Resummation for SUSY particle production at the LHC

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PHYSICS AT THE TERA SCALE
Helmholtz Alliance

AK and L. Motyka, Phys. Rev. Let. **102**, 111802 (2009)

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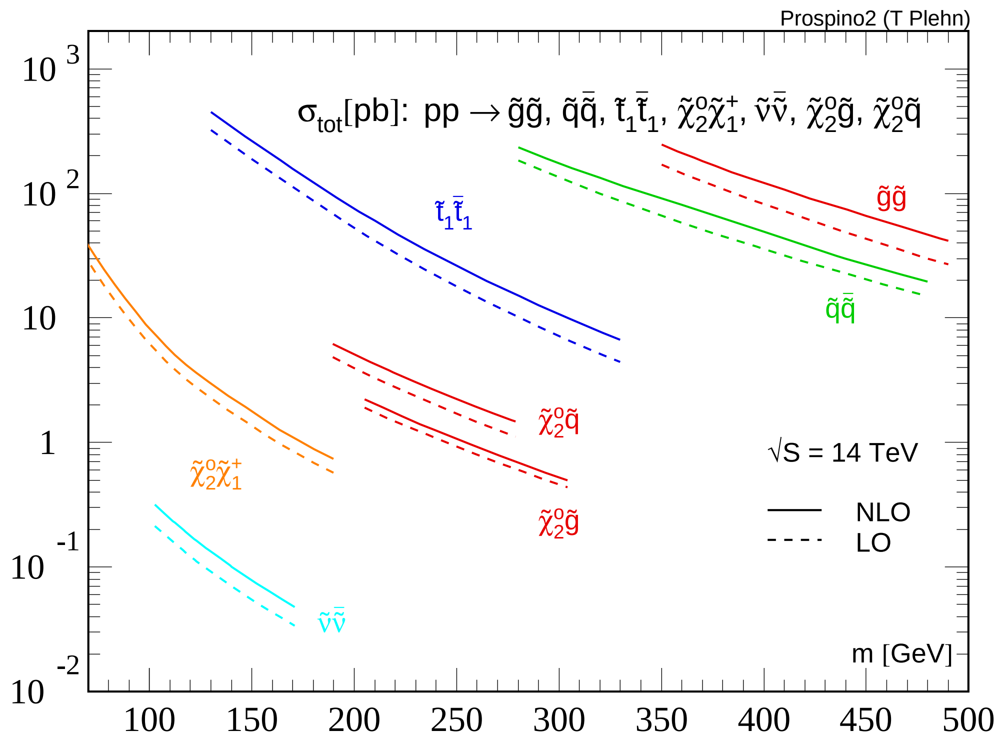
SUSY particle pair-production at the LHC

- MSSM: minimal content of SUSY particles + $R-$parity conservation
- At the LHC dominant sparticle production channels involve squarks ($\tilde{q}$) and gluinos ($\tilde{g}$) in the final state ($\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$, $\tilde{q}\tilde{g}$, $\tilde{g}\tilde{g}$ pairs)



[Plehn, Prospino2]

- Production cross sections large ⇒ "easy" SUSY discovery

Total cross sections

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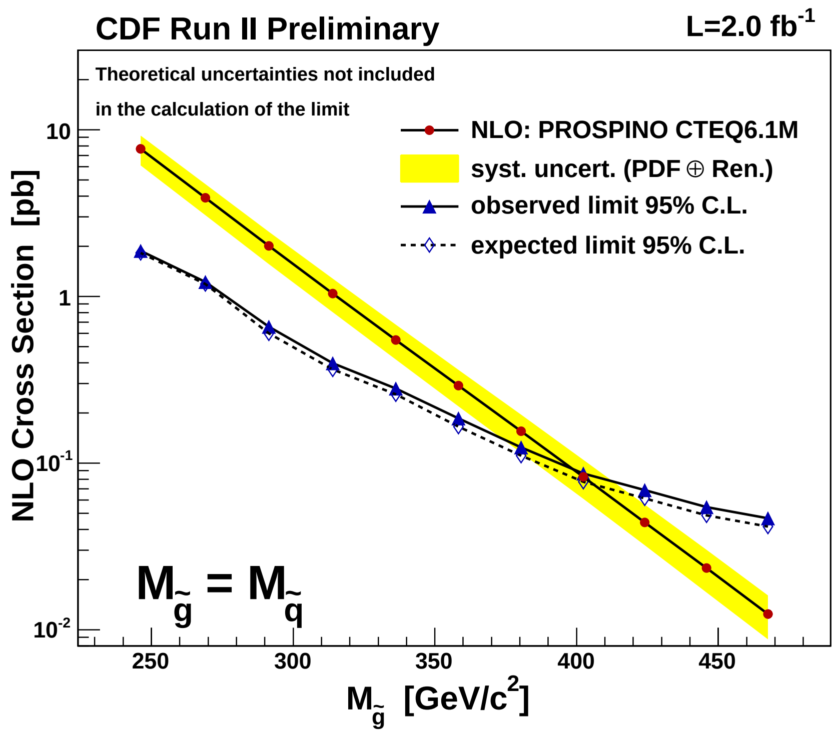

mSUGRA with $A_0 = 0$, $sgn(\mu) = -1$, $\tan\beta = 5$

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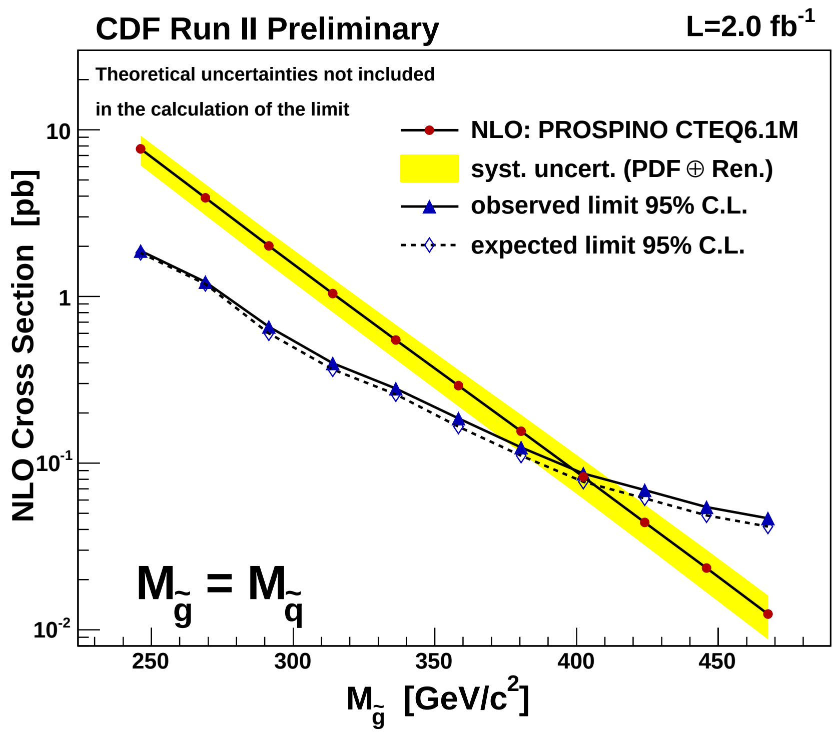


mSUGRA with $A_0 = 0$, $sgn(\mu) = -1$, $\tan\beta = 5$

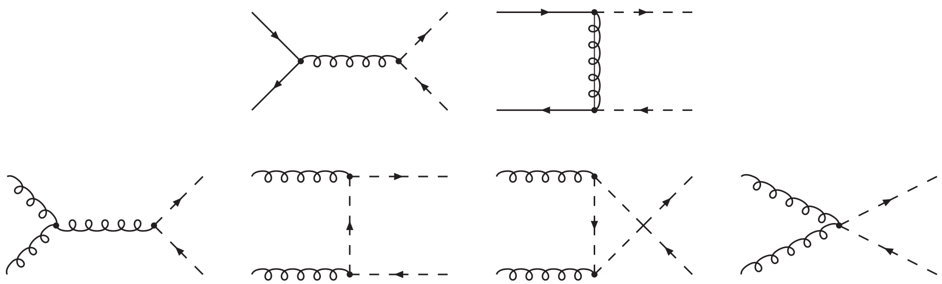
- Important to know total cross sections with high precision

Coloured sparticle production at LO

[*Kane, Leveille'82*][*Harrison, Llewellyn Smith'84*][*Dawson, Eichten, Quigg'85*]

$\mathcal{O}(\alpha_s^2)$ diagrams:

$pp \longrightarrow \tilde{q}\bar{\tilde{q}}$



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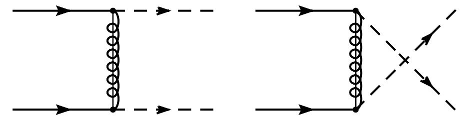

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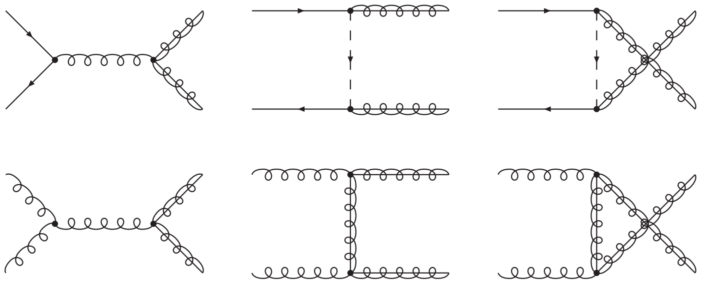

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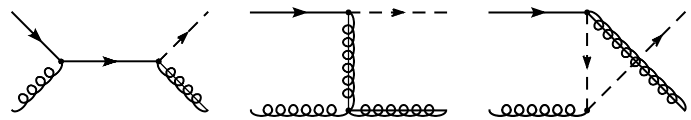

Beyond $\mathcal{O}(\alpha_s^2)$

- Higher-order corrections to $\mathcal{O}(\alpha_s^2)$ processes
  - NLO SUSY-QCD corrections $\longrightarrow \mathcal{O}(\alpha_s^3)$ [Beenakker, Höpker, Spira, Zerwas'96]
  - dominant NNLO contributions (NNLL-NNLO, Coulomb, scale dependence) $\longrightarrow \mathcal{O}(\alpha_s^4)$ [Langenfeld, Moch'09]
  - EW corrections $\longrightarrow \mathcal{O}(\alpha_s^2 \alpha)$ [Hollik, Kollar, Trenkel'07][Hollik, Mirabella'08][Hollik, Mirabella, Trenkel'08][Beccaria et al.'08]

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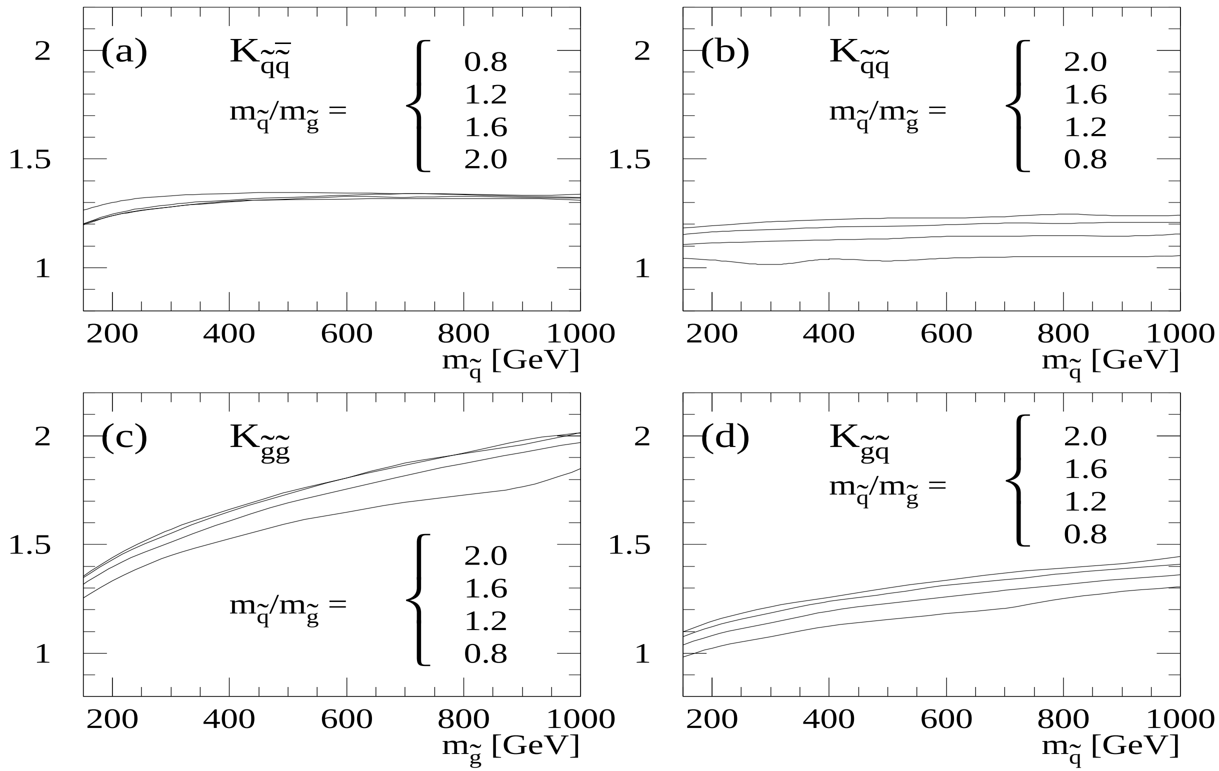
  - EW corrections $\longrightarrow \mathcal{O}(\alpha_s^2\alpha)$ [Hollik, Kollar, Trenkel'07][Hollik, Mirabella'08][Hollik, Mirabella, Trenkel'08][Beccaria et al.'08]
- Tree-level EW effects
  - QCD-EW interference and photon-induced contributions $\longrightarrow \mathcal{O}(\alpha\alpha_s)$ [Bornhauser et al.'07] [Alan, Cankocak, Demir'07][Hollik, Kollar, Trenkel'07][Hollik, Mirabella'08][Hollik, Mirabella, Trenkel'08]
  - Tree-level EW $\longrightarrow \mathcal{O}(\alpha^2)$ [Bozzi, Fuks, Klasen'05][Bornhauser et al.'07] [Alan, Cankocak, Demir'07]

Coloured sparticle production at NLO (SUSY-QCD)

[*Beenakker, Höpker, Spira, Zerwas'96*]

$$K_{ij} = \sigma_{ij}^{\rm NLO}/\sigma_{ij}^{\rm LO}$$



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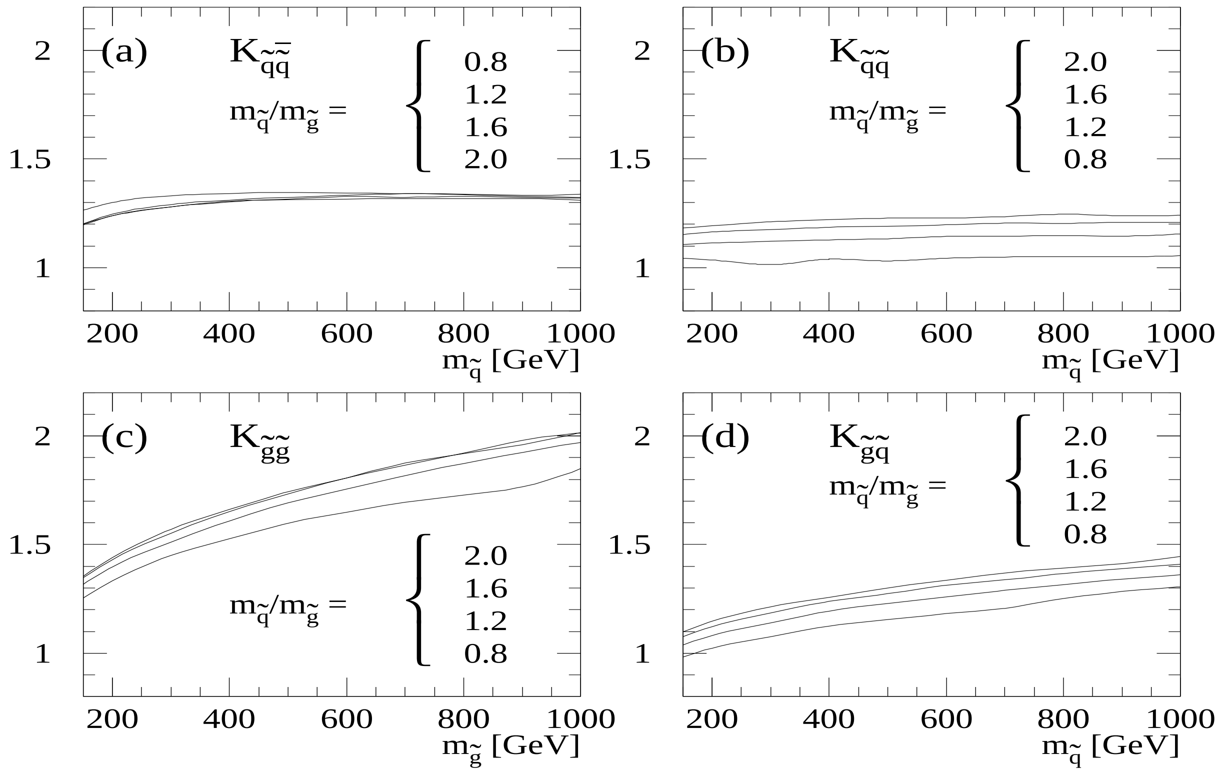

$\Rightarrow$ Increase of the cross sections due to NLO SUSY-QCD corrections

Note: assume all squarks ($\tilde{q}_L$,$\tilde{q}_R$) mass degenerate; no final state stops $\Rightarrow$

[*Beenakker, Krämer, Plehn, Spira, Zerwas'98*]

Higher-order effects

[*Beenakker, Höpker, Spira, Zerwas'96*]

- $\longrightarrow$ 100 % correction to $\sigma_{\tilde{g}\tilde{g}}$ at $m_{\tilde{g}} = 1\,\mathrm{TeV}$
  $\longrightarrow$ 40% correction to $\sigma_{\tilde{q}\tilde{g}}$ at $m_{\tilde{q}} = 1\,\mathrm{TeV}$
  $\longrightarrow$ 30% ($\sim$ 20%) correction to $\sigma_{\tilde{q}\bar{\tilde{q}}}$ ($\sigma_{\tilde{q}\tilde{q}}$) at $m_{\tilde{q}} = 1\,\mathrm{TeV}$

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- Large masses of squarks and gluons
  $\Rightarrow$ often production close to threshold $\hat{s} \sim 4m^2$ ($m = \frac{m_{\tilde{q}}+m_{\tilde{g}}}{2}$ for $\tilde{q}\tilde{g}$ production)
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- Terms with powers of $\log\left(1 - \frac{4m^2}{\hat{s}}\right)$ in the expressions for cross sections
  $\rightarrow$ the closer to threshold the more important the logarithmic terms
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- Additionally, more radiation from particles with higher colour charge
  $\longrightarrow$ $\tilde{g}\tilde{g}$ production the most affected

Soft-gluon corrections to $\tilde{q}$ and $\tilde{g}$ production

- Seen already at NLO: at threshold [Beenakker, Höpker, Spira, Zerwas'96]

$$\hat{\sigma}^{\rm NLO}_{gg\to\tilde{g}\tilde{g}} \sim 4\pi\alpha_{\rm s}\hat{\sigma}^{\rm LO}_{gg\to\tilde{g}\tilde{g}}\left\{\frac{3}{2\pi^2}\log^2(8\beta^2)-\frac{29}{4\pi^2}\log(8\beta^2)-\frac{3}{2\pi^2}\log(8\beta^2)\log\left(\frac{\mu}{m_{\tilde{g}}}\right)+\frac{1}{16}\frac{1}{\beta}\right\}$$

with $\beta^2 = 1-\frac{4m^2_{\tilde{g}}}{\hat{s}}$ and $\hat{\sigma}^{\rm LO}_{gg\to\tilde{g}\tilde{g}} \sim \frac{27}{64}\alpha_{\rm s}^2\pi\frac{\beta}{m^2_{\tilde{g}}}$

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$\boxed{\text{In the limit of } \beta \to 0 \text{ convergence of fixed-order expansion spoiled}}$

Threshold resummation

Resummation of threshold logarithms is carried out in the Mellin moment space, where the cross section factorizes:

$$\sigma^{(N)}_{h_1h_2} = \sum_{i,j} f^{(N+1)}_{i/h_1}(\mu^2) f^{(N+1)}_{j/h_2}(\mu^2) \hat{\sigma}^{(N)}_{ij}(\mu^2, \alpha_s(\mu^2))$$

with

$$g^{(N)} = \int_0^1 dz\, z^{N-1}\, g(z)$$

$$z = \frac{4m^2}{\hat{s}} \qquad \log(1-z) \longleftrightarrow \log(N)$$

The logarithmic terms exponentiate

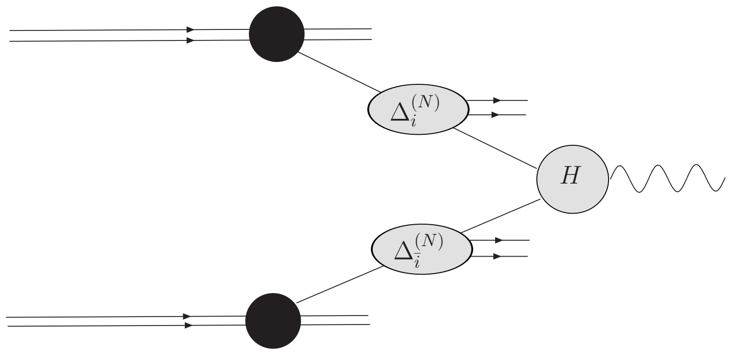
$$\hat{\sigma}^{(N)} = \hat{\sigma}^{(N)}_0 \mathcal{C} \exp(\mathcal{S})$$

($\mathcal{C}$ contains finite contributions)

$$\mathcal{S} \;=\; \underbrace{L f_1(\alpha_s L)}_{LL} + \underbrace{f_2(\alpha_s L)}_{NLL} + \underbrace{\alpha_s f_3(\alpha_s L)}_{NNLL} + \underbrace{\ldots}_{\ldots} \qquad L = \log(N)$$

Resummation for $2 \to 2$ with colour and masses (I)

Colour-singlet production: Drell-Yan, Higgs, ... [*Catani, Trentadue'89*] [*Sterman'87*]



Up to NLL:

$$\hat{\sigma}_{i\bar{i}}^{(N)} = \underbrace{H_{i\bar{i}}^{(N)}}_{\text{hard function}} \times \underbrace{\Delta_i^{(N)} \Delta_j^{(N)}}_{\text{soft-collinear radiation}}$$

universal factors; KNOWN

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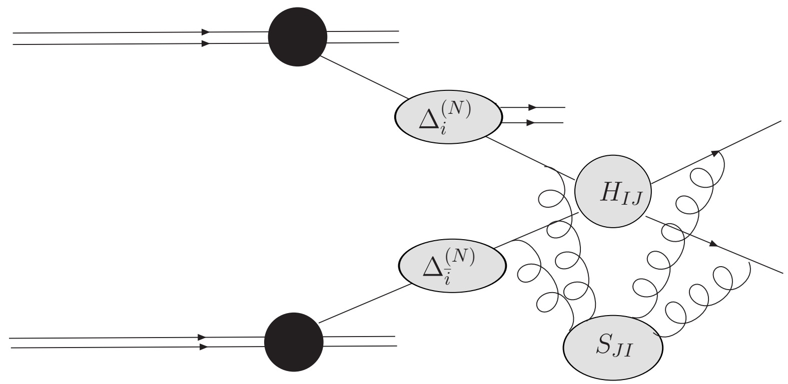
$2 \to 2$ with colour flow [Kidonakis, Sterman'96-97][Kidonakis, Oderda, Sterman'98][Bonciani et al.'03]



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$$\sigma^{(N)}_{ij\to kl} = \underbrace{H^{(N)}_{ij\to kl,IJ}}_{\text{hard function}} \times \underbrace{\Delta_i^{(N)}\Delta_j^{(N)}}_{\text{soft-collinear radiation}} \times \underbrace{S_{ij\to kl,JI}}_{\text{soft wide-angle emission}}$$

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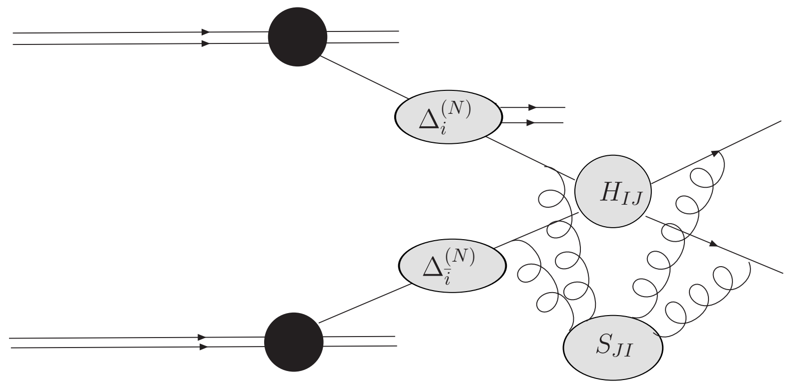

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Factorization $\Rightarrow$ RGE

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right) S^{(N)}_{JI} = -\Gamma^{\dagger}_{JK} S(N)_{KI} - S(N)_{JL}\Gamma_{LI}$$

Soft anomalous dimension $\quad \Gamma_{LK} = -\frac{g}{2}\frac{\partial}{\partial g}\mathrm{Res}_{\epsilon\to 0} Z_{LK}(g,\epsilon)$

Anomalous dimensions

- Massless $2 \to 2$ QCD processes: anomalous dimensions known at 1-loop (NLL) [*Kidonakis, Oderda, Sterman'98*][*Bonciani et al.'03*] and 2-loop (NNLL) [*Mert Aybat, Dixon, Sterman'06*]
- Massive case: anomalous dimension known for heavy quark $Q\bar{Q}$ production at 1-loop (NLL) [*Kidonakis, Sterman'96*][*Bonciani et al.'98*] and at 2-loop (NNLL) [*Moch, Uwer'08*]

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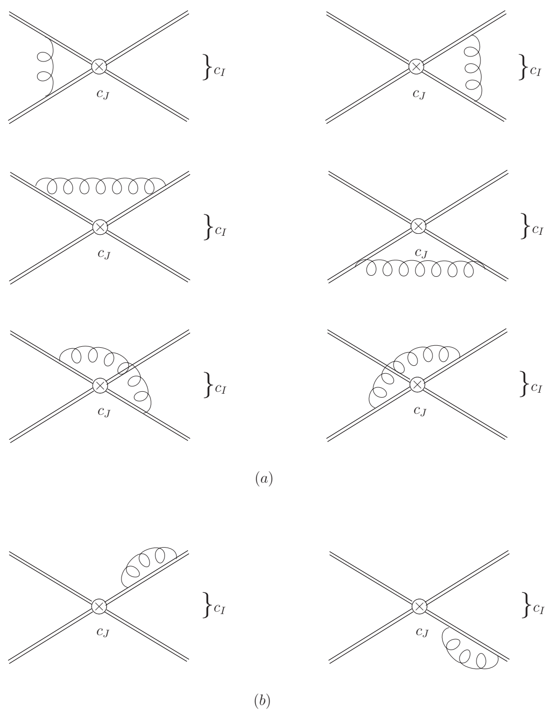

- 1-loop integrals I_d in the eikonal approximation
  $\longrightarrow$ [*Kidonakis, Sterman'96*]
- only need to know coefficients of the $1/\epsilon$ UV poles of the I_d integrals

$$\sum_{d}^{\#\text{ of diagrams}} c_J^* \mathcal{T}_k(c_I)\; I_d\big|_{\frac{1}{\epsilon}\text{ pole}}$$

Resummation for $2 \to 2$ with colour and masses (II)

- In orthogonal basis in colour space for which $\Gamma^{ij\to kl}$ is diagonal [*Kidonakis, Sterman'96-97*][*Bonciani, Catani, Mangano, Nason'98*]

$$\hat{\sigma}^{(N)}_{ij\to kl} = \sum_I \hat{\sigma}^{(N)}_{0,ij\to kl,I} \, \widetilde{C}_{ij\to kl,I} \, \Delta^i_{(N+1)} \, \Delta^j_{(N+1)} \, \Delta^{(N+1)}_{S,ij\to kl,I}$$

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- Radiative factor for soft non-collinear emission

$$\Delta^{(N+1)}_{S,ij\to kl,I} = \exp\left\{\int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{ij\to kl,I}(\alpha_s((1-z)^2Q^2))\right\}$$

related to Γ by

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- In the threshold limit $\hat{s}\to 4m^2$, $\Gamma^{ij\to kl}$ matrices for the s-channel colour bases become diagonal
- In general need to know $\hat{\sigma}^{(N)}_{0,ij\to kl}$, $D^{(1)}_{ij\to kl}$, $\widetilde{C}_{ij\to kl}$ coefficients in each colour channel

Threshold limit for $\Gamma^{ij\to kl}$

[*AK, Motyka'08*] [*Beenakker, Brensing, Krämer, AK, Laenen, Niessen*]

- $D^{(1)}$-coefficients for the full set of $\tilde{q}$ and $\tilde{g}$ production processes

$$\{D^{(1)}_{q\bar{q}\to\tilde{q}\bar{\tilde{q}},\,I}\} = \{0,-3\} \qquad I = \{\mathbf{1},\mathbf{8}\}$$

$$\{D^{(1)}_{gg\to\tilde{q}\bar{\tilde{q}},\,I}\} = \{0,-3\} \qquad I = \{\mathbf{1},\mathbf{8}\}$$

$$\{D^{(1)}_{q\bar{q}\to\tilde{g}\tilde{g},\,I}\} = \{0,-3,-3\} \qquad I = \{\mathbf{1},\mathbf{8_S},\mathbf{8_A}\}$$

$$\{D^{(1)}_{gg\to\tilde{g}\tilde{g},\,I}\} = \{0,-3,-3,-6,-8;-3,-3,-6\} \qquad I = \{\mathbf{1},\mathbf{8_S},\mathbf{8_A},\mathbf{10}\oplus\overline{\mathbf{10}},\mathbf{27},\mathbf{8}_{\mathrm{M}1},\mathbf{8}_{\mathrm{M}2},\mathbf{10}_{\mathrm{M}}\}$$

$$\{D^{(1)}_{qq\to\tilde{q}\tilde{q},\,I}\} = \{-4/3,-10/3\} \qquad I = \{\bar{\mathbf{3}},\mathbf{6}\}$$

$$\{D^{(1)}_{qg\to\tilde{q}\tilde{g},\,I}\} = \{-4/3,-10/3,-16/3\} \qquad I = \{\mathbf{3},\bar{\mathbf{6}},\mathbf{15}\}$$

Threshold limit for $\Gamma^{ij\to kl}$

[*AK, Motyka'08*] [*Beenakker, Brensing, Krämer, AK, Laenen, Niessen*]

- $D^{(1)}$-coefficients for the full set of $\tilde{q}$ and $\tilde{g}$ production processes

$$\{D^{(1)}_{q\bar{q}\to\tilde{q}\bar{\tilde{q}},\,I}\} = \{0,-3\} \qquad I = \{\mathbf{1},\mathbf{8}\}$$

$$\{D^{(1)}_{gg\to\tilde{q}\bar{\tilde{q}},\,I}\} = \{0,-3\} \qquad I = \{\mathbf{1},\mathbf{8}\}$$

$$\{D^{(1)}_{q\bar{q}\to\tilde{g}\tilde{g},\,I}\} = \{0,-3,-3\} \qquad I = \{\mathbf{1},\mathbf{8_S},\mathbf{8_A}\}$$

$$\{D^{(1)}_{gg\to\tilde{g}\tilde{g},\,I}\} = \{0,-3,-3,-6,-8;-3,-3,-6\} \qquad I = \{\mathbf{1},\mathbf{8_S},\mathbf{8_A},\mathbf{10}\oplus\overline{\mathbf{10}},\mathbf{27},\mathbf{8}_{\mathrm{M1}},\mathbf{8}_{\mathrm{M2}},\mathbf{10}_{\mathrm{M}}\}$$

$$\{D^{(1)}_{qq\to\tilde{q}\tilde{q},\,I}\} = \{-4/3,-10/3\} \qquad I = \{\bar{\mathbf{3}},\mathbf{6}\}$$

$$\{D^{(1)}_{qg\to\tilde{q}\tilde{g},\,I}\} = \{-4/3,-10/3,-16/3\} \qquad I = \{\mathbf{3},\bar{\mathbf{6}},\mathbf{15}\}$$

- Values of the quadratic Casimir operators for the SU(3) representations for the outgoing state $\longrightarrow$ soft gluon radiation from the total colour charge of the heavy-particle pair produced at threshold

Resummation-improved NLL+NLO total cross section

NLL resummed expression has to be matched with the full NLO result

$$
\begin{aligned}
\sigma^{\rm (match)}_{h_1 h_2 \to kl}(\rho, m^2, \{\mu^2\}) \quad &= \quad \sum_{i,j=q,\bar{q},g} \int_{C_{\rm MP}-i\infty}^{C_{\rm MP}+i\infty} \frac{dN}{2\pi i}\, \rho^{-N}\, f^{(N+1)}_{i/h_1}(\mu_F^2)\, f^{(N+1)}_{j/h_2}(\mu_F^2) \\
&\times \quad \left[\, \hat{\sigma}^{\rm (res)}_{ij\to kl,N}(m^2, \{\mu^2\}) \;-\; \hat{\sigma}^{\rm (res)}_{ij\to kl,N}(m^2, \{\mu^2\})\Big|_{\rm NLO}\, \right] \\
&+ \quad \sigma^{\rm NLO}_{h_1 h_2 \to kl}(\rho, m^2, \{\mu^2\})\,,
\end{aligned}
$$

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&\times \left[\hat{\sigma}^{(\mathrm{res})}_{ij\to kl,N}(m^2, \{\mu^2\}) - \hat{\sigma}^{(\mathrm{res})}_{ij\to kl,N}(m^2, \{\mu^2\}) \Big|_{\mathrm{NLO}} \right] \\
&+ \sigma^{\mathrm{NLO}}_{h_1 h_2 \to kl}(\rho, m^2, \{\mu^2\}),
\end{aligned}
$$

- Inverse Mellin transform evaluated using a contour in the complex N space according to 'Minimal Prescription' [*Catani, Mangano, Nason Trentadue'96*]
- NLO cross sections evaluated with publicly available code PROSPINO

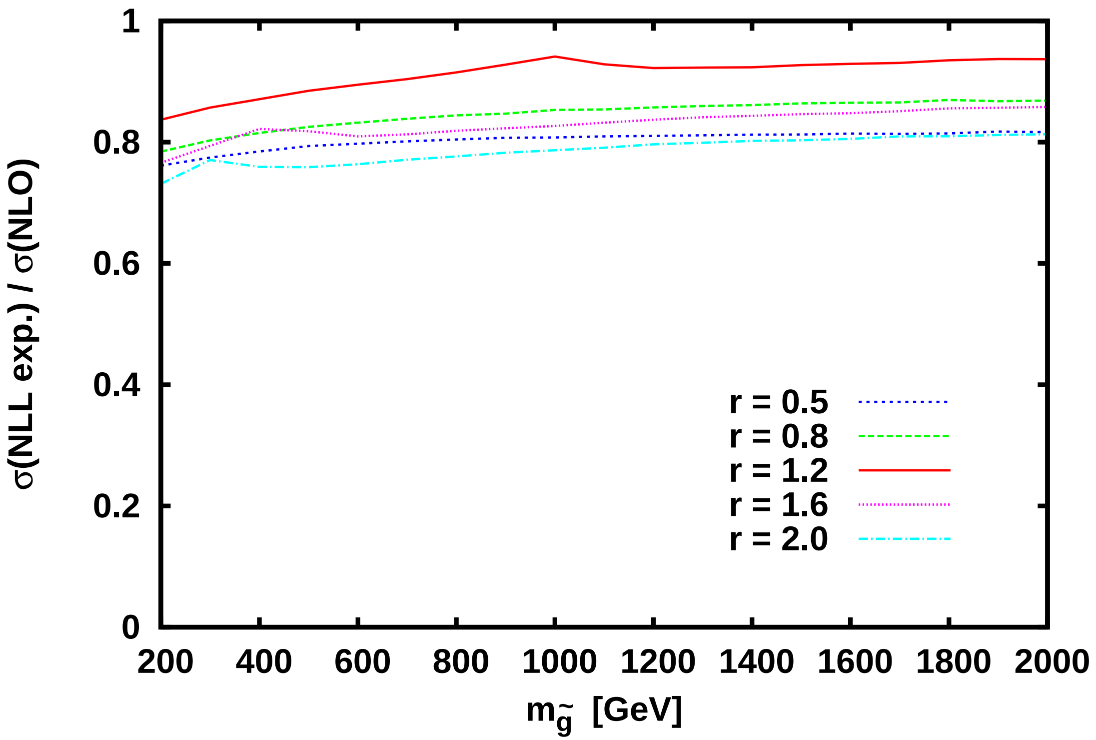
[*Beenakker, Hoepker, Krämer, Plehn, Spira, Zerwas*]

[*Plehn, http://www.ph.ed.ac.uk/ tplehn/prospino/*]

Importance of the NLL terms

[*AK, Motyka*]

Expanded NLL-resummed cross section up to $\mathcal{O}(\alpha_s^3)$ vs. NLO



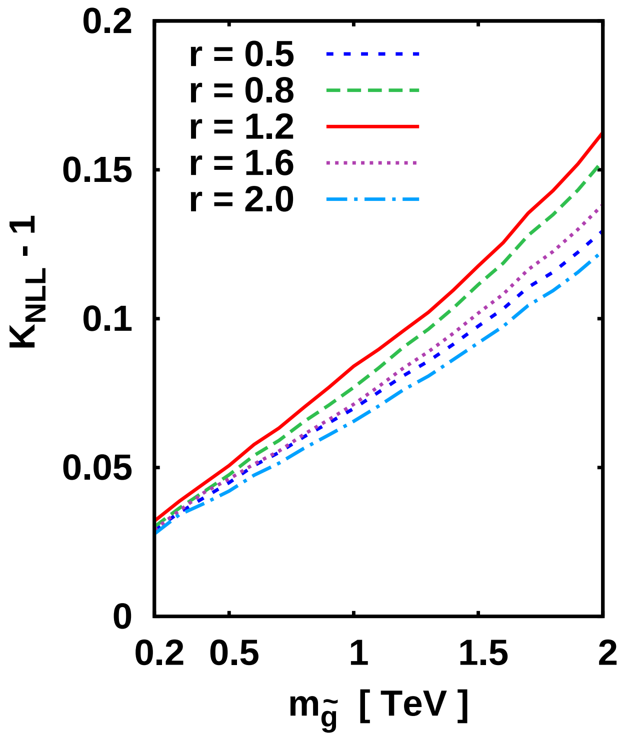
$pp \longrightarrow \tilde{g}\tilde{g}$ at the LHC

$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}$ ($\mu_F = \mu_R = m_{\tilde{g}}$, CTEQ6M)

preliminary

NLL gluino-pair production at the LHC

[*AK, Motyka'08*] [*Beenakker, Brensing, Krämer, AK, Laenen, Niessen*]



$$K^{\mathrm{NLL}} = \frac{\sigma^{\mathrm{match}}}{\sigma^{\mathrm{NLO}}}$$

$$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}$$

($\mu_F = \mu_R = m_{\tilde{g}}$, CTEQ6M)

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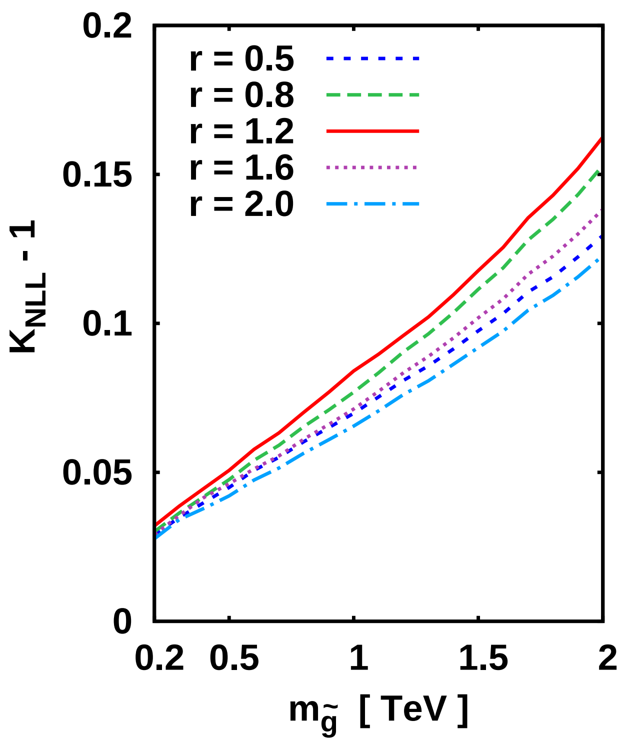
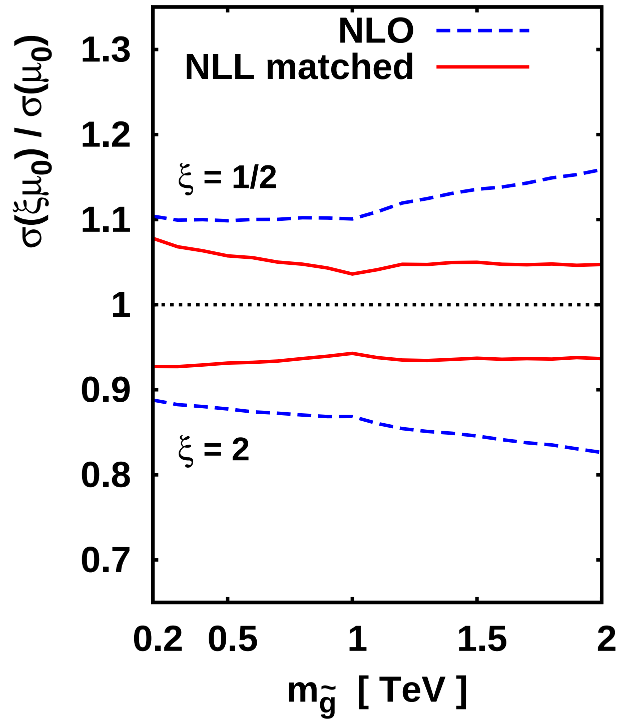
[AK, Motyka'08] [Beenakker, Brensing, Krämer, AK, Laenen, Niessen]





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$$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}$$

($\mu_F = \mu_R = m_{\tilde{g}}$, CTEQ6M)

$$\sigma^{\mathrm{NLO}}(\mu = \xi m_{\tilde{g}})/\sigma^{\mathrm{NLO}}(\mu = m_{\tilde{g}})$$

vs.

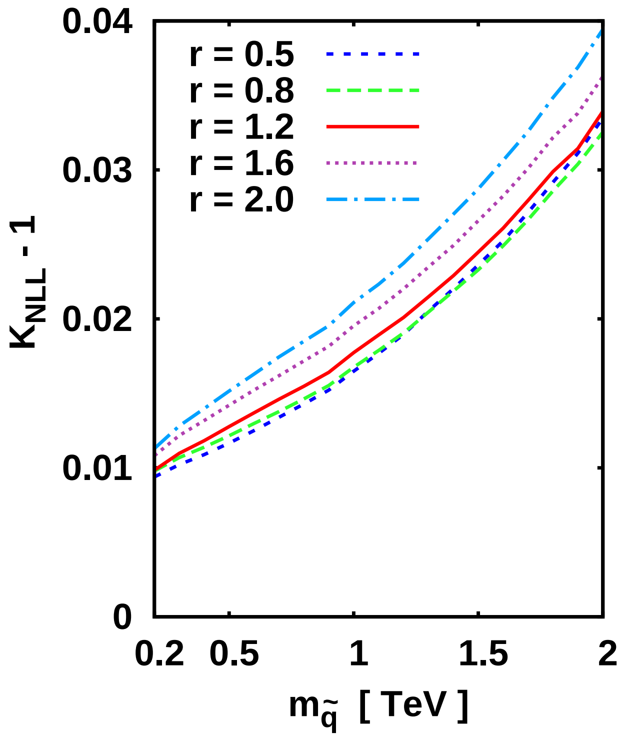
$$\sigma^{(\mathrm{match})}(\mu = \xi m_{\tilde{g}})/\sigma^{(\mathrm{match})}(\mu = m_{\tilde{g}})$$

for $\xi = 0.5$ and $\xi = 2$

($\mu = \mu_F = \mu_R; r = 1.2$, CTEQ6M)

NLL squark-antisquark production at the LHC

[*AK, Motyka'08*] [*Beenakker, Brensing, Krämer, AK, Laenen, Niessen*]



$$\mathrm{K}^{\mathrm{NLL}} = \frac{\sigma^{\mathrm{match}}}{\sigma^{\mathrm{NLO}}}$$

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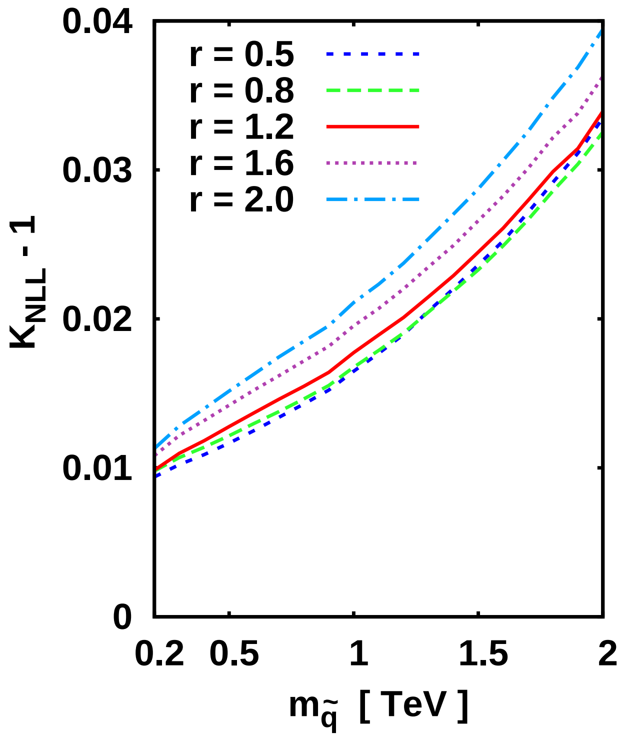
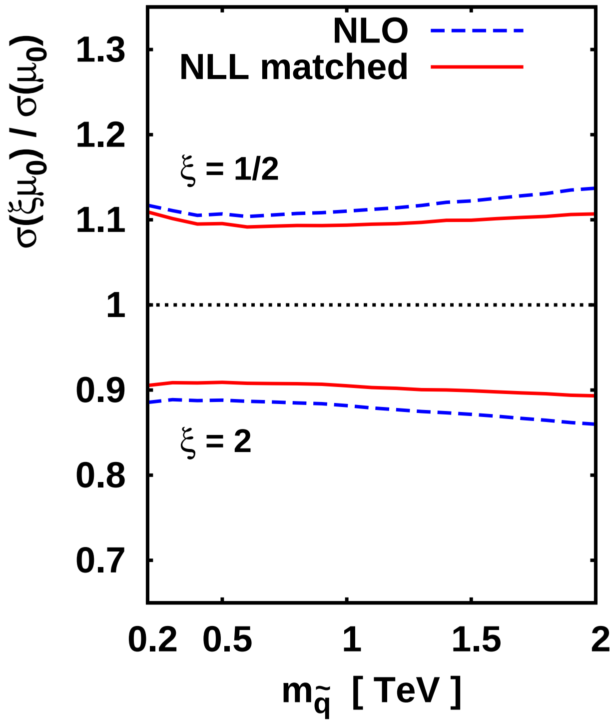


$$\mathrm{K}^{\mathrm{NLL}} = \frac{\sigma^{\mathrm{match}}}{\sigma^{\mathrm{NLO}}}$$

$$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}$$

($\mu_F = \mu_R = m_{\tilde{q}}$, CTEQ6M)



$$\sigma^{\mathrm{NLO}}(\mu = \xi m_{\tilde{q}})/\sigma^{\mathrm{NLO}}(\mu = m_{\tilde{q}})$$

vs.

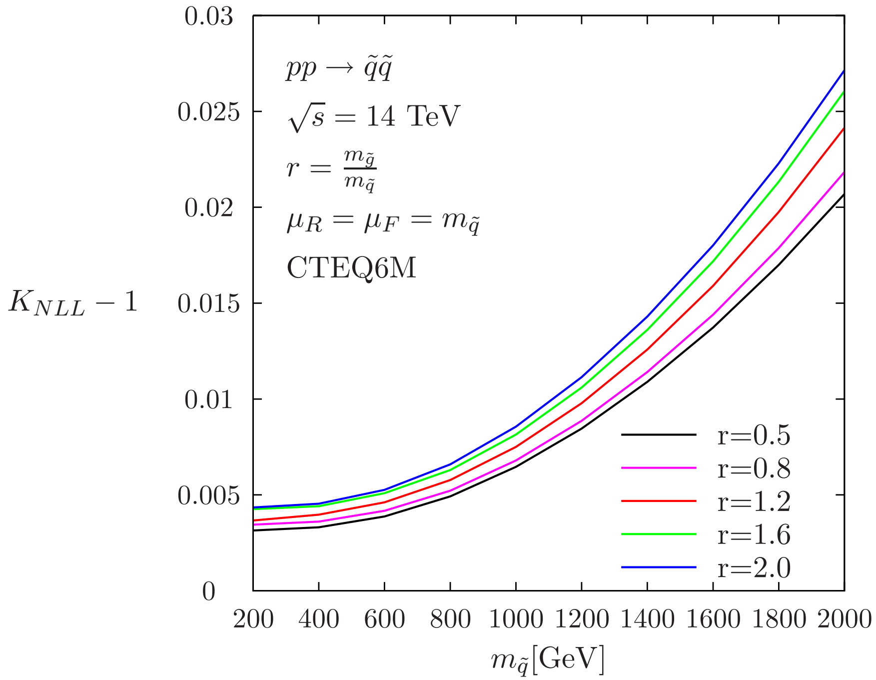
$$\sigma^{(\mathrm{match})}(\mu = \xi m_{\tilde{q}})/\sigma^{(\mathrm{match})}(\mu = m_{\tilde{q}})$$

for $\xi = 0.5$ and $\xi = 2$

($\mu = \mu_F = \mu_R; r = 1.2$, CTEQ6M)

NLL squark-squark production at the LHC

[*Beenakker, Brensing, Krämer, AK, Laenen, Niessen*]



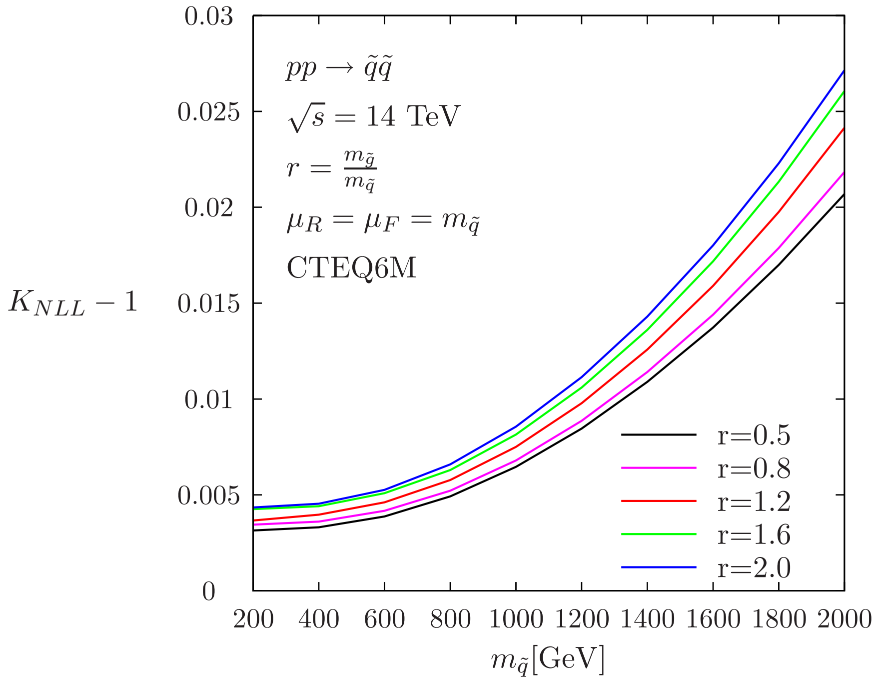
$$\mathrm{K}^{\mathrm{NLL}} = \frac{\sigma^{\mathrm{match}}}{\sigma^{\mathrm{NLO}}}$$

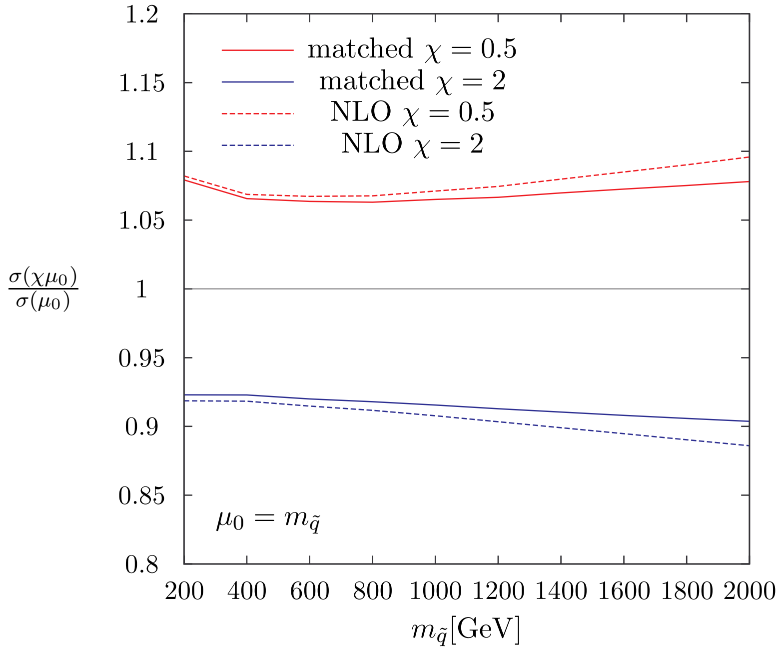
$$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}$$

$$(\mu_F = \mu_R = m_{\tilde{q}})$$

NLL squark-squark production at the LHC

[*Beenakker, Brensing, Krämer, AK, Laenen, Niessen*]





$$\mathrm{K}^{\mathrm{NLL}} = \frac{\sigma^{\mathrm{match}}}{\sigma^{\mathrm{NLO}}}$$

$$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}$$

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$$\sigma^{(\mathrm{match})}(\mu = \chi m_{\tilde{q}})/\sigma^{(\mathrm{match})}(\mu = m_{\tilde{q}})$$

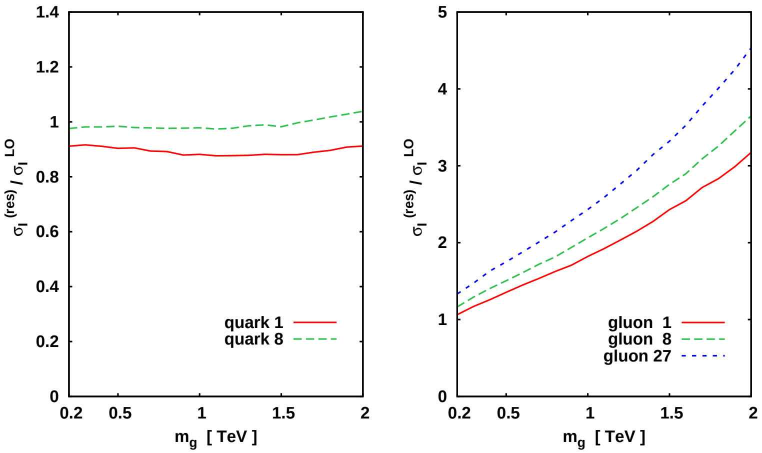
$$(\mu = \mu_F = \mu_R; r = 1.2)$$

Colour-channel decomposition of the NLL result

[*AK, Motyka*]

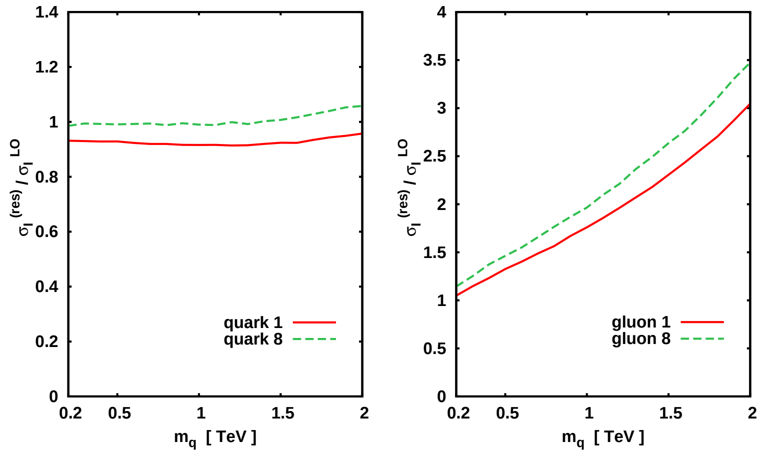
$\sigma_I^{(res)}/\sigma_I^{(LO)}$ in each colour channel I

$pp \longrightarrow \tilde{g}\tilde{g}$



quark channel | gluon channel

$pp \longrightarrow \tilde{q}\bar{\tilde{q}}$



quark channel | gluon channel

preliminary

- Larger soft-gluon correction for higher s-channel colour representations
- Gluon initial-state channels receive bigger enhancement than quark channels

Summary

- If SUSY realized in Nature, $\tilde{q}$ and $\tilde{g}$ production processes will be among the most dominant mechanisms of sparticle production at the LHC

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Summary

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- NLL resummed (matched with NLO) predictions for total cross sections: corrections due to soft gluon emissions above NLO
  - Anomalous dimensions for all pair-production processes of massive coloured sparticles
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Summary

- If SUSY realized in Nature, $\tilde{q}$ and $\tilde{g}$ production processes will be among the most dominant mechanisms of sparticle production at the LHC
- NLL resummed (matched with NLO) predictions for total cross sections: corrections due to soft gluon emissions above NLO
  - Anomalous dimensions for all pair-production processes of massive coloured sparticles
    - confirm physical picture: at threshold soft gluon radiation from the total charge of the heavy-particle pair
  - Corrections due to NLL resummation highest for $\tilde{g}\tilde{g}$ production
    - $\mathcal{O}(10\%)$ correction for at $1\,\mathrm{TeV} \lesssim m_{\tilde{g}} \lesssim 2\,\mathrm{TeV}$
    - reduction of sensitivity to scale choices: factor of $\sim 3(2)$ for at $m_{\tilde{g}} \sim 2\ (1)\,\mathrm{TeV}$
  - Soft gluon effects moderate for squark production processes