

CKM elements from squark-gluino loops

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“Supersymmetric renormalization of the CKM matrix and
new constraints on the Squark Mass Matrices of the MSSM”
[arXiv:hep-ph/0810.1613]

Outline:

- The SUSY flavour problem and the squark mass matrix
- Non-decoupling flavour-changing SQCD self-energies
- **CKM renormalization and constraints from a fine-tuning argument**
- SUSY breaking as the origin of flavour?

SUSY flavour problem

- Squark mass matrices are not necessarily diagonal in the same basis as the quark mass matrices

→ Quark-squark-gluino vertex is flavour-changing in general

Dangerously large flavour-mixing in FCNC processes involving the strong coupling constant.

Squark mass matrix

$$M_{\tilde{u}}^{w2} = \begin{pmatrix} \textcircled{\mathbf{m}_{\tilde{q}}^2} + M_z^2 \left(1 + \frac{2}{3} \sin^2(\theta_w)\right) \cos(2\beta) \mathbf{1} + \mathbf{m}_u^{(0)} \mathbf{m}_u^{(0)\dagger} & \textcircled{v_u \mathbf{A}^u} - \mathbf{m}_u^{(0)} \mu \cot(\beta) \\ \textcircled{v_u \mathbf{A}^{u\dagger}} - \mathbf{m}_u^{(0)\dagger} \mu^* \cot(\beta) & \textcircled{\mathbf{m}_{\tilde{u}}^2} + \frac{2}{3} M_z^2 \sin^2(\theta_w) \cos(2\beta) \mathbf{1} + \mathbf{m}_u^{(0)} \mathbf{m}_u^{(0)\dagger} \end{pmatrix}$$

$$M_{\tilde{d}}^{w2} = \begin{pmatrix} \textcircled{\mathbf{m}_{\tilde{q}}^2} - M_z^2 \left(1 + \frac{1}{3} \sin^2(\theta_w)\right) \cos(2\beta) \mathbf{1} + \mathbf{m}_d^{(0)} \mathbf{m}_d^{(0)\dagger} & \textcircled{v_d \mathbf{A}^d} - \textcircled{\mathbf{m}_d^{(0)} \mu \tan(\beta)} \\ \textcircled{v_d \mathbf{A}^{d\dagger}} - \textcircled{\mathbf{m}_d^{(0)\dagger} \mu^* \tan(\beta)} & \textcircled{\mathbf{m}_{\tilde{d}}^2} - \frac{1}{3} M_z^2 \sin^2(\theta_w) \cos(2\beta) \mathbf{1} + \mathbf{m}_d^{(0)} \mathbf{m}_d^{(0)\dagger} \end{pmatrix}$$



Chirality conserving, flavour non-diagonal



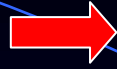
Chirality flipping and flavour non-diagonal



Chirality flipping, flavour-diagonal, but $O(1)$ for large $\tan(\beta)$

$$\tan(\beta) = v_u / v_d$$

Squark mass matrix
is hermitian



$$W^{\tilde{q}\dagger} M_{\tilde{q}}^2 W^{\tilde{q}} = M_{\tilde{q}}^{2(D)}$$

Parameterization:

In the convention of: F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini,
Nucl. Phys. B 477 (1996) 321 [arXiv:hep-ph/9604387].

$$M_{\tilde{q}}^2 = \begin{pmatrix} \left(M_{1L}^{\tilde{q}}\right)^2 & \Delta_{12}^{\tilde{q}LL} & \Delta_{13}^{\tilde{q}LL} & \Delta_{11}^{\tilde{q}LR} & \Delta_{12}^{\tilde{q}LR} & \Delta_{13}^{\tilde{q}LR} \\ \Delta_{12}^{\tilde{q}LL*} & \left(M_{2L}^{\tilde{q}}\right)^2 & \Delta_{23}^{\tilde{q}LL} & \Delta_{12}^{\tilde{q}RL*} & \Delta_{22}^{\tilde{q}LR} & \Delta_{23}^{\tilde{q}LR} \\ \Delta_{13}^{\tilde{q}LL*} & \Delta_{23}^{\tilde{q}LL*} & \left(M_{3L}^{\tilde{q}}\right)^2 & \Delta_{13}^{\tilde{q}RL*} & \Delta_{23}^{\tilde{q}RL*} & \Delta_{33}^{\tilde{q}LR} \\ \Delta_{11}^{\tilde{q}LR*} & \Delta_{12}^{\tilde{q}RL} & \Delta_{13}^{\tilde{q}RL} & \left(M_{1R}^{\tilde{q}}\right)^2 & \Delta_{12}^{\tilde{q}RR} & \Delta_{13}^{\tilde{q}RR} \\ \Delta_{12}^{\tilde{q}LR*} & \Delta_{22}^{\tilde{q}LR*} & \Delta_{23}^{\tilde{q}RL} & \Delta_{12}^{\tilde{q}RR*} & \left(M_{2R}^{\tilde{q}}\right)^2 & \Delta_{23}^{\tilde{q}RR} \\ \Delta_{13}^{\tilde{q}LR*} & \Delta_{23}^{\tilde{q}LR*} & \Delta_{33}^{\tilde{q}LR*} & \Delta_{13}^{\tilde{q}RR*} & \Delta_{23}^{\tilde{q}RR*} & \left(M_{3R}^{\tilde{q}}\right)^2 \end{pmatrix}$$

Mass insertion approximation

(L.J. Hall, V.A. Kostelecky and S. Raby, Nucl. Phys. B 267 (1986) 415.)

- Diagonalize the quark mass matrices:

$$U_L^{(0)q\dagger} m_q^{(0)} U_R^{(0)q} = m_q^{(D)}$$

- Carry out the same rotation on the squark fields

➡ **super-CKM basis**

- Treat all neutral vertices as flavour diagonal and the off-diagonal elements of the squark mass matrix as perturbations.

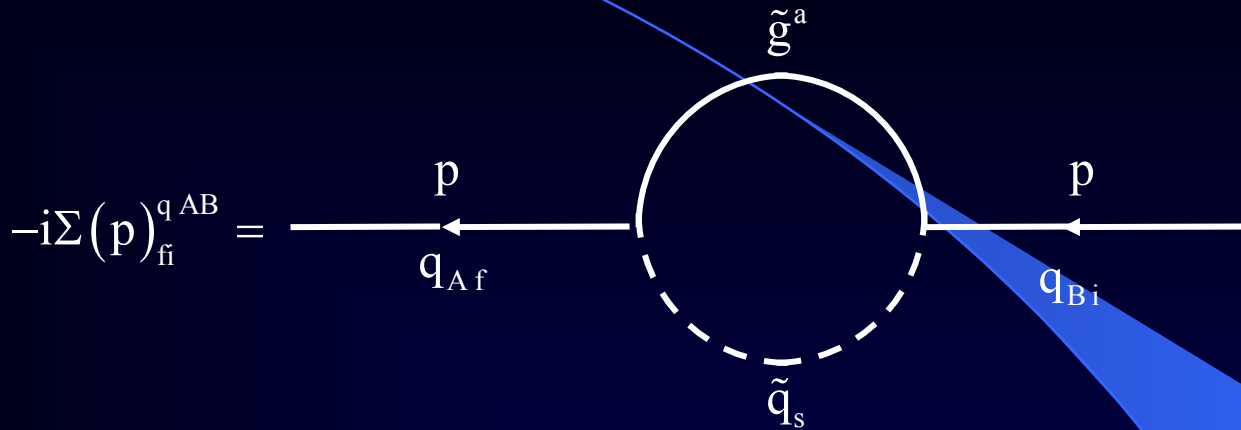


$$-i\Delta_{fi}^{\tilde{q}AB}$$

- Define dimensionless quantity:

$$\delta_{fi}^{\tilde{q}AB} \equiv \frac{\Delta_{fi}^{\tilde{q}AB}}{\sum_{s=1}^6 \left(\frac{M_{\tilde{q}}^2}{6} \right)_{ss}}$$

Flavour-changing self energy:



Mass insertion approximation

$$\Sigma(0)_{fi}^q = g_s^2 \frac{m_{\tilde{g}}}{6\pi^2} (\Delta_{fi}^{\tilde{q}LR} P_R + \Delta_{fi}^{\tilde{q}RL} P_L) C_0(m_{\tilde{g}}^2, M_{fA}^{\tilde{q}}, M_{iB}^{\tilde{q}})$$

Exact diagonalization

$$\Sigma(0)_{fi}^q = g_s^2 \frac{m_{\tilde{g}}}{6\pi^2} \sum_{s=1}^6 \left(\left(\mathbf{V}_{LR}^s \right)_{fi} P_R + \left(\mathbf{V}_{LR}^s \right)_{fi} P_L \right) B_0(m_{\tilde{g}}^2, m_{q_s}^2) \text{ with}$$

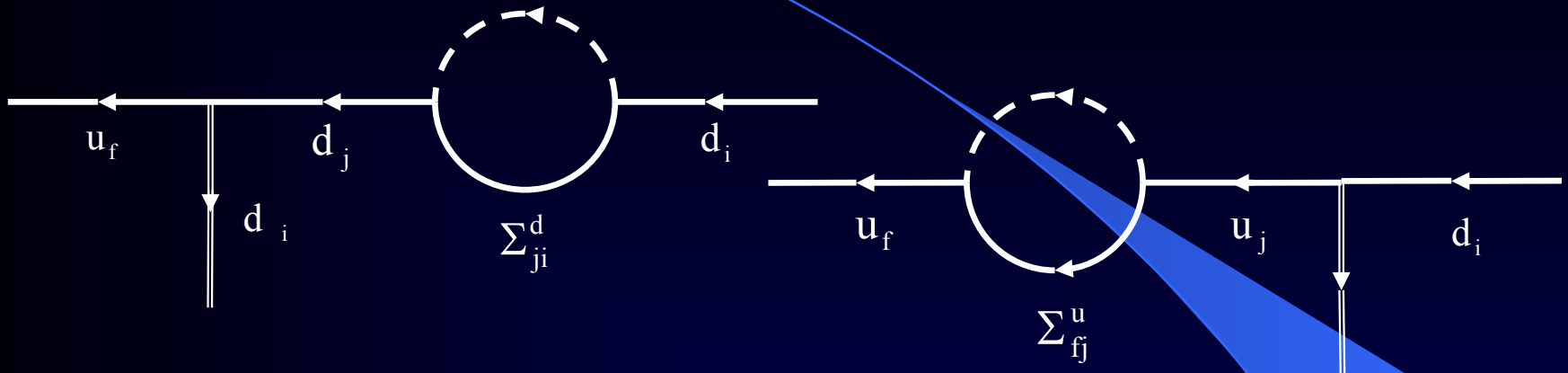
dimensionless

$$\left(\mathbf{V}_{LR}^s \right)_{fi} \equiv \sum_{j,k=1}^6 U_{jf}^{qL*} W_{js}^{\tilde{q}} U_{ki}^{qR} W_{k+3,s}^{\tilde{q}*}, \quad \left(\mathbf{V}_{RL}^s \right)_{fi} \equiv \sum_{j,k=1}^6 U_{jf}^{qR*} W_{j+3,s}^{\tilde{q}} U_{ki}^{qL} W_{ks}^{\tilde{q}*}$$



$\Sigma(0) \sim M_{\text{SUSY}}$
for constant δ

Renormalization of the CKM matrix



$$V_{\text{CKM}}^{(0)} = U_L^{u(0)\dagger} U_L^{d(0)} \rightarrow V_{\text{CKM}} = \left(1 + \Delta U_L^{u\dagger}\right) V_{\text{CKM}}^{(0)} \left(1 + \Delta U_L^d\right)$$

$$\Delta U_L^q = \begin{pmatrix} 0 & \frac{1}{m_2} \Sigma_{12}^{q\text{LR}} & \frac{1}{m_3} \Sigma_{13}^{q\text{LR}} \\ \frac{-1}{m_2} \Sigma_{21}^{q\text{RL}} & 0 & \frac{1}{m_3} \Sigma_{23}^{q\text{LR}} \\ \frac{-1}{m_3} \Sigma_{31}^{q\text{RL}} & \frac{-1}{m_3} \Sigma_{32}^{q\text{RL}} & 0 \end{pmatrix} \rightarrow \text{Antihermitian correction}$$

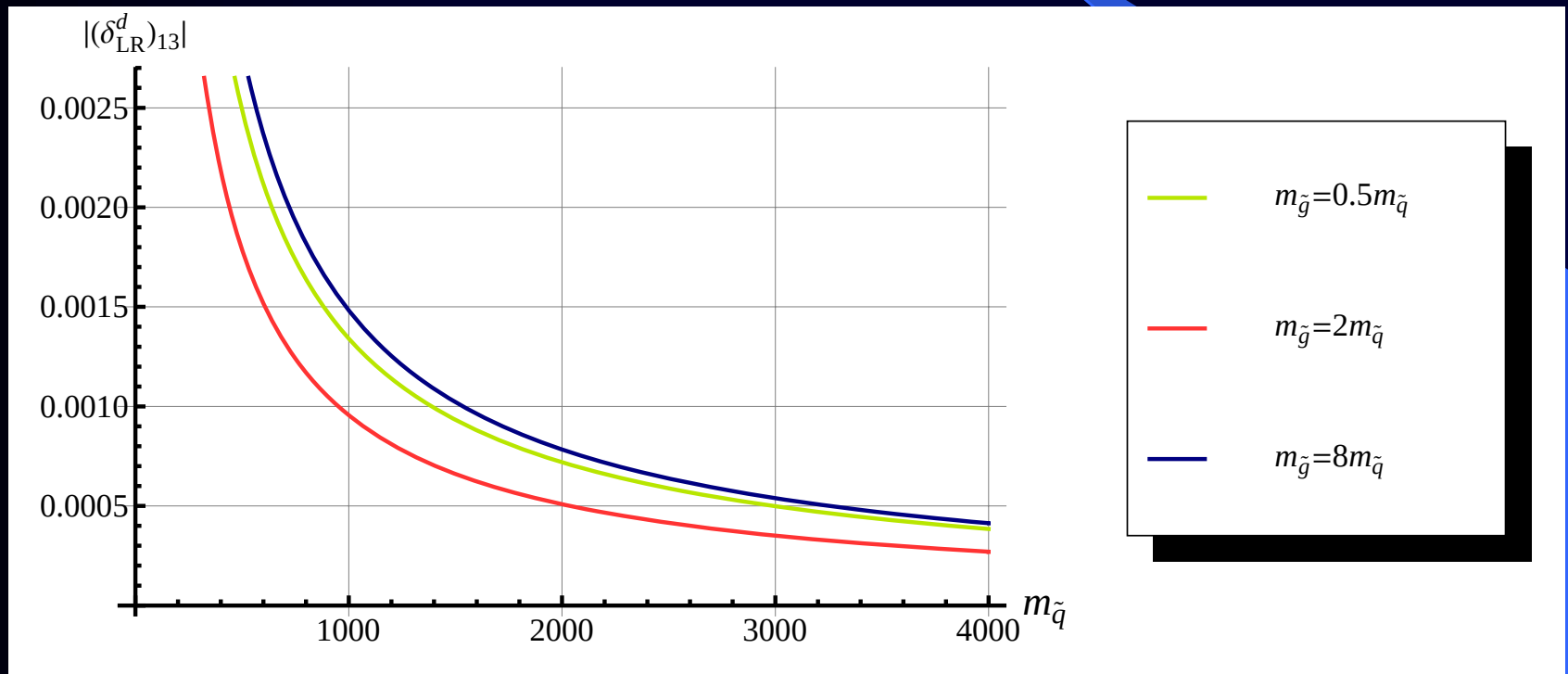
Constraints from fine tuning

Large accidental cancellations are unlikely
and from the theoretical point of view
undesirable.

→ Corrections to the CKM matrix should not
exceed the experimentally measured values.

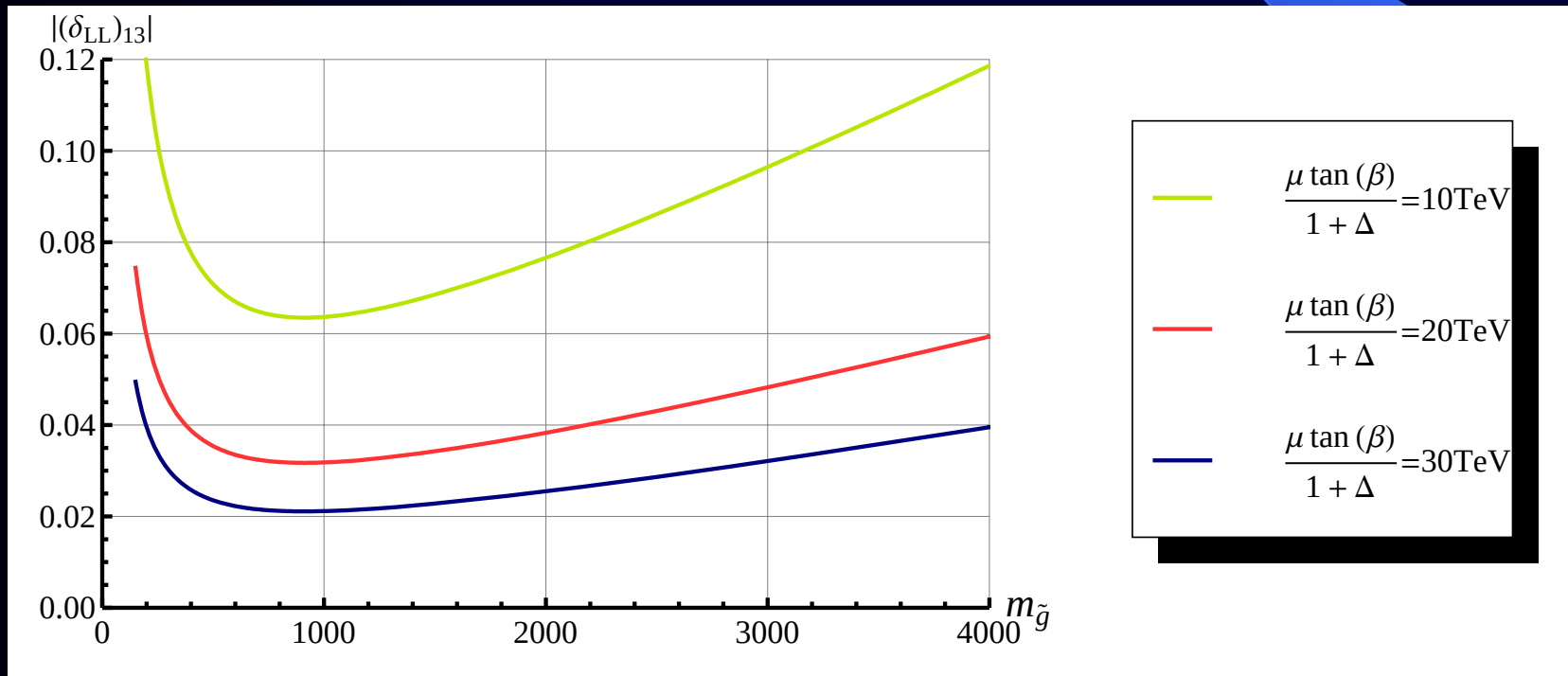
Because of the V-A structure of the W
vertex only δ_{ij}^{qLR} is directly constrained
for $j > i$.

Constraints on $\delta_{13}^{\text{d LR}}$ from V_{ub}



Constraints on δ_{13}^{dLL} from V_{ub}

For large chirality flipping elements, for example $m_b \mu \tan(\beta)$, also δ_{ij}^{qLL} can be constrained. Strongest for δ_{13}^{qLL} :



Existing bounds on δ :

FCNC bounds (decoupling):

- M. Ciuchini et al. [arXiv:hep-ph/9808328]
- F. Borzumati, C. Greub, T. Hurth and D. Wyler [arXiv:hep-ph/9911245]
- D. Becirevic et al. [arXiv:hep-ph/0112303]
- M. Ciuchini et al. [arXiv:hep-ph/0703204]
- ...

Vacuum stability bounds (non-decoupling):

J.A. Casas and S. Dimopoulos,
Stability bounds on flavor-violating trilinear soft terms in the MSSM,
Phys. Lett. B **387** (1996) 107 [arXiv:hep-ph/9606237]

Results and comparison

quantity	our bound	bound from FCNC	bound from vacuum stability
$\delta_{12}^{\text{d LR}}$	0.0011	0.006, K mixing	0.00015
$\delta_{13}^{\text{d LR}}$	0.001	0.15, B_d mixing	0.005
$\delta_{23}^{\text{d LR}}$	0.01	0.06, $b \rightarrow s\gamma$	0.05
$\delta_{13}^{\text{d LL}}$	0.032	0.5, B_d mixing	--
$\delta_{12}^{\text{u LR}}$	0.0047	0.016, D mixing	0.0012
$\delta_{13}^{\text{u LR}}$	0.027	--	0.22
$\delta_{23}^{\text{u LR}}$	0.27	--	0.22

Bounds calculated with $m_{\text{squark}} = m_{\text{gluino}} = 1000 \text{ GeV}$

SUYS as the origin of flavour?

At tree-level:

$$\mathbf{m}_q^{(0)} = v_q \mathbf{Y}^q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{V}_{CKM}^{(0)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

First two generation masses and all CKM elements are generated radiatively with A terms.

S. Weinberg, Phys. Rev. Lett. **29** (1972) 388. S. Weinberg, Phys. Rev. D **5** (1972) 1962.
[28] W. Buchmüller and D. Wyler, Phys. Lett. B **121** (1983) 321.
J. Ferrandis and N. Haba, Phys. Rev. D **70**, 055003 (2004) [arXiv:hep-ph/0404077].

Features of this model

- Higher symmetry in the Yukawa Sektor $SU(2)^3 \times U(1)$
- Natural phase alignment solves EDM (SUSY CP) problem

F. Borzumati, G. R. Farrar, N. Polonsky and S. D. Thomas, Nucl. Phys. B **555** (1999) 53 [arXiv:hep-ph/9902443]

- MFV in respect to the first two generations.
- Consistent with FCNC-constraints for SUSY masses above ~ 0.5 TeV

Conclusions

- Flavour-changing SQCD self-energies lead to antihermitian corrections to the CKM matrix
- Strong constraints on δ_{ij}^{qLR} and δ_{13}^{dLL} are obtained if the absence of accidental cancellations is imposed.
- The constraints from the CKM renormalization are still stronger than the FCNC bounds for $M_{\text{SUSY}} \approx 0.5 \text{ TeV}$
- **A model of radiative generation of light quark masses and CKM elements can solve the SUSY flavour and SUSY CP problem.**