Heavy-fermion Corrections to the Matching Coefficient of the Vector Current

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in collaboration with

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Outline

Introduction

- matching coefficient important building block for full NNNLO prediction of $\sigma(e^+e^- \to t\bar{t})$ at threshold
- contribution from diagrams with light quark loops done
  [PM,Piclum,Seidel,Steinhauser]
- next step: heavy quark loops (much more complicated)

Introduction

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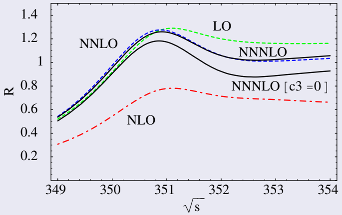
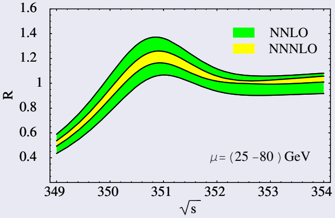
$\sigma(e^+e^- \to t\bar{t})$



scale dependence



[Beneke, Kiyo, Schuller]

Notations and Conventions

QCD

$$j^{\mu} = \bar{Q}\gamma^{\mu}Q$$

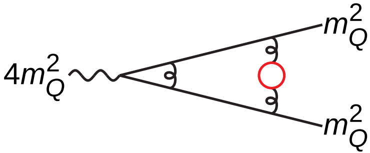
NRQCD

$$\tilde{j}^{i} = \phi^{\dagger}\sigma^{i}\chi$$

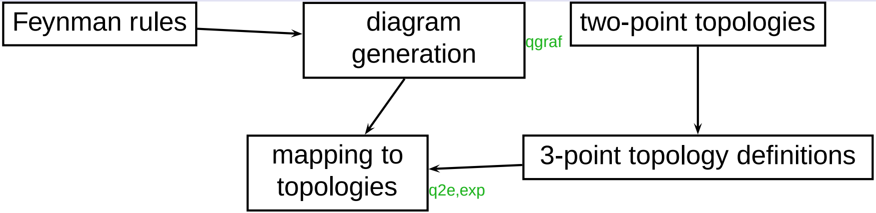
Matching

$$j^{i} = c_v(\mu)\tilde{j}^{i} + \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

- matching coefficient c_v can be obtained by calulating $\gamma \to t\bar{t}$ at threshold

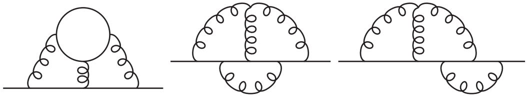


Outline of the Calculation



Generation of Three-Point Topology Definitions

- diagrams generated with `qgraf` [Nogueira]
- three-point topologies automatically generated from two-point topologies



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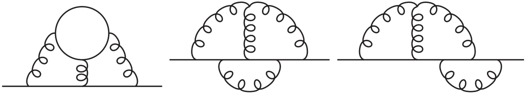
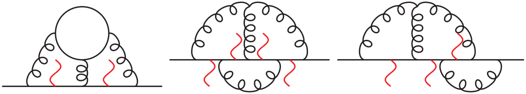


- attach photon to fermion line



- all Feynman diagrams mapped onto 20 topologies using `q2e, exp`

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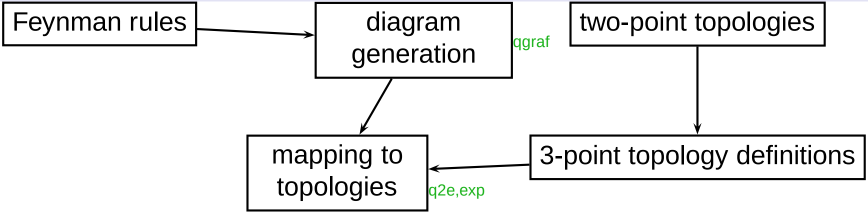

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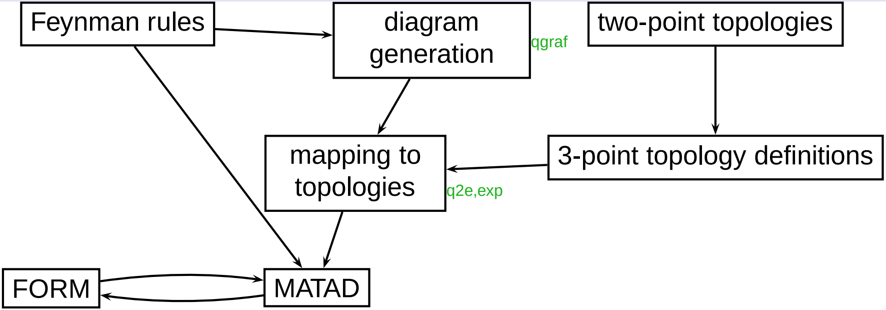

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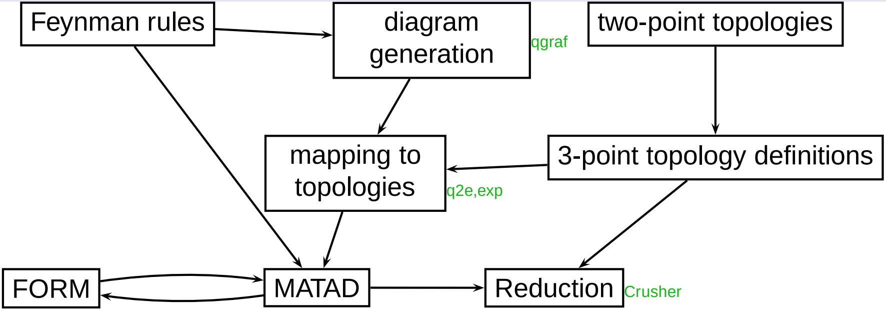

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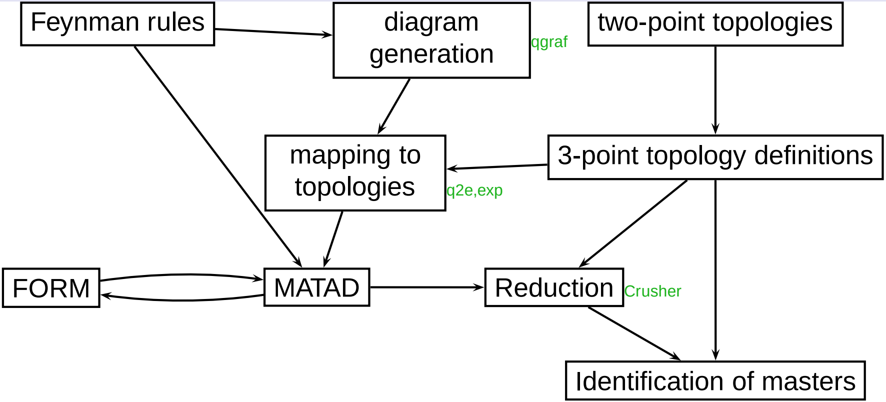

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- input for `Crusher` generated automatically
- sets of master integrals of different topologies not disjoint
- identification done automatically $\longrightarrow$ 24 master integrals

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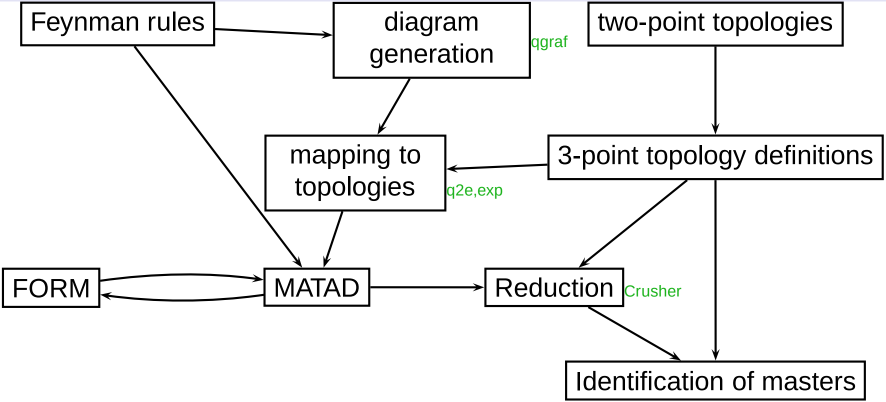

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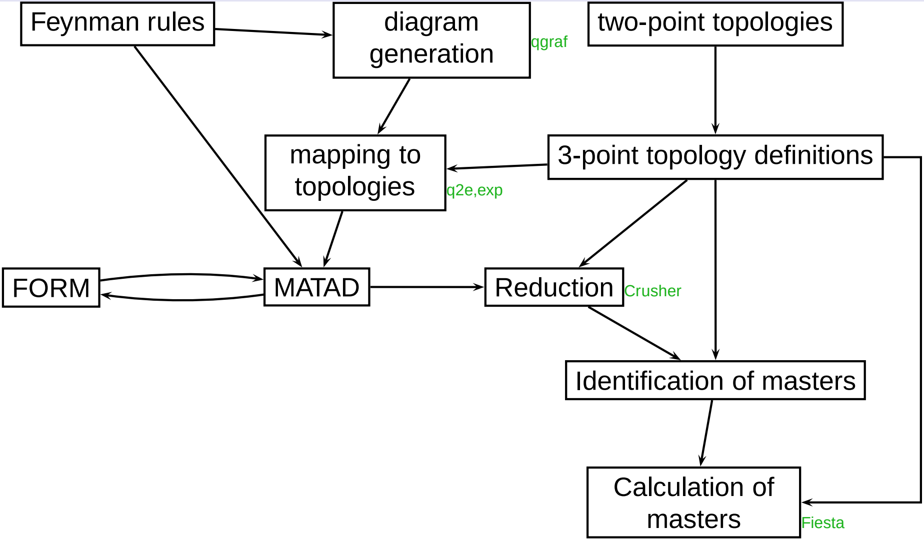

Calculation of master integrals

- 24 master integrals, 12 known analytically
- single scale integrals $\rightarrow$ limited number of tools
- evaluated using sector decomposition (FIESTA[Smirnov, Tentyukov])
  $\rightarrow$ “low” numerical precision e.g.
  $(0.059386 \pm 2 \cdot 10^{-6})/\epsilon + (0.990406 \pm 0.000013) +$
  $(1.025197 \pm 0.000092)\epsilon + (39.4366 \pm 0.000736)\epsilon^2 +$
  $(33.115019 \pm 0.006431)\epsilon^3$

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- other methods:
  - Mellin-Barnes
  - differential equations

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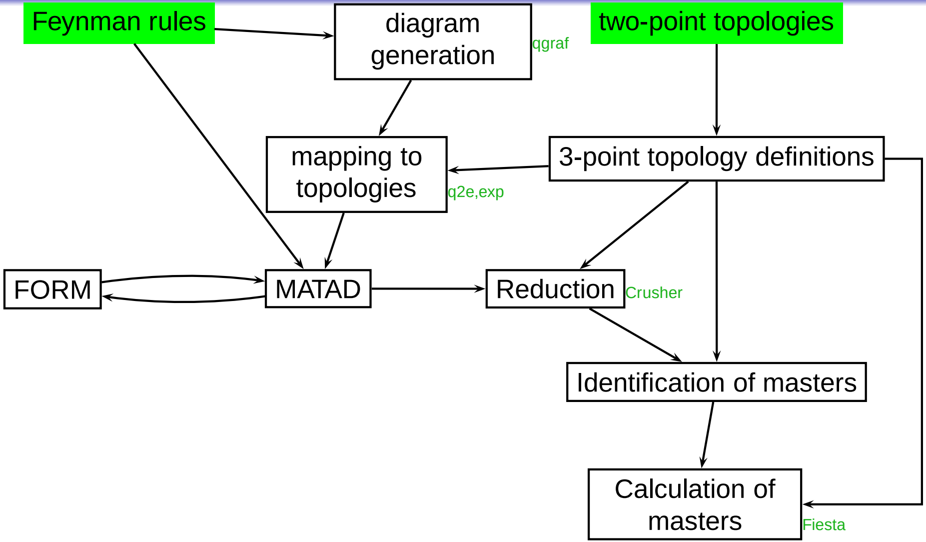

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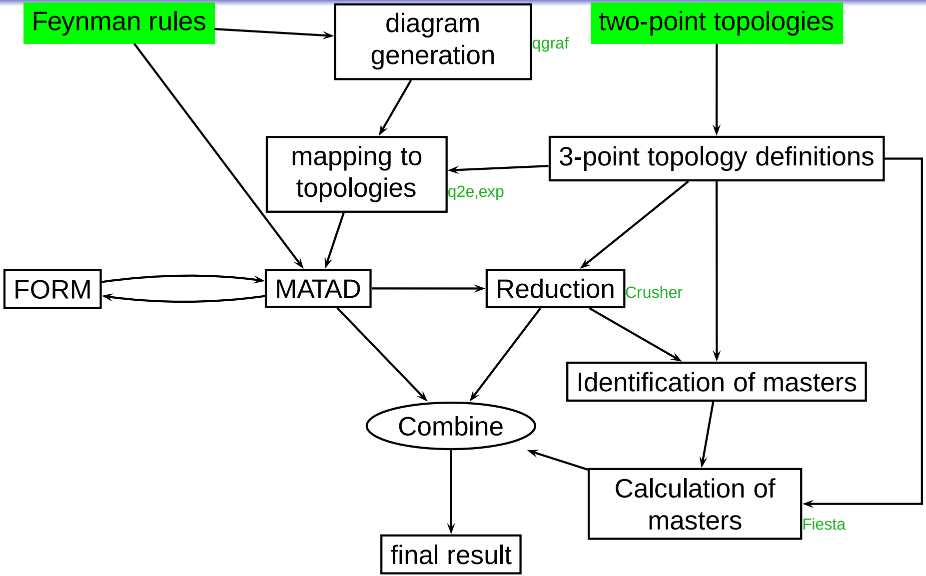

Results

$$c_v = 1 + \frac{\alpha_s^{(n_l)}(\mu)}{\pi} c_v^{(1)} + \left(\frac{\alpha_s^{(n_l)}(\mu)}{\pi} \right)^2 c_v^{(2)} + \left(\frac{\alpha_s^{(n_l)}(\mu)}{\pi} \right)^3 c_v^{(3)} + \mathcal{O}(\alpha_s^4)$$

$$\begin{aligned} c_v^{(3)} &= C_F T n_l \left(C_F c_{FFL} + C_A c_{FAL} + T n_h c_{FHL} + T n_l c_{FLL} \right) \\ &+ C_F T n_h \left(C_F c_{FFH} + C_A c_{FAH} + T n_h c_{FHH} \right) \\ &+ \text{non-fermionic and singlet terms} \end{aligned}$$

Results

$c_V|_{n_h}$

	numerical result	semi-analytical result
$C_F^2 n_h|_{\log}$	$-0.495(2)$	$-0.494(1)$
$C_F^2 n_h$	$-0.841(3)$	$-0.840(2)$
$C_F C_A n_h$	$-0.10(2)$	$-0.09(2)$
$C_F n_h^2$	$0.05126(1)$	0.05124
$C_F n_h n_l$	$-0.27029(4)$	-0.27025
$c_V^{(3)}|_{n_h}$	$-0.93(4)n_h$ $-0.09009(1)n_h n_l$	$-0.92(4)n_h$ $-0.09008 n_h n_l$

- no significant improvement by using known analytical results

Checks

Comparison with old calculation of n_l part

	new calc	old n_l part calc
2-loop	$-42.5138(2)$	-42.5140
$C_F^2 n_l$	$46.691(1)$	$46.7(1)$
$C_F C_A n_l$	$39.623(1)$	$39.6(1)$
$C_F n_l n_h$	$-0.27029(4)$	-0.2703
$C_F n_l^2$	$-2.46833(3)$	-2.4683
$c_v^{(3)}\vert_{n_l}$	$n_l\,(120.660(3) - 0.8228 n_l)$	$n_l\,(121. - 0.8228 n_l)$

- excellent agreement
- even smaller error (sector decomposition $\leftrightarrow$ Mellin-Barnes)

Checks

Change of integral basis

	basis 2	basis 3
$C_F^2 n_h\|_{\log}$	$-0.50(1)$	$-0.496(8)$
$C_F^2 n_h$	$-0.85(6)$	$-0.86(2)$
$C_F C_A n_h$	$-0.15(9)$	$-0.13(4)$
$C_F n_h^2$	$0.0513(1)$	$0.0513(1)$
$C_F n_h n_l$	$-0.27028(3)$	$-0.2703(2)$
$\tilde{c}_v^{(3)}\|_{n_h}$	$-1.04(23) n_h$ $-0.09009(1) n_h n_l$	$-1.00(11) n_h$ $-0.09008(5) n_h n_l$

- replace "standard" master integrals by (more complicated) integrals with raised powers of propagators
- requires deeper expansion in ϵ (up to ϵ^5)

Checks: Summary

- setup applied to light fermionic contribution reproduces known result
- gauge parameter independent (linear term)
- renormalizable
- numerical results stable against reparametrization of integrals
- different integral bases give compatible results
- FIESTA-parameter (ifCut) independent

Conclusion

- heavy-fermionic contributions to the matching coefficient of the vector current calculated
- (fully) automated calculation
- important step toward the full NNLO calculation
- conservative error estimate

final result

$$c_v^{(3)} \approx -0.823\, n_l^2 + 120.66(1)\, n_l - 0.93(8) + \text{non-fermionic and singlet terms}$$

Outlook

$$c_v^{(3)} \approx -0.823\, n_l^2 + 120.66(1)\, n_l - 0.93(8)$$

$+$ non-fermionic and singlet terms

possible with available setup !?

problematic (imaginary parts)

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problematic (imaginary parts)

- non-fermionic part
  - 78 topologies
  - $\approx$132 master integrals (118 done, 14 missing)
- singlet part: handle on master integrals still missing