Precision predictions for two-body hadronic B decays

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Outline

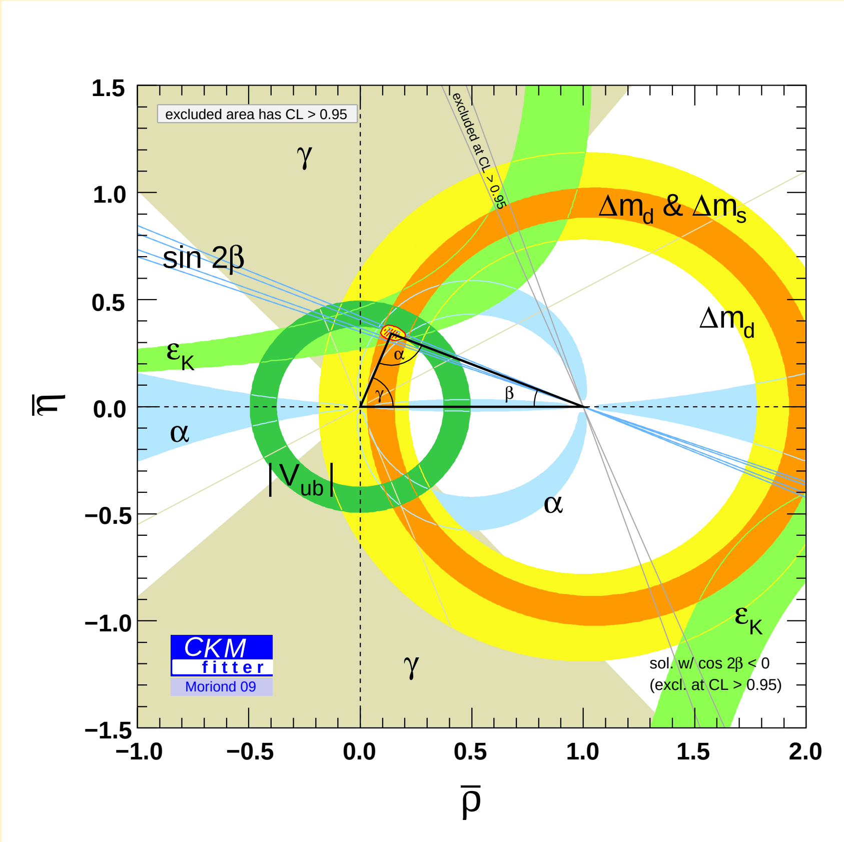
- Introduction and motivation
- Theoretical framework
- Technical details of two-loop calculation
- Results and outlook

Introduction and motivation

- CKM mechanism of quark flavour transitions well established

  [Cabibbo'63; Kobayashi,Maskawa'73; Nobel Prize 2008]

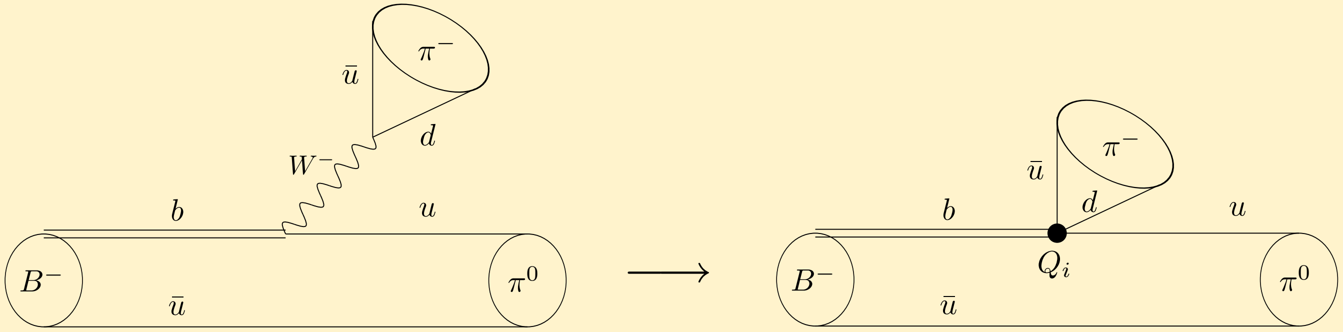
- Quark flavour sector still an active field of research, era of precision physics
  - Quantify its amount of CP violation ($\Rightarrow$ BAU)
  - Indirect search for new physics (NP). Smoking guns: β_s, $\Delta A_{\mathrm{CP}}(\pi K)$
  - Many observables: branching ratios, CP asymmetries, polarisations, ...



- Hadronic B decays important for extracting UT quantities, e.g.
  - α: $B \to \pi\pi,\, \rho\rho,\, \rho\pi$
  - β: $B \to J/\psi\, K_S,\, \phi\, K_S$
  - γ: $B \to DK$

- Need precision in theory predictions and exptl. measurements to disentangle NP from SM background

Effective theory for B decays



- M_W, M_Z, $m_t \gg m_b$: integrate out heavy gauge bosons and t-quark

- Effective Hamiltonian: *[Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]*

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{6} C_k Q_k + C_8 Q_8 \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \sum_q (\bar{q} \gamma_\mu T^a q) \qquad Q_8 = -\frac{g_s}{16\pi^2}\, m_b\, \bar{d}_L\, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

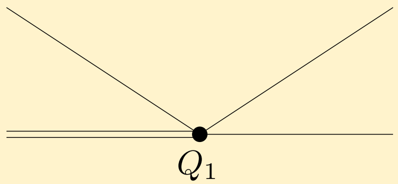
$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

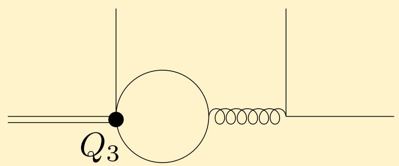
$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \qquad \lambda_p = V_{pb} V_{pd}^*$$

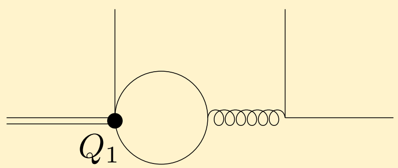
Effective theory for B decays

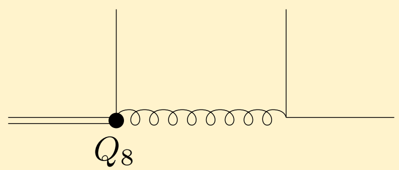
- Convenient resummation of large logarithms $L \equiv \ln(\frac{\mu_W}{\mu_b})$ via RG techniques

  LO: $\mathcal{O}(\alpha_s^n L^n)$ NLO: $\mathcal{O}(\alpha_s^n L^{n-1})$ NNLO: $\mathcal{O}(\alpha_s^n L^{n-2})$
- To be supplemented by evanescent operators
- Can use naïvely anticommuting γ_5 in CMM basis

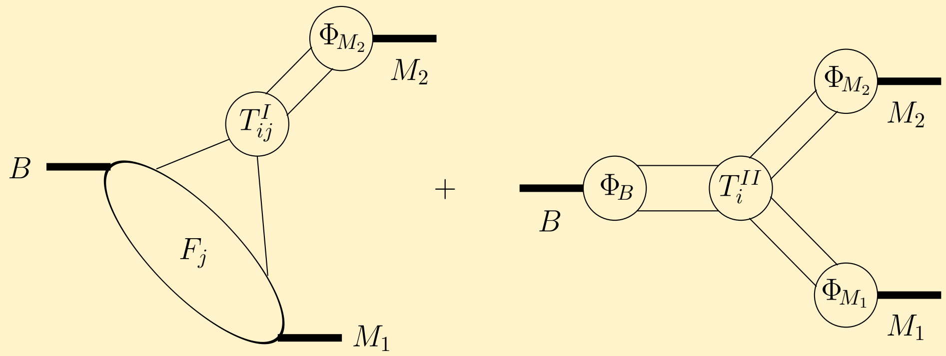








QCD factorisation

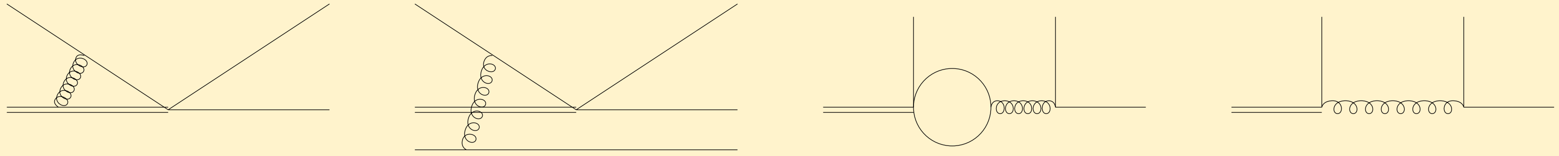


- Theoretical description of non-leptonic B decays difficult due to complicated QCD effects in the purely hadronic final state
- Simplification in the limit $m_b \gg \Lambda_{\rm QCD}$ *[Beneke,Buchalla,Neubert,Sachrajda'99-'04]*

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0) \, f_{M_2} \int_0^1 du \; T_i^I(u) \, \phi_{M_2}(u)$$

$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^1 d\omega dv du \; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

- $T^{I,II}$: Hard scattering kernels, perturbatively calculable. $T^{II} = \mathcal{O}(\alpha_s)$
- $F_+ : B \to M$ form factor. f_i : decay constants. ϕ_i : light-cone distr. amplitudes



QCD factorisation

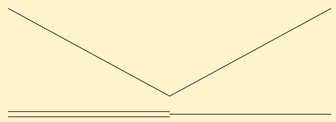
- Alternative representation of matrix elements *[Beneke,Neubert'03]*

$$\sqrt{2}\,\langle\pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle = \lambda_u\big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\big]\;A_{\pi\pi}$$

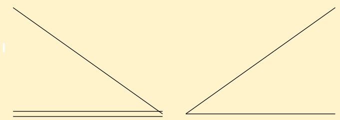
$$\langle\pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = \big\{\lambda_u\big[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\big]+\lambda_c\,\alpha_4^c(\pi\pi)\big\}\;A_{\pi\pi}$$

$$-\,\langle\pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle = \big\{\lambda_u\big[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\big]-\lambda_c\,\alpha_4^c(\pi\pi)\big\}\;A_{\pi\pi}$$

- α_1 : colour-allowed tree amplitude, “right insertion”



- α_2 : colour-suppressed tree amplitude, “wrong insertion”



- $\alpha_4^{u/c}$: Penguin amplitudes

$$\alpha_1(\pi\pi) = 1.015 + [0.025+0.012i]_V + [??+0.027i]_{VV} - \left[\tfrac{r_{\rm sp}}{0.485}\right]\big\{[0.020]_{\rm LO}+[0.034+0.029i]_{HV}+[0.012]_{\rm tw3}\big\}$$

$$= 0.975^{+0.034}_{-0.072} + (0.010^{+0.025}_{-0.051})i$$

$$\alpha_2(\pi\pi) = 0.184 - [0.153+0.077i]_V + [??-0.049i]_{VV} + \left[\tfrac{r_{\rm sp}}{0.485}\right]\big\{[0.122]_{\rm LO}+[0.050+0.053i]_{HV}+[0.071]_{\rm tw3}\big\}$$

$$= 0.275^{+0.228}_{-0.135} + (-0.073^{+0.115}_{-0.082})i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01]

[Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

- Goal: $\mathcal{O}(\alpha_s^2)$ vertex corrections to α_1 and α_2 $\Leftrightarrow$ 2-loop matrix elements of Q_1, Q_2

SCET operator basis

Right insertion

$$O_1 = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)h_v\right]$$

$$O_2 = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)\gamma^\perp_\beta\gamma^\perp_\alpha h_v\right]$$

$$O_3 = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\chi\right]\left[\bar{\xi}\,\not{n}_+(1-\gamma_5)\gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta\gamma^\perp_\alpha h_v\right]$$

In addition, for a massive final state

$$O_1' = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\chi\right]\left[\bar{\xi}\,\not{n}_+(1+\gamma_5)h_v\right]$$

$$O_2' = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\chi\right]\left[\bar{\xi}\,\not{n}_+(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\beta h_v\right]$$

$$O_3' = \left[\bar{\chi}\,\frac{\not{n}_-}{2}(1-\gamma_5)\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\chi\right]\left[\bar{\xi}\,\not{n}_+(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\beta\gamma^\perp_\gamma\gamma^\perp_\delta h_v\right]$$

Wrong insertion (only massless final state)

$$\tilde{O}_1 = \left[\bar{\xi}\,\gamma_\perp^\alpha(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha h_v\right]$$

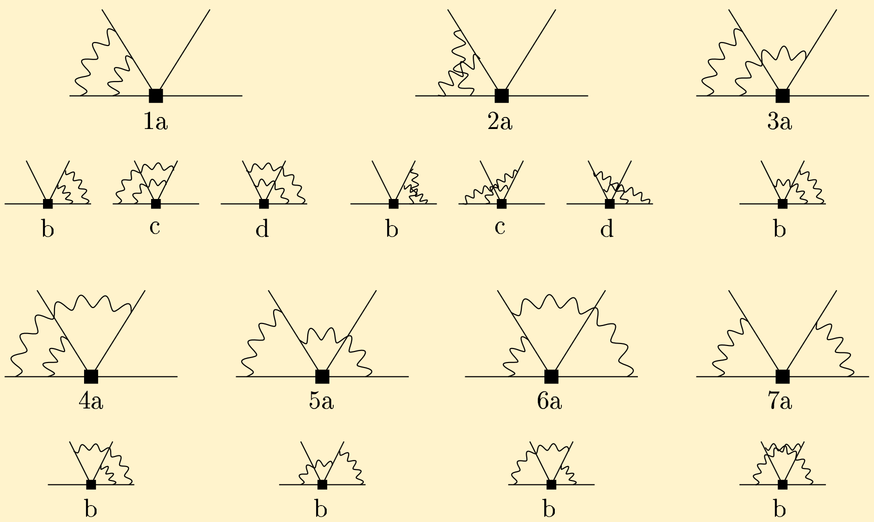
$$\tilde{O}_2 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right]$$

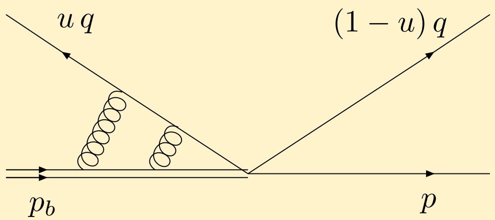
$$\tilde{O}_3 = \left[\bar{\xi}\,\gamma_\perp^\alpha\gamma_\perp^\beta\gamma_\perp^\gamma\gamma_\perp^\delta\gamma_\perp^\epsilon(1-\gamma_5)\chi\right]\left[\bar{\chi}(1+\gamma_5)\gamma^\perp_\alpha\gamma^\perp_\epsilon\gamma^\perp_\delta\gamma^\perp_\gamma\gamma^\perp_\beta h_v\right]$$

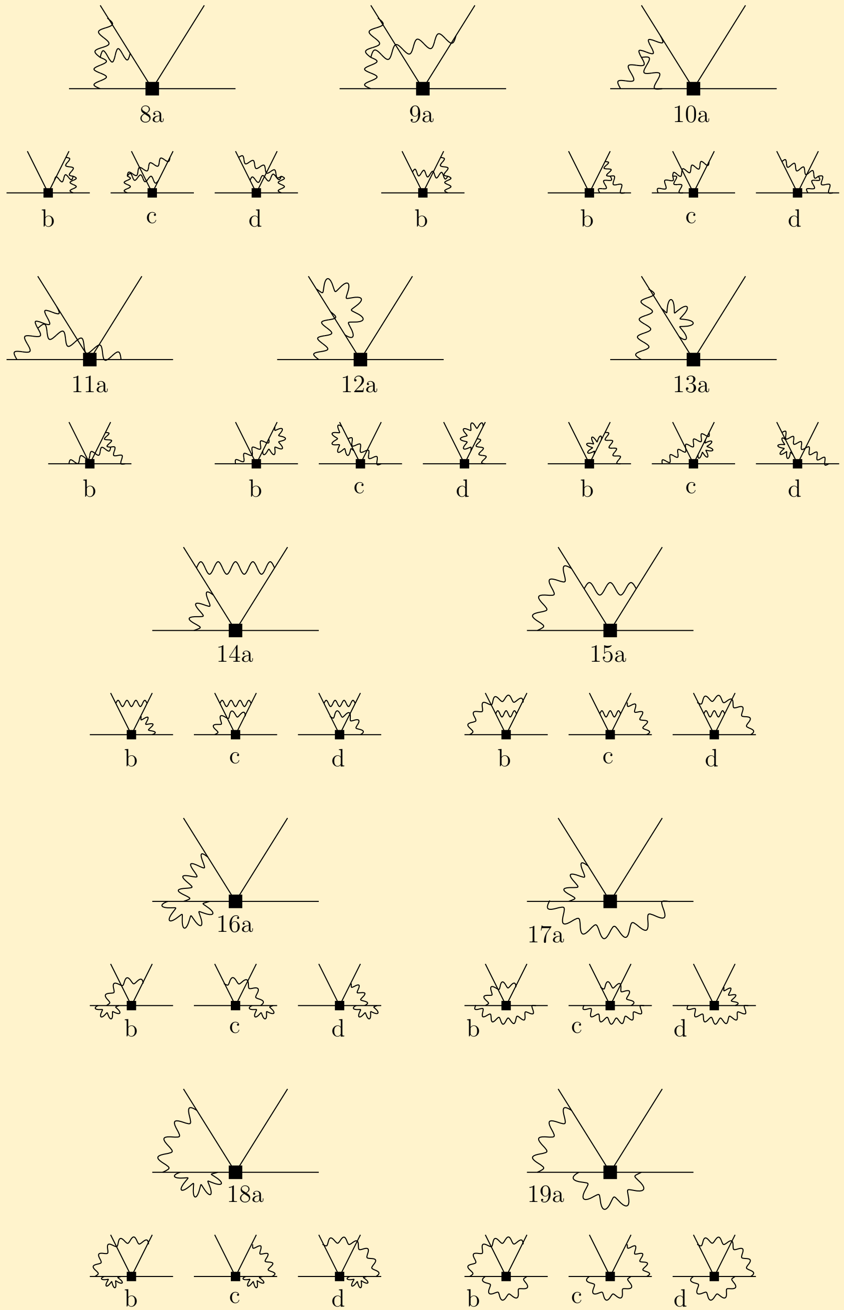
All operators with indices 2 and 3 are evanescent. Moreover: Fierz$(\tilde{O}_1) = O_1$ in $D = 4$.

Two-loop diagrams

- Non-factorizable two-loop diagrams for non-leptonic B-decays *[Beneke,Buchalla,Neubert,Sachrajda'00]*







- Kinematics: $p_b^2 = m_b^2$, $q^2 = 0$, $p^2 = 0$ or $p^2 = m_c^2$

Reduction methods

- Work in dimensional regularisation with $D = 4 - 2\epsilon$, to regulate UV and IR divergences. Poles up to $1/\epsilon^4$.

- Elimination of tensor structure via a Passarino-Veltman ansatz *[Passarino,Veltman'79]*

- Yields scalar integrals with irreducible scalar products in the numerator

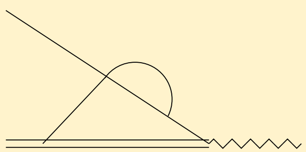
- Integration-by-parts (IBP) identities, 8 per diagram *[Tkachov'81; Chetyrkin,Tkachov'81]*

$$\int \frac{d^D k}{(2\pi)^D} \int \frac{d^D l}{(2\pi)^D} \frac{\partial}{\partial a^\mu} \left[b^\mu \, f(k,l,p_i) \right] = 0 \; ; \qquad a^\mu = k^\mu, \, l^\mu \; ; \qquad b^\mu = k^\mu, \, l^\mu, \, p_i^\mu$$

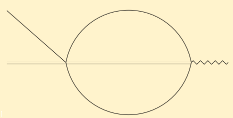
- Lorentz-Invarianz (LI) identities, 1 per diagram *[Gehrmann,Remiddi'99]*

- Solve system of equations by means of Laporta algorithm *[Laporta'01; Anastasiou,Lazopoulos'04; Smirnov'08]*

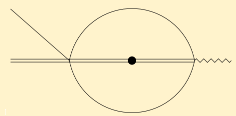
- Obtain scalar integrals as a linear combination of master integrals



$$= \frac{(8-3D)(7uD - 8D - 24u + 28)}{3(D-4)^2 \, m_b^4 \, u^3}$$

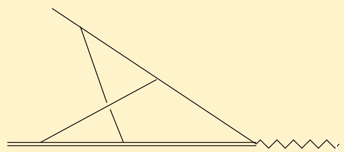


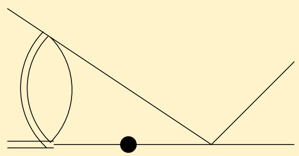
$$- \frac{2[u^2(D-4) + (16D-56)(1-u)]}{3(D-4)^2 \, m_b^2 \, u^3}$$



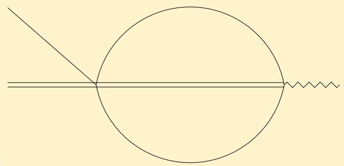
Master Integrals

- Reduction yields 42 master integrals for $m_c = 0$. For finite m_c, this roughly doubles.
- Poles up to $1/\epsilon^4$. Analytic calculation of coefficient functions for $m_c = 0$.
  Harmonic polylogarithms up to weight 4 of argument u or $1-u$. *[Remiddi,Vermaseren'99]*
- Several calculations in agreement *[Bell'07; Bonciani,Ferroglia'08; Asatrian,Greub,Pecjak'08; Beneke,Li,TH'08; TH'09]*
- Sample integrals





- Applied techniques
  - Hypergeometric functions



$$= \frac{(m_b^2)^{1-2\epsilon}}{(4\pi)^{4-2\epsilon}} \frac{\Gamma^2(1-\epsilon)\Gamma(\epsilon)\Gamma(2\,\epsilon-1)}{\Gamma(2-\epsilon)} \, {}_2F_1(\epsilon, 2\,\epsilon-1\,;\,2-\epsilon\,;\,1-u)$$

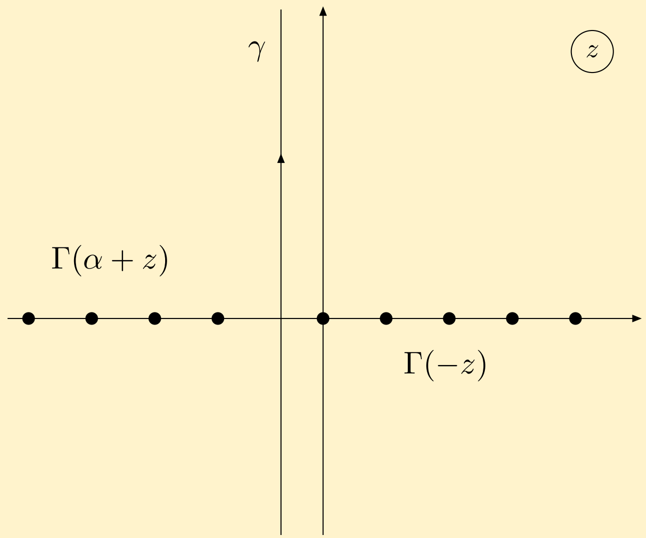
  * ϵ-expansion: XSummer (Form), `HypExp` (Mathematica) *[Moch,Uwer'05; Maitre,TH'05,'07]*

Master Integrals (cont'd.)

- Applied techniques (cont'd.)

  – Mellin-Barnes representation *[Smirnov'99; Tausk'99]*

$$\frac{1}{(A_1+A_2)^{\alpha}} = \int_{\gamma} \frac{dz}{2\pi i}\; A_1^{z}\, A_2^{-\alpha-z}\, \frac{\Gamma(-z)\,\Gamma(\alpha+z)}{\Gamma(\alpha)}$$



  ∗ partially automated

  ∗ Numerical cross checks possible

  [Czakon'05; Gluza,Kajda,Riemann'07]

  – Differential equations *[Kotikov'91; Remiddi'97]*

$$\frac{\partial}{\partial u}\mathrm{MI}_i(u) = f(u,\epsilon)\,\mathrm{MI}_i(u) + \sum_{j\neq i} g_j(u,\epsilon)\,\mathrm{MI}_j(u)$$

  ∗ Requires result of Laporta reduction.

  ∗ Boundary condition in $u=0$ or $u=1$ from Mellin-Barnes representation

Master formula

- Master formula for the hard scattering kernel (right insertion)

$$
\begin{aligned}
T_i^{(1)} &= A_{i1}^{(1),nf} + Z_{ij}^{(1)}\, A_{j1}^{(0)} \\
T_i^{(2)} &= A_{i1}^{(2),nf} + Z_{ij}^{(1)}\, A_{j1}^{(1)} + Z_{ij}^{(2)}\, A_{j1}^{(0)} \\
&\quad + Z_\alpha^{(1)}\, A_{i1}^{(1),nf} + (-i)\, \delta_m^{(1)}\, A_{i1}^{\prime(1),nf} \\
&\quad + T_i^{(1)} \left[\xi_{45}^{(1)} - C_{FF}^{(1)} - Z_J^{(1)} - Z_{BL}^{(1)} + Z_{ext}^{(1)}\right] \\
&\quad - \sum_{b>1} H_{ib}^{(1)}\, Y_{b1}^{(1)}
\end{aligned}
$$

- Important check: $T_i^{(1)}$ and $T_i^{(2)}$ free of poles ✓
- Higher order ϵ terms in $T_i^{(1)}$ required for $T_i^{(2)}$
- $A_{j1}^{(0)}$ and $A_{j1}^{(1)}$ include ME of evanescent operators
- Terms like $T_i^{(1)}\, Z_{BL}^{(1)}$ contain a convolution
- $Y_{b1}^{(1)}$: renormalization constants of the SCET operator basis

Results

- Topological tree amplitude to NNLO (right insertion)

$$
\begin{aligned}
\alpha_1(M_1 M_2) &= C_2 + \frac{\alpha_s}{4\pi}\frac{C_F}{2N_c}\left\{C_1\, V^{(1)} + \frac{\alpha_s}{4\pi}\left[C_1\, V_1^{(2)} + C_2\, V_2^{(2)}\right] + \mathcal{O}(\alpha_s^2)\right\} + \ldots \\
\frac{C_F}{2N_c} V_i^{(j)} &= \int_0^1 du\, T_i^{(j)}(u)\, \phi_{M_2}(u) \\
\phi_{M_2}(u) &= 6u(1-u)\left[1 + \sum_{n=1}^{\infty} a_n^{M_2}\, C_n^{(3/2)}(2u-1)\right]
\end{aligned}
$$

- We obtain at $\mu = m_b$ (all numbers preliminary!)

$$
\begin{aligned}
V^{(1)} &= (-22.500 - 9.425\, i) + (5.500 - 9.425\, i)\, a_1^{M_2} + (-1.050)\, a_2^{M_2} \\
V_1^{(2)} &= (-178.39 - 349.44\, i) + (641.65 - 119.36 i)\, a_1^{M_2} + (-85.39 - 62.63\, i)\, a_2^{M_2} \\
V_2^{(2)} &= (322.19 + 320.94\, i) + (-212.97 + 154.41\, i)\, a_1^{M_2} + (3.8146 - 34.0626\, i)\, a_2^{M_2} \\
\alpha_1(\pi\pi) &= [1.008]_{|V^{(0)}} + [0.022 + 0.009 i]_{|V^{(1)}} + [0.026 + 0.028 i]_{|V^{(2)}} + \ldots
\end{aligned}
$$

Outlook

To do

- Two-loop color suppressed amplitude (wrong insertion) of vertex correction
- Comparison with results of G. Bell *[Bell'09]*
- Massive final state ($B \to D\pi$)
- Penguin amplitudes

$$\alpha_4^u(\pi\pi) = -0.029 - [0.002 + 0.001i]_V + [0.003 - 0.013i]_P + [?? + ??\, i]_{\mathcal{O}(\alpha_s^2)} + \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO} + [0.001 + 0.000i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.024^{+0.004}_{-0.002} + (-0.012^{+0.003}_{-0.002})i$$

$$\alpha_4^c(\pi\pi) = -0.029 - [0.002 + 0.001i]_V - [0.001 + 0.007i]_P + [?? + ??\, i]_{\mathcal{O}(\alpha_s^2)} + \left[\frac{r_{\rm sp}}{0.485}\right]\left\{[0.001]_{\rm LO} + [0.001 + 0.001i]_{HV+HP} + [0.001]_{\rm tw3}\right\} = -0.028^{+0.005}_{-0.003} + (-0.006^{+0.003}_{-0.002})i$$

[Beneke,Buchalla,Neubert,Sachrajda'99,'01; Beneke,Neubert'03; Beneke,Jäger'05,'06; Kivel'06; Pilipp'07; Bell'07]

[Hill,Becher,Lee,Neubert'04; Becher,Hill'04; Kirilin'05; Beneke,Yang'05]

- Phenomenological analysis

Backup slides

Definitions and exptl. numbers

$$A_{\pi\pi} = i\,\frac{G_F}{\sqrt{2}}\,m_B^2 F_+^{B\to\pi}(0) f_\pi$$

$$r_{\rm sp} = \frac{9 f_\pi \hat{f}_B}{m_b \lambda_B F_+^{B\to\pi}(0)}$$

$$\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega}\,\phi_B(\omega,\mu)$$

$$\mathcal{B}(B^- \to \pi^-\pi^0) = (5.7 \pm 0.5) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^+\pi^-) = (5.13 \pm 0.24) \times 10^{-6}$$

$$\mathcal{B}(\bar{B}^0 \to \pi^0\pi^0) = (1.62 \pm 0.31) \times 10^{-6}$$

[PDG'08]