Determination of CKM matrix elements (mainly at the B-factories)

Heiko Lacker Humboldt University Berlin

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The CKM matrix

1. Origin

Relation to the Yukawa sector:

$$L_{quark\ masses} = \overline{u}_{L} M_{u} u_{R} + \overline{d}_{L} M_{d} d_{R} + h.c., \quad u \equiv \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad d \equiv \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$M_{u, diag} = U_{L}^{+} M_{u} U_{R}$$

$$V_{CKM} = U_{L}^{+} D_{L}$$

2. CP violation through the CKM Matrix

- a) Beyond strong CP violation the only source of CP violation in SM
- b) Extremely economic: Only 1 CP violating parameter for 3 generations

c) <u>#generations</u>	Observed baryon asymmetry
3	CP violation & quark masses too small
4	CP violation & quark masses sufficiently large (Hou (2008)) e.w. Phase transition?

Why measuring CKM matrix elements ?

1. Fundamental parameters of the SM

Point to Beyond Standard Model (BSM) physics (like fermion mass values)

<u>Metrology</u>: Determine them as best as possible

=> Model building & model testing (link to e.g. neutrino sector)

2. Consistency/Unitarity

Any measured deviation from unitarity points to BSM physics

- E.g.: * More generations
 - * Additional particles & couplings beyond the SM (H[±], SUSY, ...)

Consistency checks:

- * Compare observables constraining same CKM parameter (e.g. tree vs loop)
- * directly measured CKM parameter with its prediction from global fit
- * Overall unitarity of the matrix

Parametrisation of the CKM matrix

Three Euler angles and one phase (Chau, Keung 1984):

$$\begin{split} V &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\ c_{ij} &= \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}, i < j = 1, 2, 3 \qquad c_{ij} > 0, s_{ij} > 0 \quad (0 \le \theta_{ij} \le \pi/2) \end{split}$$

Freedom of phase re-definition => phase can appear at other places!

A statement like "The phase of the CKM matrix is 60°." only makes sense in a given phase convention.

Not always clearly specified in the literature.

Parametrisation of the CKM matrix



Wolfenstein parametrisation (Expansion in $\lambda = \sin \theta_c \approx V_{us}$) \approx 0.225):

$$V_{CKM} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

=> η =0 <=> No CP-violation in SM

Define to all orders in λ (Buras, Lautenbacher, Ostermaier, 1994):

 $s_{12} \equiv \lambda$ $s_{23} \equiv A \lambda^{2}$ $s_{13} e^{-i\delta} \equiv A \lambda^{3} (\rho - i \eta)$ Unitary holds to all orders!

Unitarity Triangle



All sides and angles can be determined from the B-meson system alone (but does not contain the full information)!

Hunting for New Physics in Loop-m ediated processes



λ from tree-level processes

$|V_{ud}|$: 1) Super-allowed nuclear β -decays (SFD)2) Neutron β -decay3) Pionic β -decay V_{CKM} = $|V_{us}|$: 1) Semileptonic Kaon decays2) Leptonic Kaon & Pion decay3) τ -decays

4) Semileptonic Hyperon decays

 $V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

$$\approx \begin{vmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A \lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\ A \lambda^3 (1 - \rho - i\eta) & -A \lambda^2 & 1 \end{vmatrix}$$

|V_{cd}|, |V_{cs}|: 1) Dimuon production from neutrinos on nuclei
 2) Semileptonic D-meson decays

At present, the most stringent constraint on $\lambda\,$ comes $\,\text{from}\,|V_{_{\text{ud}}}|_{_{\text{SFD}}}\,!$

λ from tree-level processes

$$1 = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$$



V_{cb} : Exclusive sem ileptonic B-decays



Heavy Quark-Limit: Only one FF (ξ)

For light degrees of freedom (q, g): no change in interactions with Q for w=1 if m_Q is very large => $\xi(w=1)=1$ <u>For finite masses:</u>

Luke's theorem $B \rightarrow D^* l\nu$: $F(w=1) = \eta_A \eta_{QD} \cdot (1 + 0 \cdot \frac{\Lambda_{QD}}{m_Q} + b \cdot \frac{\Lambda_{QD}^2}{m_Q^2} + \dots)$ $B \rightarrow D l\nu$: $V_1(w=1) = \eta_A \eta_{QD} \cdot (1 + a \cdot \frac{\Lambda_{QD}}{m_Q} + b \cdot \frac{\Lambda_{QD}^2}{m_Q^2} + \dots)$

 $B \rightarrow D^* l v \ preferred$:

* Theoretical uncertainties in FF calculation
* Phase space (w→1)
* Experimental Background

V_{cb} : hclusive sem ileptonic B-decays

Including non-perturbative effects (Heavy Quark Expansion): 'Kinetic Mass scheme' (there are others, e.g.: 'Y(1S) scheme'):

$$\Gamma(B \to X_c \ l \nu) = 1.014 \cdot \left| V_{cb} \right|^2 \frac{G_F^2 m_b^5}{192 \ \pi^3} A^{pert} \left(\frac{m_c}{m_b} \right) \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2 \ m_b^2} \right) - 2 \left(1 - \frac{m_c^2}{m_b^2} \right)^4 \frac{\mu_G^2}{m_b^2} + O\left(\frac{1}{m_b^3} \right) \right)$$

No contribution at order $1/m_b =>$ Decay rate close to quark-level decay rate For precision determination of V_{cb}: $1/m_b^2$ and $1/m_b^3$ to be taken into account

Lifetimes of B mesons $\tau(B) = (1.530 \pm 0.008) \text{ ps}$

Semileptonic BF

BF($B^+/B^0 \rightarrow X I \nu$) = (10.74 ± 0.16)%

 τ (B) = (1.639 ± 0.009) ps

V_{cb} : hclusive sem ileptonic B-decays

- * Theory Error on $|V_{cb}|$ dominates: O(5%) !
- * Reason: Theory parameters difficult to calculate
- * Idea: Shape of differential spectrum is function of theory parameters

$$\frac{d\Gamma}{dE}(E) = f(E|m_b, m_c; \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, \dots; \alpha_s)$$

- * Measure Moments of differential spectra
 - => Determine theory parameters from data

$$B \rightarrow X_c l \nu$$
$$M_n^l(E_0) = \frac{\int_{E_0} E_l^n \frac{d\Gamma}{dE_l}(E_l) dE_l}{\int_{E_0} \frac{d\Gamma}{dE_l}(E_l) dE_l}$$



V_{cb} : hclusive sem ileptonic B-decays

Experiments: BABAR, BELLE, CDF, CLEO, DELPHI

Hadronic mass and leptonic energy moments in $B \rightarrow X_c l_V$

+ 1. and 2. photon energy moment in $B \rightarrow X_s \gamma$



 $|V_{cb}|(incl) = (41.67 \pm 0.43 \pm 0.08 \pm 0.58) \times 10^{-3}$ Below 2% precision

V_{ub} : Exclusive sem ileptonic B-decays



$$BF(B \to h \, l \, \nu) \propto \left| V_{ub} \right|^2 F^2(q^2)$$

In contrast to $B \rightarrow D^{(*)}$ transitions: No Heavy Quark Symmetry providing form factor normalization

Form Factors from:

- * Light-Cone Sum Rules (LCSR): only valid at small q²
- * Lattice QCD: only valid at high q²

FF uncertainties enter at two different places:

1. FF shape => $BF(B \rightarrow h l \nu) = \frac{N_{sig}}{\epsilon N_{sig}}$

2. FF normalization => $|V_{ub}|$ =

$$BF(B \to h l \nu) = \underbrace{N_{sig}}_{V_{sig}}$$
$$V_{ub} = \sqrt{\frac{BF(B \to h l \nu)}{\tau_{b} \Gamma_{thv}}}$$

V_{ub} : hclusive sem ileptonic B-decays





⇒ Extrapolation: Fraction of accepted signal events?

V ... hclusive sem ileptonic B-decays







Summary: |V_{ub}| and |V_{cb}|

INCLUSIVE

 $(41.67 \pm 0.43 \pm 0.08 \pm 0.58) \times 10^{-3}$

moments in B->X_cI v & B->X_s γ

EXCLUSIVE

 $(38.18 \pm 0.56_{exp} \pm 0.54_{theostat} \pm 0.83_{theosys}) \times 10^{-3}$

B->D*I v using FF at zero recoil: $0.921 \pm 0.013_{theostat} \pm 0.020_{theosys}$ 0808.2519 [hep-lat]







 $|V_{cb}|$

Mixing and CP violation: Observables

Mass eigenstates:

$$B_{H} = p B^{0} + q B^{0}$$
$$B_{L} = p B^{0} - q B^{0}$$

Oscillations: $B^0 \to B^0 \to B^0$ with frequency $\Delta m \equiv M_H - M_L$





Measurement of a time-dependent CP asymmetry



Experimental inputs with "no" theory machinery - β



Experimental inputs with "no" theory machinery - β



Experimental inputs with "no" theory machinery - ß



Experimental inputs with "no" theory machinery - ß



Experimental inputs with "no" theory machinery - γ

CP violation in charged B- decays: $b \rightarrow c$

 $b \to c \,\overline{u} \, s, \, u \,\overline{c} \, s$



CKM phase	strong phase
difference: γ	difference: δ

 r_{B} , δ : for each final state D^(*)K^(*)

Experimental inputs with "no" theory machinery - γ

CP violation in charged B- decays:

 $b \rightarrow c \overline{u} s, u \overline{c} s$



CKM phase	strong phase
difference: γ	difference: δ

D⁰-decay into CP-eigenstates «GLW»

Gronau-London; Gronau-Wyler (1991)

 $\overline{D}{}^{0} \rightarrow K^{+} \pi$ (fav.) & $D^{0} \rightarrow K^{+} \pi$ (disfav.) «ADS» Atwood-Dunietz-Soni (1997)

«GGSZ» $D^0 \rightarrow K_S \pi^+ \pi^-$ (Dalitz plot)

Bondar (Belle, 2002) Giri-Grossman-Soffer-Zupan (2003)

=> Syst. in Dalitz model; CP in D-decays

Experimental inputs with "no" theory machinery - γ

CP violation in charged B- decays:

 $b \to c \,\overline{u} \, s, \, u \,\overline{c} \, s$



CKM phase	strong phase
difference: γ	difference: δ

 r_{R}, δ : for each final state D^(*)K^(*)



=> Syst. in Dalitz model; CP in D-decays



Experimental inputs with modest theory machinery - α

Assumption: Isospin symmetry of strong interactions in B $\rightarrow \pi\pi$, $\rho\rho$, $\rho\pi/\pi\pi\pi$



Issues:

- * e.w. penguins, π - $\eta^{(\prime)}$ -/ ρ - ω -Mixing, other isospin violations, finite ρ -width
- * Control of charmless BG, Dalitz plot structure ($\pi\pi\pi$)
 - => Difficult to go below a few degrees precision

Experimental Inputs relying heavily on theory machinery



Experimental Inputs relying heavily on theory machinery



Experimental Inputs relying heavily on theory machinery



Need for consistent "averages" for LQCD parameters

However:

- 1. Not clear how to do it
- 2. Subjective
- 3. Meaning of errors not clear

f_{B_s}	$(228 \pm 3 \pm 17) MeV$
f_{B_s}/f_{B_d}	$1.196 \pm 0.008 \pm 0.023$
B_{s}	$1.23 \pm 0.03 \pm 0.05$
B_{s}/B_{d}	$1.05 \pm 0.02 \pm 0.05$

 \hat{B}_{K} 0.721±0.005±0.040

Here:

only 2 & (1+1) unquenched calculations

"Algorithmic average" preserving smallest systematic uncertainty

The overall picture



- Good agreement on 2σ-level
- Big success for:
 - * B-factories
 - * Tevatron (Δm_s)
 - * KM mechanism



• NP can still be around due to sizeable non-perturbative QCD uncertainties

The role of leptonic B-decays, esp. B→τv

1. Helicity suppression => $B \rightarrow ev$, $B \rightarrow \mu v$ not measurable at B-factories

=> $B \rightarrow \tau v$ (10⁻⁴): Evidence at B-factories (first @ Belle)

$${\cal B}(B^- o \ell^- ar
u) = rac{G_F^2 m_B}{8\pi} m_l^2 \left(1 - rac{m_l^2}{m_B^2}
ight)^2 f_B^2 |V_{ub}|^2 au_B$$

- 2. Uncertainties in prediction: f_{B} and $|V_{ub}|$
- 3. $\Delta m_d \& BF(B \rightarrow \tau v)$ removes dependence on f_B (but not on B_d): $\Delta m_d \propto f_B^2 B_d$
- 4. Sensitive to H[±] contributions $^{-0.5}$ $B(B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau}) = B_{\mathcal{M}} (B^{-} \rightarrow \tau^{-} \bar{\nu}_{\tau}) \times \left(1 - \tan^{2} \beta \frac{m_{B}^{2}}{m_{H^{\pm}}^{2}}\right)^{2} - 1.0$ $^{-1.0}$



Interesting effect in $B \rightarrow \tau v$ (or in sin2 β)?



$$\frac{\mathrm{BR} \ (\mathrm{B} \to \tau \, \mathrm{v})}{\mathrm{Amd}} = \frac{3 \ \pi}{4} \ \frac{\mathrm{m} \tau^2}{\mathrm{m} \mathrm{W}^2 \ \mathrm{S} \ (\mathrm{xt})} \left(1 - \frac{\mathrm{m} \tau^2}{\mathrm{m} \mathrm{B}^2}\right)^2 \tau \mathrm{B}^+ \ \frac{1}{\mathrm{B}_{\mathrm{Ed}}} \ \frac{1}{|\mathrm{Vud}|^2} \ \left(\frac{\mathrm{sin}\beta}{\mathrm{sin}\gamma}\right)^2$$

$B \rightarrow \tau v \& \Delta m_d$: B_d prediction

Deviation may be caused by: 1. Statistical fluctuation or

- 2. NP (H[±], sin2 β) or
- 3. Problem in B_d -value



NP in B_d-Mixing ?





- NP space still considerably large
- Deviation from SM: 2.1σ



 Deviation from SM: 1.9σ
 (driven by CP violation in B_s mixing: Tevatron)

Conclusion & Outlook

- KM mechanism dominant source of observed CP violation but: significant NP effects may hide behind non-perturbative QCD uncertainties ($\Delta m_d, \Delta m_s, \epsilon_{\kappa}, V_{ub}, V_{cb}$) Important improvements have to come from theory
- β , α & γ can be significantly improved @ a Super-B-factory; there are limitations (α) or dependence on external input (γ : Dalitz plot \rightarrow charm factory (CLEO-c, Super-F?))
- Rare decays @ a Super-B-factory: e.g. $B \rightarrow I\nu$, sin2 β (b \rightarrow s peng), $B \rightarrow V\gamma$, , $B \rightarrow X_{d}\gamma$
- Charm-/B-/Super-F-factory: $V_{cd} \& V_{cs}$ from (semi)leptonic D and D_s-decays; D-mixing
- LHCb: * improvements in the area of angle measurements (β , esp. γ ; α challenging)
 - * B_s-sector (β_s from B_s $\rightarrow \psi \phi$, B_s $\rightarrow \phi \phi$ (b \rightarrow s penguin), rare decays like B_s $\rightarrow \mu \mu$)



Detailed comparisons (incl. B→τv, not shown)



V_{cb} : Exclusive sem ileptonic B-decays



V ... Exclusive sem ileptonic B-decays



Experimental inputs with modest theory machinery - α



1.4.4 CP violation generated by the CKM matrix

$$L_{weak} \propto W_{\mu}^{+} \left(\overline{u} \, \overline{c} \, \overline{t} \right) \gamma^{\mu} (1 - \gamma^{5}) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} \qquad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



 $u b \rightarrow d c$ $\overline{u} \,\overline{b} \rightarrow \overline{d} \,\overline{c}$

$$M \underbrace{\propto V_{ud}^* V_{cb}(\bar{u_d} \Gamma^{\mu} u_u)(\bar{u_c} \Gamma_{\mu} u_b)}_{CP} \qquad M^+ \underbrace{\propto V_{ud} V_{cb}^*(\bar{u_u} \Gamma^{\mu} u_d)(\bar{u_b} \Gamma_{\mu} u_c)}_{H^*} \qquad \Gamma^{\mu} \equiv \chi^{\mu} (1 - \chi^5)$$

$$\begin{array}{c} || ?\\ M_{CP} \underbrace{\propto V_{ud}^* V_{cb}(\bar{u_u} \Gamma^{\mu} u_d)(\bar{u_b} \Gamma_{\mu} u_c)}_{H^*} \\ \end{array}$$

$$\begin{array}{c} CP \text{ invariance:} \qquad M^+ \equiv M_{CP} \Rightarrow V_{CKM} \text{ real} \end{array}$$

Interesting effects in leptonic decays?



- Deviation: 2.9 σ
- Prediction driven by angles (not V_{ub})
- Either hint for
 - 1. NP (H[±], in sin2 β , ...) or
 - 2. problem in B_d-value
- Deviation: 2.9 σ
- Prediction driven by one LQCD result for f_{Ds} ((241±3) MeV, 0706.1726)
- If LQCD value confirmed by other calculations with similar errors:

Hint for NP, but not: H[±] or 4th gen. (Kronfeld & Dobrescu, 2008)

sin2β(b→s penguin)

