

Heavy quarks on the lattice: future perspectives



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DESY, A Research Centre of the Helmholtz Association



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The principle

First principle “solution” of QCD

experiments, hadrons

$$m_p = 938.272 \text{ MeV}$$

$$M_\pi = 139.570 \text{ MeV}$$

$$m_K = 493.7 \text{ MeV}$$

$$m_D = 1896 \text{ MeV}$$

$$m_B = 5279 \text{ MeV}$$

fundamental parameters
& hadronic matrix elements

$$\alpha(\mu)$$

$$m_u(\mu), m_s(\mu)$$

$$m_c(\mu), m_b(\mu)$$

$$F_B, F_{B_s}, \xi \dots$$

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- Non-perturbative regulator:
lattice with spacing a

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continuum limit $a \rightarrow 0$

Some sample results from the literature

Review of E. Gamiz lattice 2008

examples of results

$m_c^{\overline{\text{MS}}}(3 \text{ GeV})$	=	0.986(10) GeV	HPQCD
$m_b^{\overline{\text{MS}}}(m_b)$	=	4.20(4) GeV	HPQCD
$\xi = \frac{F_{B_s} \sqrt{m_{B_s}}}{F_B \sqrt{m_B}}$	=	1.211(38)(24)	FNAL/MILC
F_{B_s}	=	243(11) MeV	FNAL/MILC
F_{D_s}	=	241(3) MeV	HPQCD

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Precision up to 1 % is claimed

The machinery



The present numbers quoted for phenomenology with small errors are dominated by “rooted staggered” sea quark computations [MILC-collaboration]

rooting: (sea quarks)

- ▶ $\rightarrow$ non-local
 - ▶ locality (= renormalizability = correctness) argued to be recovered as $a \rightarrow 0$
[Bernard, Golterman, Sharpe]
- series of ingredients: Symanzik effective theory – chiral PT, replica trick

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NRQCD (or Fermilab action) for b-quarks

- ▶ power law divergences
($a \propto e^{-1/2b_0 g_0^2}$)

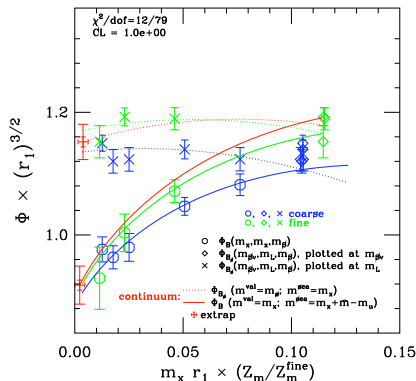
$$\frac{g_0^{2k}}{a m_b} \sim \frac{1}{a [\log(a)]^k m_b} \xrightarrow{a \rightarrow 0} \infty$$

delicate analysis of continuum limit

The machinery

staggered chiral perturbation theory

$m \rightarrow (m_u + m_d)/2$ & $a \rightarrow 0$ in one (necessary due to rooting)

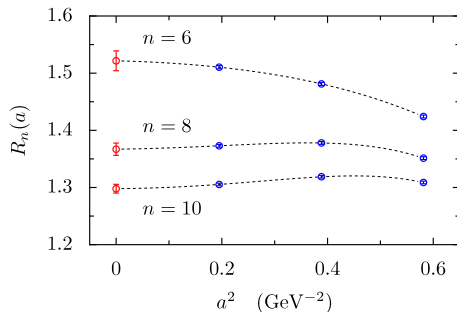


many parameter fits

The machinery

Bayesian fits

$a \rightarrow 0$ from high order polynomial in a with few points



The machinery



It appears good to perform independent computations with an independent technology

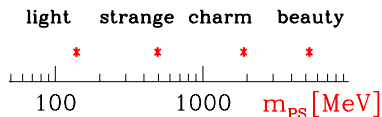
- ▶ manifestly local
- ▶ non-perturbative subtraction of power law divergences

Such computations are in progress

... but first let us understand that LQCD is a challenge

The challenge

multiple scale problem
always difficult
for a numerical treatment



The challenge

multiple scale problem

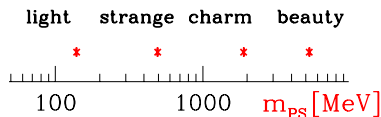
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lattice cutoffs:

$$\Lambda_{UV} = a^{-1}$$

$$\Lambda_{IR} = L^{-1}$$

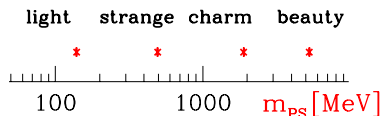


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lattice cutoffs:

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$$L^{-1} \ll m_{\pi}, \dots, m_D, m_B \ll a^{-1}$$

$$O(e^{-LM_{\pi}})$$

↓

$$L \gtrsim 4/M_{\pi} \sim 6 \text{ fm}$$

$$m_D a \lesssim 1/2$$

↓

$$a \approx 0.05 \text{ fm}$$

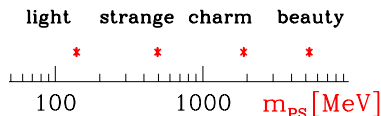
$$L/a \gtrsim 120$$

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beauty not yet accommodated: effective theory, Λ_{QCD}/m_b expansion

Perspectives

First let me state, there is ChPT so $m_\pi = 280 \text{ MeV}$ and a theory-guided extrapolation seems good enough

→ $L \approx 3 \text{ fm}$, $L/a = 64$

In addition there is ChPT including $O(a^2)$ -effects

[Singleton & Sharpe; O. Bär, G.Rupak & N.Shores]

and

- ▶ new algorithms
- ▶ new machines
- ▶ development / demonstration of **effective field theory strategies**

Perspectives: algorithms for dynamical fermions

- ▶ mass preconditioning [M. Hasenbusch]
- ▶ multiple time scale integrators [Sexton & Weingarten; C. Urbach et al.]
- ▶ odd number of flavours, $m_s \neq m_c$:
RHMC [M. Clark & A. Kennedy]
- ▶ Domain decomposition + deflation [M. Lüscher]

performance improved enormously:

from $\text{time} \propto m_{\text{quark}}^{-n}$, $n \gtrsim 3$ to

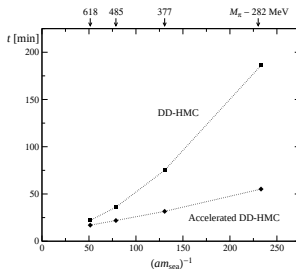
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Perspectives: machines

For illustration: the German situation (roughly)

year	machine	speed/Tflops	share for a typical collaboration
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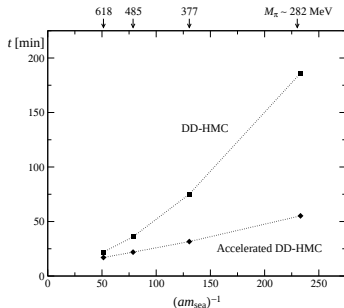


Growth (recently) stronger than Moore's law

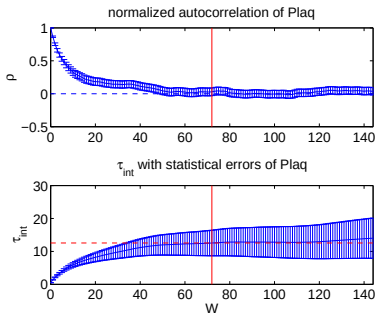
Perspectives: algorithms & machines

Example: 128×64^3 , $a = 0.04$ fm, $L = 2.6$ fm

at $m_q = m_s/2$: (2 trajectories)/hour=(1 MD unit)/hour
on 1024 node BG/P (1/64 Pflops)



execution time of
accelerated DD-HMC



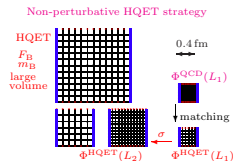
$2\tau_{int} = \#$ MD unit per
effective independent measurement

256×128^3 at the physical point ($M_\pi = M_\pi^{\text{physical}}$) seems in reach

Perspectives: strategies for b quark

- ▶ Non-perturbative HQET (expansion in Λ_{QCD}/m_b)
[Heitger & S., 2001]
- ▶ Step scaling strategy
[G.M. de Divitiis, M. Guagnelli, F. Palombi, R. Petronzio & N. Tantalo]

Very much related and may be combined!

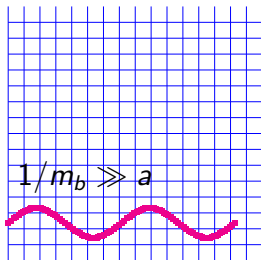


Non-perturbative matching of HQET and QCD

- The trick: start in small volume,
 $L = L_1 \approx 0.4 \text{ fm}$, $a = 0.01 \text{ fm}$

Φ_k finite volume masses,
decay constants ...

QCD

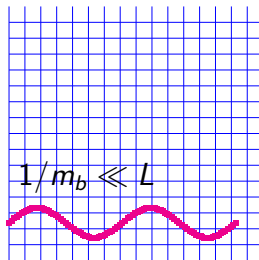


$$\Phi_k^{\text{QCD}} = \Phi_k^{\text{HQET}}$$

$$k = 1, 2, \dots, N_{\text{HQET}}$$

$$N_{\text{HQET}} = \text{\# of parameters}$$

HQET

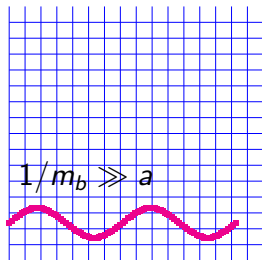


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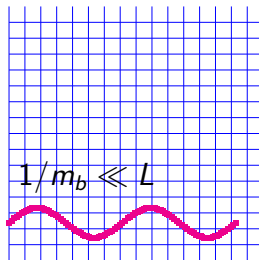


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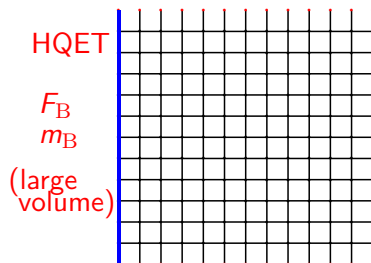
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HQET



- HQET-parameters from QCD-observables in small volume
 – at small lattice spacing $L^{-1} \ll m_b \ll a^{-1}$
 power divergences subtracted non-perturbatively

The HQET strategy: first view



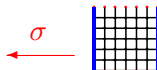
ω_k : coefficients in effective Lagrangian ...
e.g.

$$\omega_2 \bar{b} \sigma \cdot \mathbf{B} b$$

$$\omega_2 \sim 1/(2m_b)$$

$\Phi^{\text{QCD}}(L_1)$

↓ matching



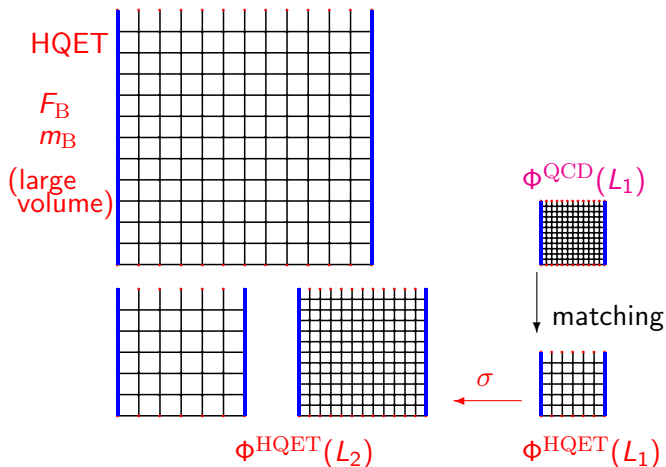
$\omega_k(a_2)$

$a_2 = 0.05 \dots 0.1 \text{ fm}$

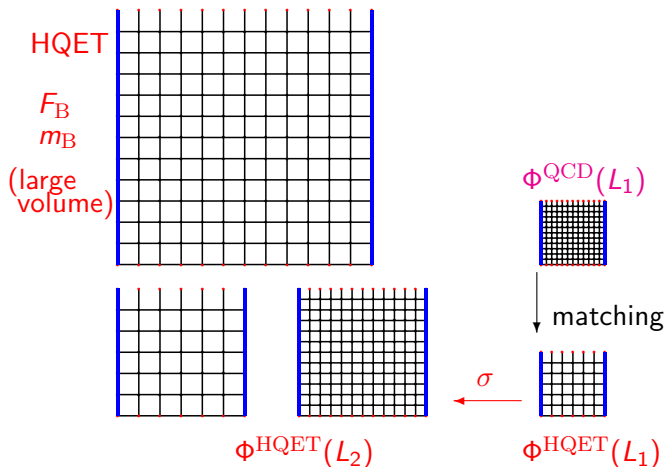
$\omega_k(a_1)$

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The HQET strategy: second view



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- **continuum limit** can be taken in all steps

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- ▶ All in all 12 different matching conditions in volume $L = L_1$

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		$\theta_1 = 0$ $\theta_2 = 1/2$	$\theta_1 = 1/2$ $\theta_2 = 1$	$\theta_1 = 1$ $\theta_2 = 0$
Main strategy				
0	17.25(20)	17.12(22)	17.12(22)	17.12(22)
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1/2	17.01(22)	17.23(28)	17.21(27)	17.22(28)
1	16.78(28)	17.17(32)	17.14(30)	17.15(30)
3 % $= O(\Lambda^2/m_b^2)$		0.6% $\ll$ total error $= O(\Lambda^3/m_b^3)$		

$\ln F_B:$
 $O(\Lambda^2/m_b^2) =$
 $2(1)\%$
 [Blossier, Della
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- $\bar{m}_b^{\overline{MS}}(\bar{m}_b) = 4.347(48)\text{GeV}$ quenched, $r_0 = 0.5\text{ fm}$ (4-loop RGE)
 [Della Morte, Garron, Papinutto & S]

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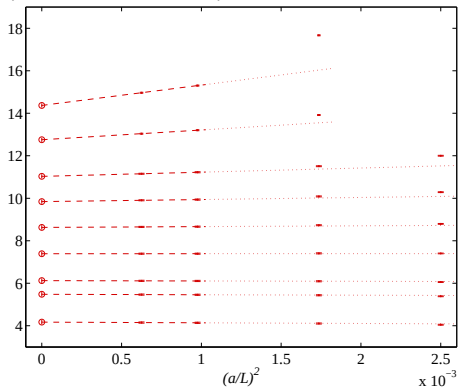
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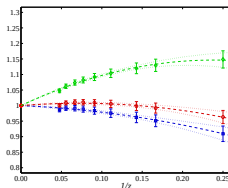
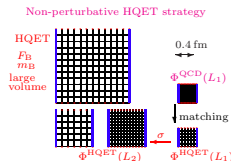
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[Della Morte, Garron, Papinutto & S]
- And with previous strategy (static with $x \propto 1/m_b$ interpolations)
 $\bar{m}_b^{\overline{MS}}(\bar{m}_b) = 4.421(67)\text{GeV}$ [Guazzini, S., Tantalo]
- 1–1.5% precision; uncertainty dominated by ΔZ_M , $M = Z_M m_0$

Perspectives: dynamical fermions are absolutely needed

At present this strategy is being applied to $N_f = 2$ QCD
(just up and down sea)



in progress within **CLS**

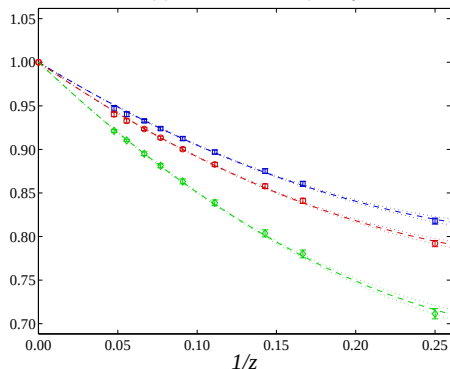


$$\uparrow 1/z = 1/(M_b L)$$

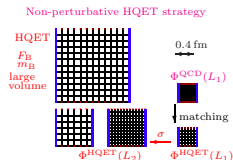
B. Blossier, G. De Divitiis, M. Della Morte, P. Fritzsch, N. Garron, J. Heitger, G. von Hippel, T. Mendes, R. Petronzio, S. Schäfer, H. Simma, R.S., N. Tantalo

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Verification of approach to the spin-symmetric limit

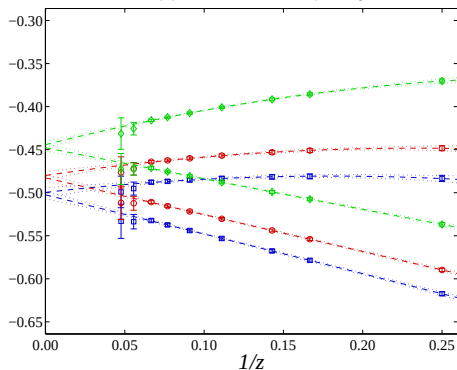


Ratio of axial-vector and vector correl. fct. No.1



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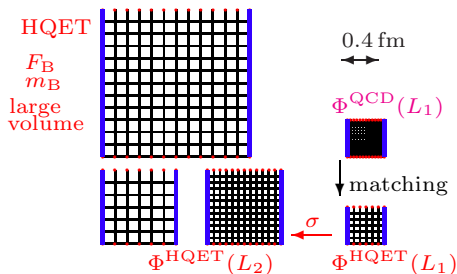


Ratio of axial-vector and vector correl. fct. No.2

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Verification of approach to the spin-symmetric limit

Non-perturbative HQET strategy



Step scaling functions in progress, large volume, see next talk.

$N_f = 2$ QCD: Coordinated Lattice Simulations

Teams

- * Berlin (team leader Ulli Wolff)
- * CERN (L. Giusti, M. Lüscher)
- * DESY-Zeuthen (Rainer Sommer)
- * Madrid (Carlos Pena)
- * Mainz (Hartmut Wittig)
- * Rome (Roberto Petronzio)
- * Valencia (Pilar Hernández)

Physics planned at present

- * Fundamental parameters up to M_b
 - * Pion interactions
 - * Baryon physics
 - * Kaon physics
- also with mixed actions

β	$a[\text{fm}]$	lattice	$L[\text{fm}]$	masses	
5.30	0.08	48×24^3	1.9	6 masses	CERN, Rome
5.30	0.08	64×32^3	2.6	6 masses	CERN, Rome
5.50	0.06	64×32^3	1.9	5 masses	DESY, Berlin, Madrid
5.70	0.04	96×48^3	1.9	2 masses	DESY, Berlin
5.70	0.04	128×64^3	2.6	2 masses	DESY, Berlin, started

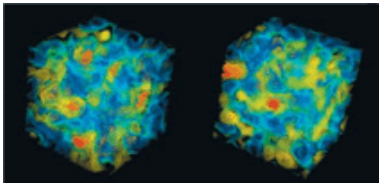
Promising for charm and beauty

Conclusions: the present mood

BREAKTHROUGHS OF THE YEAR

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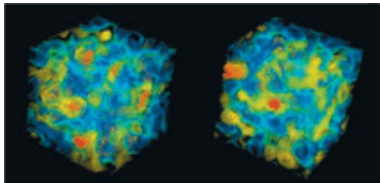


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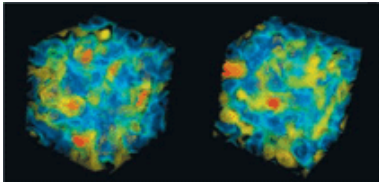
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Berliner Zeitung

this is going a bit overboard but there are **good perspectives** for matching experimental **precisions** on

(semi-) leptonic B decays, B-mixing; not on purely hadronic decays!

...

by the time superB comes

Tight Schedule Toward Upgrade

