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DESY Zeuthen

B meson phenomenology

An introductory lecture

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Outline

1. Basics
2. Meson mixing: CP violation
3. Meson mixing: mass difference
4. CKM phenomenology
5. B_s mixing and new physics
6. Summary

Focus: Meson mixing,

- the best tool for CKM metrology,
- the prime source of information on CP violation,
- an efficient analyser of physics beyond the SM. (Other player: rare decays.)

1. Basics

The neutral K, D, B_d and B_s mesons mix with their antiparticles, $\overline{K}$, $\overline{D}$, $\overline{B}_d$ and $\overline{B}_s$:

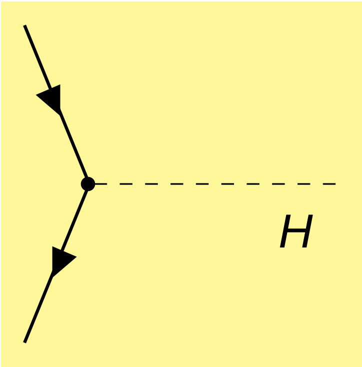
$$K \sim \overline{s}d, \qquad D \sim c\overline{u}, \qquad B_d \sim \overline{b}d, \qquad B_s \sim \overline{b}s,$$
$$\overline{K} \sim s\overline{d}, \qquad \overline{D} \sim \overline{c}u, \qquad \overline{B}_d \sim b\overline{d}, \qquad \overline{B}_s \sim b\overline{s},$$

These are eigenstates of flavour and of $H^{\rm QCD} + H^{\rm QED}$, but not of $H^{\rm weak}$.

Generic notation: M, $\overline{M}$.
CPT theorem: $M_M = M_{\overline{M}}$ and $\Gamma_M = \Gamma_{\overline{M}}$.

Yukawa sector: Yukawa coupling of the Higgs field:

$$y_{ij}\overline{f}_i f_j (v + H)$$



$\Rightarrow$ quark mass matrix: $\quad m_{ij} = y_{ij} v$

diagonalisation $\Rightarrow$ fermion masses and CKM matrix V_{CKM}.

$V_{CKM} \neq 1$

$\Rightarrow$ couplings of the W-Bosons to quarks of different generations, flavor physics

y_{ij}, V_{CKM} complex $\Rightarrow$ CP violation

10 parameters in the quark sektor,

10 or 12 parameters in the lepton sector.

We can diagonalise the up–type Yukawa matrix Y^u by unitary rotations of $Q_j \equiv (u_{jL}, d_{jL})^T$ and u_{jR} (with quark family index $j = 1, 2, 3$) in flavour space.

In this basis only a single Yukawa coupling is larger than 0.05, $y_t \approx 1$. Moreover, the off–diagonal elements of the down–type Yukawa matrix Y^d range from $|Y^d_{31}| \sim 10^{-7}$ to $|Y^d_{23}| \sim 6 \cdot 10^{-4}$ (at the scale 500 GeV) (SM flavour puzzle).

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$\Rightarrow$ The $SU(2)_L$ symmetry is no more manifest, The unitary 3×3 *Cabibbo-Kobayashi-Maskawa (CKM) matrix* appears in the W couplings to quarks:

$$\mathcal{L}_W = \frac{g_w}{\sqrt{2}} \sum_{j,k=1,2,3} \left[V_{jk}\, \overline{u}_{jL}\, \gamma^\mu d_{kL}\, W^+_\mu + V^*_{jk}\, \overline{d}_{kL}\, \gamma^\mu u_{jL}\, W^-_\mu \right]$$

Here g_w is the weak coupling constant and V is the CKM matrix.

Small Yukawa couplings

⇒ Flavour-changing neutral current (FCNC) processes are not only loop-suppressed, but are further suppressed by small CKM elements and/or the Glashow-Iliopoulos-Maiani (GIM) mechanism. Typical situation: Internal loops with light quarks cancel each other up to terms of order $(m_c^2 - m_u^2)/M_W^2$ (in K and B physics) or $(m_s^2 - m_d^2)/M_W^2$ (in D physics).

⇒ FCNC processes are highly sensitive to short-distance physics, e.g. virtual effects of the top quark and new physics at the TeV scale.

Wolfenstein expansion

Expand in $V_{us} \simeq \lambda = 0.2246$:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(1+\frac{\lambda^2}{2}\right)(\overline{\rho}-i\overline{\eta}) \\ -\lambda - iA^2\lambda^5\overline{\eta} & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 - iA\lambda^4\overline{\eta} & 1 \end{pmatrix}$$

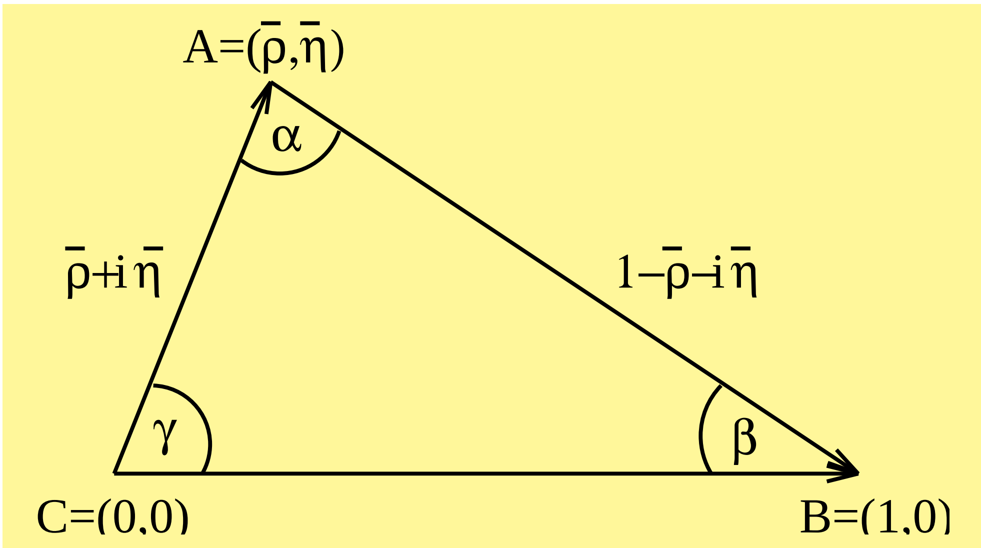
with the Wolfenstein parameters λ, A, $\overline{\rho}$, $\overline{\eta}$

CP violation $\Leftrightarrow \overline{\eta} \neq 0$

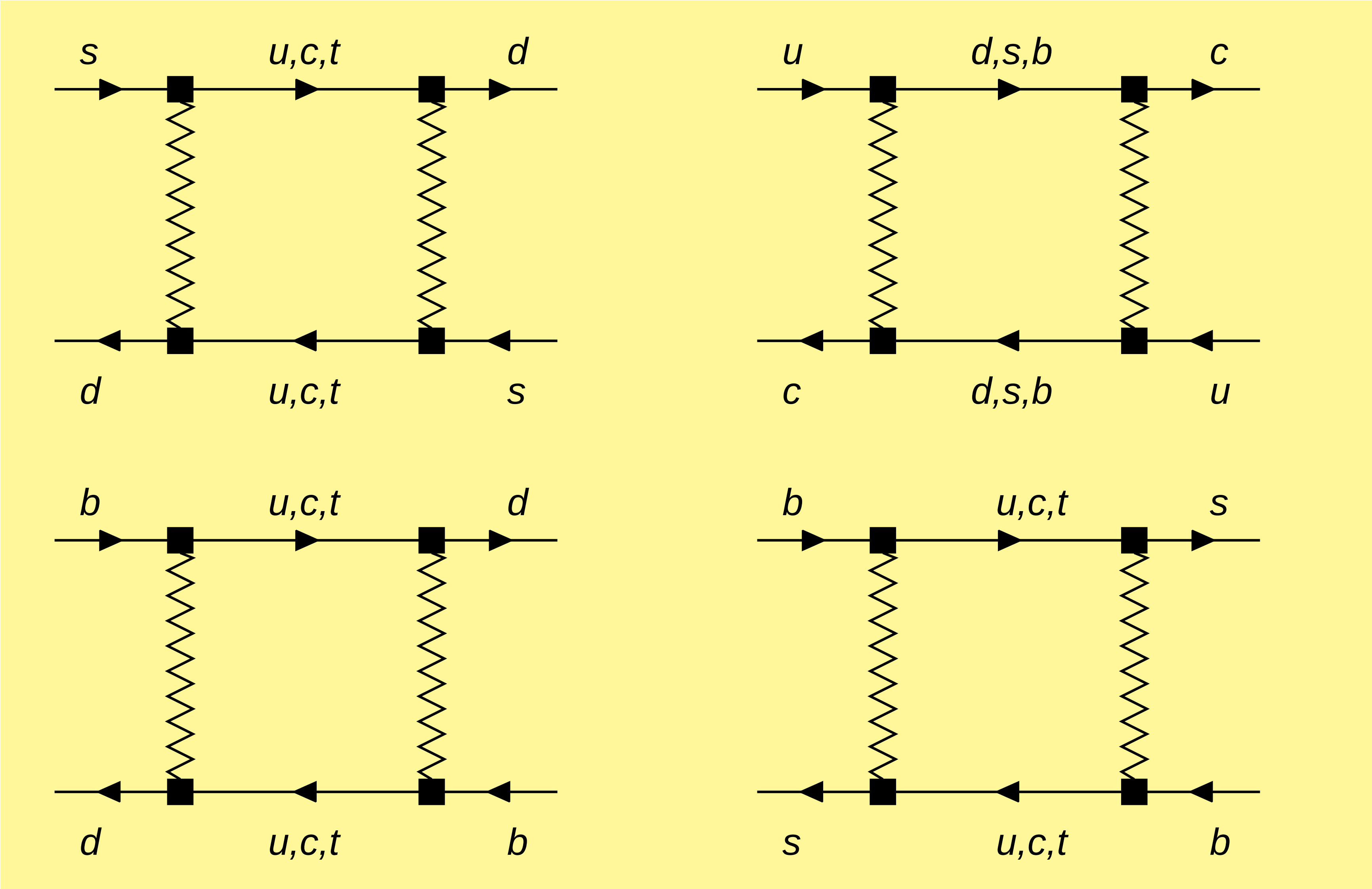
Unitarity triangle:

Exact definition:

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = \left|\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right| e^{i\gamma}$$



Box diagrams for $K - \overline{K}$, $D - \overline{D}$, $B_d - \overline{B}_d$ and $B_s - \overline{B}_s$ mixing:



For each process there is also a second box diagram, obtained by a 90° rotation.

Time evolution

Schrödinger picture: $|\,\psi,t\,\rangle = \mathcal{U}(t,0)|\,\psi\,\rangle$, with the unitary time-evolution operator $\mathcal{U}(t,0)$.

Consider first: charged meson (i.e. K^+, D^+ or B^+), which cannot mix with other states. The corresponding state at $t=0$, $|\,M^+\,\rangle$, will evolve into a superposition of all states allowed by energy-momentum conservation, i.e. $|\,M^+\,\rangle$ and all final states $|\,f\,\rangle$ into which M^+ can decay. With

$$|\,M^+(t)\,\rangle \;=\; |\,M^+\,\rangle\langle\,M^+\,|\mathcal{U}(t,0)|\,M^+\,\rangle$$

we can write

$$\mathcal{U}(t,0)|\,M^+\,\rangle \;=\; |\,M^+(t)\,\rangle + \sum_f |\,f\,\rangle\langle\,f\,|\mathcal{U}(t,0)|\,M^+\,\rangle.$$

In order to find $|\,M^+(t)\,\rangle$ we take a shortcut, by employing the exponential decay law to deduce

$$|\,M^+(t)\,\rangle \;=\; e^{-iM_M t} e^{-\Gamma t/2}|\,M^+\,\rangle$$

in the meson rest frame, where $E=M$.

$$| M^+(t) \rangle = e^{-iM_M t} e^{-\Gamma t/2} | M^+ \rangle$$

Probability of survival:

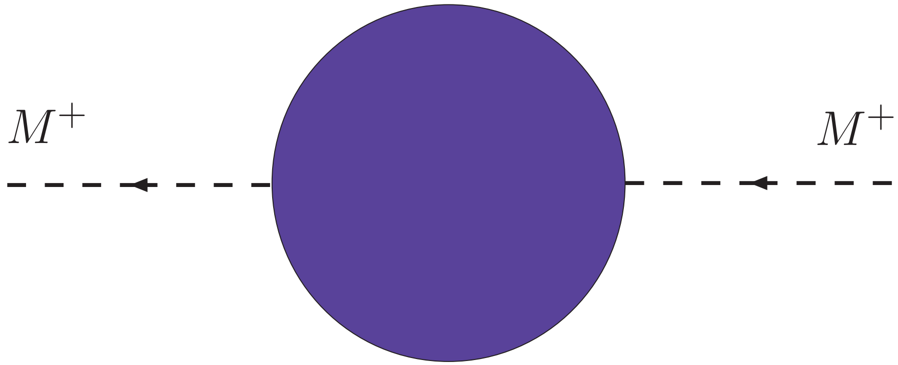
$$\left| \langle M^+ | M^+(t) \rangle \right|^2 = e^{-\Gamma t}$$

Then the desired Schrödinger equation reads:

$$i\frac{d}{dt}|\,M^+(t)\,\rangle = \left(M_M - i\frac{\Gamma}{2}\right)|\,M^+(t)\,\rangle.$$

The optical theorem tells us that M_M and $-\Gamma/2$ are given by the real and imaginary parts of the self-energy $2M_M\Sigma$:

$$-i(2\pi)^4\delta^{(4)}(\vec{p}\,' - \vec{p})\Sigma = \frac{\langle\, M^+(\vec{p}\,')\,|S|\,M^+(\vec{p})\,\rangle}{2M_M}$$



Two-state system: Schrödinger equation:

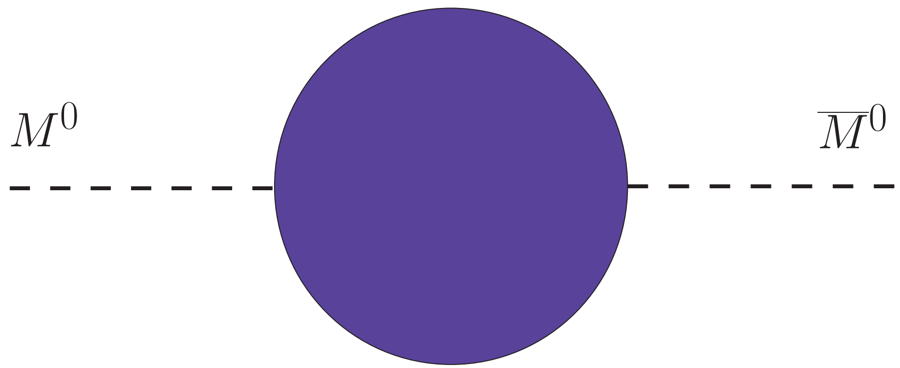
$$i \frac{d}{dt} \begin{pmatrix} |\, M(t) \,\rangle \\ |\, \overline{M}(t) \,\rangle \end{pmatrix} = \Sigma \begin{pmatrix} |\, M(t) \,\rangle \\ |\, \overline{M}(t) \,\rangle \end{pmatrix}$$

where now Σ is the 2×2 matrix

$$-i(2\pi)^4 \delta^{(4)}(p_i' - p_j)\Sigma_{ij} \;=\; \frac{\langle\, i, \vec{p}_i{}' \,|S^{\rm SM}|\, j, \vec{p}_j \,\rangle}{2 M_M}$$

with $|\, 1, \vec{p}_1 \,\rangle = |\, M(\vec{p}_1) \,\rangle$ and $|\, 2, \vec{p}_2 \,\rangle = |\, \overline{M}(\vec{p}_2) \,\rangle$.

Σ_{12}:



Without shortcut: Need time-dependent perturbation theory and Wigner-Weisskopf approximation, see e.g appendix of Nachtmann's book.

Any matrix can be written as the sum of a hermitian and an antihermitian matrix. We write

$$\Sigma = M - i\,\frac{\Gamma}{2}$$

with the mass matrix $M = M^\dagger$ and the decay matrix $\Gamma = \Gamma^\dagger$. Then

$$M_{12} = \frac{\Sigma_{12} + \Sigma_{21}^*}{2}, \qquad \frac{\Gamma_{12}}{2} = i\,\frac{\Sigma_{12} - \Sigma_{21}^*}{2}.$$

These are the dispersive and absorptive parts of Σ_{12}.

In perturbation theory: dispersive and absorptive parts are calculated by retaining all complex couplings (CKM elements!), but replacing the loop integrals by their real and imaginary parts, respectively.

CPT theorem: $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$.

Solving the Schrödinger equation

Diagonalise $\Sigma = M - i\Gamma/2$: Mass eigenstates:

lighter eigenstate: $|\, M_L \,\rangle = p|\, M \,\rangle + q|\, \overline{M} \,\rangle \,,$

heavier eigenstate: $|\, M_H \,\rangle = p|\, M \,\rangle - q|\, \overline{M} \,\rangle \,,$

with $|p|^2 + |q|^2 = 1$.

With

$$Q = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \qquad \text{and} \qquad Q^{-1} = \frac{1}{2pq} \begin{pmatrix} q & p \\ q & -p \end{pmatrix}.$$

find

$$\begin{pmatrix} |\, M(t)\, \rangle \\ |\, \overline{M}(t)\, \rangle \end{pmatrix} = Q \begin{pmatrix} e^{-iM_L t - \Gamma_L t/2} & 0 \\ 0 & e^{-iM_H t - \Gamma_H t/2} \end{pmatrix} Q^{-1} \begin{pmatrix} |\, M\, \rangle \\ |\, \overline{M}\, \rangle \end{pmatrix}$$

$$= \begin{pmatrix} g_+(t) & \frac{q}{p} g_-(t) \\ \frac{p}{q} g_-(t) & g_+(t) \end{pmatrix} \begin{pmatrix} |\, M\, \rangle \\ |\, \overline{M}\, \rangle \end{pmatrix}$$

It is easy to express $g_\pm(t)$ in terms of

$$m = \frac{M_H + M_L}{2} = M_{11} = M_{22}\,, \qquad \Gamma = \frac{\Gamma_L + \Gamma_H}{2} = \Gamma_{11} = \Gamma_{22}\,,$$

$$\Delta m = M_H - M_L\,, \qquad \Delta\Gamma = \Gamma_L - \Gamma_H\,.$$

Three physical quantities in meson-antimeson mixing:

$$|M_{12}|, \qquad |\Gamma_{12}|, \qquad \text{and} \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right).$$

Useful:

$$\left|\frac{q}{p}\right|^2 \equiv 1 - a.$$

For B mesons one has $|\Gamma_{12}| \ll |M_{12}|$ and

$$a \simeq \text{Im}\,\frac{\Gamma_{12}}{M_{12}} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi\,, = \begin{cases} -5\cdot 10^{-4} & \text{for } B_d\!-\!\overline{B}_d \text{ mixing} \\ 2\cdot 10^{-5} & \text{for } B_s\!-\!\overline{B}_s \text{ mixing} \end{cases}$$

Effective hamiltonians

Concept: Remove ("integrate out") heavy particles:

$$\langle f \,|\mathbf{T} e^{-i\int d^4x H^{\rm SM}_{\rm int}(x)}|\, i\,\rangle = \langle f \,|\mathbf{T} e^{-i\int d^4x H^{\rm eff}(x)}|\, i\,\rangle \left[1+\mathcal{O}\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^n\right]$$

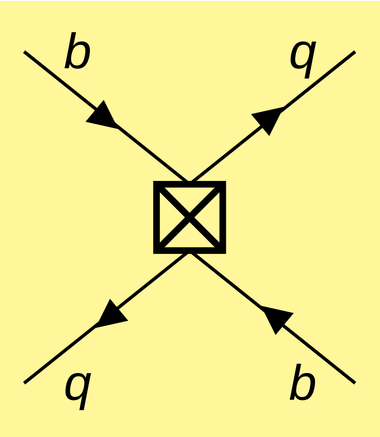
Effective $\Delta B = 2$ hamiltonian $H^{|\Delta B|=2}$:

$$H^{|\Delta B|=2} = \frac{G_F^2}{4\pi^2}\,(V_{tb}V_{tq}^*)^2\, C^{|\Delta B|=2}(m_t, M_W, \mu)\, Q(\mu) + h.c.$$

with the four-quark operator

$$Q = \overline{q}_L \gamma_\nu b_L \, \overline{q}_L \gamma^\nu b_L$$

with $q = d$ or s.



All short-distance information resides in the Wilson coefficient $C^{|\Delta B|=2}$.

To describe meson decays we need effective $\Delta F = 1$ hamiltonians, e.g. $H^{|\Delta B|=1}$ for B decays.

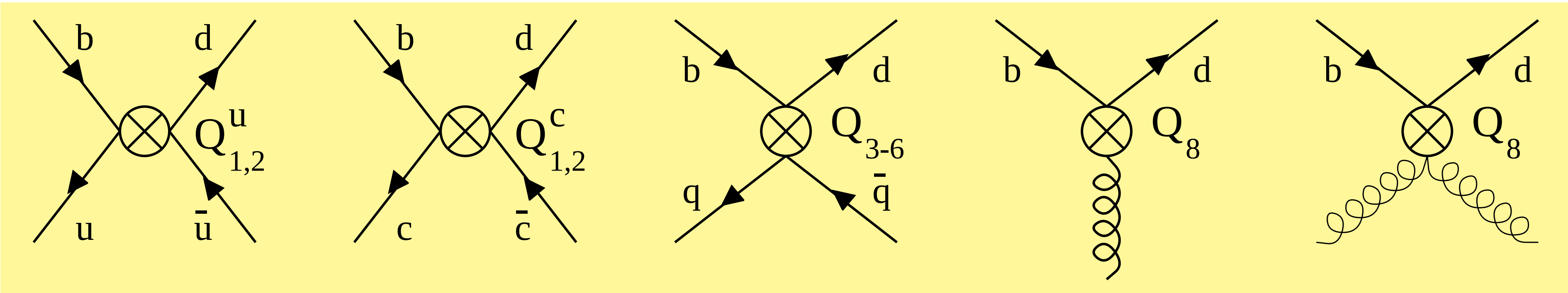
$$H^{|\Delta B|=1} = \frac{G_F}{\sqrt{2}} \left[\sum_{i=1}^{2} C_i \left(V_{CKM} Q_i^u + V'_{CKM} Q_i^c \right) + V''_{CKM} \sum_{i \geq 3} C_i Q_i \right]$$

yields an expansion in $(m_b/M_W)^2$ with

Q_i: effective $|\Delta B| = 1$ operators, e.g. $Q_2^c = \overline{c}\gamma_\mu(1-\gamma_5)b\,\overline{d}\gamma^\mu(1-\gamma_5)c$

C_i: Wilson coefficients = effective couplings, contain short distance structure, perturbative QCD corrections, depend on m_t/M_W.

$V_{CKM}^{('')}$: product of CKM elements



Opposite side tagging:

b-flavoured hadrons are produced in pairs. Determine the flavour of some meson at the time $t=0$ (to distinguish $|\,M(t)\,\rangle$ from $|\,\overline{M}(t)\,\rangle$) through a flavour-specific decay of the other hadron, such as $B \to X\ell^- \overline{\nu}_\ell$.

B factories: The $(B_d, \overline{B}_d)$ pairs are produced in an entangled state. Opposite side tagging uses the Einstein-Podolski-Rosen effect: The flavour-specific decay "starts the clock" at $t=0$.

2. Meson mixing: CP violation

Time-dependent decay rate:

$$\Gamma(M(t) \to f) = \frac{1}{N_M} \frac{d\,N(M(t) \to f)}{d\,t},$$

where $d\,N(M(t) \to f)$ is the number of $M(t) \to f$ decays within the time interval $[t, t + d\,t]$.
N_M is the number of M's present at time $t = 0$.

Key quantity:

$$\lambda_f = \frac{q}{p} \frac{\overline{A}_f}{A_f}$$

with

$$\begin{aligned} A_f &= \langle f | M \rangle = \langle f | H^{|\Delta F|=1} | M \rangle, \\ \overline{A}_f &= \langle f | \overline{M} \rangle = \langle f | H^{|\Delta F|=1} | \overline{M} \rangle. \end{aligned} \qquad (1)$$

Master formulae:

$$\Gamma(M(t) \to f) = \mathcal{N}_f \left|\langle f | \, M(t) \, \rangle\right|^2$$
$$= \mathcal{N}_f \, |A_f|^2 \, e^{-\Gamma t} \left\{ \frac{1+|\lambda_f|^2}{2} \cosh \frac{\Delta\Gamma \, t}{2} + \frac{1-|\lambda_f|^2}{2} \cos(\Delta m \, t) \right.$$
$$\left. - \mathrm{Re}\, \lambda_f \, \sinh \frac{\Delta\Gamma \, t}{2} - \mathrm{Im}\, \lambda_f \, \sin(\Delta m \, t) \right\},$$

$$\Gamma(\overline{M}(t) \to f) = \mathcal{N}_f \left|\langle f | \, \overline{M}(t) \, \rangle\right|^2 \tag{2}$$
$$= \mathcal{N}_f \, |A_f|^2 \, (1+a) \, e^{-\Gamma t} \left\{ \frac{1+|\lambda_f|^2}{2} \cosh \frac{\Delta\Gamma \, t}{2} - \frac{1-|\lambda_f|^2}{2} \cos(\Delta m \, t) \right.$$
$$\left. - \mathrm{Re}\, \lambda_f \, \sinh \frac{\Delta\Gamma \, t}{2} + \mathrm{Im}\, \lambda_f \, \sin(\Delta m \, t) \right\}.$$

CP-conjugate state:

$$| \overline{f} \rangle = CP | f \rangle$$

$$\begin{aligned}
\Gamma(M(t) \to \overline{f}) &= \mathcal{N}_f \left| \langle \overline{f} | \, M(t) \, \rangle \right|^2 \\
&= \mathcal{N}_f \left| \overline{A}_{\overline{f}} \right|^2 e^{-\Gamma t} \, (1-a) \left\{ \frac{1 + |\lambda_{\overline{f}}|^{-2}}{2} \cosh \frac{\Delta\Gamma \, t}{2} - \frac{1 - |\lambda_{\overline{f}}|^{-2}}{2} \cos(\Delta m \, t) \right. \\
&\qquad \left. - \mathrm{Re} \, \frac{1}{\lambda_{\overline{f}}} \sinh \frac{\Delta\Gamma \, t}{2} + \mathrm{Im} \, \frac{1}{\lambda_{\overline{f}}} \sin(\Delta m \, t) \right\}, \\
\Gamma(\overline{M}(t) \to \overline{f}) &= \mathcal{N}_f \left| \langle \overline{f} | \, \overline{M}(t) \, \rangle \right|^2 \qquad (3) \\
&= \mathcal{N}_f \left| \overline{A}_{\overline{f}} \right|^2 e^{-\Gamma t} \left\{ \frac{1 + |\lambda_{\overline{f}}|^{-2}}{2} \cosh \frac{\Delta\Gamma \, t}{2} + \frac{1 - |\lambda_{\overline{f}}|^{-2}}{2} \cos(\Delta m \, t) \right. \\
&\qquad \left. - \mathrm{Re} \, \frac{1}{\lambda_{\overline{f}}} \sinh \frac{\Delta\Gamma \, t}{2} - \mathrm{Im} \, \frac{1}{\lambda_{\overline{f}}} \sin(\Delta m \, t) \right\}.
\end{aligned}$$

Application 1: Flavour-specific decay (tagging mode) of a B meson

$$\overline{A}_f = \lambda_f = 0$$

We consider in addition $|\overline{A}_{\overline{f}}| = |A_f|$, i.e. no direct CP violation.
Examples: $B_{d,s} \to \ell^+ \nu_\ell X^-$, $B_s \to D_s^- \pi^+$.

(i) With Eq. (2) one easily finds the mixing asymmetry:

$$\mathcal{A}_0(t) = \frac{\Gamma(B(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(B(t) \to f) + \Gamma(B(t) \to \overline{f})}$$
$$= \frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma\, t/2)} + \frac{a}{2}\left[1 - \frac{\cos^2(\Delta m\, t)}{\cosh^2(\Delta\Gamma\, t/2)}\right],$$

(ii) In the same way find the CP asymmetry in flavour-specific $B \to f$ decays (semileptonic CP asymmetry):

$$a_{\rm fs} = \frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to \overline{f})} = a + \mathcal{O}(a^2).$$

No tagging is needed to measure $a_{\rm fs}$! Define the untagged decay rate

$$\Gamma[f,t] = \Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to f). \qquad (4)$$

to find:

$$a_{\rm fs,unt}(t) = \frac{\Gamma[f,t] - \Gamma[\overline{f},t]}{\Gamma[f,t] + \Gamma[\overline{f},t]} = \frac{a_{\rm fs}}{2} - \frac{a_{\rm fs}}{2}\,\frac{\cos(\Delta m\, t)}{\cosh(\Delta\Gamma t/2)}.$$

To measure $a_{\rm fs}$ just count the number of f's and $\overline{f}$'s in the final state.

SM predictions:

K	B_d	B_s
$a = (6 \pm 1) \cdot 10^{-3}$	$a = (-5 \pm 1) \cdot 10^{-4}$	$a = (2.0 \pm 0.6) \cdot 10^{-5}$

Application 2: B decay into a CP eigenstate $f_{\rm CP}$ which involves only a single CKM factor ($\Rightarrow |A_{f_{\rm CP}}| = |\overline{A}_{f_{\rm CP}}|$) (golden mode).

$$CP|\, f_{\rm CP}\,\rangle = \eta_{f_{\rm CP}}|\, f_{\rm CP}\,\rangle$$

with $\eta_{f_{\rm CP}} = \pm 1$.

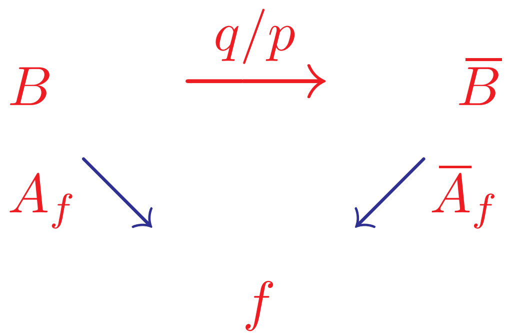
Time-dependent CP asymmetry:

$$a_f(t) = \frac{\Gamma(\overline{B}(t) \to f) - \Gamma(B(t) \to \overline{f})}{\Gamma(\overline{B}(t) \to f) + \Gamma(B(t) \to \overline{f})} \,.$$

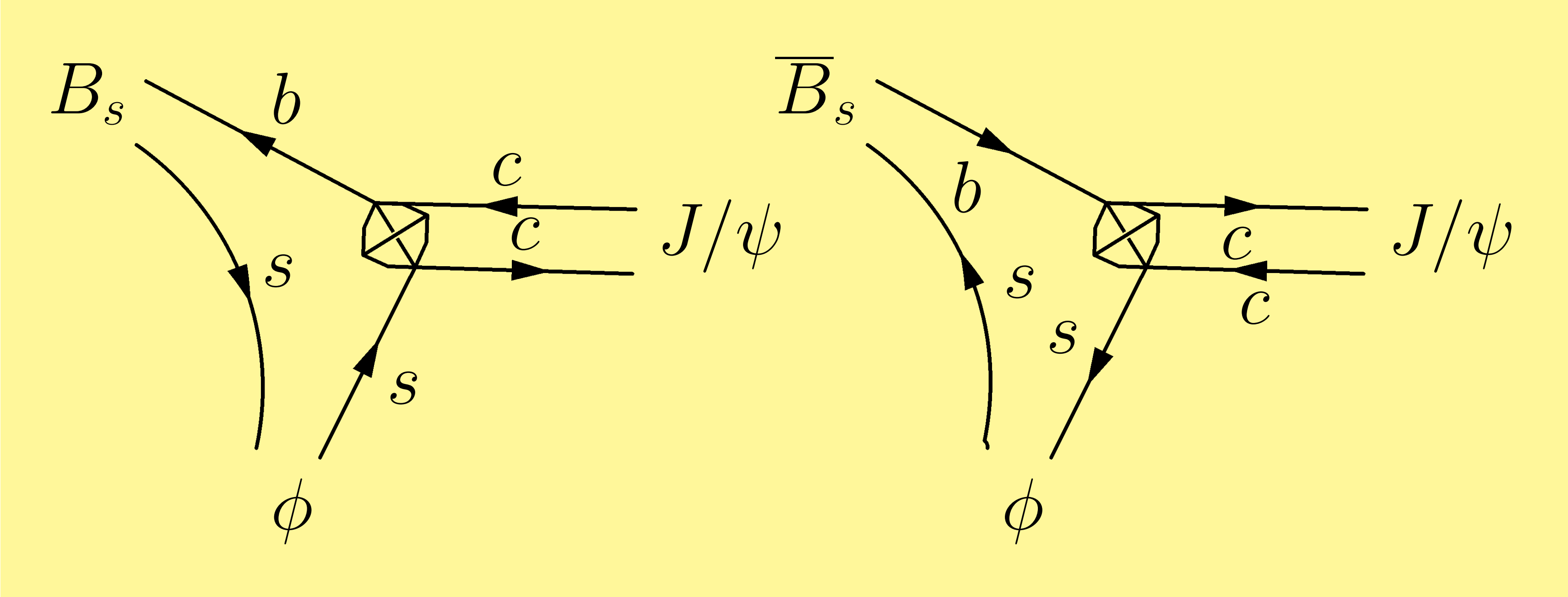
Using Eq. (2)

$$a_{f_{\rm CP}}(t) = -\frac{\operatorname{Im} \lambda_{f_{\rm CP}} \sin(\Delta m t)}{\cosh(\Delta\Gamma t/2) - \operatorname{Re} \lambda_{f_{\rm CP}} \sinh(\Delta\Gamma t/2)} + \mathcal{O}(a)\,,$$

$\operatorname{Im} \lambda_f$ quantifies the CP violation in the interference between mixing and decay:



Example 1: $B_s \to (J/\psi\phi)_{L=0} \quad \Rightarrow \quad \eta_{f_{\rm CP}} = +1$



$$\frac{\overline{A}_{J/\psi\phi}}{A_{J/\psi\phi}} = -\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}$$

$\uparrow$

from $CP|\,B_s\,\rangle = -|\,\overline{B}_s\,\rangle$

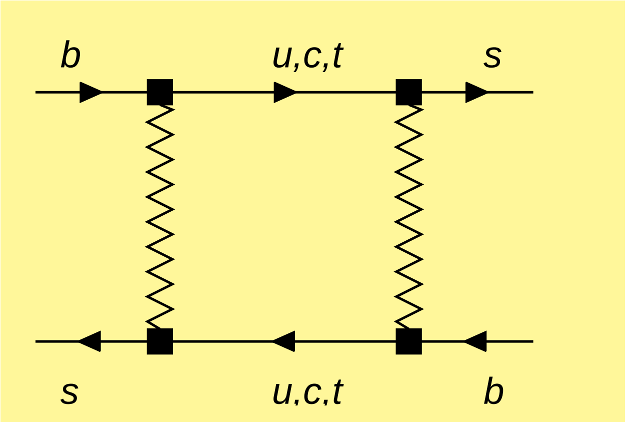
$$\frac{q}{p} = -\frac{m_{12}^*}{|m_{12}|} + \mathcal{O}(a, |\Gamma_{12}/M_{12}|^2)$$

$\Rightarrow$ Only need the phase of m_{12}^*:

$$\frac{q}{p} = -\frac{V_{tb}^{*2} V_{ts}^2}{|V_{tb}^2 V_{ts}^2|} = -\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}$$

$\uparrow$

from $CP|\, B_s \,\rangle = -|\, \overline{B}_s \,\rangle$



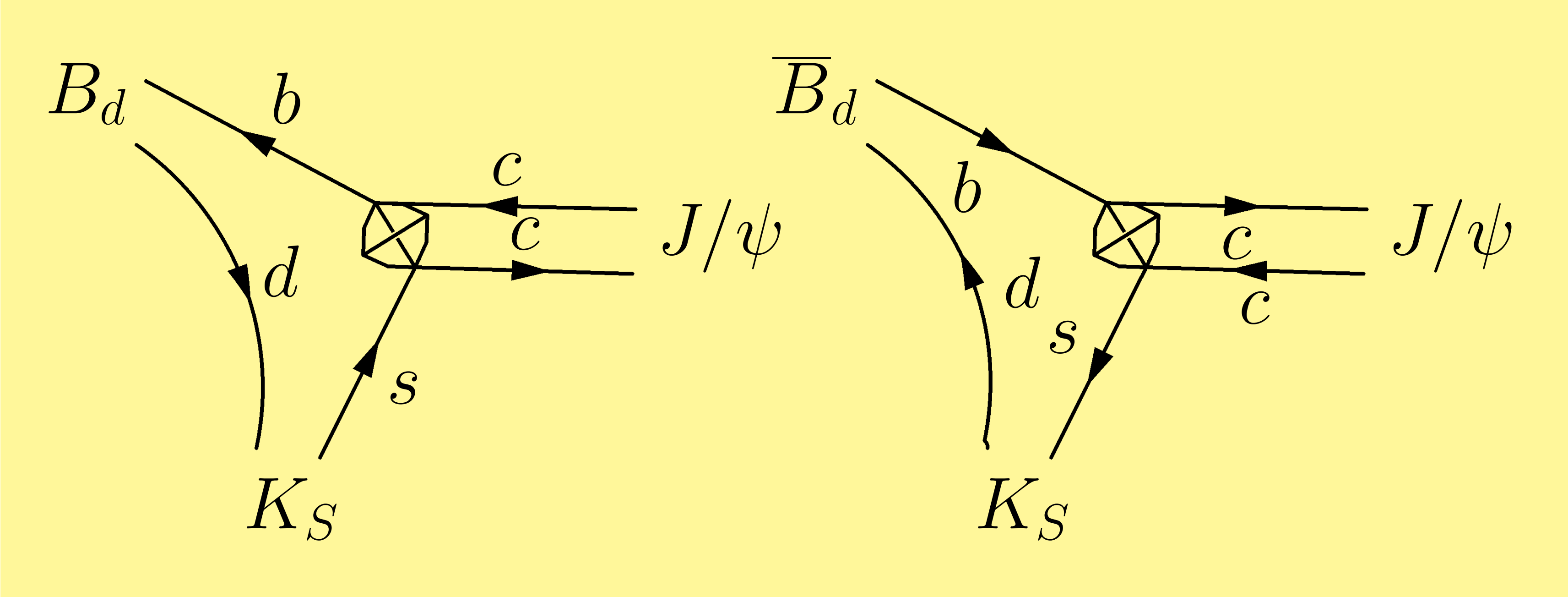
$$\lambda_{J/\psi\phi} = \frac{q}{p} \frac{\overline{A}_{J/\psi\phi}}{A_{J/\psi\phi}} = \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \equiv e^{2i\beta_s}$$

$$\Rightarrow \qquad \mathrm{Im}\, \lambda_{J/\psi\phi} = \sin(2\beta_s) = 2\lambda^2\eta \approx 0.04$$

with the Wolfenstein parameters $\lambda \simeq 0.2246$ and $\eta \approx 0.4$.

Example 2: $B_d \to J/\psi K_S$

$L = 1 \qquad \Rightarrow \qquad \eta_{f_{\rm CP}} = -1$



$$\lambda_{J/\psi K_S} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \frac{V_{us} V_{ud}^*}{V_{us}^* V_{ud}} \simeq -e^{-2i\beta}$$

$$\Rightarrow \qquad {\rm Im}\, \lambda_{J/\psi K_S} = \sin(2\beta) \approx 0.68$$

More golden modes ($|A_{f_{\rm CP}}| = |\overline{A}_{f_{\rm CP}}|$):

- $B_d \to \phi K_S$, $B_d \to K_S K_S$
- $B_s \to J/\psi \eta^{(\prime)}$
- $K_{\rm long} \to \pi^0 \nu \overline{\nu}$
- $D^0 \to K_S \rho^0$, $D^0 \to K_S \pi^0$

Remark: $K_{\rm long} = K_H$

3. Meson mixing: Mass difference

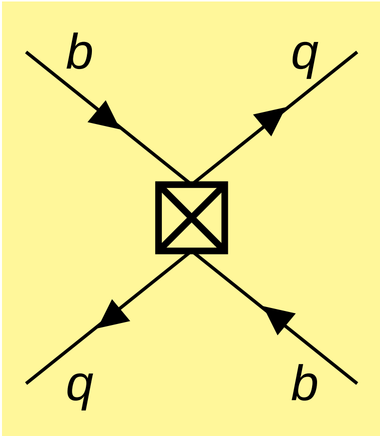
Discuss first: Δm in the $\mathrm{B}-\overline{\mathrm{B}}$ system.

Effective $|\Delta B| = 2$ hamiltonian ($q = d$ or $q = s$):

$$H^{|\Delta B|=2} = \frac{G_F^2}{4\pi^2} (V_{tb} V_{tq}^*)^2 \, C^{|\Delta B|=2}(m_t, M_W, \mu) \, Q(\mu) + h.c.$$

$C^{|\Delta B|=2}$ is the Wilson coefficient of the local four-quark operator

$$Q = \overline{q}_L \gamma_\nu b_L \, \overline{q}_L \gamma^\nu b_L .$$



The short-distance physics is contained in

$$C^{|\Delta B|=2}(m_t, M_W, \mu) = M_W^2 \, S\left(\frac{m_t^2}{M_W^2}\right) \eta_B \, b_B(\mu) \, .$$

where the result of the box diagram is contained in the Inami-Lim function

$$S(x) = x\left[\frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2}\right] - \frac{3}{2}\left[\frac{x}{1-x}\right]^3 \ln x \, .$$

The next-to-leading-order QCD corrections are contained in $\eta_B = 0.55$ and $b_B(\mu)$, where μ is the renormalisation scale. The μ-dependence of $b_B(\mu)$ cancels the μ-dependence of the matrix element to the calculated order.

The dominant theoretical uncertainty comes from the matrix element:

$$\langle \mathrm{B}^0 \, | \mathrm{Q} | \, \overline{\mathrm{B}}^0 \rangle \; = \; \frac{2}{3} m_{B_q}^2 \, f_{B_q}^2 \, \frac{\widehat{B}}{b_B(\mu)}$$

$$f_{B_d}\sqrt{\widehat{B}} \; = \; (227 \pm 39) \text{ MeV}, \qquad f_{B_s}\sqrt{\widehat{B}} \; = \; (272 \pm 47) \text{ MeV}$$

Putting everything together:

$$\Delta m_{B_q} = 2|M_{12}| = \frac{|\langle B_q | H^{|\Delta B|=2} | \overline{B}_q \rangle|}{m_{B_q}}$$

$$= \frac{G_F^2}{6\pi^2} \eta_B \, m_{B_q} \, \widehat{B} f_{B_q}^2 \, M_W^2 \, S\left(\frac{m_t^2}{M_W^2}\right) \left|V_{tb} V_{tq}^*\right|^2 .$$

where $q = s$ or d.

Δm_{B_d} determines $|V_{td}|$.

$|V_{ts}|$ entering Δm_{B_s} is fixed by CKM unitarity to $|V_{ts}| \simeq |V_{cb}|$.

Combining Δm_{B_d} and Δm_{B_s}:

The ratio $\Delta m_{B_d}/\Delta m_{B_s}$ determines $|V_{td}|/|V_{ts}|$ via

$$\left|\frac{V_{td}}{V_{ts}}\right| = \sqrt{\frac{\Delta m_{B_d}}{\Delta m_{B_s}}}\sqrt{\frac{M_{B_s}}{M_{B_d}}}\,\xi$$

with the hadronic quantity

$$\xi = \frac{f_{B_s}\sqrt{\widehat{B}_{B_s}}}{f_{B_d}\sqrt{\widehat{B}_{B_d}}} = 1.20 \pm 0.06.$$

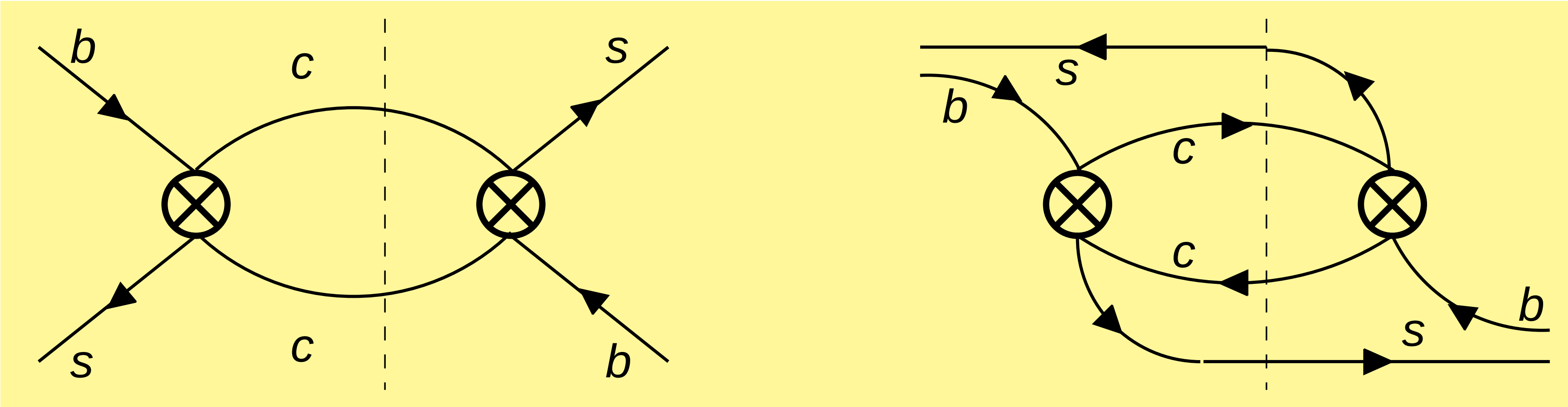
In the limit of exact $\mathrm{SU(3)_F}$ one has $\xi = 1$.

Both Δm_{B_d} and $\Delta m_{B_d}/\Delta m_{B_s}$ constrain one side of the unitarity triangle:

$$R_t \equiv \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2} = \left|\frac{V_{td}}{V_{ts}\lambda}\right|$$

Contributions from $H^{|\Delta F|=1}$

Box diagrams with light internal quarks are not reproduced by $H^{|\Delta F|=2}$, but involve two insertions of $H^{|\Delta F|=1}$. For example:



$\Delta\Gamma$ in all four mixed neutral meson systems stems from such diagrams involving two insertions of $H^{|\Delta F|=1}$.

The contributions from $H^{|\Delta B|=1}$ to Δm_{B_d} and Δm_{B_s} are suppressed by m_b^2/m_t^2 and are therefore negligible.

Δm_K receives an uncalculable $\mathcal{O}(30\%)$ correction from $H^{|\Delta S|=1}$. The contributions from $H^{|\Delta C|=1}$ totally dominate Δm_D and cannot be reliably calculated because of delicate GIM cancellations proportional to $(m_s^4 - m_d^4)/(m_c^2 M_W^2)$.

4. CKM phenomenology

The apex of the standard unitarity triangle is $(\overline{\rho}, \overline{\eta})$, where

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = R_u e^{i\gamma} \tag{5}$$

Trading $V_{ub}^* V_{ud}$ for $V_{tb}^* V_{td}$ one finds:

$$1 - \overline{\rho} - i\overline{\eta} = -\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = R_t e^{-i\beta} \tag{6}$$

That is, R_u and R_t are two sides of the triangle.

The unitarity relation $V_{tb}^* V_{td} + V_{cb}^* V_{cd} + V_{ub}^* V_{ud} = 0$ is hard-wired into Eqs. (5) and (6). Note that $R_u e^{i\gamma} + R_t e^{-i\beta} = 1$.

The CKM matrix... ...expanded in $\lambda \simeq 0.22$:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(1+\frac{\lambda^2}{2}\right)(\overline{\rho}-i\overline{\eta}) \\ -\lambda - iA^2\lambda^5\overline{\eta} & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 - iA\lambda^4\overline{\eta} & 1 \end{pmatrix}$$

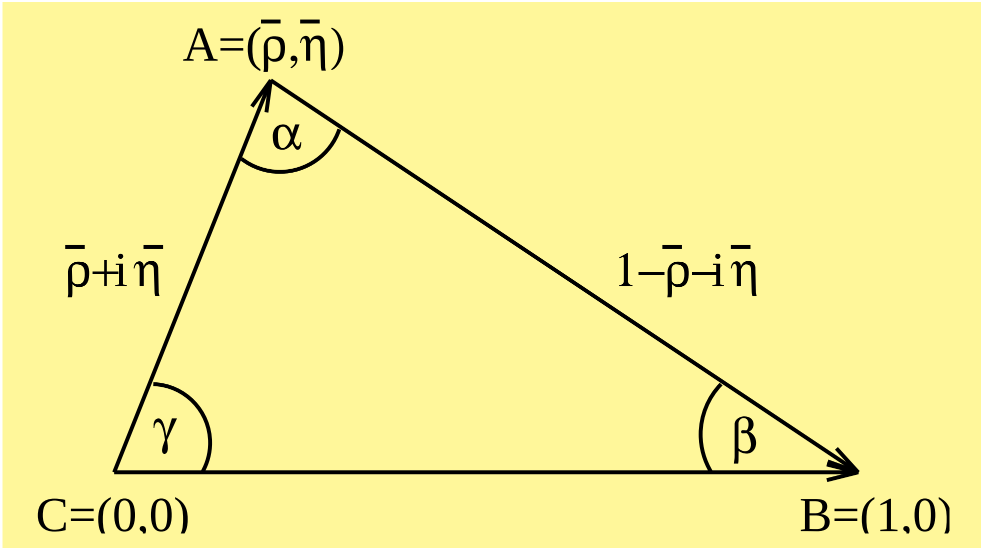
with the Wolfenstein parameters λ, A, $\overline{\rho}$, $\overline{\eta}$

CP violation $\Leftrightarrow \overline{\eta} \neq 0$

Unitarity triangle:

Exact definition:

$$\overline{\rho} + i\overline{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} = \left|\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right| e^{i\gamma}$$



V_{ud}, V_{us}, V_{cd}, V_{cs} and V_{cb} are determined from tree-level decays, which are insensitive to new physics.

All information on V_{td} and V_{ts} is from flavour-changing neutral current (FCNC) loop diagrams.

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In order to constrain new physics it is useful to disentangle the tree-level and the loop information on $(\overline{\rho}, \overline{\eta})$: The tree-level unitarity triangle gives the true apex $(\overline{\rho}, \overline{\eta})$, these values for $\overline{\rho}$ and $\overline{\eta}$ are then inserted into the SM predictions of the loop quantites which are then confronted with experiment.

Tree-level unitarity triangle

$b \to u\ell\overline{\nu}_\ell$ determines $|V_{ub}|$ and thereby $R_u = \sqrt{\overline{\rho}^2 + \overline{\eta}^2}$

There are two ways of tree-level determinations of $\arg V_{ub}^* = \gamma$:

(1) $B \to \overset{(}{\overline{D}}\overset{)}{} X$ decays:
Use the interference of the two tree amplitudes $b \to c\overline{u}q$ and $b \to u\overline{c}q$ (with $q = d$ or $q = s$) to get $\gamma = \arg V_{ub}^*$.

(2) Combine the mixing-induced CP asymmetries in $b \to u\overline{u}d$ decays of B_d with β measured in $B_d \to J/\psi K_S$:

$a_f(t)$ measured in $B_d \to \pi\pi$, $B_d \to \rho\rho$ or $B_d \to \rho\pi$ determines $2\beta^{\rm exp} + 2\gamma$ (usually called $2\pi - 2\alpha^{\rm exp}$). Potentially $2\beta^{\rm exp}$ differs from 2β by new physics contributions to $\mathrm{B}-\overline{\mathrm{B}}$ mixing. Taking $2\beta^{\rm exp}$ from $a_{J/\psi K_S}$ eliminates the mixing effects in

$$\gamma = \pi - \alpha^{\rm exp} - \beta^{\rm exp}. \tag{7}$$

Data (@68% CL, from CKMFitter):

Measurement through $B \to \overset{(-)}{D} X$ decays:

$$\gamma = 67^{\circ}{}^{+32^{\circ}}_{-25^{\circ}} \tag{8}$$

Measurement via $\alpha^{\exp}$ using Eq. (7):

$$\alpha^{\exp} = 88.3^{\circ}{}^{+5.7^{\circ}}_{-4.8^{\circ}}$$

$$\beta^{\exp} = 21.15^{\circ}{}^{+0.90^{\circ}}_{-0.88^{\circ}}$$

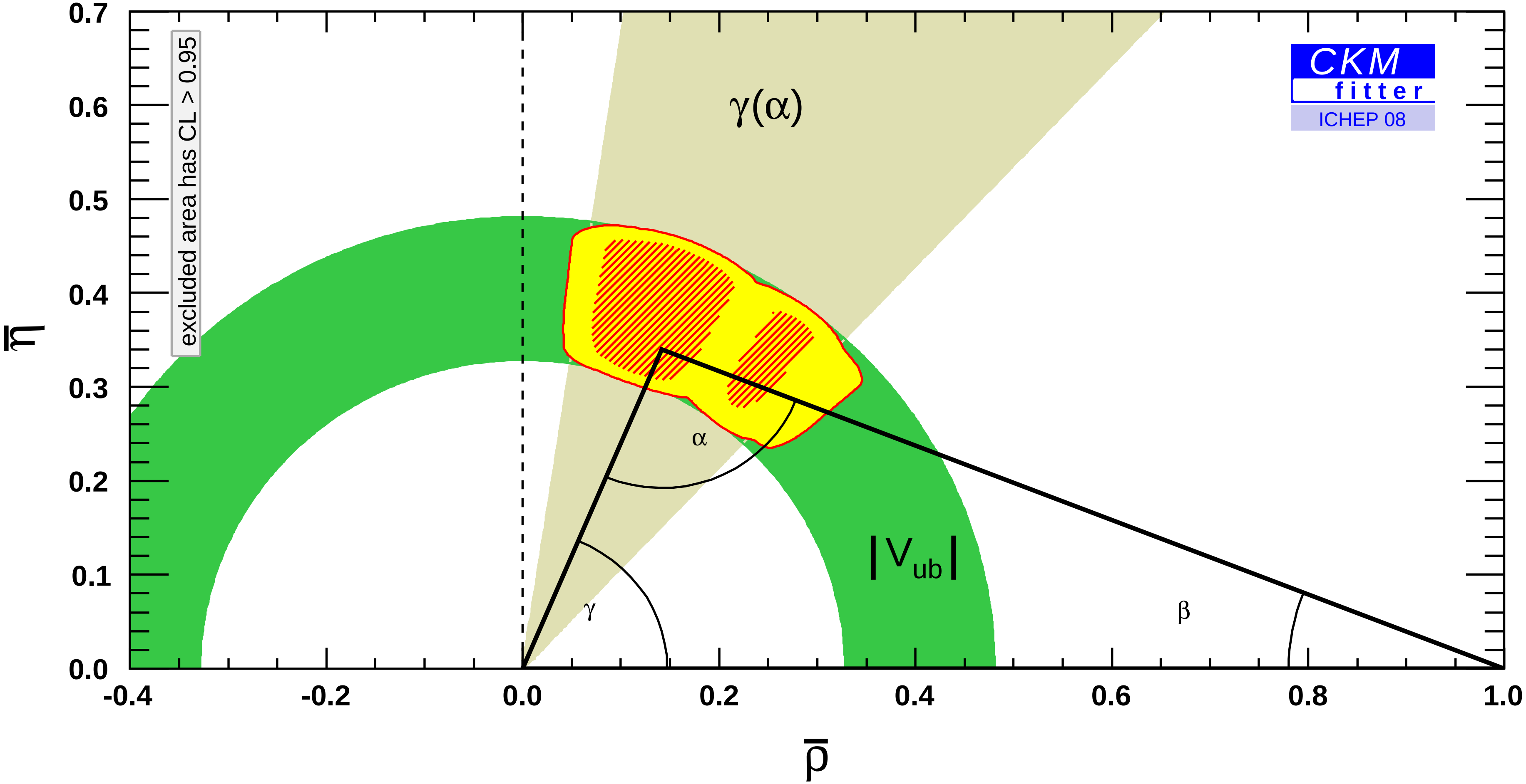
$$\Rightarrow \qquad \gamma = 71^{\circ}{}^{+5^{\circ}}_{-6^{\circ}} \tag{9}$$

At 90% CL the results from the two methods are

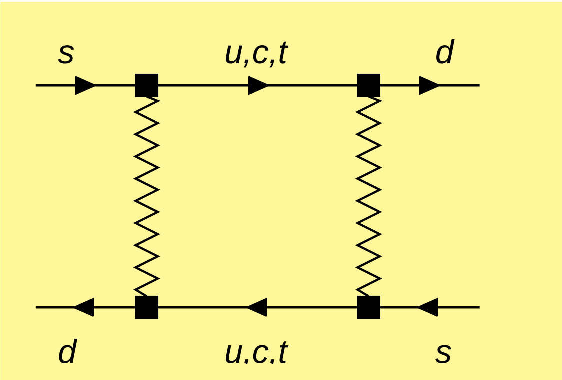
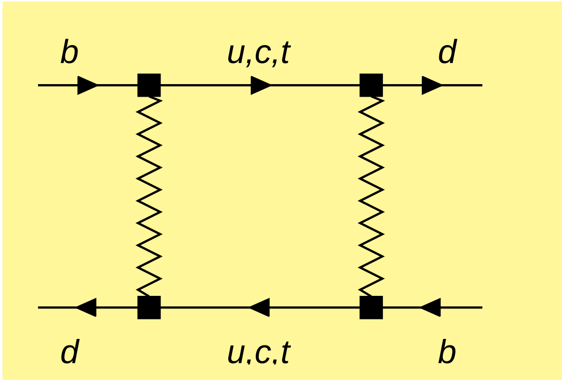
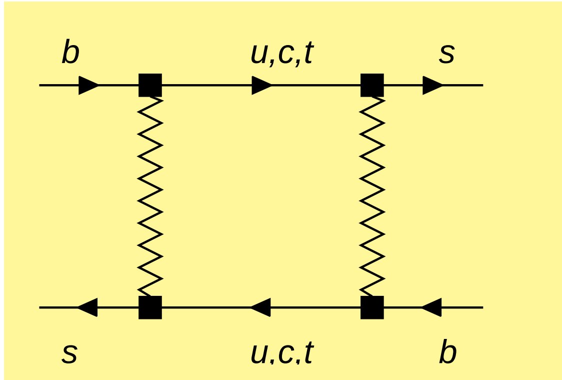
$\gamma = 67^{\circ}{}^{+50^{\circ}}_{-36^{\circ}}$ and $\gamma = 71^{\circ}{}^{+11^{\circ}}_{-24^{\circ}}$,

respectively.

Unitarity triangle from tree decays



UT constraints from meson-antimeson mixing

$\mathrm{K}-\overline{\mathrm{K}}$ mixing	$B_d-\overline{B}_d$ mixing	$B_s-\overline{B}_s$ mixing
		

Observables:

	$\Delta m_{B_d} \propto \lvert V_{td}\rvert^2$	$\Delta m_{B_s} \propto \lvert V_{ts}\rvert^2$
	$\sqrt{(1-\overline{\rho})^2+\overline{\eta}^2}$	

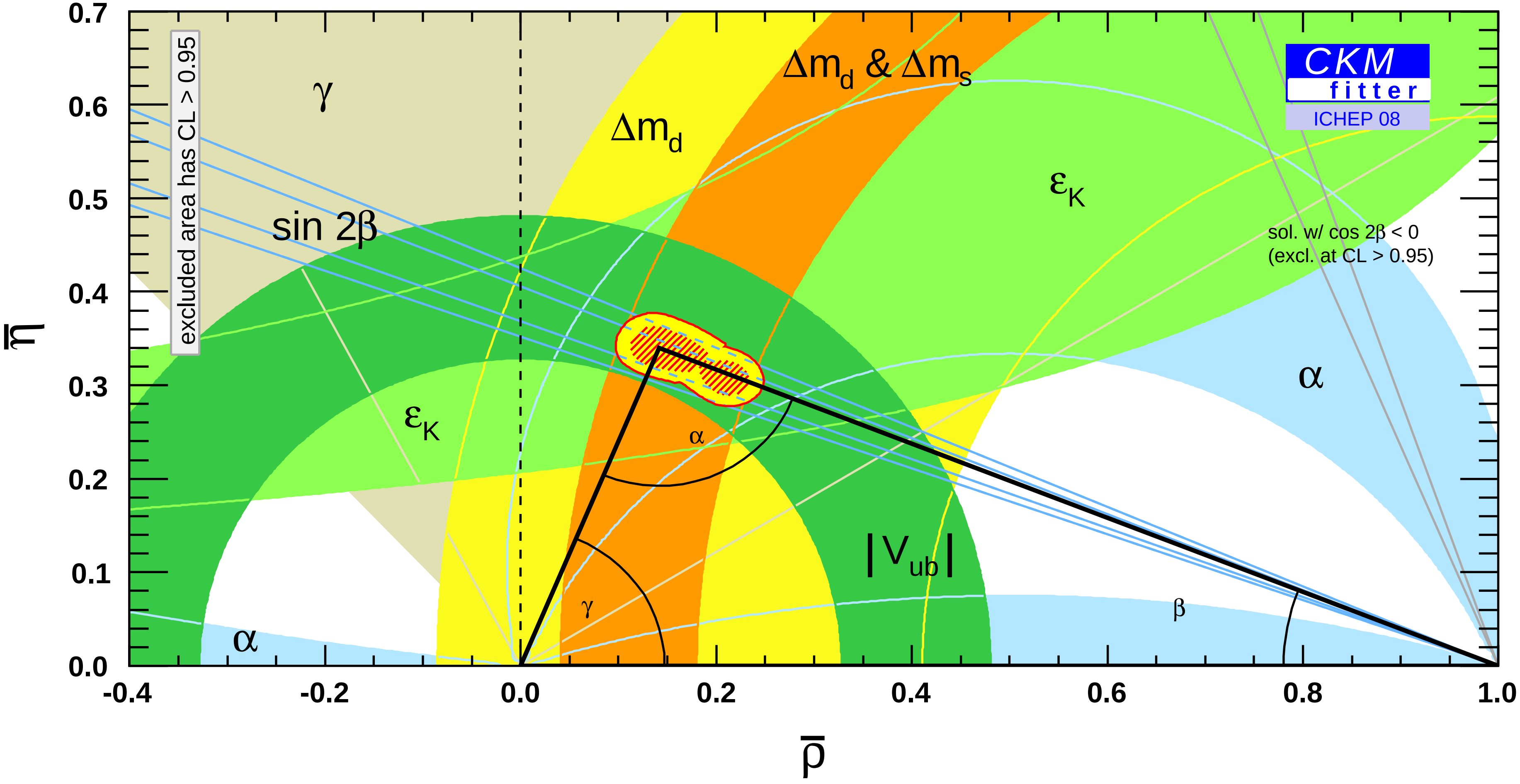
~~CP~~:

ϵ_K gives $\mathrm{Im}\,(V_{ts}V_{td}^*)^2$	$\sin(2\beta)$, $\beta \simeq \arg V_{td}^*$	$\sin(2\beta_s)$, $\beta_s \simeq \arg(-V_{ts})$
$\overline{\eta}[(1-\overline{\rho})+\mathrm{const.}]$	$\dfrac{\overline{\eta}}{1-\overline{\rho}}$	$\overline{\eta}$

Hadronic matrix elements:

$\langle\, \overline{K}^0\, \lvert \overline{s} d_{V-A} \overline{s} d_{V-A} \rvert\, K^0\, \rangle$	$\langle\, \overline{B}^0\, \lvert \overline{b} d_{V-A} \overline{b} d_{V-A} \rvert\, B^0\, \rangle$	$\langle\, \overline{B}^0_s\, \lvert \overline{b} s_{V-A} \overline{b} s_{V-A} \rvert\, B^0_s\, \rangle$

Unitarity triangle, all constraints



5. B_s mixing and new physics

Recall:

$$\Delta m = M_H - M_L \simeq 2|M_{12}|,$$
$$\Delta\Gamma = \Gamma_L - \Gamma_H = 2|\Gamma_{12}|\cos\phi$$

and

$$a_{\rm fs} = \left|\frac{\Gamma_{12}}{M_{12}}\right|\sin\phi.$$

New physics: M_{12}^s is very sensitive to virtual effects of new heavy particles.

$\Rightarrow$ $\Delta m_s \simeq 2|M_{12}^s|$ and ϕ_s can change.

$\Rightarrow$ $|\Delta\Gamma_s| = \Delta\Gamma_{s,\rm SM}\,|\cos\phi_s|$ is depleted

and $|a_{\rm fs}^s|$ is enhanced, by up to a factor of 200 in the B_s system.

To identify or constrain new physics one wants to measure both the magnitude and phase of M_{12}^s.

$$\rightarrow \qquad \Delta m_s = 2|M_{12}^s|$$

Three untagged measurements are sensitive to $\arg M_{12}^s$:

1. $|\Delta\Gamma_s| = \Delta\Gamma_{s,\mathrm{SM}} \, |\cos\phi_s|$
2. $a_{\mathrm{fs}}^s = \left|\dfrac{\Gamma_{12}^s}{M_{12}^s}\right| \sin\phi_s$
3. the angular distribution of $\overset{(-)}{B}_s \rightarrow VV'$, where V, V' are vector bosons.

Gold-plated tagged measurement of $\arg M_{12}^s$:

Mixing-induced CP asymmetry $a_{J/\psi\phi}(t)$ (with angular analysis)

Highlight of winter 2007/2008: First measurements of $a_{J/\psi\phi}(t)$ by the CDF and DØ experiments.

Generic new physics

Define the complex parameter Δ_s through

$$M_{12}^s \equiv M_{12}^{\mathrm{SM,s}} \cdot \Delta_s \,, \qquad \Delta_s \equiv |\Delta_s| e^{i\phi_s^\Delta} .$$

In the Standard Model $\Delta_s = 1$. Frequently used alternative notation:

$$\Delta_s = r_{\mathrm{s}}^2 \cdot e^{i\,2\theta_s}$$

The CDF measurement

$$\Delta m_s = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1}$$

and ($B = \widehat{B}/b_B(\mu = m_b)$)

$$f_{B_s}\sqrt{B} = 221 \pm 38 \text{ MeV}$$

imply

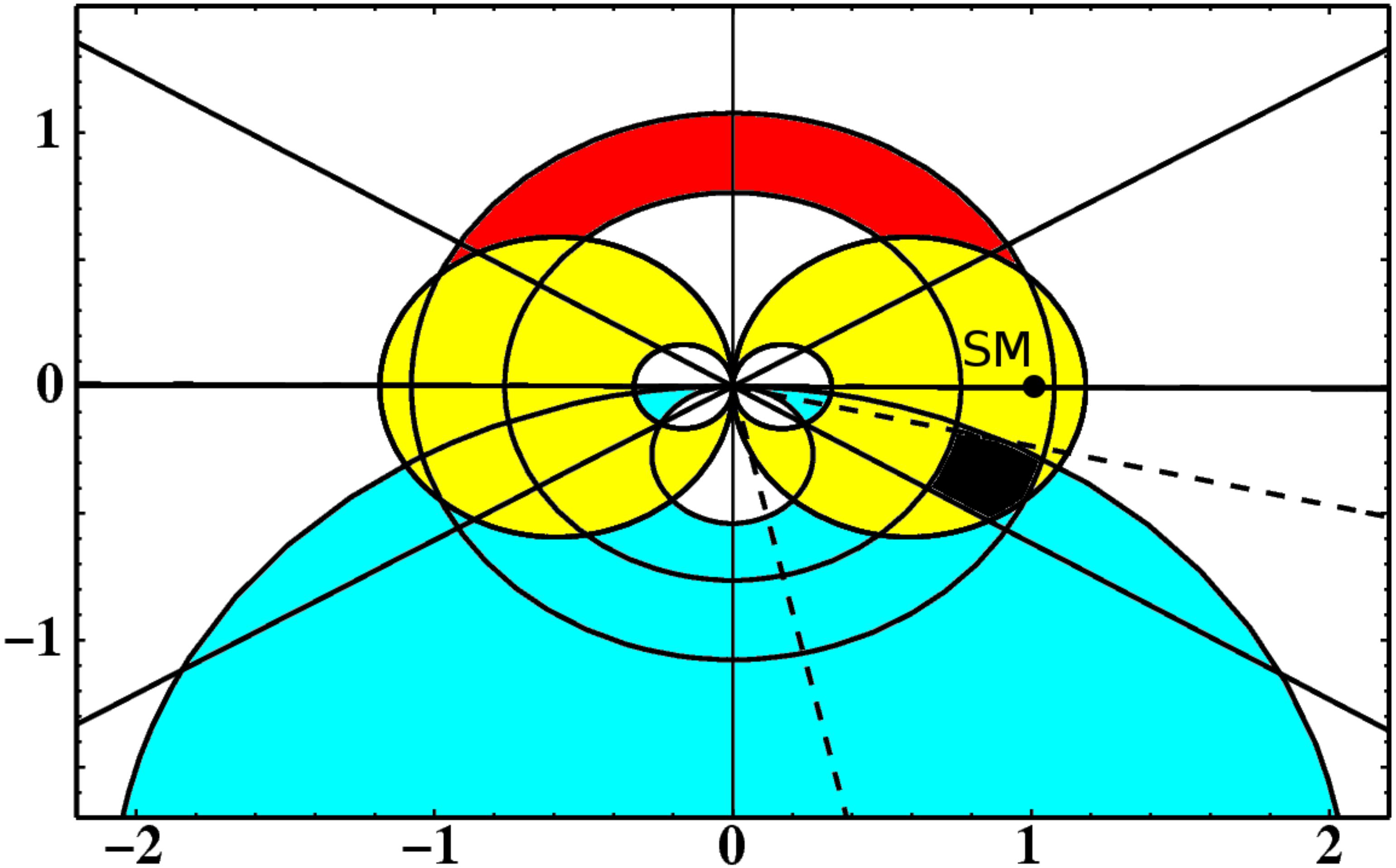
$$|\Delta_s| = 0.92 \pm 0.32_{\text{(th)}} \pm 0.01_{\text{(exp)}}$$

Status of December 2006: CDF or DØ data available for

- mass difference Δm_s,
- the semileptonic CP asymmetry $a_{\rm fs}^s$,
- the angular distribution in $\overset{(-)}{B}_s \to J/\psi\phi$ and
- $\Delta\Gamma_s$

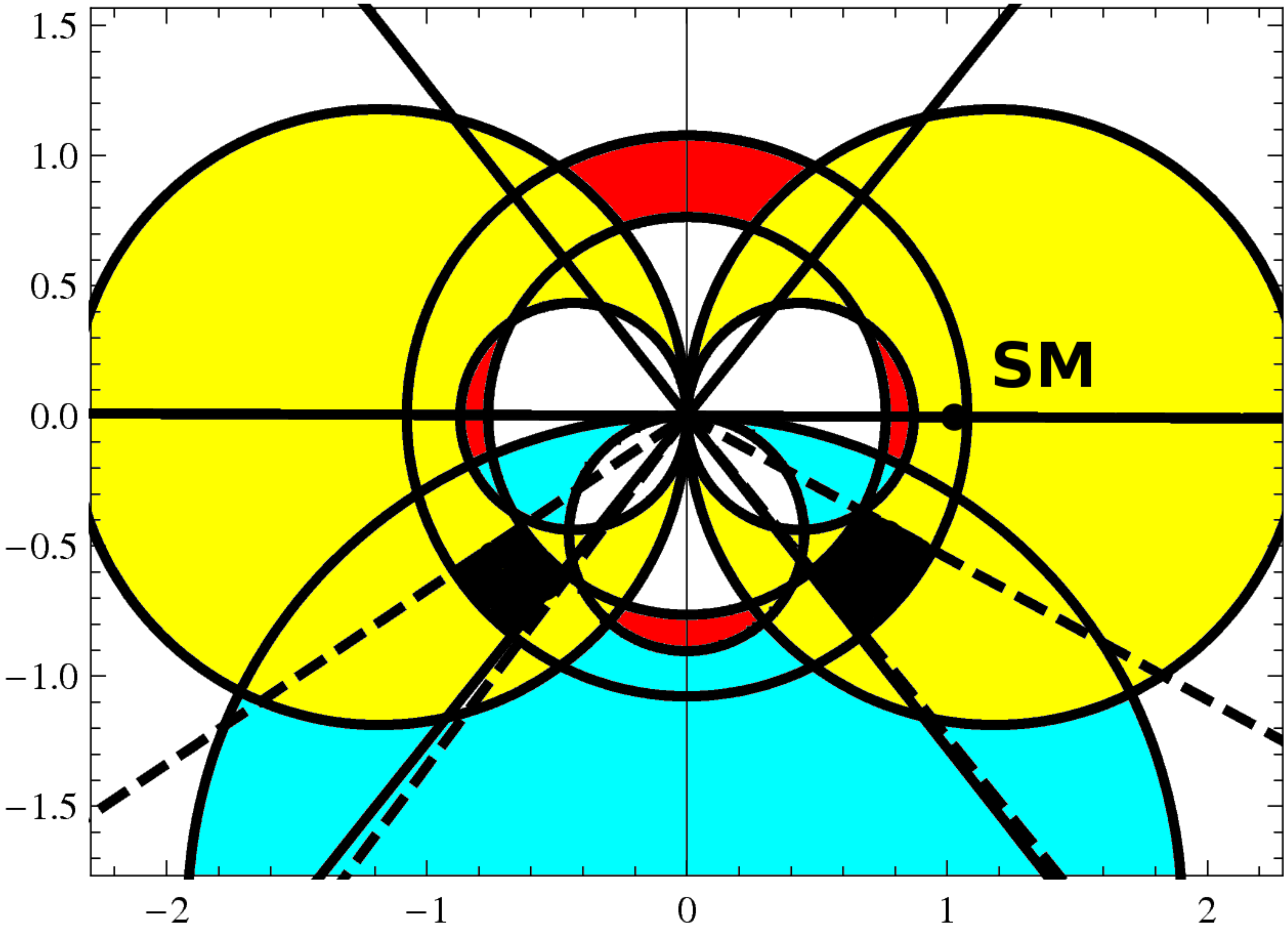
to constrain Δ_s.

The complex Δ_s plane in 2006:



We black area shown corresponds to a deviation from the Standard Model by 2σ. The area delimited by the dashed lines has mirror solutions in the other three quadrants. Alex Lenz, UN, hep-ph/0612167

Spring 2008: Adding the results from the tagged CDF and DØ analyses (and updating $a_{\rm fs}^s$):



New data presented at ICHEP 2008: Discrepancy with SM increased.

6. Summary

- Meson-antimeson mixing is a suberb field to study short-distance physics, probing the virtual effects of heavy particles.
- Mixing-induced CP asymmetries allow for determinations of fundamental CP phases without hadronic uncertainties.
- UT phenomenology: $\mathrm{B}-\overline{\mathrm{B}}$ mixing is crucial for the precise present-day determinations of R_t, β and α. $\mathrm{K}-\overline{\mathrm{K}}$ mixing defines a hyperbola in the $(\overline{\rho}, \overline{\eta})$ plane.
- New physics in $B_d-\overline{B}_d$ mixing and $\mathrm{K}-\overline{\mathrm{K}}$ mixing can be tested by overconstraining the UT triangle.
- $B_s-\overline{B}_s$ mixing directly probes new physics. There are (statistically insignificant) hints of physics beyond the Standard Model in the CP phase ϕ_s.

A pinch of new physics in $B_s - \overline{B}_s$ mixing?