quad\Rightarrow\quad \frac{g^2_{\text{eff}}(\mu)}{g^2(\mu)} = 1 + \frac{g^2}{16\pi^2}T(R)\cdot\frac{4}{3}\log\frac{M^2}{\mu^2}$$

More examples…

can be found in 1610.00710 and my upcoming paper(s)

- General recipe for one-loop matching:
  - Extract quadratic pieces of the UV theory Lagrangian.
  - Draw covariant diagrams and read off their expressions.
  - Add them up to obtain the EFT Lagrangian.
- See Section 3.5 of 1610.00710 for detailed recipe.

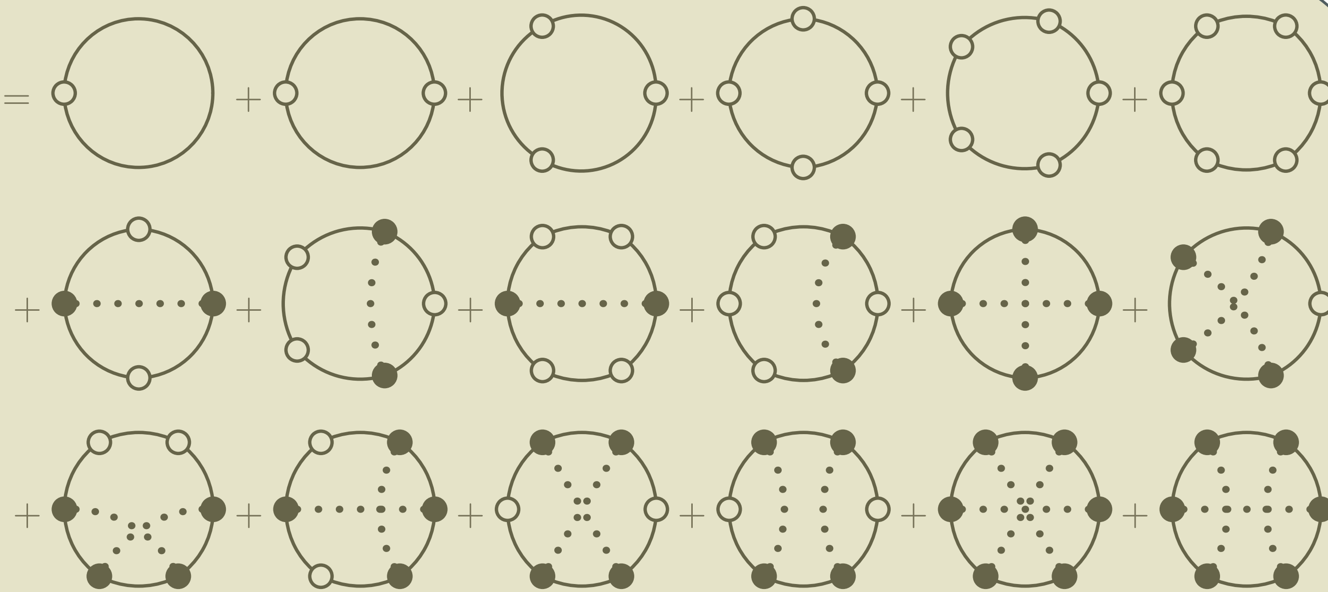
Summary

- EFT matching can be done in more elegant and simpler ways than using conventional Feynman diagrams.
- Covariant diagrams keep track of one-loop functional matching calculations, which
  - preserve gauge covariance in intermediate steps,
  - allow universal results (master formulas) to be easily derived,
  - can deal with additional structures in straightforward ways,
  - you can use as soon as you have learned the (simple) rules!

The end

Feel the power!

$$\mathcal{L}^{\text{1-loop}}_{\text{EFT}} =$$



$$= \sum \left(\text{universal coefficient} \times \text{tr(operator)}\right)$$

Thank you for your attention!