Parton Distributions from Lattice QCD

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Motivation

- non-perturbative calculation of low moments of nucleon PDFs

$$\langle x^n \rangle_q = \int_0^1 dx\, x^n \{q(x) - (-1)^n \overline{q}(x)\}$$

- comparison with precise unpolarized results ($\text{err}_{\text{ex}} \approx 0.1 \cdot \text{err}_{\text{lat}}$)

- nearly competitive with helicity measurements ($\text{err}_{\text{ex}} \approx \text{err}_{\text{lat}}$)

- potential for predictions of transversity moments ($\text{ex} = \text{unknown}$)

- nucleon GPDs: form factors, spin content, transverse structure

Moments of Parton Distributions

- light-cone expansion generates twist-two operators

$$O_q^{\mu_1\cdots\mu_n} = \overline{q} iD^{\{\mu_1} \ldots iD^{\mu_{n-1}}\gamma^{\mu_n\}} q$$

- moments of parton distributions from forward matrix elements

$$\langle P | O_q^{\mu_1\cdots\mu_n} | P \rangle = 2\, \langle x^{n-1} \rangle_q \, P^{\{\mu_1} \ldots P^{\mu_n\}}$$

- unpolarized, helicity and transversity moments

$$\begin{aligned} \langle x^n \rangle_q &= \int_0^1 dx\, x^n \left\{ q(x) - (-1)^n \overline{q}(x) \right\} \\ \langle x^n \rangle_{\Delta q} &= \int_0^1 dx\, x^n \left\{ \Delta q(x) + (-1)^n \Delta \overline{q}(x) \right\} \\ \langle x^n \rangle_{\delta q} &= \int_0^1 dx\, x^n \left\{ \delta q(x) - (-1)^n \delta \overline{q}(x) \right\} \end{aligned}$$

- matrix elements are calculated in Euclidean space on the lattice

Lattice QCD with Twisted Fermions

- continuum twisted QCD is equivalent to QCD upto a field redefinition

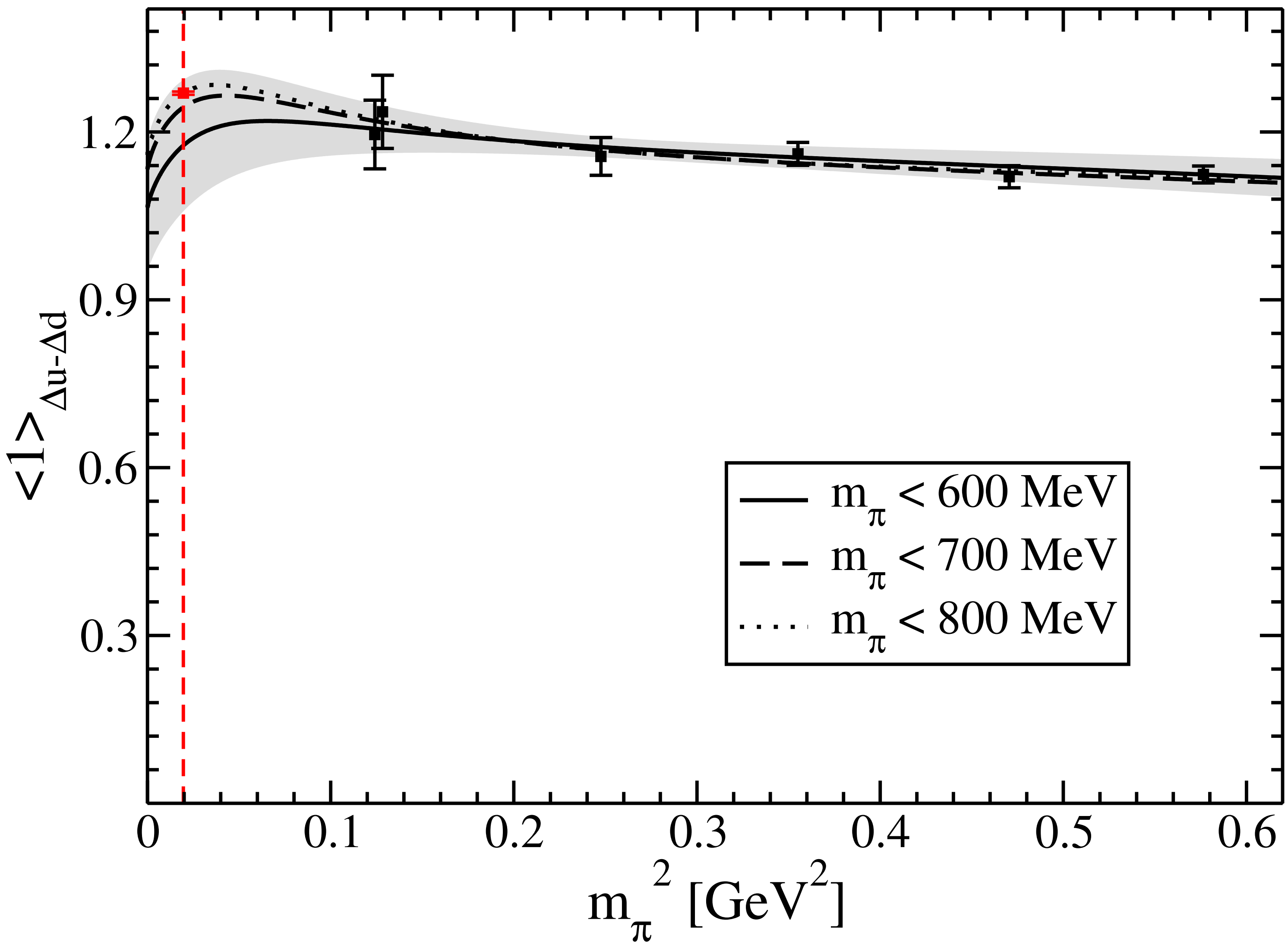
$$\chi^{\mathrm{tw}} = \exp(i\gamma_5\tau_3\theta)\chi^{\mathrm{ph}} \qquad \overline{\chi}^{\mathrm{tw}} = \overline{\chi}^{\mathrm{ph}}\exp(i\gamma_5\tau_3\theta)$$

- twisted quark mass μ provides an infared regulator: $\det(D^\dagger D) \geq \mu^2$
- we use the maximally twisted Wilson action: $\theta = \pi/4$
- physical observables are accurate to $\mathcal{O}(a^2)$ at maximal twist
- 270 MeV $< m_\pi <$ 600 MeV and $a = 0.053$, 0.067 and 0.085 fm

Benchmark Calculations: g_A and $\langle x \rangle_{u-d}$

- axial charge of the nucleon, $g_A = \langle 1 \rangle_{\Delta u - \Delta d}$
  - most accurate moment of PDFs to calculate
  - precise experimental measurement, $g_A = 1.2695 \pm 0.0029$ (PDG)
  - strong chiral and volume dependence provides stringent test
- momentum fraction of the nucleon, $\langle x \rangle_{u-d}$
  - also measured accurately
  - more demanding test of renormalization

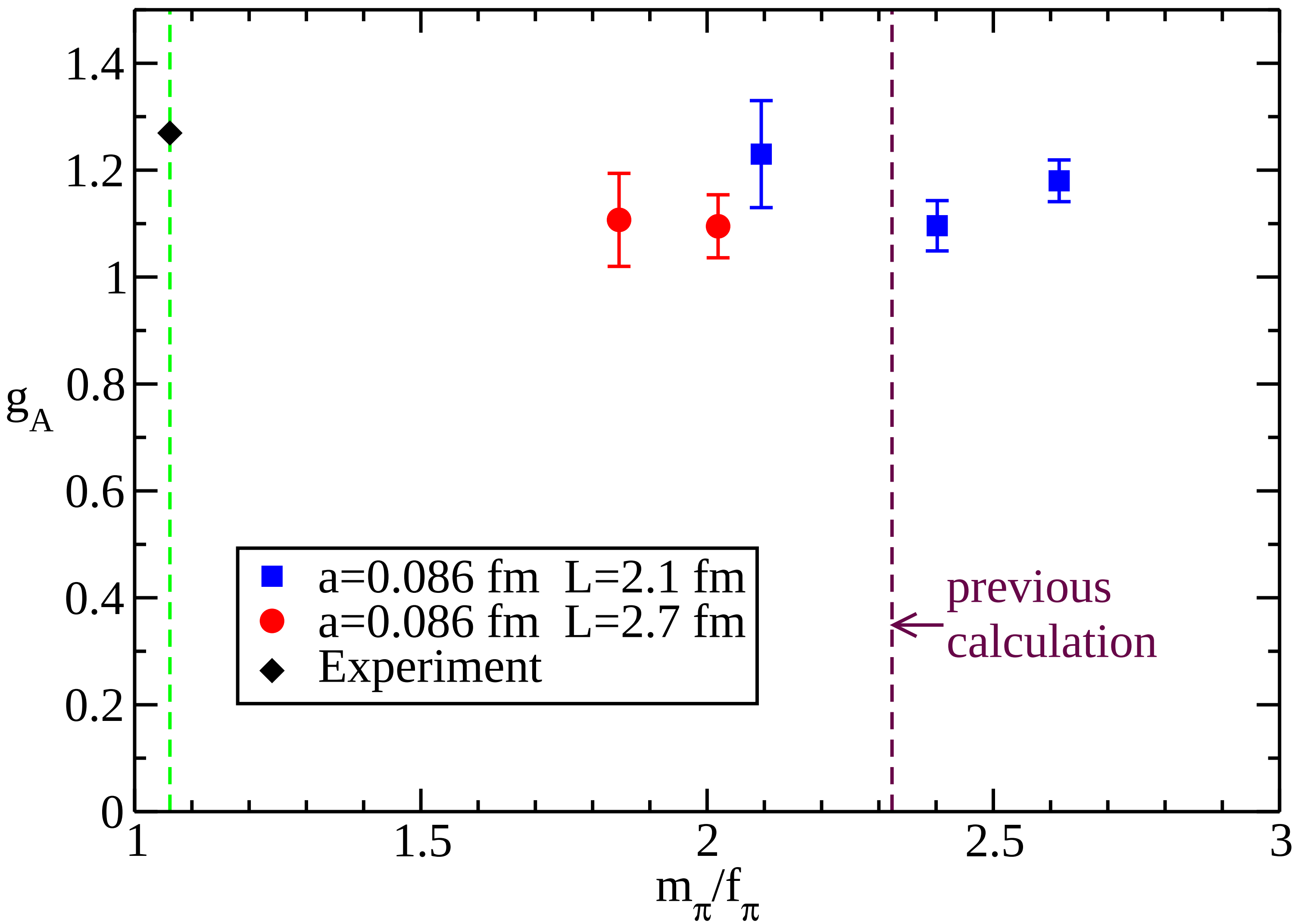
Axial Charge: Previous Calculation



- recent results have $m_\pi^2|_{\text{min}} \approx 0.07$ GeV^2

work done with LHPC: hep-lat/0710.1373

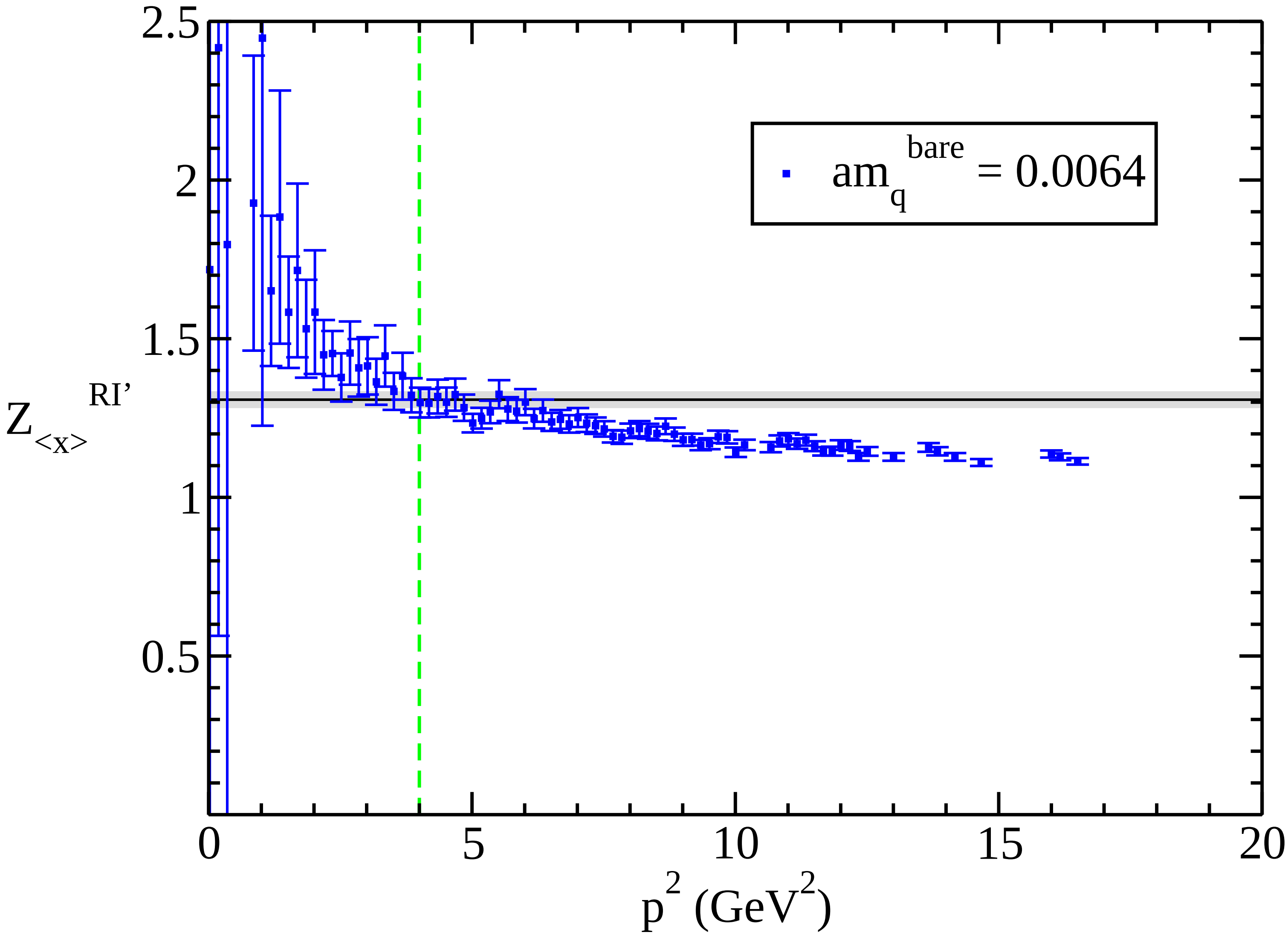
Axial Charge: Recent Results



- $m_\pi/f_\pi|_{\text{cur}} \approx 0.8\, m_\pi/f_\pi|_{\text{prev}}$ and NNLO terms are $(0.8)^4 \approx 0.4$

results from: T. Korzec

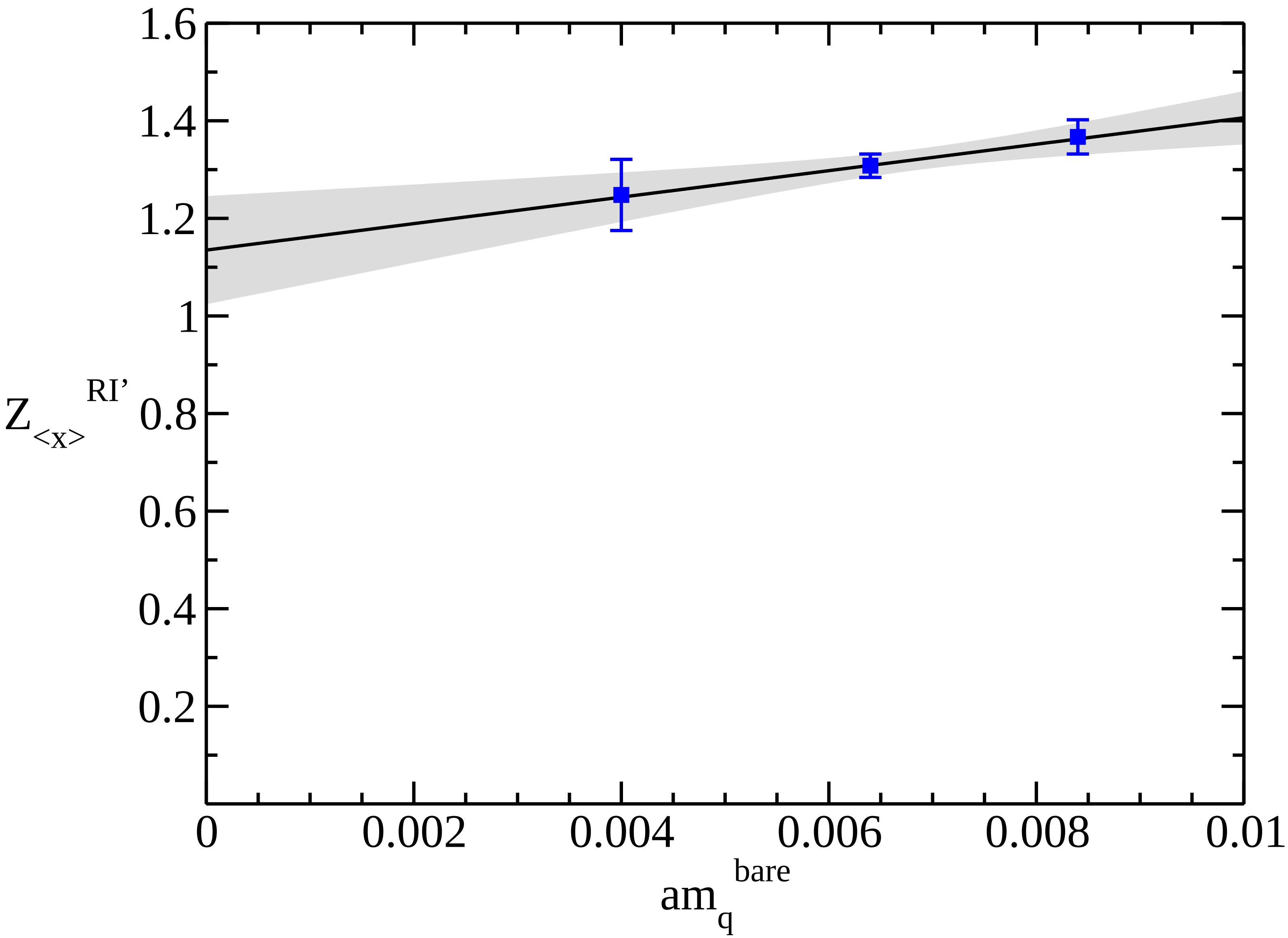
Renormalization of $\langle x \rangle_{u-d}$



- RI-MOM imposes $Z^{RI'}\langle p|O|p\rangle|_{p^2=\mu^2} = \langle p|O|p\rangle|_{\text{tree}}$ for $\Lambda_{\text{QCD}} \ll \mu \ll 1/a$

results from: Z. Liu

Matching of $\langle x \rangle_{u-d}$



- $Z^{RI'}_{\langle x \rangle} = 1.14(11)$ and $Z^{\overline{MS}}_{\langle x \rangle} = Z^{\overline{MS}}_{RI'}\, Z^{RI'}_{\langle x \rangle} = 1.03(10)$ for $\mu = 4$ GeV^2

Perturbative Renormalization: Cross Check

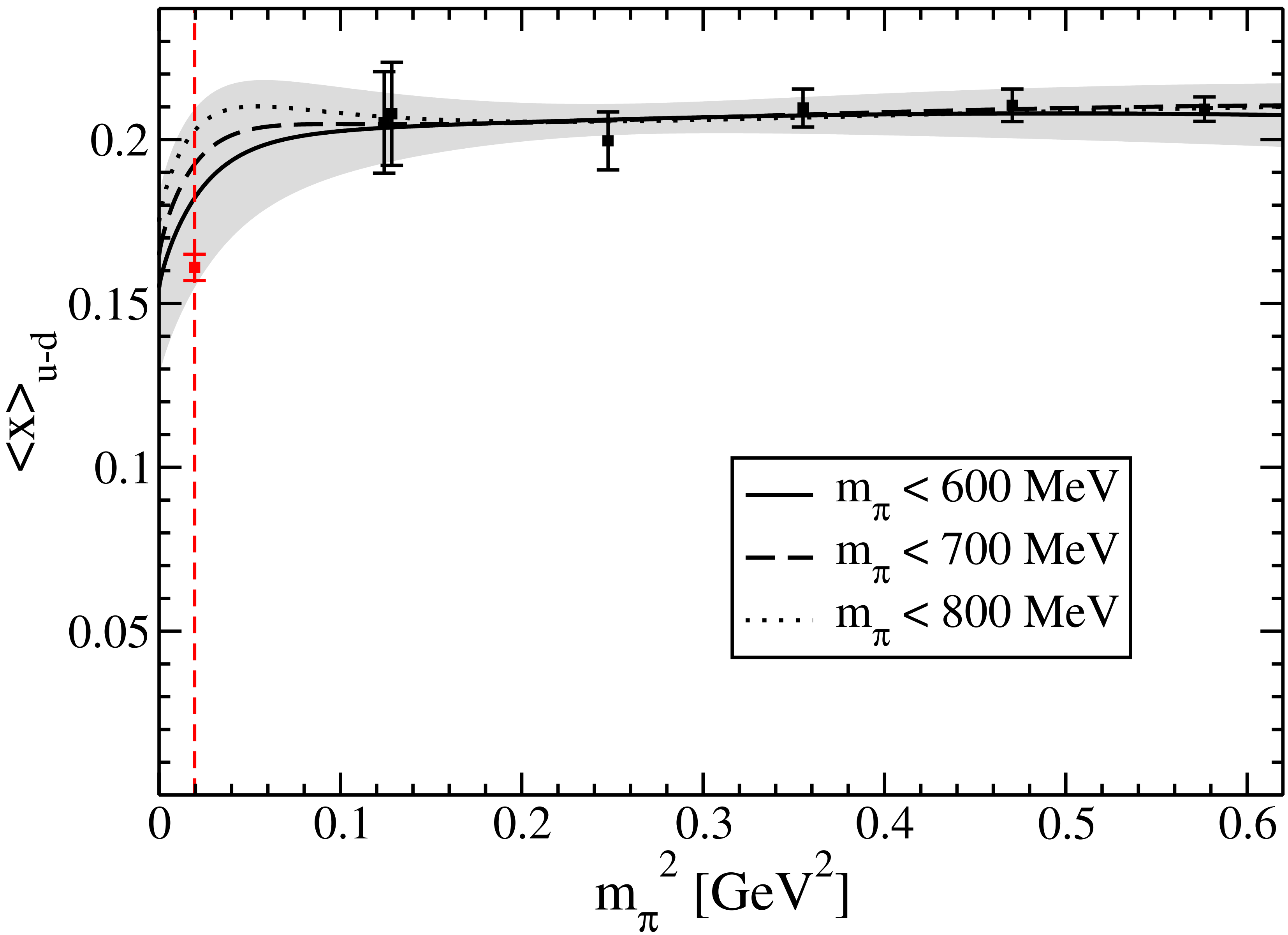
- lattice perturbation theory is being used as a cross-check
- initial results for $Z_q(\mu = 1/a)$ look promising

a	$Z_q^{\text{non-pert.}}(\mu = 1/a)$	$Z_q^{\text{pert.}}(\mu = 1/a)$
0.0995	0.755(4)	0.7499
0.0855	0.757(3)	0.7620
0.0667	0.777(5)	0.7781

- eventual cross checks from the Schrödinger functional (J. Lopez)

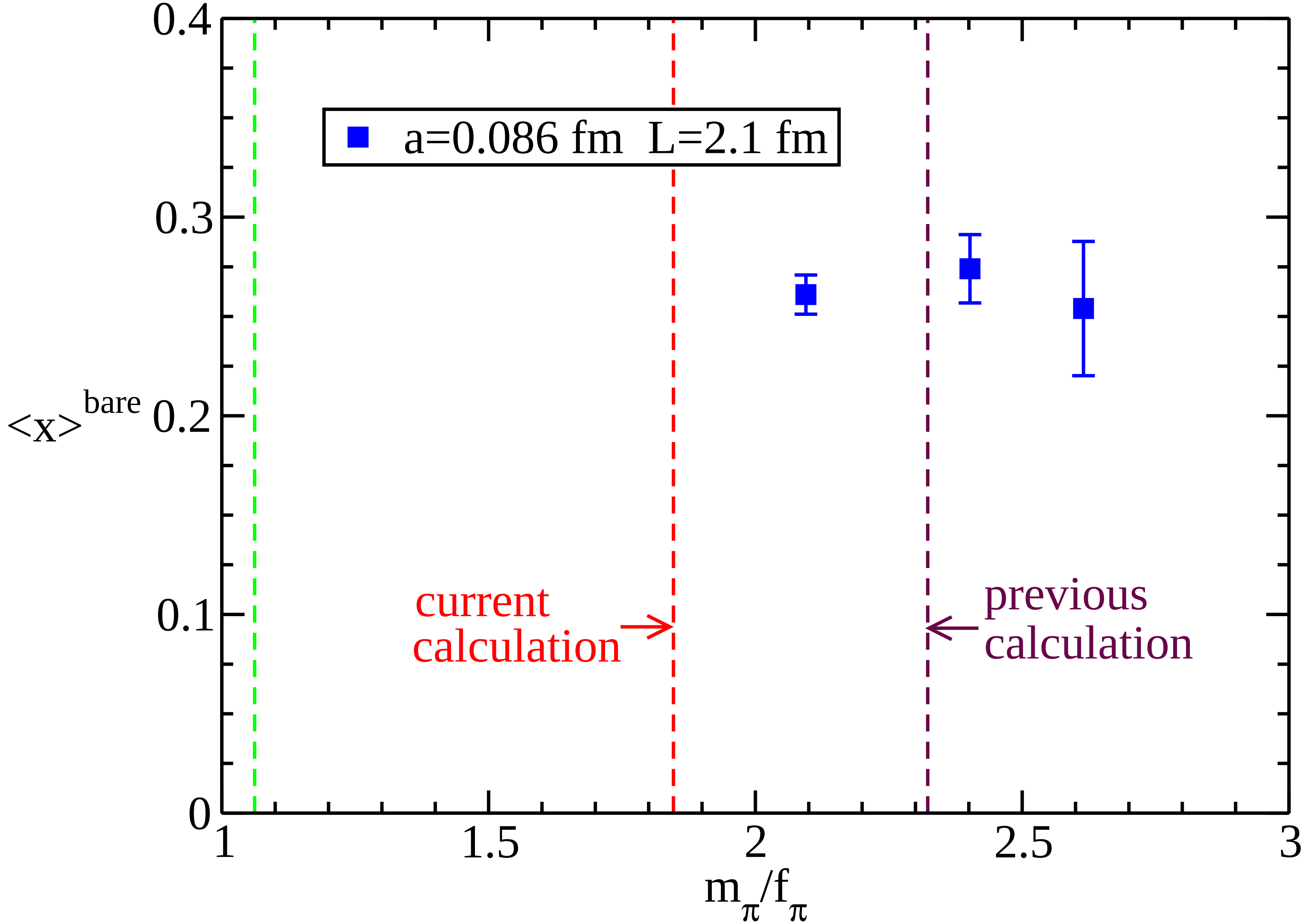
results from: M. Constantinou

Momentum Fraction: Previous Calculation



work done with LHPC: hep-lat/0710.1373

Momentum Fraction: Recent Results



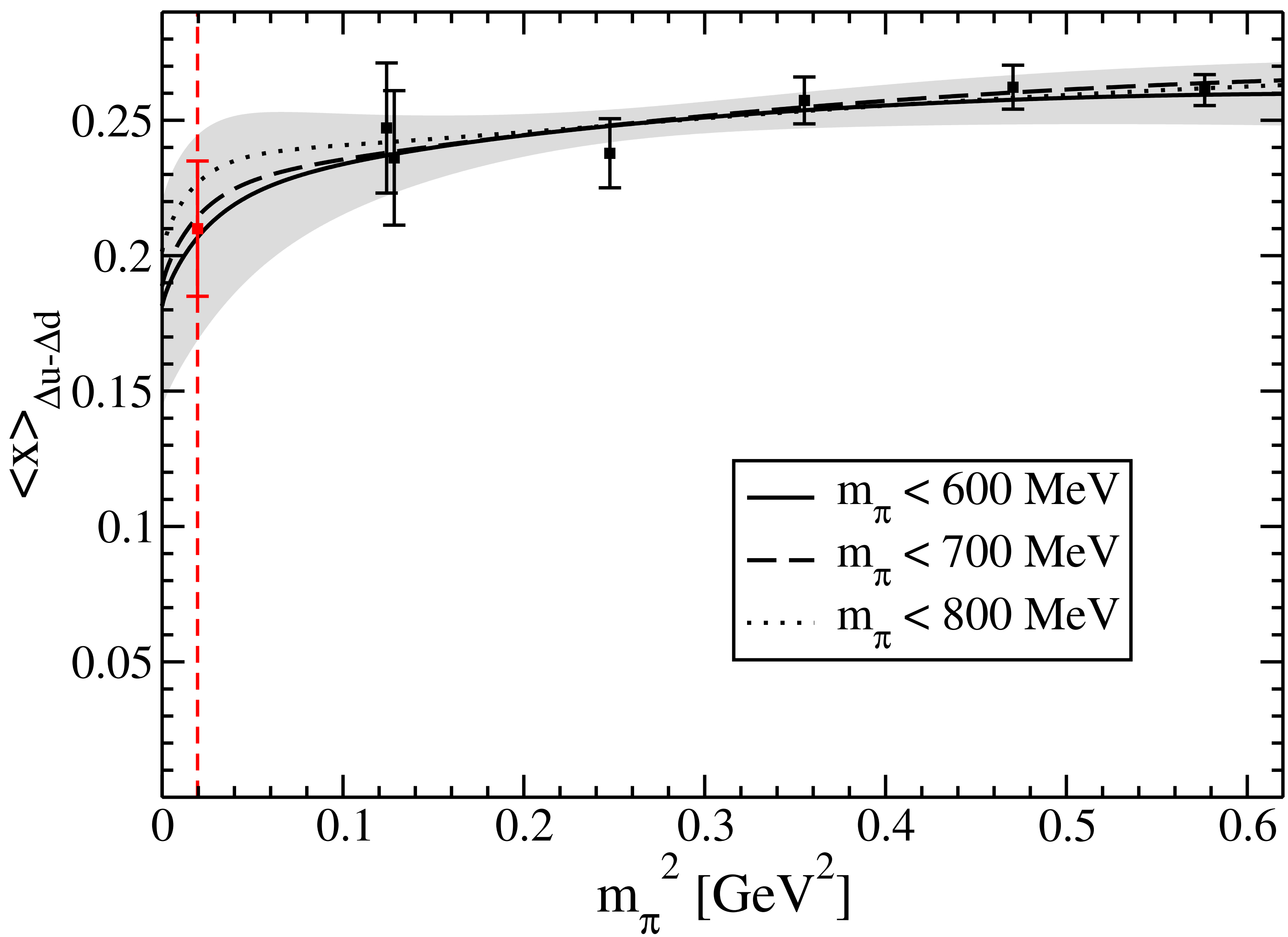
results from: D. Renner

Outlook

- we are using a twisted formulation of QCD on the lattice
- we can examine the chiral limit more deeply: $m_\pi|_{\text{min}} = 270$ MeV
- two more lattice spacings will allow a continuum limit: $a|_{\text{min}} = 0.053$ fm
- the accuracy of our calculation will improve significantly
- perturbative and non-pert. renormalization of remaining operators
- extension to off-forward matrix elements to study GPDs
- a new calculation is beginning with u, d, s and c quarks

Extra Slides

Previous $\langle x \rangle_{\Delta u - \Delta d}$



Moments of Parton Distributions

- the unpolarized PDF in light-cone coordinates and light-cone gauge

$$q(x) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle P|\overline{q}(0)\gamma^+ q(y^-)|P\rangle$$

- x-moments convert non-local operator into the local twist-two operators

$$\begin{aligned}
\int dx\, x^n q(x) &= \int \frac{dy^-}{4\pi} \int dx\, x^n e^{ixP^+y^-} \langle P|\overline{q}(0)\gamma^+ q(y^-)|P\rangle \\
&= \int \frac{dy^-}{4\pi} \delta(P^+y^-)(-1)^n (iP^+)^{-n} (\partial_-)^n \langle P|\overline{q}(0)\gamma^+ q(y^-)|P\rangle \\
&= 1/2\,(P^+)^{-(n+1)} \langle P|\overline{q}\gamma^+ (i\partial_-)^n q|P\rangle
\end{aligned}$$

- this relates moments to local operators

$$2(P^+)^{n+1} \langle x^n \rangle_q = \langle P|\overline{q}\gamma^+ (i\partial_-)^n q|P\rangle$$

- restoring gauge invariance and Lorentz indices

$$2P^{\{\mu_1} \ldots P^{\mu_{n+1}\}} \langle x^n \rangle_q = \langle P|\overline{q}\gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_{n+1}\}} q|P\rangle$$

Lattice Calculation of Matrix Elements

- lattice calculations calculate only correlation functions

$$\sum_{\vec{x}',\vec{x}} e^{-i\vec{p}\cdot\vec{x}'} e^{i\vec{p}\cdot\vec{x}} \langle P(t',\vec{x}\,')O(\tau,\vec{y}\,)\overline{P}(t,\vec{x}\,)\rangle =$$

- lattice field theory provides a finite, unitary quantization of QCD

$$= \sum_{\vec{x}',\vec{x}} e^{-i\vec{p}\cdot\vec{x}'} e^{i\vec{p}\cdot\vec{x}} \langle\Omega|\hat{P}(\vec{x}\,')e^{-\hat{H}(t'-\tau)}\hat{O}(\vec{y}\,)e^{-\hat{H}(\tau-t)}\hat{\overline{P}}(\vec{x}\,)|\Omega\rangle$$

- there is a proper Hilbert space and a positive definite Hamiltonian

$$= \sum_{n',n} \langle\Omega|\hat{P}|n',\vec{p}\rangle e^{-E_{n'}(t'-\tau)}\langle n',\vec{p}|\hat{O}(\vec{y}\,)|n,\vec{p}\rangle e^{-E_n(\tau-t)}\langle n,\vec{p}|\hat{\overline{P}}|\Omega\rangle$$

- Euclidean time evolution acts as a "low energy" filter, $e^{-\hat{H}t}$

$$\rightarrow |\langle\Omega|\hat{P}|\vec{p}\rangle|^2 e^{-E_{\vec{p}}(t'-t)}\langle\vec{p}|\hat{O}(\vec{y}\,)|\vec{p}\rangle$$

- ratio of 3-point and 2-point cancel factor, leaving only matrix element

Chiral Perturbation Theory

- chiral perturbation theory is an expansion in $m_\pi/(4\pi f_\pi)$
- the LO expression for g_A is a constant, $\mathring{g}_A$
- NLO adds $m_\pi^2 \ln m_\pi^2$ with a unique coefficient and a counter-term $c(\mu)$

$$g_A = \mathring{g}_A - \mathring{g}_A \frac{(2\mathring{g}_A^2 + 1)}{(4\pi \mathring{f}_\pi)^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right) + c(\mu) m_\pi^2$$

- fix the scale $\mu = \mathring{f}_\pi$ and replace $\mathring{g}_A \to g_A$ and $\mathring{f}_\pi \to f_\pi$ in NLO piece

$$g_A(1 + (2g_A^2 + 1) m_\pi^2/(4\pi f_\pi)^2 \ln(m_\pi^2/f_\pi^2)) = \mathring{g}_A + c\, m_\pi^2$$