LATTICE FIELD THEORY

Stefan Schaefer
for the NIC group

82nd PRC meeting, Zeuthen

DESY NIC HELMHOLTZ ASSOCIATION

Overview

Participation and leadership in two large European collaborations with rich QCD research programme

Currently particular interest in

ALPHA	ETMC
non-perturbative matching of heavy-light currents HQET	structure functions
$B \to \pi$ form factor	neutron EDM
α_s	$(g-2)_\mu$
	hadronic contributions to electroweak observables

Effects of the charm quark

NLO SU(2) ChPT constants

...

Different discretizations and analysis methods

$\rightarrow$ understanding of systematics

Highlights

In this presentation focus on three topics

ALPHA

Running coupling α_s

ETMC

Nucleon σ terms

Algorithms

Addressing the exponential signal-to-noise problem

Running coupling constant

Running coupling constant fundamental quantity in QCD analysis.

The ALPHA collaboration has developed a unique computational strategy.

Fully non-perturbative up to very high scales.

Use perturbation theory only where safe.

Unprecedented level of control of systematics.

Master formula: (simplified)

$$\Lambda^{(N_\mathrm{f})}_{\overline{\mathrm{MS}}} = \frac{\Lambda^{(N_f)}_{\overline{\mathrm{MS}}}}{\Lambda^{(\mathrm{lat})}} \times L_0\Lambda^{(\mathrm{lat})} \times \frac{1}{L_0 H^{(\mathrm{lat})}} \times H^{(\mathrm{phys})}$$

conversion	running	scale	PDG
PT	PT + NP	NP	
	small V	large V	

Running coupling constant

Non-perturbative running from hadronic scales up to 200 GeV.

Two different couplings

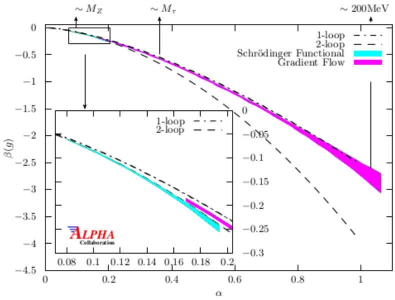
Schrödinger Functional ALPHA '90s

- small statistical errors at high energy
- small discretization effects
- two-loop perturbation theory known

Gradient Flow Fritzsch, Ramos'13

- small statistical errors at low energy
- larger discretization effects
- only basic perturbation theory

Running coupling



Use GF at strong coupling, SF at weak coupling.

β function as a function of coupling constant.

Comparison to universal part of β function.

Scale setting

Leading the generation of the CLS $N_f = 2 + 1$ ensembles

Non-perturbatively improved Wilson fermions

5 small lattice spacings $a = 0.04\,\mathrm{fm}, \ldots\ 0.08\,\mathrm{fm}$

Many quark masses to control chiral effects

Multi-purpose configurations used by researchers all over Europe

Denmark: Odense; Germany: DESY, Mainz, Münster, Regensburg, Wuppertal; Spain: Madrid; Italy: Milano, Rome; Ireland: Dublin; CERN

Key publications

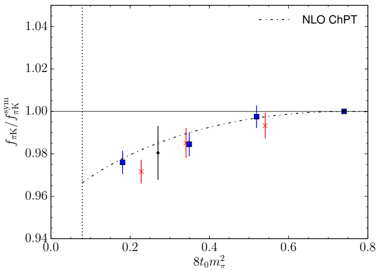
Code and algorithmic setup: M. Lüscher, S.S. CPC 184 (2013) 519

Determination of the action: J. Bulava, S.S., NPB 874 (2013), 188

Description of the ensembles: Bruno et al, JHEP 1502 (2015) 043

Scale setting: M. Bruno, T. Korzec, S.S., arXiv:1608.08900

Scale setting



Dimensionful quantity to set the scale

$$f_{\pi K} = \frac{2}{3}(f_K + \frac{1}{2} f_\pi)$$

Small chiral corrections along chosen trajectory

Small deviation from parameter free ChPT prediction

Running coupling constant

$$\Lambda^{(3)}_{\overline{\rm MS}} = \frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\Lambda} \times \Lambda L_0 \times \frac{2L_0}{L_0} \times \frac{L_{had}}{2L_0} \times \frac{\sqrt{t_0}}{L_{had}} \times \frac{1}{F_{\pi K}\sqrt{t_0}} \times F_{\pi K}$$

	$\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\Lambda}$	ΛL_0	$\frac{2L_0}{L_0}$	$\frac{L_{had}}{2L_0}$	$\frac{\sqrt{t_0}}{L_{had}}$	$\frac{1}{F_{\pi K}\sqrt{t_0}}$	$F_{\pi K}$
	PT	SF running 1604.06193	scheme change	GF running 1607.06423	CLS	scale setting 1608.08900	PDG
rel.err.	0%	2.6%	0.8%		2.6%	1.1%	

Conversion of Λ parameters: (with $m_c(m_c)$ and $m_b(m_b)$ from PDG)

$$\Lambda^{(3)}_{\overline{\rm MS}} = 332(14)\,{\rm MeV}\,, \quad \Lambda^{(4)}_{\overline{\rm MS}} = 289(14)\,{\rm MeV}\,, \quad \Lambda^{(5)}_{\overline{\rm MS}} = 207(11)\,{\rm MeV}$$

Estimate of conversion error:

loops	α_n	$\alpha_n - \alpha_{n-1}$
2	0.11670	–
3	0.11771	0.00109
4	0.11787	0.00016
5 (β)	0.11794	0.0007

Prelimiary result

$$\alpha_{\overline{\rm MS}}(m_Z) = 0.1179(10)(2)$$

ALPHA Collaboration

Summary: strong coupling constant

$$\alpha_s^{\overline{\mathrm{MS}}}(M_Z) = 0.1179(10)(2)$$

Combination of many large parts

Running in Schrödinger Functional (with two couplings)

Precise renormalization constants (Z_{A})

Scale setting on large volume CLS lattices

New quality of results with unprecedented control of systematics.

Many pieces published

Preliminary result presented at conferences

Perturbation theory misleading even at $\alpha = 0.2$.

Non-perturbative approach necessary.

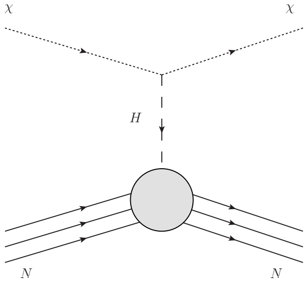
Nucleon sigma term

Nucleon sigma terms

$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle$$
$$\sigma_{\rm s} = m_s\langle N|\bar{s}s|N\rangle$$

Strange content

$$y = \frac{2\sigma_s}{\sigma_{\pi N}} = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$



- Mass fraction of the nucleon
- π–N scattering
- Search for BSM physics

Lattice determinations

Method I: Feynman-Hellmann theorem

$$\sigma_{\pi N} = m_{ud} \frac{\partial}{\partial m_{ud}} m_N \Big|_{phys} \approx \frac{1}{2} m_\pi \frac{\partial}{\partial m_\pi} m_N$$

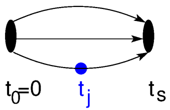
Pro	**Con**
Simple observable	Need many simulations with different quark masses
2pt functions	Take derivative numerically
	Large steps in quark mass needed $\rightarrow$ systematic error?

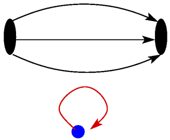
Example: BMWc PRL 116 (2016)

$$\sigma_{\pi N} = 38(3)(3)\,\mathrm{MeV}$$

Lattice determination

Method II: Direct evaluation





Pro

No derivative approx

Better systematics

Con

3pt functions

Disconnected diagrams
$\rightarrow$ numerically challenging

ETMC computation

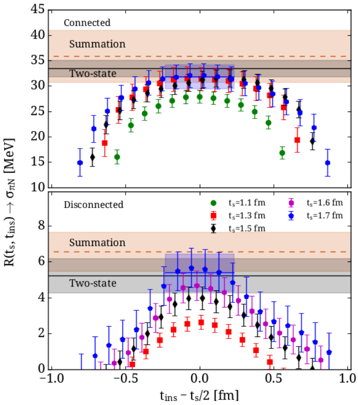
PRL 116 (2016) no.25, 252001

$N_f = 2$ twisted mass fermions with clover term

$a = 0.093\,\text{fm},\ L/a = 48$

Physical pion mass

Results of ETMC computation



$$R(t_s, t_j) \equiv \frac{C_{3pt}(t_s, t_j)}{C_{2pt}(t_s)}$$

$$\sigma_{\pi N} = 37.22(2.57)(^{+0.99}_{-0.6})\ \mathrm{MeV}$$

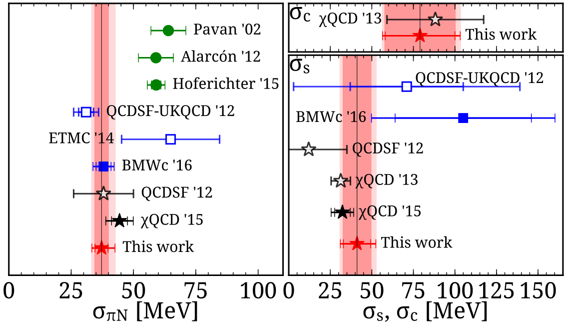
$$\sigma_s = 41.05(8.25)(^{+1.09}_{-0.69})\ \mathrm{MeV}$$

$$\sigma_c = 79(21)(^{+2.1}_{-1.3})\ \mathrm{MeV}$$

Different separation between nucleon source/sink

Variation as function of operator insertion $\to$ look for plateau

Summary nucleon sigma term



Direct evaluation ETMC results agree with Feynman-Hellmann results.

Very different systematics $\rightarrow$ good check.

Tension between phenomenological and lattice determinations.

Majority of physics observables extracted from long-distance behavior of n-point functions

Generic problem! Parisi'82

Most notable exception: pion correlation functions.

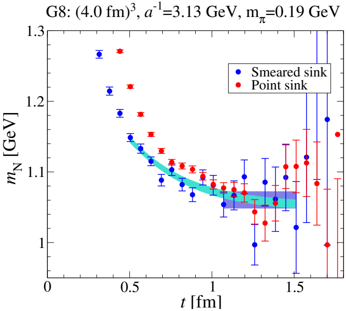
Signal falling exponentially

$$C_\mathrm{N}(x_0) = \sum_{\vec{x}} \langle N(x_0,\vec{x})\bar{N}(0,0)\rangle$$
$$= A\,e^{-m_\mathrm{N} x_0}$$

Signal-to-noise ratio decreasing exponentially

$$\frac{C_\mathrm{N}(x_0)}{\delta C_\mathrm{N}(x_0)} \propto e^{(m_\mathrm{N}-\frac{3}{2}m_\pi)x_0}$$

$$m_\mathrm{eff} = -\partial_{x_0} \log C_\mathrm{N}(x_0)$$



Plot:von Hippel et al, arXiv:1605.00564

Monte Carlo

$$\langle A \rangle = \frac{1}{Z} \int [dU] \, e^{-S[U]} \, A[U]$$

Standard Markov Chain Monte Carlo

Generate N field configurations $U_1, U_2, \cdots, U_N$

Compute observables $A[U_1], \cdots, A[U_N]$

Estimate of expectation value and its uncertainty

$$\bar{A} = (\frac{1}{N} \sum_i A[U_i]) \pm \delta\bar{A} \qquad \text{with} \qquad \delta\bar{A} = \sqrt{\frac{\text{var}(A)}{N}}$$

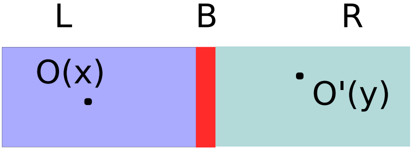
Strategies for improvement

Increase N, exponentially in the distance.

Find a better $A \rightarrow$ variance reduction

Do something about the $\sqrt{N}$ scaling. $\rightarrow$ **Use locality of the theory.**

Multilevel



Domain decomposition of lattice.

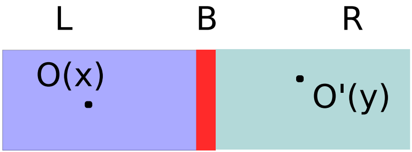
Active domains separated by boundary B.

$$\langle \{O(x) - \bar{O}\}\{O'(y) - \bar{O}'\}\rangle$$
$$= \frac{1}{Z_B}\int [dU_B] e^{-S_B[U_B]} [\{O(x) - \bar{O}\}]_L(U_B) [\{O'(y) - \bar{O}'\}]_R(U_B)$$

Estimate integrals over variables in L and R with N_1 configs per U_B

$$[O(x)]_L(U_B) = \frac{1}{Z_L}\int [dU_L] e^{-S(U_B,U_L)} O(x)$$
$$[O(y)]_R(U_B) = \frac{1}{Z_R}\int [dU_R] e^{-S(U_B,U_R)} O'(y)$$

Multilevel



Start with set of N_0 level-0 gauge field configurations

Used to define the fixed **boundary**.

N_1 independent updates in region L and R.

$N_0 \times N_1$ configurations on level 1.

Yields effectively $N_0 \times N_1^2$ configurations drawn from correct probability distribution.

Long tradition: Multihit (Parisi et al'83), Lüscher-Weisz'01,. . .

New: Methods for fermions

Multilevel for fermions

Need **factorized observable** and **domain decomposed action**

$$\langle P^{uu}(x)P^{dd}(y)\rangle = \frac{1}{Z}\int [dU]\det D\, e^{-S_g[U]}\mathrm{tr}\Big[\frac{1}{D_{m_u}}(x,x)\gamma_5\Big]\mathrm{tr}\Big[\frac{1}{D_{m_d}}(y,y)\gamma_5\Big]$$

Valence sector

Cè, Giusti, S.S., Phys.Rev. D93 (2016) 094507

Gluonic flow observables

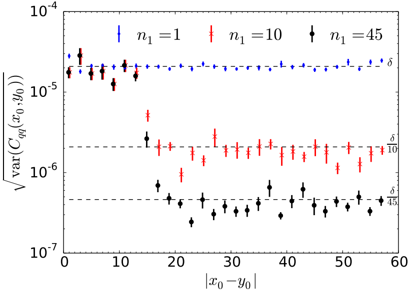
García Vera, S.S., Phys.Rev. D93 (2016) 074502

Sea quarks

Cè, Giusti, S.S., arXiv:1609.02419

Building blocks for fermion observables in full QCD available.
First extensive tests passed very successfully.

Example



Topological charge density correlation function

$$C_{qq}(x_0, y_0) = \frac{1}{L^3} \langle \bar{q}(x_0) \bar{q}(y_0) \rangle$$

with

$$\bar{q}(x_0) = \frac{1}{64\pi^2} \sum_{\vec{x}} F^a_{\mu\nu}(x) \tilde{F}^a_{\mu\nu}(x)$$

$N_f = 2$ dynamical flavors, $a = 0.067$ fm

Reduction of simulation cost by a factor of n_1 up to $n_1 = 45$ confirmed.

Many more topics

Hadron physics

Hadronic spectrum and structure (Sigma terms, PDF, ...)
Hadronic contribution to muon Anomalous Magnetic Moment
Heavy Flavour physics (HQET, $B \to \pi$, $B_s \to K$ form factors)
Non-pertrubative decoupling

Fundamental parameters

Strong coupling constant α_s
Quark masses

Beyond QCD

Higgs-Yukawa model
Large-N limit of SU(N)
Turbulence

Algorithms

Integration methods (quasi-MC and polynomial exact methods)
Variance reduction and multilevel methods
Hamiltonian approach to field theory

Summary

Large diverse research programme

Good progress in many areas

$\Lambda_{\overline{\mathrm{MS}}}^{(3)}$**: new level of rigor reached**

Nucleon σ terms with alternative method

$\rightarrow$ check of systematics

Algorithmic developments for next level of accuracy

$\rightarrow$ huge potential for baryon physics