Hadronic B decays in the MSSM with large $\tan\beta$

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in collaboration with

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based on our recent work: arXiv:0901.4841.

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Outline

Current status of B physics

- B-meson weak decays play a very important role in:
  - testing the Standard Model;
  - probing the origin of CP violation;
  - searching for indirect signals of New Physics.
  - ...
- Exp.: the precision of measurements has been and will be improved significantly due to:
  - BaBar at SLAC, and Belle at KEK;
  - Tevatron at Fermilab, and LHCb at CERN;
  - the proposed Super-B factory.
  - ...
- Theo.: various theoretical frameworks have been proposed:
  - based on flavour symmetries of QCD;
  - based on factorization of the evolved QCD dynamics;
  - the QCD factorization (or BBNS) framework;

MSSM with large tan β

- MFV hypothesis: no significant deviations from the SM; the pattern of flavour-changing interactions are governed by the SM Yukawa couplings.

G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, hep-ph/0207036;
A. J. Buras, hep-ph/0310208
...

- the MFV MSSM with large tan β scenario:
  - consistent with approximate unification of top and bottom Yukawa couplings at high energies predicted by some SO(10) models;
  - the FCNC processes mediated by Higgs scalars in the down-quark sector can be significantly enhanced at large tan β;
  - still exhibit sizeable deviations from the SM with specific signatures: increase $\mathrm{Br}(B^0_{d,s} \to \mu^+\mu^-)$ by few orders; decrease $\mathrm{Br}(B_u \to \tau\nu)$ by 20-30%;

A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, hep-ph/0210145;
G. Isidori and P. Paradisi, hep-ph/0605012;
...

Their effects on non-leptonic B decays

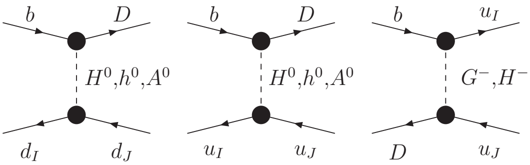
- the Higgs exchanges can also generate scalar four-quark operators, which contribute to non-leptonic B decays:

C. S. Huang, P. Ko, X. H. Wu and Y. D. Yang, hep-ph/0511129;
H. Hatanaka and K. C. Yang, arXiv:0711.3086;
J. F. Cheng, C. S. Huang and X. H. Wu, hep-ph/0404055;
....

- in connection with transverse polarization in $B \to VV$ decays, and for specific decay modes; large deviations from SM expectations are found;
- Question: *given the present strong constraints from leptonic* $B_s \to \mu^+\mu^-$ *and* $B_u \to \tau\nu$ *decays, is it possible to gain further insight on this specific scenario from hadronic B decays?*

Working assumptions:

- to simplify the following discussion, we assume:
  - the large $\tan\beta$ scenario: $\tan\beta \simeq 50$, $\sin\beta \approx 1$, $1/\cos\beta \approx \tan\beta$;
  - the "decoupling limit": the super-partner particles are somewhat heavier than the EW gauge bosons and the Higgs bosons;
- Thus, the leading effect is due to Higgs exchanges for both neutral and charged interactions:



- the effective couplings originate from loop-induced Higgs couplings and are enhanced by several powers of $\tan\beta$ (see Buras *et al.*), hep-ph/0210145.

The NP effective Hamiltonian

- The Higgs-induced new effective Hamiltonian can be written as

$$\mathcal{H}_{\text{eff}}^{\text{Higgs}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \left(C_{11}^D Q_{11}^p + C_{12}^D Q_{12}^p + \sum_{i=13}^{14} \sum_{q=d,s,b} C_i^q Q_i^q \right) + \text{h.c.},$$

- the "current-current" operators from charged Higgs exchange:

$$Q_{11}^p = (\bar{p}_i b_i)_{S+P} \, (\bar{D}_j p_j)_{S-P}, \qquad Q_{12}^p = (\bar{p}_i b_j)_{S+P} \, (\bar{D}_j p_i)_{S-P}, \qquad (p = u, c)$$

- the "penguin" operators from neutral Higgs-fermion vertices:

$$Q_{13}^q = (\bar{D}_i b_i)_{S+P} \, (\bar{q}_j q_j)_{S-P}, \qquad Q_{14}^q = (\bar{D}_i b_j)_{S+P} \, (\bar{q}_j q_i)_{S-P}, \qquad (q = d, s, b)$$

- contrary to the SM case, all of these new operators have the $(S+P) \times (S-P)$ Dirac structure, rather than $(V-A) \times (V \pm A)$ form.

The short-distance WCs at the initial scale

- we use the effective Higgs couplings in the decoupling and large $\tan\beta$ limit given in hep-ph/0110121, hep-ph/0210145.
- combining a flavour-changing and a flavour-conserving coupling, we get:

$$C_{13}^{d_J}(\mu_H) = \frac{1}{2}\frac{\bar{m}_{d_J}\bar{m}_b\epsilon_Y y_t^2 \tan^3\beta}{(1+\tilde{\epsilon}_3\tan\beta)(1+\epsilon_0\tan\beta)(1+\tilde{\epsilon}_J\tan\beta)}\,\mathcal{F}_{2,J}^{-}, \qquad C_{14}^{d_J}(\mu_H) = 0.$$

$$C_{11}^{D}(\mu_H) = -\frac{\bar{m}_b\bar{m}_D}{M_{H^+}^2}\frac{\tan^2\beta}{(1+\epsilon_0\tan\beta)^2}, \qquad C_{12}^{D}(\mu_H) = 0.$$

- the ϵ-coefficients denote the loop-induced Higgs-fermion couplings;
- in the MFV MSSM with large $\tan\beta$, only these operators $Q_{11,12}^{p}$, $Q_{13,14}^{s,b}$ are relevant, while all the others are negligible due to various suppression factors.

Renormalization group evolution

- we perform the evolution from $\mu_H = 200\,\text{GeV}$ down to $m_b = 4.2\,\text{GeV}$;
- including the penguin diagrams, the scalar operators mix into the SM penguin operators and their "mirror" copies: six SM operators $Q^p_{1,2}$, Q_{3-6}, their mirror copies $Q'^p_{1,2}$, Q'_{3-6}, and six scalar operators $Q^p_{11,12}$, $Q^{D,b}_{13,14}$,
  - for the scalar operators:

$$C^D_{11}(m_b)/C^D_{11}(\mu_H) = C^q_{13}(m_b)/C^q_{13}(\mu_H) \approx 2.35\ (2.20)\,,$$

$$C^D_{12}(m_b)/C^D_{11}(\mu_H) = C^q_{14}(m_b)/C^q_{13}(\mu_H) \approx 0.088\ (0)\,.$$

  - for the mirror QCD penguin operators:

$$C'^D_i(m_b) \approx -0.71\,C^{\text{SM}}_i(m_b) \times [C^D_{11}(\mu_H) + C^D_{13}(\mu_H)], \quad i = 3\ldots 6,$$

  - the SM QCD penguin operators are modified by ($C_i = C^{\text{SM}}_i + \delta C_i$):

$$\delta C_i(m_b) \approx -0.71\,C^{\text{SM}}_i(m_b) \times C^b_{13}(\mu_H), \quad i = 3\ldots 6.$$

Naive estimate of the scalar operator coefficients

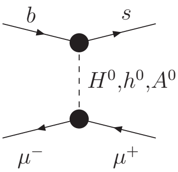
- assuming ϵ_0, ϵ_Y, $\tilde{\epsilon}_J$ to be of order 0.01, and $M_{A^0} = 200\,\text{GeV}$ and $\tan\beta = 50$, then the scalar penguin operator coefficients $C_{13}^s \simeq 0.01$, $C_{13}^b \simeq 0.5$;

  A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, hep-ph/0210145

- while in the SM, we have $C_3 \simeq 0.014$, $C_4 \simeq -0.036$, $C_5 \simeq 0.009$, $C_6 \simeq -0.042$;
- Thus, they are comparable, and hence large effects from the scalar operators on hadronic B decays are possible;
- but how about the situation after taking into account the constraints from the other decay processes, such as $B_s \to \mu^+\mu^-$ and $B^+ \to \tau^+\nu_\tau$?

Constraints from $B_s \to \mu^+\mu^-$

- this decay proceeds via an interaction similar to the $b \to s\bar{q}q$ transitions, with the $\bar{q}q$ pair replaced by a muon pair:



- using the present upper limit, $\text{Br}(B_s \to \mu^+\mu^-) \leq 5.8 \cdot 10^{-8}$@95% C.L., $f_{B_s} = 240\,\text{MeV}$, $\bar{m}_s(2\,\text{GeV}) = 90\,\text{MeV}$ and $\bar{m}_b(\bar{m}_b) = 4.2\,\text{GeV}$, we obtain:

$$|C_{13}^s(m_b)| \leq 0.001\ (0.01), \qquad |C_{13}^b(m_b)| \leq 0.05\ (0.5),$$

- C_{13}^q are constrained to be a factor of 10 smaller than the naive estimates.

Constraints from $B^+ \to \tau^+\nu_\tau$

- the $C_{11}^D(\mu_H)$ can be related to $B^+ \to \tau^+\nu_\tau$ in a similar way:

$$R_{\tau\nu_\tau} \equiv \frac{\mathrm{Br}(B^+ \to \tau^+\nu_\tau)_{\mathrm{MSSM}}}{\mathrm{Br}(B^+ \to \tau^+\nu_\tau)_{\mathrm{SM}}} = \left(1 + C_{11}^D(\mu_H)\,\frac{m_B^2(1+\epsilon_0\tan\beta)}{\bar{m}_D(\mu_H)\bar{m}_b(\mu_H)}\right)^2 .$$

- using $\mathrm{Br}(B^+ \to \tau^+\nu_\tau) = (1.51 \pm 0.33)\cdot 10^{-4}$, $|V_{ub}|f_{B_d} = 7.4\cdot 10^{-4}\,\mathrm{GeV}$, we get:

$$-0.08 < C_{11}^s(m_b) < -0.06, \quad \text{or} \quad -0.005 < C_{11}^s(m_b) < 0.018.$$

- this constraint on C_{11}^s is not as stringent as the one on C_{13}^s; However, we should keep in mind that the charged Higgs contribution must compete with the large SM tree operators.
- So, the largest values for these new coefficients are constrained to be:

$$C_{11}^s(m_b) = -0.08, \qquad C_{13}^s(m_b) = 0.001, \qquad C_{13}^b(m_b) = 0.05.$$

Hadronic matrix elements for $B \longrightarrow M_1 M_2$ decays

- to calculate the decay amplitude, we employ the QCD factorization framework:

M. Beneke, G. Buchalla, M. Neubert, C. T Sachrajda, 1999, 2000, 2001,
M. Beneke, M. Neubert, 2002, 2003,
M. Beneke, J. Rohrer, D. S. Yang, 2006, 2007.

- the hadronic matrix element of the effective Hamiltonian is written as:

$$\langle M_1' M_2' | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \sum_{p=u,c} \lambda_p^{(D)} \langle M_1' M_2' | \mathcal{T}_A^p + \mathcal{T}_B^p | \bar{B} \rangle,$$

- $\mathcal{T}_A^p$: LO, vertex, penguin and spectator-scattering terms;
- $\mathcal{T}_B^p$: the weak annihilation amplitudes.

Hadronic matrix elements (continued)

- now including the scalar and the mirror QCD penguin operators, we have:

$$\begin{aligned}
\mathcal{T}_A^p \quad = \quad & \delta_{pu}\left[\alpha_1(M_1M_2) + \alpha_{11}^D(M_1M_2)\right] A([\bar{q}_s u][\bar{u}D]) \\
& + \delta_{pu}\left[\alpha_2(M_1M_2) + \alpha_{12}^D(M_1M_2)\right] A([\bar{q}_s D][\bar{u}u]) \\
& + \left[\alpha_3^p(M_1M_2) + \alpha_3^{\prime pD}(M_1M_2)\right] \sum_{q=u,d,s} A([\bar{q}_s D][\bar{q}q]) \\
& + \left[\alpha_4^p(M_1M_2) + \alpha_4^{\prime pD}(M_1M_2)\right] \sum_{q=u,d,s} A([\bar{q}_s q][\bar{q}D]) \\
& + \sum_{q=d,s} \alpha_{3q}^p(M_1M_2)\, A([\bar{q}_s D][\bar{q}q]) + \sum_{q=d,s} \alpha_{4q}^p(M_1M_2)\, A([\bar{q}_s q][\bar{q}D]) \\
& + \ldots .
\end{aligned}$$

- $\alpha_{11,12}^D$: charged Higgs effects; $\alpha_{3,4}^{\prime pD}$: mirror QCD penguins; $\alpha_{3q,4q}^p$: neutral Higgs effects; modifications of the standard QCD penguin amplitudes.

Hadronic matrix elements (continued)

- since $V \pm A$ and $S \pm P$ operators contribute differently to pseudoscalar (P) and vector (V) final states, we have to distinguish between different final states.
- the new operator contributions to $\bar{B} \to VV$ decays obey a different hierarchy:

$$\bar{\mathcal{A}}_0 : \bar{\mathcal{A}}_- : \bar{\mathcal{A}}_+ = 1 : \frac{\Lambda_{\rm QCD}^2}{m_b^2} : \frac{\Lambda_{\rm QCD}}{m_b} \; \left(1 : \frac{\Lambda_{\rm QCD}}{m_b} : \frac{\Lambda_{\rm QCD}^2}{m_b^2}\right)$$

- for example, we have:

$$\alpha_4'^{\,p}(M_1M_2) = \begin{cases} -a_4'^{\,p}(M_1M_2) - r_\chi^{M_2}\, a_6'^{\,p}(M_1M_2), & \text{if } M_1M_2 = PP, \\ a_4'^{\,p}(M_1M_2) + r_\chi^{M_2}\, a_6'^{\,p}(M_1M_2), & \text{if } M_1M_2 = PV, \\ a_4'^{\,p}(M_1M_2) - r_\chi^{M_2}\, a_6'^{\,p}(M_1M_2), & \text{if } M_1M_2 = VP, \\ -a_4'^{\,p}(M_1M_2) + r_\chi^{M_2}\, a_6'^{\,p}(M_1M_2), & \text{if } M_1M_2 = V^0V^0, \\ f_\pm^{M_1}\left(-a_4'^{\,p}(M_1M_2) + r_\chi^{M_2}\, a_6'^{\,p}(M_1M_2)\right), & \text{if } M_1M_2 = V^\pm V^\pm, \end{cases}$$

- $f_\pm^{M_1} = F_\mp^{B\to M_1}(0)/F_\pm^{B\to M_1}(0)$: $f_+^{M_1} \sim m_B/\Lambda_{\rm QCD}$ and $f_-^{M_1} \sim \Lambda_{\rm QCD}/m_B$.

Hadronic matrix elements (continued)

- within the QCDF framework, the $a_{iq}^{(\prime)\,p}$ coefficients have such general form:

$$
\begin{aligned}
a_{iq}^{(\prime)\,p}(M_1M_2) &= \left(C_i^{(\prime)\,q} + \frac{C_{i\pm1}^{(\prime)\,q}}{N_c}\right) N_i^{(\prime)}(M_2) \\
&\quad + \frac{C_{i\pm1}^{(\prime)\,q}}{N_c}\,\frac{C_F\alpha_s}{4\pi}\left[V_i^{(\prime)}(M_2) + \frac{4\pi^2}{N_c}\,H_i^{(\prime)}(M_1M_2)\right] + P_i^{(\prime)\,p}(M_2),
\end{aligned}
$$

- $N_i^{(\prime)}(M_2)$: the tree-level result; $V_i^{(\prime)}(M_2)$: the 1-loop vertex correction; $H_i^{(\prime)}(M_1M_2)$: the spectator scattering; $P_i^{(\prime)\,p}(M_2)$: the penguin diagrams.
- for the scalar operators, setting C_{12}^{D}=C_{14}^{q}=0, we have explicitly:

$$
\begin{aligned}
a_{13q}^{p}(M_1M_2) &= C_{13}^{q}\,N_{13q}, \\
a_{14q}^{p}(M_1M_2) &= \frac{C_{13}^{q}}{N_c} + \frac{C_{13}^{q}}{N_c}\,\frac{C_F\alpha_s}{4\pi}\left[V_5(M_2) + \frac{4\pi^2}{N_c}\,H_5(M_1M_2)\right],
\end{aligned}
$$

Hadronic matrix elements (continued)

- the scalar operators also contribute to the penguin terms $P_{4,6}^{(\prime)\,p}(M_2)$ with:

$$\begin{aligned}
\delta P_4^p(M_1 M_2) &= \frac{C_F \alpha_s}{4\pi N_c}\left(-\frac{1}{2}\right) C_{13}^b \left[\frac{4}{3}\log\frac{m_b}{\mu} - G_{M_2}^f(1)\right], \\
\delta P_4^{\prime\,p}(M_1 M_2) &= \frac{C_F \alpha_s}{4\pi N_c}\left(-\frac{1}{2}\right) \left\{ C_{13}^D \left[\frac{4}{3}\log\frac{m_b}{\mu} - G_{M_2}^f(0)\right] \right. \\
&\quad \left. + C_{11}^D \left[\frac{4}{3}\log\frac{m_b}{\mu} - G_{M_2}^f(s_p)\right] \right\}
\end{aligned}$$

- the explicit scale dependence in $\delta P_{4,6}^{(\prime)\,p}(M_2)$ cancels the extra scale dependence of the (mirror) QCD penguin coefficients at LL accuracy;
- both Q_{11}^p and Q_{13}^D contribute the mirror penguin operators, while only Q_{13}^b mixes into the SM penguins.

$B \longrightarrow PP, PV$ decays:

- to evaluate the Higgs contributions, the largest values of the coefficient functions allowed by the constraints from leptonic decays are assumed:

$$C_{11}^{s}(m_b) = -0.08, \qquad C_{13}^{s}(m_b) = 0.001, \qquad C_{13}^{b}(m_b) = 0.05.$$

α_1	$0.966 + 0.021i\ [\pi \bar{K}]$	$0.981 + 0.021i\ [\rho \bar{K}]$	$0.973 + 0.021i\ [\pi \bar{K}^*]$
α_2	$0.351 - 0.084i\ [\bar{K} \pi]$	$0.260 - 0.084i\ [\bar{K}^* \pi]$	$0.323 - 0.084i\ [\bar{K} \rho]$
α_{11}^{s}	$-0.059\ [\pi \bar{K}]$	$-0.059\ [\rho \bar{K}]$	$0\ [\pi \bar{K}^*]$
α_{12}^{s}	$0.003 + 0.003i\ [\bar{K} \pi]$	$-0.006 - 0.003i\ [\bar{K}^* \pi]$	$0.004 + 0.003i\ [\bar{K} \rho]$

- the charged Higgs exchange ($\alpha_{11,12}^{D}$) contributes directly to tree-dominated decays (such as $B \rightarrow \pi\pi, \pi\rho, \rho\rho$), but must compete with the sizeable SM tree amplitudes $\alpha_{1,2}$.
- since $\alpha_{11,12}^{D} \propto \bar{m}_D$, only $b \rightarrow s\bar{u}u$ transitions are of interest, but no such kind of decays, since in this case the tree amplitudes are doubly CKM-suppressed, $\lambda_u^{(s)} \ll \lambda_c^{(s)}$.

$B \longrightarrow PP, PV$ decays:

	$\bar{K}\eta_s$	$\bar{K}^*\eta_s$	$\bar{K}\phi$
α_3^u	$-0.0013 + 0.0046i$	$0.0027 + 0.0046i$	$0.0006 - 0.0005i$
α_4^u	$-0.095 - 0.040i$	$0.038 + 0.008i$	$-0.031 - 0.017i$
$\alpha_3'^{u}$	$7.8 \cdot 10^{-5} - 0.0001i$	$2.8 \cdot 10^{-5} + 0.0001i$	$(1.4 - 1.3i) \cdot 10^{-5}$
$\alpha_4'^{u}$	$0.0035 + 0.0015i$	$0.0011 + 0.0003i$	$-0.0013 - 0.0006i$
$\alpha_{3,\mathrm{EW}}^u$	$-0.0089 - 0.0002i$	$-0.0091 - 0.0002i$	$-0.0082 - 0.0001i$
$\alpha_{4,\mathrm{EW}}^u$	$-0.0016 + 0.0006i$	$-0.0025 + 0.0008i$	$-0.0024 + 0.0007i$
α_{3s}^u	0.00078	0.00078	0
α_{4s}^u	$(-6.3 - 3.7i) \cdot 10^{-5}$	$(9.7 + 3.7i) \cdot 10^{-5}$	$(-6.3 - 3.7i) \cdot 10^{-5}$

- the effects from the mirror QCD penguin operators ($\alpha_{3,4}'^{p}$) must compete with the SM penguin amplitudes, which requires the scalar WCs to be of order 1.
- the direct FCNC Higgs coupling contributions ($\alpha_{3q,4q}^p$) are an isospin-violating effect, and must compete only with the small SM EW penguins.
- since $\alpha_{3q,4q}^p \propto \bar{m}_q$, only the $q = s$ case is of interest. This singles out the decay modes: $\bar{B} \to \bar{K}^{(*)}(\eta^{(\prime)}, \phi)$, $\bar{B}_s \to (\eta^{(\prime)}, \phi)(\eta^{(\prime)}, \phi)$, $\bar{B} \to \bar{K}^{(*)}K^{(*)}$, $\bar{B}_s \to K^{(*)}\phi$.

$B \longrightarrow VV$ decays: $F_+^{B \to V_1} = 0.06$

	$(\bar{K}^* \phi)^{00}$	$(\bar{K}^* \phi)^{--}$	$(\bar{K}^* \phi)^{++}$
$\alpha_1\ [\rho \bar{K}^*]$ $\alpha_2\ [\bar{K}^* \rho]$	$0.987 + 0.021i$ $0.240 - 0.084i$	$1.101 + 0.041i$ $-0.173 - 0.169i$	1.018 0.170
$\alpha_{11}^s\ [\rho \bar{K}^*]$ $\alpha_{12}^s\ [\bar{K}^* \rho]$	0 $-0.007 - 0.003i$	0 -0.002	0 $-0.247 - 0.068i$
α_3^u α_4^u	$0.0001 - 0.0005i$ $-0.026 - 0.015i$	$-0.0023 - 0.0010i$ $-0.044 - 0.017i$	-0.0035 -0.031
δP_4^u δP_6^u	$1.4 \cdot 10^{-5}$ $-1.5 \cdot 10^{-5}$	$0.7 \cdot 10^{-5}$ 0	$2.2 \cdot 10^{-5}$ 0
$\alpha_3'^{u}$ $\alpha_4'^{u}$	$(-0.2 + 1.3i) \cdot 10^{-5}$ $0.0011 + 0.0006i$	$(5.1 + 2.3i) \cdot 10^{-6}$ $0.0001 + 5.5 \cdot 10^{-5} i$	0.0010 $0.0173 + 0.0074i$
$\delta P_4'^{u}$ $\delta P_6'^{u}$	$-0.0005 - 0.0007i$ -0.0003	$-0.0004 - 0.0007i$ 0	$-0.0007 - 0.0007i$ 0
$\alpha_{3,\mathrm{EW}}^u$ $\alpha_{4,\mathrm{EW}}^u$	$-0.0084 - 0.0001i$ $-0.0017 + 0.0007i$	$0.0044 - 0.0003i$ $0.0015 + 0.0014i$	-0.009 -0.0015
α_{3s}^u α_{4s}^u	0 $(9.7 + 3.7i) \cdot 10^{-5}$	0 $2.0 \cdot 10^{-5}$	0 $0.0031 + 0.0008i$

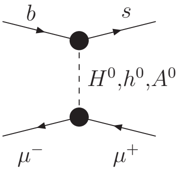
Conclusion

- Motivated by the interest in the MFV MSSM with large tan β, we have reanalyzed hadronic B decays in this model.
- The hadronic and leptonic flavour-changing interactions are closely related, and we translate the present limit on the $\mathrm{Br}(B_s \to \mu^+\mu^-)$, and the observation of $B^+ \to \tau^+\nu_\tau$ into a constraint on the relevant scalar operator coefficients.
- We then calculated the hadronic matrix elements of scalar operators and mirror QCD penguin operators in the QCDF framework.
- The current limits on leptonic B decays exclude any visible effects in two-body hadronic B decays.
- Although the positive-helicity amplitude of $\bar{B} \to VV$ may receive order one modifications relative to the SM, this amplitude is too small to be detected at present or planned B factories.

Back up

Constraints from $B_s \to \mu^+\mu^-$

- this decay proceeds via an interaction similar to the $b \to s\bar{q}q$ transitions, except with the $\bar{q}q$ pair replaced by a muon pair:



- for large $\tan\beta$, a single scalar operator $(\bar{D}b)_{S+P}\,(\bar{\mu}\mu)_{S-P}$ dominates the decay amplitude, while the SM contribution is negligible, with the coefficient:

$$C_{\mu\mu}(\mu_H) = -\frac{1}{2}\frac{\bar{m}_b m_\mu \epsilon_Y y_t^2 \tan^3\beta}{(1+\tilde{\epsilon}_3\tan\beta)(1+\epsilon_0\tan\beta)}\mathcal{F}_{2l}^-,$$

$$\mathcal{F}_{2l}^- = \frac{s_{\alpha-\beta}(c_\alpha)}{M_{H_0}^2} + \frac{c_{\alpha-\beta}(-s_\alpha)}{M_{h_0}^2} - \frac{1}{M_{A_0}^2} \approx \mathcal{F}_{2,J}^-.$$

Constraints from $B_s \to \mu^+\mu^-$ (continued)

- using the relation between $C_{\mu\mu}(\mu_H)$ and C_{13}^q, we get:

$$(1+\tilde{\epsilon}_J \tan\beta)\,|C_{13}^{d_J}(\mu_H)| = \frac{2\sqrt{2\pi}(\bar{m}_b+\bar{m}_s)(\mu_H)}{G_F f_{B_s} m_{B_s}^{5/2} \tau_{B_s}^{1/2}\,|\lambda_t^{(s)}|}\,\frac{\bar{m}_{d_J}(\mu_H)}{m_\mu}\left[\mathrm{Br}(B_s \to \mu^+\mu^-)\right]^{1/2}.$$

- using the present exp. limit, $\mathrm{Br}(B_s \to \mu^+\mu^-) \leq 5.8 \cdot 10^{-8}$@95% C.L., $f_{B_s} = 240\,\mathrm{MeV}$, $\bar{m}_s(2\,\mathrm{GeV}) = 90\,\mathrm{MeV}$ and $\bar{m}_b(\bar{m}_b) = 4.2\,\mathrm{GeV}$, we obtain:

$$(1+\epsilon_0\tan\beta)\,|C_{13}^s(\mu_H)| \leq 1.4\cdot 10^{-4}, \quad (1+\tilde{\epsilon}_3\tan\beta)\,|C_{13}^b(\mu_H)| \leq 7.9\cdot 10^{-3}.$$

- requiring $(1+\tilde{\epsilon}_J\tan\beta) \geq 1/3$ and including the evolution factor lead to:

$$|C_{13}^s(m_b)| \leq 0.001\ (0.01), \qquad |C_{13}^b(m_b)| \leq 0.05\ (0.5),$$

- C_{13}^q are constrained to be a factor of 10 smaller than the naive estimates.

Constraints from $B^+ \to \tau^+ \nu_\tau$

- the $C_{11}^D(\mu_H)$ can be related to $B^+ \to \tau^+ \nu_\tau$ in a similar way:

$$R_{\tau\nu_\tau} \equiv \frac{\mathrm{Br}(B^+ \to \tau^+ \nu_\tau)_{\mathrm{MSSM}}}{\mathrm{Br}(B^+ \to \tau^+ \nu_\tau)_{\mathrm{SM}}} = \left(1 + C_{11}^D(\mu_H) \frac{m_B^2(1+\epsilon_0 \tan\beta)}{\bar{m}_D(\mu_H)\bar{m}_b(\mu_H)}\right)^2 .$$

- using $\mathrm{Br}(B^+ \to \tau^+ \nu_\tau) = (1.51 \pm 0.33) \cdot 10^{-4}$, $|V_{ub}| f_{B_d} = 7.4 \cdot 10^{-4}\,\mathrm{GeV}$, and assigning a conservative 50% uncertainty to the SM prediction, we get:

$$-0.012 < (1+\epsilon_0 \tan\beta)\, C_{11}^s(\mu_H) < -0.009,$$
$$-0.001 < (1+\epsilon_0 \tan\beta)\, C_{11}^s(\mu_H) < 0.003.$$

- requiring $1+\epsilon_0 \tan\beta > 1/3$ and including the RG evolution results in:

$$-0.08 < C_{11}^s(m_b) < -0.06, \quad \text{or} \quad -0.005 < C_{11}^s(m_b) < 0.018.$$

- this constraint on C_{11}^s is not as stringent as the one on C_{13}^s; However, the charged Higgs contribution must compete with the SM tree operators.