Top quark threshold production at LHC

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- resummation of threshold-logs
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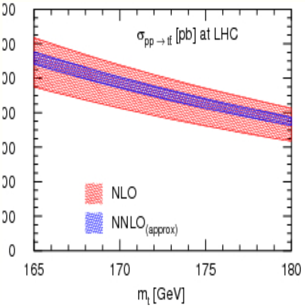
Millions of Top@LHC

LHC will produce 8 million $t\bar{t}$/year $(10/(\text{fb year}))$

- studies of top quark properties:
  $\delta m_t^{\text{exp}} \sim 1\text{GeV}$, $\delta\langle s_t \cdot s_{\bar{t}}\rangle_{\text{exp}} \sim 10\%$, tWb-coupling, $\cdots$
- a possibility to be standard candle
- current theory status; $\delta\sigma_{\text{NNLO}'} \approx 8\%$



NLO Nason-Dawson-Ellis '88, Mangano-Nason-Ridolfi (HVQMNR)'92

NLL Laenen-Smith-vanNeeerven '94, Berger-Contopanagos '95, Catani-Mangano-Nason-Trentadue '96

NNLL Bonciani-Catani-Mangano-Nason '98

NNLO' Moch-Uwer 08, Kidonakis-Vogt 08, Cacciari-Frixione-Mangano-Nason-Ridolfi 08

(Fig. from Moch-Uwer 08)

Top@LHC near threshold

There appeared paper on a top-mass measurement at LHC near threshold Hagiwara-Sumino-Yokoya (08)

We perform a detail study in NRQCD assembling existing knowledge@NLO/NLL:

Accumulation of knowledge

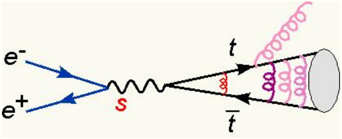
- Color-singlet result before dawn of NRQCD: Kühn-Mirkes '93
- Advent of NRQCD factorization: Bodwin-Braaten-Lepage '95
- Complete' NLO NRQCD result known since 1997: Petrelli-Cacciari-Greco-Maltoni-Mangano '97;

  Hagiwara-Sumino-Yokoya found a new corr 08
- Threshold-logs Kodaira-Trentadue '82, Catani-d'Emilio-Trentadue '88

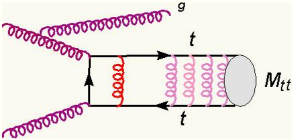
Part I

Threshold X section @NLO in NRQCD

Threshold @ILC vs LHC



- fixed s: threshold scan
- QCD vacuum $\rightarrow t\bar{t}(^3S_1^{[1]})$
- No-ISR/soft-FSR



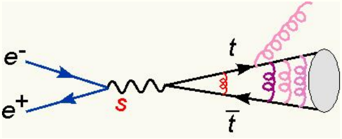
- $\hat{s}$ integrated out:$[\frac{d\mathcal{L}}{d\tau}](\tau)$
- color combination $\rightarrow t\bar{t}(^{2s+1}S_J^{[1,8]})$
- ISR/FSR soft-collinear dynamics

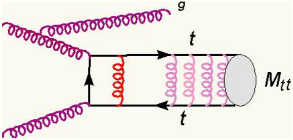
NRQCD Factorization

Threshold: $\sqrt{s} \approx 2m_t$

$$\sigma_{\rm ILC}(s) = \ C^2(s) \ \ G(\sqrt{s})$$

Threshold @ILC vs LHC





- fixed s: threshold scan
- QCD vacuum $\to t\bar{t}(^3S_1^{[1]})$
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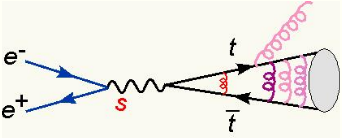
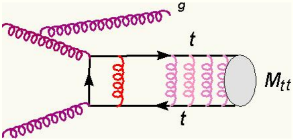
NRQCD Factorization

$$\text{Threshold: } M \approx 2m_t \qquad (M^2 = (p_t + p_{\bar{t}})^2)$$

$$\sigma_{\rm LHC}(S) = \frac{d\mathcal{L}_{ij}}{d\tau}\left(\frac{\hat{s}}{S}\right) \otimes F_{ij\to T}(\hat{s}, M) \otimes G(M)$$

Threshold @ILC vs LHC





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- $\hat{s}$ integrated out:$[\frac{d\mathcal{L}}{d\tau}](\tau)$
- color combination $\rightarrow t\bar{t}(^{2s+1}S_J^{[1,8]})$
- ISR/FSR soft-collinear dynamics

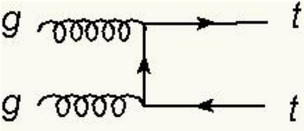
NRQCD Factorization

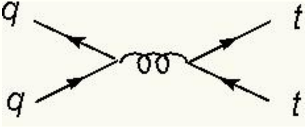
Threshold: $M \approx 2m_t \quad (M^2 \equiv (p_t + p_{\bar{t}})^2)$

$$\frac{d\sigma_{\rm LHC}}{dM} = \frac{d\mathcal{L}_{ij}}{d\tau}\left(\frac{\hat{s}}{S}\right) \otimes F_{ij\to T}\left(\hat{s}, M\right) \times G(M)$$

Threshold X section; $M^2 \equiv (p_t + p_{\bar{t}})^2 \sim (2m_t)^2$

Relative velocity of $t\bar{t}$ is $v \sim 0 \rightarrow$ S-wave dominance.





$$|\mathcal{M}(q\bar{q} \rightarrow t\bar{t})|^2 = \int dM^2 \underbrace{\delta(M^2 - \hat{s})\, |\bar{v}_q \gamma^\mu\, T^a\, u_q|^2\, \frac{1}{\hat{s}^2}}\, \underbrace{|\,\psi^\dagger(p_t)\sigma^i T^b\, \chi(p_{\bar{t}})\,|^2}$$

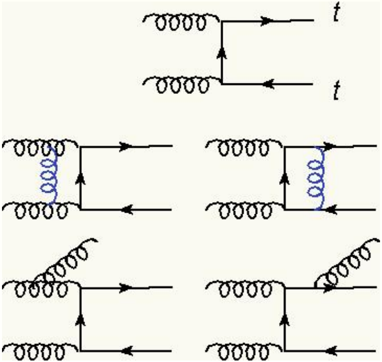
$$\frac{d\sigma_{ij\rightarrow T}}{dM^2}(\hat{s}, M, \mu_f) \sim \quad F_{ij\rightarrow T}\,(\hat{s}, M) \quad \mathrm{Im}\, G^{[1,8]}\,(M)$$

F: Free $t\bar{t}$ production rate

G: $t\bar{t}$ evolution factor into "boundstate" (Green Func)

QCD corr to the X section

$$M\frac{d\sigma_{ij\to T}}{dM}(\hat{s}, M, \mu_f) = F_{ij\to T}(z)\,\frac{1}{m_t^2}\mathrm{Im}\,G^{[1,8]}(M)$$



- 1-loop virtual corr to $F_{ij\to T}$ $\supset 1/\epsilon_{IR}^2, 1/\epsilon_{IR}, \cdots$
- 1-loop coulomb singularity: α_s/v absorbed into $G(M)$ (NRQCD factorization)
- real emission: soft $1/\epsilon_{IR}$ to cancel those of virtual corrections, collinear $1/\epsilon$ absorbed into PDFs: $f_i(x_1)\otimes f_j(x_2)$
- F, G are separately μ-independent @NLO

QCD corr to the X section

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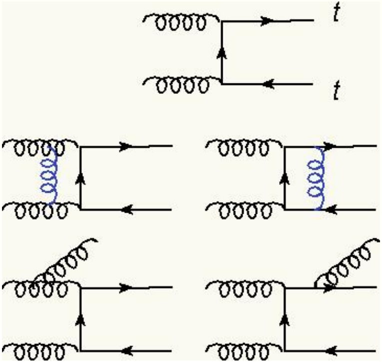
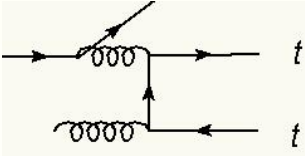




- sub-leading processes can be collinear singular, whose $1/\epsilon$ will be absorbed into PDF $\to$ quark-gluon mixing in AP-evolution.

Free $t\bar{t}$ production rate $F_{ij\to T}$

Kühn-Mirkes'93,Petrelli-Cacciari-Greco-Maltoni-Mangano'97

Free $t\bar{t}$ production rate: $F_{ij\to T} = \sigma_V \times R(z)$

- Hard virtual Corr: $\sigma_V = \mathcal{N}_{ij\to T}\dfrac{\pi^2\alpha_s(\mu)}{3\hat{s}}(1+\dfrac{\alpha_s(\mu)}{\pi}C_h)$
- Real emission: $R(z) = \delta_{ij\to T}\ \delta(1-z) + \dfrac{\alpha_s}{\pi}\left(A_c(z) + A_{nc}(z)\right)$

- $\delta_{ij\to T} = 1$ for $gg \to {}^1S_0^{[1,8]}, q\bar{q} \to {}^3S_1^{[8]}$ otherwise zero
- $z = M^2/\hat{s}$: momentum fraction of a parton
- C_h: color-singlet (Hagiwara-Kim-Yoshino '89) [non-decoupling top loop (Hagiwara-Sumino-Yokoya 08)]

Numerics: Free $t\bar{t}$ production rate@NLO

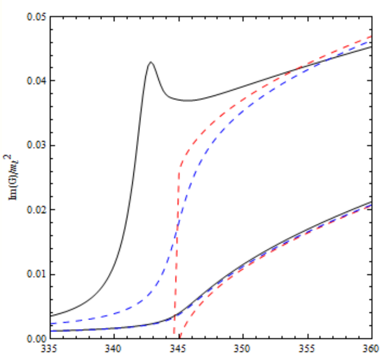
- $\mathcal{L} \otimes F$ in unit of $10^{-6}/\mathrm{GeV}^2 \sim 0.4 \times 10^3$pb for LHC at reference point $M = 2m_t$ with $\mu = M/2, M, 2M$

	$\mathcal{L} \otimes F[ij \to T^{[1]}]$			$\mathcal{L} \otimes F[ij \to T^{[8]}]$		
$gg \to {}^1S_0$	20.7	21.2	20.9	63.2	62.7	60.2
$gq \to {}^1S_0$	-0.795	-1.74	-2.19	-1.99	-4.36	-5.47
$q\bar{q} \to {}^1S_0$	0.00664	0.00509	0.00398	0.0166	0.0127	0.00995
$gg \to {}^3S_1$	0.175	0.127	0.0936	6.06	4.26	3.07
$gq \to {}^3S_1$		—		3.99	1.68	0.279
$q\bar{q} \to {}^3S_1$		—		23.1	23.8	23.6
total	20.1	19.6	18.8	94.3	88.1	81.8

- Dominance of leading processes: $gg \to {}^1S_0^{[1,8]}, q\bar{q} \to {}^3S_1^{[8]}$
- Collinear factorization (μ_f-cancelation)
  $gq \to {}^1S_0^{[1,8]} \Leftrightarrow gg \to {}^1S_0^{[1,8]}; \quad gq \to {}^3S_1^{[8]} \Leftrightarrow q\bar{q} \to {}^3S_1^{[8]}$

Green Functions @NLO: Coulomb resummation

- $\left\{\left[2m_t - \frac{\partial^2}{m_t} + V^{[1,8]}(r)\right] - (M + i\Gamma_t)\right\} G^{[1,8]}(\vec{r}; M + i\Gamma_t) = \delta^{(3)}(\vec{r})$
- $V^{[1,8]} = -\frac{\mathcal{C}^{[1,8]}\alpha_s}{r}\left(1 + \delta^{NLO} + \cdots\right) \qquad (\mathcal{C}^{[1]} = C_F,\ \mathcal{C}^{[8]} < 0)$
- $G^{[1,8]} = \frac{v}{4\pi}\left(i + \frac{\alpha_s \mathcal{C}^{[1,8]}}{v}\left[\frac{i\pi}{2} - \ln v\right]\right) + \mathcal{O}(\alpha_s^2)$ v:relative velocity



- solid: C-Resum with Γ_t
- red-dash: 1-loop (stable top)
  $\longrightarrow$ fixed order perturbation
- blue-dash: 1-loop with Γ_t

Green Functions @NLO: Coulomb resummation

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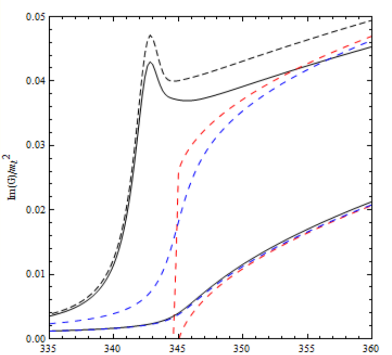


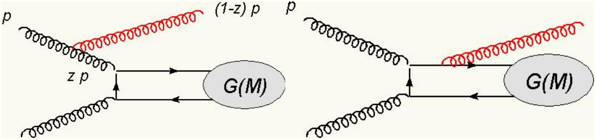
- solid: C-Resum with Γ_t
- red-dash: 1-loop (stable top)
  $\rightarrow$ fixed order perturbation
- blue-dash: 1-loop with Γ_t
- $G^{[1]}_{\mathrm{NNLO}}$ (for error estimate)

Part II

Threshold soft/collinear gluon resummation

Real emission: $R(z) = \delta_{ij\to T}\ \delta(1-z) + \frac{\alpha_s}{\pi}\left(A_c(z) + A_{nc}(z)\right)$

Kühn-Mirkes'93,Petrelli-Cacciari-Greco-Maltoni-Mangano'97



Collinear/Non-Coll Func

- $A_c[gg \to {}^1S_0^{[1,8]}] = (1-z)P_{gg}(z)\mathcal{S}_+(z) - \frac{\beta_0}{2}\ln\frac{\mu_f^2}{M^2}\delta(1-z)$
- $A_{nc}[gg \to {}^1S_0^{[8]}] = -2C_A\left[\frac{1}{1-z}\right]_+ + \cdots$
- $A_{nc}[gg \to {}^1S_0^{[1]}] =$ (rational + log) func in z

- Altarelli-Parisi splitting Func :$P_{gg}(z)$
- Soft Func: $\mathcal{S}_+(z) = \left[\frac{2\ln(1-z)}{1-z}\right]_+ + \left[\frac{1}{1-z}\right]_+ \ln\frac{\mu_f^2}{zM^2}$

End point (z=1) log V.S. non-log @NLO

- Three lines for $\mu = \mu_f = (m_t, 2m_t, 4m_t)$
- Three terms for LO, singular and regular terms at z=1
- Hard corr C_h as multiplicative common factor to all terms

$$\left(\mathcal{L} \otimes F_{\rm NLO}\right)\left[gg \to {}^1S_0^{[1]}\right] = \left\{ \begin{array}{c} 14.5 + [4.53]_{\mathcal{A}_+} + \left(1.68\right)_{\mathcal{A}} \\ 14.0 + [5.66]_{\mathcal{A}_+} + \left(1.58\right)_{\mathcal{A}} \\ 13.0 + [6.37]_{\mathcal{A}_+} + \left(1.48\right)_{\mathcal{A}} \end{array} \right\}$$

$$\left(\mathcal{L} \otimes F_{\rm NLO}\right)\left[gg \to {}^1S_0^{[8]}\right] = \left\{ \begin{array}{c} 39.3 + [16.6]_{\mathcal{A}_+} + \left(7.26\right)_{\mathcal{A}} \\ 37.4 + [18.8]_{\mathcal{A}_+} + \left(6.52\right)_{\mathcal{A}} \\ 34.4 + [20.0]_{\mathcal{A}_+} + \left(5.83\right)_{\mathcal{A}} \end{array} \right\}$$

$$\left(\mathcal{L} \otimes F_{\rm NLO}\right)\left[q\bar{q} \to {}^3S_1^{[8]}\right] = \left\{ \begin{array}{c} 16.7 + [3.50]_{\mathcal{A}_+} + \left(2.91\right)_{\mathcal{A}} \\ 16.8 + [3.41]_{\mathcal{A}_+} + \left(3.56\right)_{\mathcal{A}} \\ 16.4 + [3.28]_{\mathcal{A}_+} + \left(3.97\right)_{\mathcal{A}} \end{array} \right\}$$

Resummation of soft/collinear threshold logs

Resummed free quark production rate ($\lambda = \alpha_s(\mu)\beta_0/(4\pi)\ln N$):

$$\widetilde{F}_{ij\to T}(N, M, \mu_f) = \underbrace{g^0}_{\text{Matching term}} \exp \underbrace{\left\{\ln N \cdot g^1_{ij\to T}(\lambda) + g^2_{ij\to T}(\lambda) + \cdots\right\}}_{\text{Universal exponent}}$$

Factorization in N-space: $\mathcal{M}_N(f) \equiv \int_0^1 z^{N-1} f(z) dz$

- $\mathcal{M}_N\left((f \otimes g)[x]\right) = \widetilde{f}_N \times \widetilde{g}_N$
- $\mathcal{M}_N\left(\left[\frac{\ln^k(1-z)}{1-z}\right]_+\right) \sim \ln^{k+1} N$

- Resummation is done invoking Factorization Theorem
- $F_{ij\to T}$ can be obtained by Mellin back-trans numerically

Numerics: Free $t\bar{t}$ production rate@NLL

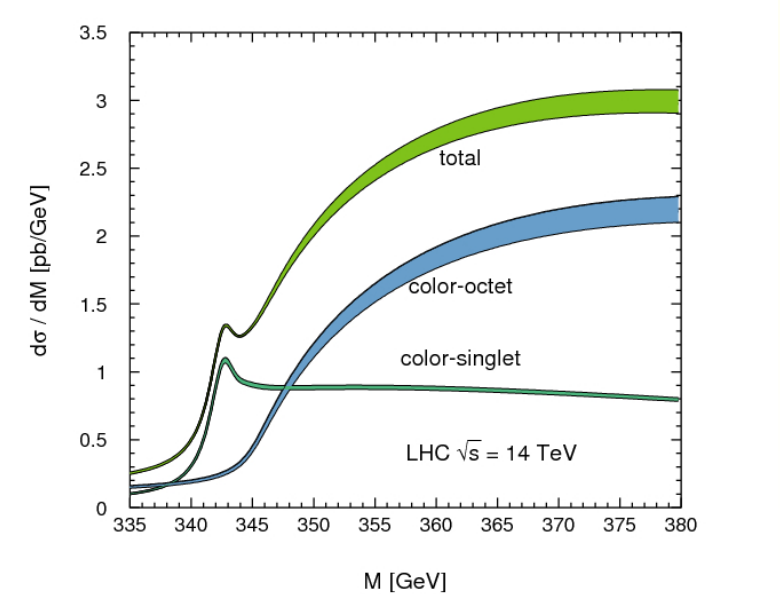
	NLO $\mathcal{L} \otimes F^{\mathrm{NLO}}_{ij\to T[1]}$			$\mathcal{L} \otimes F^{\mathrm{NLL}}_{ij\to T[8]}$		
$gg \to {}^1S_0^{[1]}$	20.7	21.2	20.9	22.0	23.2	24.0
$gg \to {}^1S_0^{[8]}$	63.2	62.7	60.2	67.8	69.7	70.6
$q\bar{q} \to {}^3S_1^{[8]}$	23.1	23.8	23.6	23.8	24.0	23.6

- few % enhancement
- how about scale dep?
  $\hookrightarrow$ needs to combine sub-leading processes

Part III

Phenomenology

Hadronic cross section $d\sigma/dM_{tt}$@LHC



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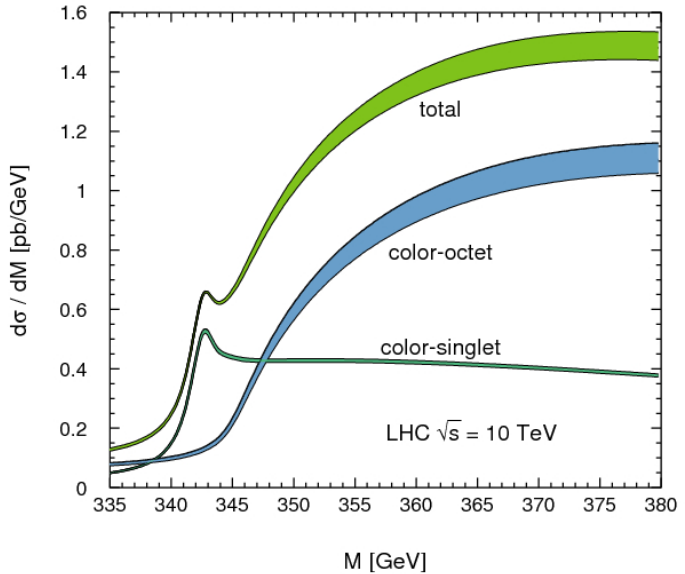
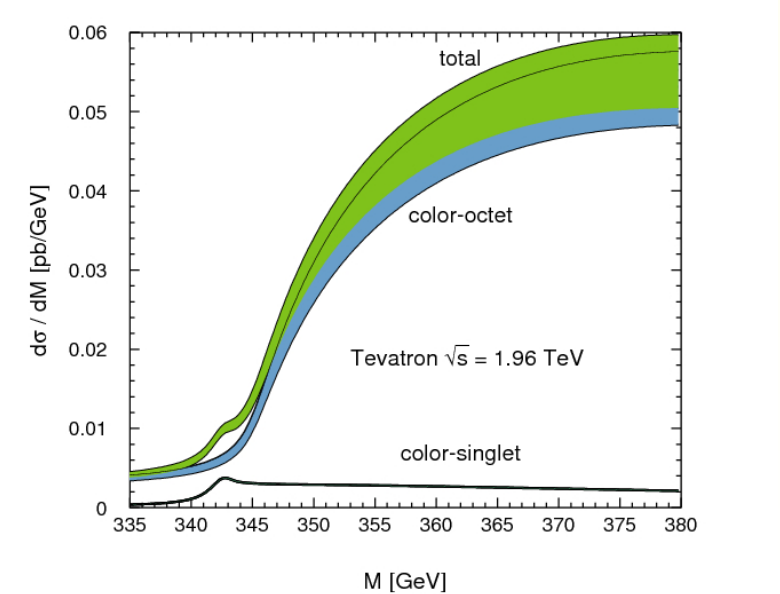
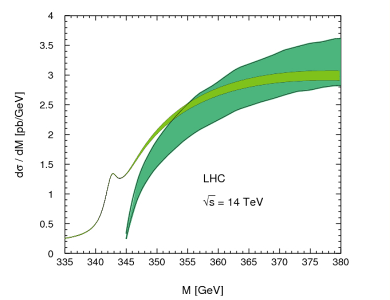
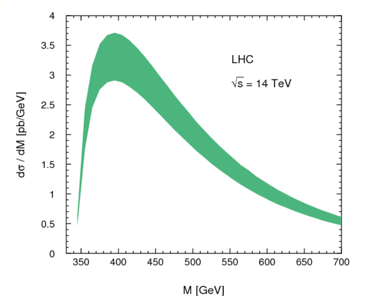


Tevatron case



Fixed order vs. NRQCD

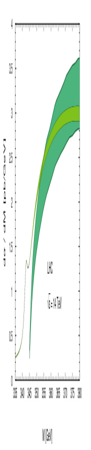


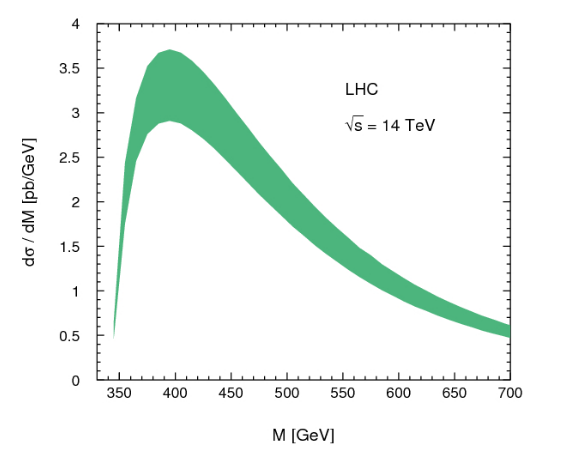


NLO comparison between fixed order(HVQMNR) and NLL NRQCD

- total cross section $\sim 840\text{pb}\ (10^7 t\bar{t}/\text{year})$
- consistency check: fixed order and NRQCD matching is O.K.
- $\Delta\sigma(gg \to t\bar{t}[^1S_0^{[1]}]) \simeq 1\%$, important for m_t measurement (agree with HSY08)

Fixed order vs. NRQCD





NLO comparison between fixed order(HVQMNR) and NLL NRQCD

m_t from M-distribution can be affected by the bound-state effect.

(recent study of M-distribution: Frederix-Maltoni 08)

Summary

Phenomenological study @NLO NRQCD / @NLL were performed for $t\bar{t}$ production near threshold at LHC :

- Resonance enhancement to the total cross section of order 1%
  - possibility of threshold scan@LHC for m_t
  - $\langle M_{tt} \rangle$ will be shifted to threshold
    $\hookrightarrow m_t$ fit from $\langle M_{tt} \rangle$ affected
- Resummation of Threshold logs
  - at most $+10\%$ shift near threshold for $d\sigma/dM$
  - Reduction of scale dependence
- Remaining Theory uncertainties
  - Bound-state dynamics G for color singlet has largest error: 20%
  - Scale dependence of hard correction is small.