Heavy Quarks on the Lattice – Current Status

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Introduction

Recent progress and perspectives in heavy quark physics on the lattice:

- Generalised Eigenvalue Problem for HQET to $\mathrm{O}(1/m)$ (cf. Aachen talk)
- Completed (quenched and unquenched) HQET/QCD matching (cf. previous talk)
- Now calculating (quenched) f_{B_s}, m_b, B_s spectra, ...
- Looking towards unquenched case for the near future

The Generalised Eigenvalue Problem

For a matrix of correlation functions on an infinite time lattice

$$C_{ij}(t) = \langle O_i(t) O_j(0) \rangle = \sum_{n=1}^{\infty} \mathrm{e}^{-E_n t} \psi_{ni} \psi_{nj} \,, \quad i,j = 1,\ldots,N$$

$$\psi_{ni} \equiv (\psi_n)_i = \langle n | \hat{O}_i | 0 \rangle = \psi_{ni}^* \quad E_n \leq E_{n+1}$$

the GEVP is

$$C(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0) \,, \quad n = 1,\ldots,N \quad t > t_0,$$

Define effective energy levels and matrix elements[Blossier et al., 2008]

$$E_n^{\mathrm{eff}} \equiv \frac{1}{a} \log \frac{\lambda_n(t,t_0)}{\lambda_n(t+a,t_0)} = E_n + \mathrm{O}(\mathrm{e}^{-(E_{N+1}-E_n)t})$$

$$\psi_n^{\mathrm{eff}} \equiv \frac{\langle P(t) O_j(0) \rangle\, v_n(t,t_0)_j}{(v_n(t,t_0)\,,\ C(t)\, v_n(t,t_0))^{1/2}} \frac{\lambda_n(t_0+t/2,t_0)}{\lambda_n(t_0+t,t_0)}$$

$$= \langle 0 | \hat{P} | n \rangle + \mathrm{O}(\mathrm{e}^{-(E_{N+1}-E_n)t_0})$$

The Generalised Eigenvalue Problem

Can construct creation operator

$$
\begin{aligned}
\hat{\mathcal{A}}_n^{\mathrm{eff}\,\dagger}(t,t_0) &= \mathrm{e}^{-\hat{H}t}\,\hat{\mathcal{Q}}_n^{\mathrm{eff}\,\dagger}(t,t_0) \\
\hat{\mathcal{Q}}_n^{\mathrm{eff}}(t,t_0) &= R_n\,(\hat{O}\,,\ v_n(t,t_0)\,) \\
R_n &= (v_n(t,t_0)\,,\ C(t)\,v_n(t,t_0))^{-1/2}\,\frac{\lambda_n(t_0+t/2,t_0)}{\lambda_n(t_0+t,t_0)}
\end{aligned}
$$

for n^{th} excited state:

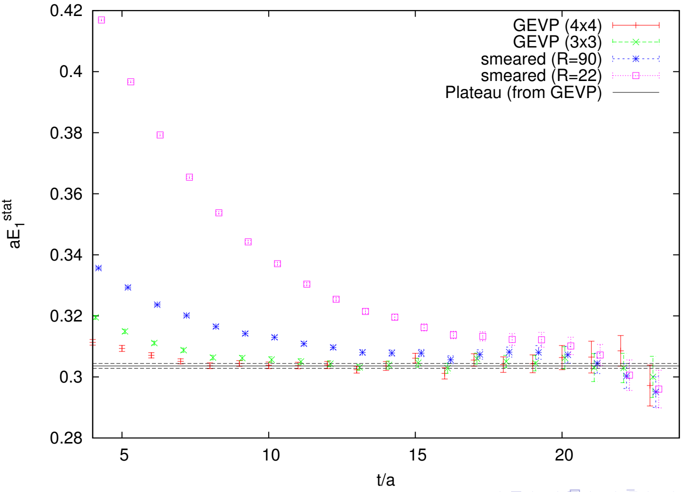
$$
\hat{\mathcal{A}}_n^{\mathrm{eff}\,\dagger}|0\rangle = |n\rangle + \sum_{n'=1}^{\infty}\pi_{nn'}(t,t_0)|n'\rangle
$$

where

$$
\pi_{nn'}(t,t_0) = \mathrm{O}(\mathrm{e}^{-(E_{N+1}-E_n)\,t_0})\,, \quad \text{at fixed } t-t_0
$$

GEVP vs. non-GEVP effective masses

Control statistical **and** systematic errors:



Effective theory to first order

For heavy quarks, expand QCD action non-relativistically

$$S_{QCD} \mapsto S^{\mathrm{stat}} + \omega\, S^{1/\mathrm{m}} + \mathrm{O}(\omega^2)$$

$$S^{\mathrm{stat}} = \sum_x \psi^\dagger D_t \psi$$

$$S^{1/\mathrm{m}} = \sum_x \psi^\dagger \mathbf{D}^2 \psi + c_\sigma \psi^\dagger \sigma \cdot \mathbf{B} \psi$$

Expansion of exponential of action gives expansion of correlation functions

$$\begin{aligned} C_{ij}(t) &= \int D\Phi \left\{ \frac{1}{V} \sum_{\vec{x}} O_i(x) O_j(0) \left(1 + \omega\, S^{1/\mathrm{m}} + \mathrm{O}(\omega^2)\right) \right\} \mathrm{e}^{-S^{\mathrm{stat}}} \\ &= C_{ij}^{\mathrm{stat}}(t) + \omega C_{ij}^{1/\mathrm{m}}(t) + \mathrm{O}(\omega^2) \end{aligned}$$

Effective theory to first order

Expand in ω

$$C_{ij}(t) = C_{ij}^{\rm stat}(t) + \omega C_{ij}^{1/{\rm m}}(t) + {\rm O}(\omega^2)$$

and find to first order in ω [Blossier et al., 2008]

$$E_n^{\rm eff,stat}(t,t_0) = \log \frac{\lambda_n^{\rm stat}(t,t_0)}{\lambda_n^{\rm stat}(t+1,t_0)} = E_n^{\rm stat} + {\rm O}({\rm e}^{-\Delta E_{N+1,n}^{\rm stat}\,t}),$$

$$E_n^{\rm eff,1/m}(t,t_0) = \frac{\lambda_n^{1/{\rm m}}(t,t_0)}{\lambda_n^{\rm stat}(t,t_0)} - \frac{\lambda_n^{1/{\rm m}}(t+1,t_0)}{\lambda_n^{\rm stat}(t+1,t_0)}$$

$$= E_n^{1/{\rm m}} + {\rm O}(t\,{\rm e}^{-\Delta E_{N+1,n}^{\rm stat}\,t}).$$

where

$$C^{\rm stat}(t)\, v_n^{\rm stat}(t,t_0) = \lambda_n^{\rm stat}(t,t_0)\, C^{\rm stat}(t_0)\, v_n^{\rm stat}(t,t_0),$$

$$\lambda_n^{1/{\rm m}}(t,t_0) = \left(v_n^{\rm stat}(t,t_0),\, [C^{1/{\rm m}}(t) - \lambda_n^{\rm stat}(t,t_0) C^{1/{\rm m}}(t_0)] v_n^{\rm stat}(t,t_0)\right)$$

Note that the GEVP is only ever solved for $C_{ij}^{\rm stat}$

Simulation setup

- $L^3 \times T = (1.5\,\mathrm{fm})^3 \times 3\,\mathrm{fm}$ lattice with periodic b.c. (antiperiodic in time for quarks)
- ensembles of 100 quenched configs each
- $L/a = 16,\ 24,\ 32$, i.e. $\approx 0.1,\ 0.07, 0.05\,\mathrm{fm}$ lattice spacing, with κ tuned to the strange quark mass
- all-to-all propagators for the strange quark [Dublin, 2005] using 50 (approximate) low modes and 2 noise sources (no low modes and 4 noise sources at smallest lattice spacing)
- $N_{\mathrm{max}} = 8$ different levels of covariant Gaussian quark smearing [Wuppertal, 1989] after APE gauge-field smearing [Basak et al., 2006]
- Truncate to an $N \times N$ matrix by projecting with the N eigenvectors of $C(a)$ having the largest eigenvalues [Niedermayer, Rufenacht, Wenger, 2000]

Observables

Energies contain divergences – look at energy differences here

$$E_n = E_n^{\rm stat} + \omega_{\rm kin} E_n^{\rm kin} + \omega_{\rm spin} E_n^{\rm spin} + \delta m$$

Decay constant is obtained from

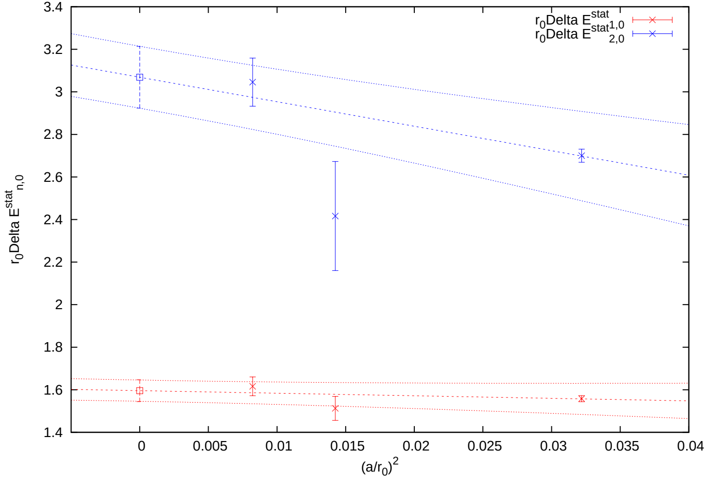
$$f_B^{(n)} \sqrt{M_B^{(n)}} = Z_A^{\rm HQET} p_n^{\rm stat} (1 + \omega_{\rm kin} p_n^{\rm kin} + \omega_{\rm spin} p_n^{\rm spin} + c_A^{\rm HQET} p_n^{\delta A})$$

where

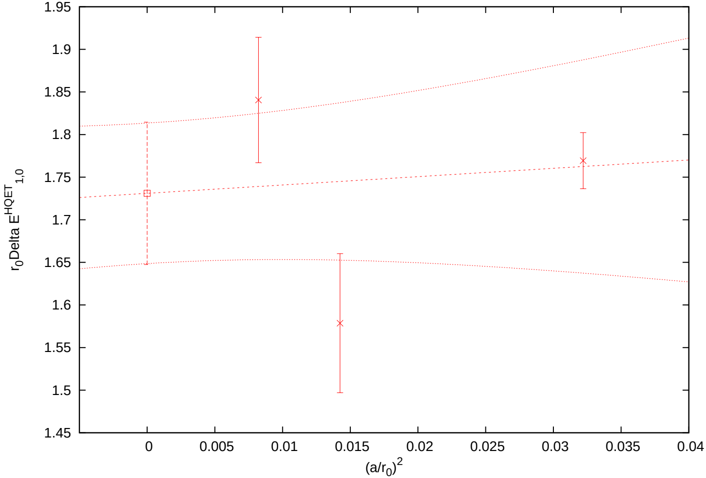
$$\begin{aligned} O_{\delta A}(\vec{x},t) &= \overline{\psi}_{\rm h}(\vec{x},t)\gamma_0\gamma_5[\vec{\gamma}\cdot\vec{D}\psi_{\rm l}^{(0)}](\vec{x},t), \\ O_{\rm kin}(\vec{x},t) &= \overline{\psi}_{\rm h}(\vec{x},t)[\vec{D}^2\psi_{\rm h}](\vec{x},t), \\ O_{\rm spin}(\vec{x},t) &= \overline{\psi}_{\rm h}(\vec{x},t)[\vec{\sigma}\cdot\vec{B}\psi_{\rm h}](\vec{x},t). \end{aligned}$$

Presented here is a preliminary first analysis – needs cross-checking and further verification

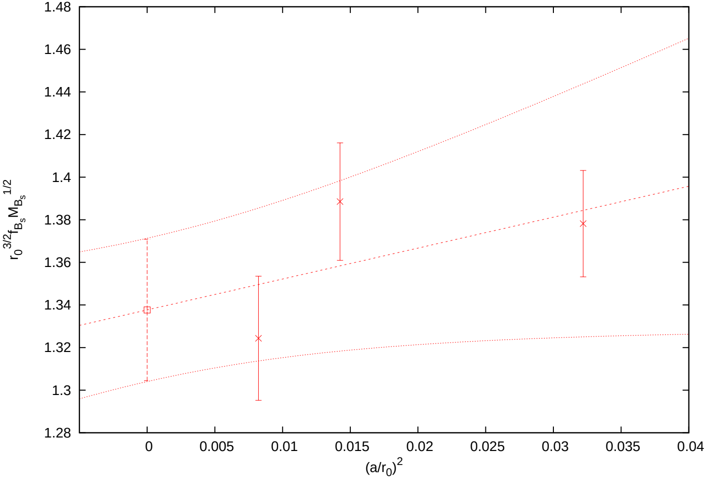
The static B_s spectrum



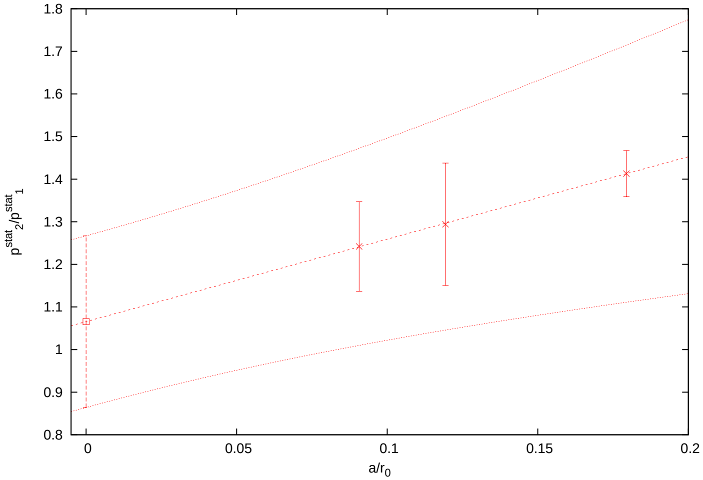
The B_s spectrum at $O(1/m)$



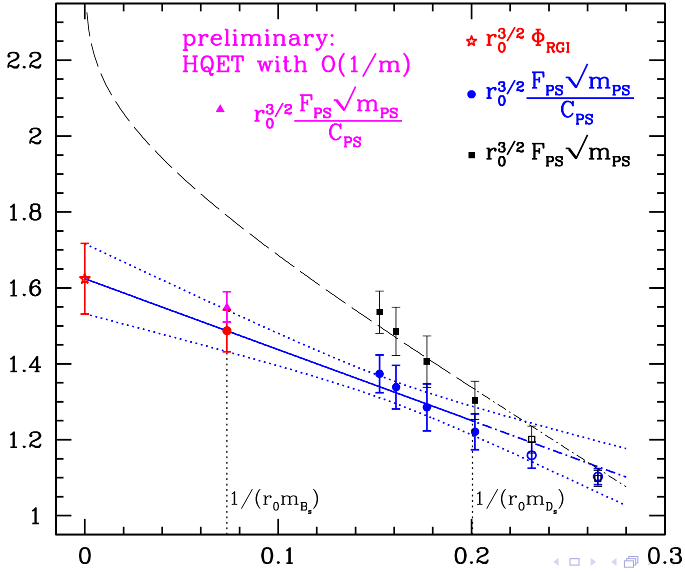
The decay constant f_{B_s}



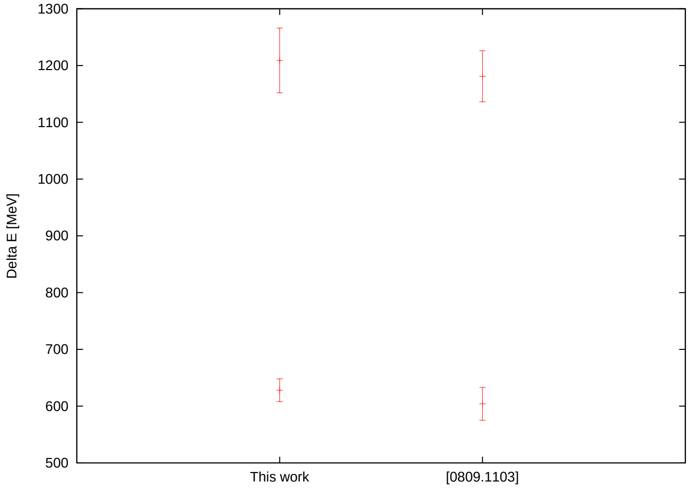
Excited state matrix element ratio



Comparison with [JHEP 0802:078,2008]



Comparison with [0809.1103]



Conclusions

- A thorough understanding of the GEVP enables us to obtain results for heavy flavour observables at $\mathrm{O}(1/m)$ from HQET
- We have presented (preliminary) results for the B_s spectrum and for the decay constant f_{B_s}
- Our results compare favourably with previous studies in static approximation
- $\mathrm{O}(1/m)$ corrections are relatively small – expansion under control
- Now looking towards the unquenched ($N_f = 2$) case

The end

Thank you for your attention