Virtual and real IR-singularities in 5-point integrals

V. Yundin (project B1)

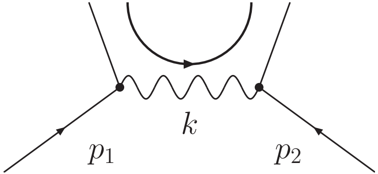
DESY, Zeuthen

SFB Meeting, 23–25 March 2009

Outline

- Introduction
  - Virtual IR divergence
  - Real soft IR divergence
  - Mellin-Barnes method
- Examples
  - QED muon-pair production 5-point function
  - QCD massless 5-point function
  - QCD massive 5-point function
- Conclusions

IR divergence in Feynman integrals

$\int d^4k \Big($  $\Big)$

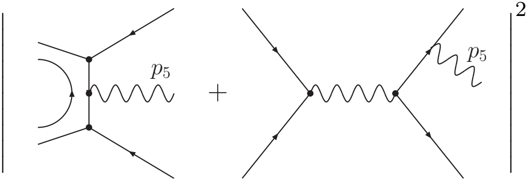
Virtual corrections IR singularity:

$$\int \frac{d^4k}{\left((k-p_1)^2 - m_1^2\right) k^2 \left((k+p_2)^2 - m_2^2\right)} =$$
$$= \int \frac{d^4k}{\left(k^2 - 2kp_1\right) k^2 \left(k^2 + 2kp_2\right)} \sim \int \frac{dk}{k} \longrightarrow \text{IR divergent}$$

- Contribute to $1/\epsilon^n$ terms in Laurent expansion.
- Naturally separated from the finite parts.

IR singularities in Feynman integrals

$$\int \frac{d^3p_5}{2E_5}$$



Real soft massless emission IR singularity:

$$\int \frac{d^3p_5}{2E_5}\frac{A}{E_5}\frac{B(E_5)}{E_5} \longrightarrow \int_0^\omega \frac{dE_5}{E_5} \longrightarrow \text{IR divergent}$$

$$\int_0^\omega \frac{dE_5}{E_5^{4-d}} \overbrace{\left(\frac{a}{\epsilon E_5} + \frac{b\ln(E_5)}{E_5} + \frac{c}{E_5}\right)} = -\frac{2a+b}{4\epsilon^2} - \frac{c-2a\ln(\omega)}{2\epsilon} + O(1)$$

- Parts $\sim 1/E_5$, $\ln(E_5)/E_5$ — contribute to $1/\epsilon^n$ terms **after** phase-space integration.
- Hidden in ϵ-finite parts of matrix element.

Intermediate summary

Two kinds of IR singularities:

“Virtual”

- Localized in $1/\epsilon^n$ terms.
- Can be calculated with any method.

“Real”

- **Not** localized
  (may be found in constant term!).
- May change order of virtual singularities.
- Separation from the finite part is **not** obvious.

Solution: Mellin-Barnes method

Example of mixed IR singularity

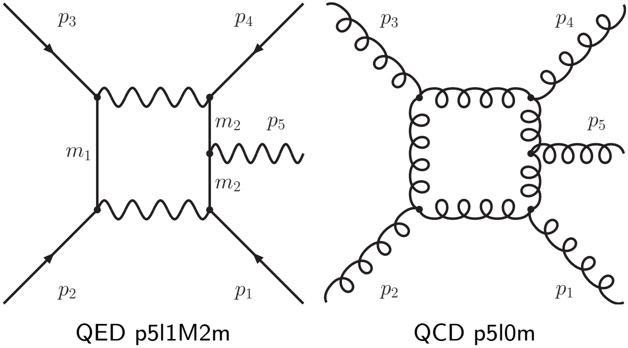


Figure: Mixed virtual/real IR singular 5-point functions

5 independent kinematic invariants:

$$p_1 p_2 = s'_{12}/2 \qquad p_1 p_5 = v_{15}/2 \sim E_5$$
$$p_3 p_4 = s'_{34}/2 \qquad p_4 p_5 = v_{45}/2 \sim E_5$$
$$(p_2 + p_3)^2 = s_{23}$$

Definitions

Massive one-loop n-point scalar Feynman integral:

$$I = \frac{e^{\epsilon\gamma_E}}{i\pi^{d/2}} \int \frac{d^d k}{(q_1^2 - m_1^2)^{\nu_1}(q_2^2 - m_2^2)^{\nu_2} \dots (q_n^2 - m_n^2)^{\nu_n}}$$

Feynman parameters representation ($\nu = \sum_{i=1}^{N} \nu_i$):

$$I = \frac{e^{\epsilon\gamma_E}(-1)^{\nu}\Gamma(\nu - d/2)}{\prod_{i=1}^{N}\Gamma(\nu_i)} \left(\prod_{j=1}^{N}\int_0^1 dx_j x_j^{\nu_j - 1}\right)\delta\Big(1 - \sum_{i=1}^{N} x_i\Big)\frac{U^{\nu-d}}{F^{\nu-d/2}}$$

In one-loop $U = \sum_{i=1}^{N} x_i = 1$,
and F could be made bilinear in x_i.

Mellin-Barnes formula

Muon pair production p5l1M2m F-form:

$$F_{\mathsf{p5l1M2m}} = m_2^2(x_1+x_5)^2 + m_1^2 x_3^2 - \\ - s'_{12}x_1x_3 - s'_{34}x_3x_5 - v_{45}x_1x_4 - v_{15}x_2x_5 - s_{23}x_2x_4$$

Massless QCD p5l0m F-form:

$$F_{\mathsf{p5l0m}} = -s_{12}x_1x_3 - s_{34}x_3x_5 - v_{45}x_1x_4 - v_{15}x_2x_5 - s_{23}x_2x_4$$

Mellin-Barnes formula

Muon pair production p5l1M2m F-form:

$$F_{\mathsf{p5l1M2m}} = m_2^2(x_1+x_5)^2 + m_1^2 x_3^2 - \\ - s'_{12}x_1x_3 - s'_{34}x_3x_5 - v_{45}x_1x_4 - v_{15}x_2x_5 - s_{23}x_2x_4$$

Massless QCD p5l0m F-form:

$$F_{\mathsf{p5l0m}} = -s_{12}x_1x_3 - s_{34}x_3x_5 - v_{45}x_1x_4 - v_{15}x_2x_5 - s_{23}x_2x_4$$

Mellin-Barnes formula and generalized Beta-function

$$\frac{1}{(X+Y)^\lambda} = \frac{1}{\Gamma(\lambda)}\frac{1}{2\pi i}\int_{-i\infty+R}^{+i\infty+R} dz\,\Gamma(\lambda+z)\Gamma(-z)\frac{Y^z}{X^{\lambda+z}}$$

$$\prod_{j=1}^{N}\int_0^1 dx_j x_j^{\nu_j-1}\delta\left(1-x_1-\cdots-x_N\right) = \frac{\Gamma(\nu_1)\Gamma(\nu_2)\cdots\Gamma(\nu_N)}{\Gamma(\nu_1+\nu_2+\cdots+\nu_N)}$$

Mellin-Barnes formula

Muon pair production p5l1M2m F-form: (**7-d integral** $\rightarrow$ **6-d**)

$$F_{\text{p5l1M2m}} = m_2^2(x_1+x_5)^2 + m_1^2 x_3^2 - \\ - s'_{12}x_1x_3 - s'_{34}x_3x_5 - v_{45}x_1x_4 - v_{15}x_2x_5 - s_{23}x_2x_4$$

Massless QCD p5l0m F-form: (**4-d integral**)

$$F_{\text{p5l0m}} = -s_{12}x_1x_3 - s_{34}x_3x_5 - v_{45}x_1x_4 - v_{15}x_2x_5 - s_{23}x_2x_4$$

Mellin-Barnes formula and generalized Beta-function

$$\frac{1}{(X+Y)^\lambda} = \frac{1}{\Gamma(\lambda)}\frac{1}{2\pi i}\int_{-i\infty+R}^{+i\infty+R} dz\, \Gamma(\lambda+z)\Gamma(-z)\frac{Y^z}{X^{\lambda+z}}$$

$$\prod_{j=1}^{N}\int_0^1 dx_j x_j^{\nu_j-1}\delta\left(1-x_1-\cdots-x_N\right) = \frac{\Gamma(\nu_1)\Gamma(\nu_2)\cdots\Gamma(\nu_N)}{\Gamma(\nu_1+\nu_2+\cdots+\nu_N)}$$

MB representation for p5l1M2m

- MB integrals with *AMBRE* (J. Gluza, K. Kajda, T. Riemann).
- Continuation algorithm by J. B. Tausk (hep-ph/9909506).
- Algorithm implemented in *MB.m* [1] (M. Czakon).

IR kinematics $s'_{12} \approx s'_{34}$ + Barnes 1st lemma $\rightarrow -1$ integration.

$$I = ({m_1}^2)^{z_3} ({m_2}^2)^{z_1} (-s_{23})^{-\epsilon-z_1-z_3-z_4-z_5-z_6} (-s'_{12})^{z_5} (-v_{15})^{z_6} (-v_{45})^{z_4}$$
$$\Gamma(-z_1)\Gamma(-z_3)\Gamma(-z_4)\Gamma(-z_5)\Gamma(-z_6)\Gamma(z_4+1)\Gamma(z_6+1)\Gamma(2z_3+z_5+1)$$
$$\Gamma(-\epsilon-z_1-z_3-z_5-z_6-2)\Gamma(-\epsilon-z_1-z_3-z_4-z_5-2)\Gamma(2z_1+z_4+z_5+z_6+2)$$
$$\Gamma(\epsilon+z_1+z_3+z_4+z_5+z_6+3)/\left({s_{23}}^3\Gamma(-2\epsilon-1)\Gamma(z_4+z_6+2)\right)$$

Integration contours parallel to imaginary axis:

$\epsilon = -3/4$	$\Re z_1 = -1/2$	$\Re z_3 = -3/16$
$\Re z_4 = -3/32$	$\Re z_5 = -7/16$	$\Re z_6 = -31/64$

[1] with modifications

Continuation for p5l1M2m

Default parameters result (in MB.m notation):

$$\Big\{\text{MBint}\Big[(m_1{}^2)^{z_3}(m_2{}^2)^{z_3-z_6}(-s_{23})^{-z_6-2}s_{23}(-s'_{12})^{-2z_3-1}(-v_{15})^{z_6}(-v_{45})^{z_6-1}\Gamma(-z_3)$$

$$\Gamma(2z_3+1)\Gamma(1-z_6)\Gamma(-z_6)\Gamma(z_6)\Gamma(z_6+1)^2\Gamma(z_6-z_3)/(\Gamma(2z_6+1)),\Big\{\{\epsilon\to 0\},\Big\{z_3\to -\tfrac{3}{16},z_6\to -\tfrac{11}{32}\Big\}\Big\}\Big],$$

$$\text{MBint}\Big[(m_1{}^2)^{z_6}(-s_{23})^{-z_6-2}s_{23}(-s'_{12})^{-2z_6-1}(-v_{15})^{z_6}(-v_{45})^{z_6-1}\Gamma(1-z_6)\Gamma(-z_6)^2\Gamma(z_6)$$

$$\Gamma(z_6+1)^2,\Big\{\{\epsilon\to 0\},\Big\{z_6\to -\tfrac{11}{32}\Big\}\Big\}\Big],\text{MBint}\Big[\Big((m_1{}^2)^{z_3}(m_2{}^2)^{z_3}(-s'_{12})^{-2z_3-1}\Gamma(-z_3)^2\Gamma(2z_3+1)$$

$$(-\epsilon\ln(-s_{23})v_{15}+2\epsilon\ln(-v_{15})v_{15}+\epsilon\gamma v_{15}+v_{15}-\epsilon\gamma v_{45}+v_{45}+\epsilon(v_{45}-v_{15})\ln(m_2{}^2)+\epsilon v_{45}\ln(-s_{23})-$$

$$-2\epsilon v_{45}\ln(-v_{15})+\epsilon(v_{15}-v_{45})\psi(-z_3))\Big)/(2\epsilon s_{23}v_{15}v_{45}),\Big\{\{\epsilon\to 0\},\Big\{z_3\to -\tfrac{3}{16}\Big\}\Big\}\Big]\Big\}$$

Continuation for p5l1M2m

Default parameters result (in MB.m notation):

$$\Big\{\text{MBint}\Big[(m_1{}^2)^{z_3}(m_2{}^2)^{z_3-z_6}(-s_{23})^{-z_6-2}s_{23}(-s'_{12})^{-2z_3-1}(-v_{15})^{z_6}(-v_{45})^{z_6-1}\Gamma(-z_3)$$
$$\Gamma(2z_3+1)\Gamma(1-z_6)\Gamma(-z_6)\Gamma(z_6)\Gamma(z_6+1)^2\Gamma(z_6-z_3)/(\Gamma(2z_6+1)),\Big\{\{\epsilon\to 0\},\Big\{z_3\to -\tfrac{3}{16},z_6\to -\tfrac{11}{32}\Big\}\Big\}\Big],$$
$$\text{MBint}\Big[(m_1{}^2)^{z_6}(-s_{23})^{-z_6-2}s_{23}(-s'_{12})^{-2z_6-1}(-v_{15})^{z_6}(-v_{45})^{z_6-1}\Gamma(1-z_6)\Gamma(-z_6)^2\Gamma(z_6)$$
$$\Gamma(z_6+1)^2,\Big\{\{\epsilon\to 0\},\Big\{z_6\to -\tfrac{11}{32}\Big\}\Big\}\Big],\text{MBint}\Big[\Big((m_1{}^2)^{z_3}(m_2{}^2)^{z_3}(-s'_{12})^{-2z_3-1}\Gamma(-z_3)^2\Gamma(2z_3+1)$$
$$(-\epsilon\ln(-s_{23})v_{15}+2\epsilon\ln(-v_{15})v_{15}+\epsilon\gamma v_{15}+v_{15}-\epsilon\gamma v_{45}+v_{45}+\epsilon(v_{45}-v_{15})\ln(m_2{}^2)+\epsilon v_{45}\ln(-s_{23})-$$
$$-2\epsilon v_{45}\ln(-v_{15})+\epsilon(v_{15}-v_{45})\psi(-z_3))\Big)/(2\epsilon s_{23}v_{15}v_{45}),\Big\{\{\epsilon\to 0\},\Big\{z_3\to -\tfrac{3}{16}\Big\}\Big\}\Big]\Big\}$$

"Optimized continuation" result:

$$I_{\text{p5l1M2m(IR)}} = -(s_{23}s'_{12})^{-1}(m_2^2)^{\epsilon}\left((-v_{45})^{-2\epsilon-1}+(-v_{15})^{-2\epsilon-1}\right)\times$$
$$\times\frac{1}{2}\int dz_3(m_1^2)^{z_3}(m_2^2)^{z_3}(-s'_{12})^{-2z_3}\times$$
$$\times\Gamma(-z_3)^2\Gamma(2z_3+1)\big(-1/\epsilon+\gamma+\psi(-z_3)\big)$$

where $\Re z_3 = -3/16$

Both representations are equivalent (checked numerically).

QED p5l1M2m IR part

$$I_{\text{p5l1M2m(IR)}} = -(s_{23}s'_{12})^{-1}(m_2^2)^{\epsilon} \times$$
$$\times \left((-v_{45})^{-2\epsilon-1} + (-v_{15})^{-2\epsilon-1}\right)\left(\frac{J_{-1}}{\epsilon} + J_0\right)$$

Sums: hep-th/0303162v4 (A. I. Davydychev, M. Yu. Kalmykov).

$$J_{-1} = \sum_{n=0}^{\infty} u^n \binom{2n}{n} \Big(S_1(2n) - S_1(n) + \ln(u)/2\Big)$$
$$J_0 = \sum_{n=0}^{\infty} u^n \binom{2n}{n} \Big(S_2(2n) - S_1(2n)^2 + S_1(2n)S_1(n) -$$
$$- S_1(2n)\ln(u) + S_1(n)\ln(u)/2 - (\ln^2(u)/4 + \zeta_2)\Big)$$

where $u = m_1^2 m_2^2 / {s'_{12}}^2$

QED p5l1M2m IR part

$$I_{\mathsf{p5l1M2m(IR)}} = -\,(s_{23}s'_{12})^{-1}(m_2^2)^{\epsilon} \times$$
$$\times \left((-v_{45})^{-2\epsilon-1} + (-v_{15})^{-2\epsilon-1}\right)\left(\frac{J_{-1}}{\epsilon} + J_0\right)$$

Analytical result for IR-part of p5l1M2m:

$$J_{-1} = \frac{1}{2}\frac{(1+\chi)\ln(\chi)}{1-\chi}, \qquad \chi = \frac{1-\sqrt{1-4{m_1}^2{m_2}^2/{s'_{12}}^2}}{1+\sqrt{1-4{m_1}^2{m_2}^2/{s'_{12}}^2}},$$

$$J_0 = \frac{1}{4(1-\chi)}\Big(2(1-\chi)\mathsf{Li}_2(\chi^2) + 8\chi\mathsf{Li}_2(\chi) - 4(1-\chi)\mathsf{Li}_2(-\chi) +$$
$$+\,4(1+\chi)\ln(1-\chi)\ln(\chi) - (1+\chi)\ln^2(\chi) - 4(1+\chi)\zeta_2\Big)$$

Hypergeometric representation

Integral before ϵ-expansion:

$$I_{\mathsf{p5l1M2m(IR)}} = -(s_{23}s'_{12})^{-1}(m_2^2)^{\epsilon}\left((-v_{45})^{-2\epsilon-1} + (-v_{15})^{-2\epsilon-1}\right) \times$$
$$\times \int dz_3 (m_1^2)^{z_3}(m_2^2)^{z_3}(-s'_{12})^{-2z_3} \times$$
$$\times \frac{\Gamma(-2\epsilon)\Gamma(1+2\epsilon)\Gamma(-\epsilon-z_3)\Gamma(-z_3)\Gamma(1+2z_3)}{\Gamma(1-2\epsilon)}$$

Hypergeometric representation (only IR part), $u = m_1^2 m_2^2 / {s'_{12}}^2$

$$I_{\mathsf{p5l1M2m(IR)}} = -(s_{23}s'_{12})^{-1}(m_2^2)^{\epsilon}\left((-v_{45})^{-2\epsilon-1} + (-v_{15})^{-2\epsilon-1}\right) \times$$
$$\times \frac{1}{2}\Gamma(2\epsilon)\left[-2^{1-2\epsilon}u^{-\epsilon}(4u-1)^{\epsilon-\frac{1}{2}}\sqrt{\pi}\,\Gamma\left(1/2-\epsilon\right) + \right.$$
$$\left. + u^{-1}\Gamma(1-\epsilon)\,{}_2F_1\left(1, 1-\epsilon; 3/2; 1/(4u)\right)\right]$$

Automatic expansion with *HypExp2* (T. Huber, D. Maitre).

Massless QCD 5l0m

MB representation is 3-dimensional

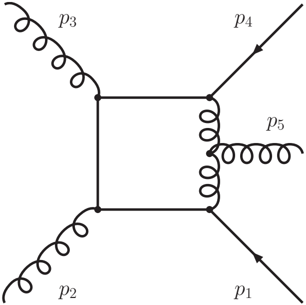
$$I_{\mathsf{p5l0m}} = (-s_{12})^{z_3}(-s_{23})^{-z_2-z_3-z_4}(-v_{15})^{z_4}(-v_{45})^{z_2}\Gamma(-z_2)\Gamma(z_2+1)$$
$$\Gamma(-z_3)\Gamma(z_3+1)\Gamma(-z_4)\Gamma(z_4+1)\Gamma(z_2+z_3+z_4+2)\Gamma(-z_2-z_3-\epsilon-2)$$
$$\Gamma(-z_3-z_4-\epsilon-2)\Gamma(z_2+z_3+z_4+\epsilon+3)/({s_{23}}^3\Gamma(z_2+z_4+2)\Gamma(-2\epsilon-1))$$

$$\epsilon = -1, \quad \Re z_2 = -1/2, \quad \Re z_3 = -13/16, \quad \Re z_4 = -7/16$$

The result for IR part:

$$I_{\mathsf{p5l0m(IR)}} = -\frac{1}{s_{12}}(-s_{23})^{-1-\epsilon}\Bigg[(-s_{23})^{1+2\epsilon}(-v_{15})^{-1-\epsilon}(-v_{45})^{-1-\epsilon}\left(\frac{2}{\epsilon^2}+\zeta_2+\epsilon\frac{14}{3}\zeta_3\right)$$
$$+\left(\frac{1}{\epsilon^2}+\frac{5\zeta_2}{2}\right)\left((-v_{45})^{-1-\epsilon}(-v_{15})^{\epsilon}+(-v_{15})^{-1-\epsilon}(-v_{45})^{\epsilon}\right)+\frac{1}{\epsilon^2}(-s_{12})^{-1-2\epsilon}(-v_{15})^{\epsilon}(-v_{45})^{\epsilon}$$
$$+\frac{\left(v_{15}\ln^2\left(\frac{v_{15}}{s_{23}}\right)+2(s_{23}-v_{15})\ln\left(1-\frac{v_{15}}{s_{23}}\right)\ln\left(\frac{v_{15}}{s_{23}}\right)+4\zeta_2 v_{15}+2(s_{23}-v_{15})\mathrm{Li}_2\left(\frac{v_{15}}{s_{23}}\right)\right)}{v_{15}v_{45}}$$
$$+\frac{\left(v_{45}\ln^2\left(\frac{v_{45}}{s_{23}}\right)+2(s_{23}-v_{45})\ln\left(1-\frac{v_{45}}{s_{23}}\right)\ln\left(\frac{v_{45}}{s_{23}}\right)+4\zeta_2 v_{45}+2(s_{23}-v_{45})\mathrm{Li}_2\left(\frac{v_{45}}{s_{23}}\right)\right)}{v_{15}v_{45}}\Bigg]$$

QCD p5l3m



$$F = m^2(x_2 + x_3 + x_4)^2 - s'_{12}x_1x_3 - s'_{34}x_3x_5 -$$
$$- v_{45}x_1x_4 - v_{15}x_2x_5 - s_{23}x_2x_4$$

$$I_{\mathsf{p5l3m}} = \left(m^2\right)^{z_1}(-s_{23})^{-3-z_1-z_4-z_5-z_6}(-s'_{12})^{z_5}(-v_{15})^{z_6}(-v_{45})^{z_4}\Gamma(-z_1)$$
$$\Gamma(-z_4)\Gamma(z_4+1)\Gamma(-z_5)\Gamma(z_5+1)\Gamma(-z_6)\Gamma(z_6+1)\Gamma(-z_1-z_5-z_6-\epsilon-2)$$
$$\Gamma(-z_4-z_5-z_6-2\epsilon-3)\Gamma(-z_1-z_4-z_5-\epsilon-2)\Gamma(z_1+z_4+z_5+z_6+\epsilon+3)$$
$$\Gamma(z_4+z_5+z_6+2)/(\Gamma(z_4+z_6+2)\Gamma(-2\epsilon-1)\Gamma(-2z_1-z_4-z_5-z_6-2\epsilon-3))$$

QCD p5l3m

$$I = \text{MBint}\Big(-2\,(m^2)^{z_1}\,(-s_{23})^{-z_1}(-v_{15})^{z_6}(-v_{45})^{-2-2\epsilon-z_6}\Gamma(-z_1)$$
$$\Gamma_B(-z_6-1)\Gamma_A(-z_1-z_6-1)\Gamma(-z_6)\Gamma(z_6+1)\Gamma(z_6+2)\Gamma(-z_1+z_6+1)$$
$$\Gamma(z_1)/(s'_{12}\Gamma(-2z_1)), \{\{\epsilon \to 0\}, \{z_1 \to -87/128, z_6 \to -5/64\}\}\Big)$$

"Bad" power of v_{45}:

$$(-v_{45})^{-2-2\epsilon-z_6} \longrightarrow (-v_{45})^{-1-2\epsilon-(-5/64)}$$

Shift contour and take residues in $z_6 = -1 - z_1$ and $z_6 = -1$.

$$I = \text{Res}_A + \text{Res}_B + I_{\text{shifted}}$$

I_{shifted} — IR safe: $(-v_{45})^{-1-2\epsilon-(-1-5/64)}$
Res_1 and Res_2 — one dimension less.

IR part of QCD p5l3m

Analytical result for IR part of QCD p5l3m function *(preliminary)*.

$$
\begin{aligned}
I_{\text{p5l3m(IR)}} = &-\frac{4(-s_{23})^{2\epsilon}\sin^{-1}\left(\frac{\sqrt{s_{23}}}{2m}\right)^2(-v_{15})^{-2\epsilon-1}}{s'_{12}v_{45}} + \frac{\left(m^2\right)^{\epsilon}(-s_{23})^{\epsilon}\ln\left(\frac{v_{45}}{v_{15}}\right)(-v_{15})^{-2\epsilon-1}}{s'_{12}v_{45}\epsilon} \\
&+\frac{7\left(m^2\right)^{\epsilon}\pi^2(-s_{23})^{\epsilon}(-v_{15})^{-2\epsilon-1}}{12s'_{12}v_{45}} - \frac{\left(m^2\right)^{\epsilon}(-s_{23})^{\epsilon}(-v_{15})^{-2\epsilon-1}}{s'_{12}v_{45}\epsilon^2} \\
&-\frac{\epsilon\ln^3\left(-\frac{s_{23}}{m^2}\right)}{3s'_{12}v_{15}v_{45}} - \frac{\epsilon\ln^3\left(\frac{v_{15}}{v_{45}}\right)}{s'_{12}v_{15}v_{45}} - \frac{\epsilon\ln\left(-\frac{s_{23}}{m^2}\right)\ln^2\left(\frac{v_{15}}{v_{45}}\right)}{s'_{12}v_{15}v_{45}} - \frac{\epsilon\ln^2\left(-\frac{s_{23}}{m^2}\right)\ln\left(\frac{v_{15}}{v_{45}}\right)}{s'_{12}v_{15}v_{45}} - \frac{3\pi^2\epsilon\ln\left(\frac{v_{15}}{v_{45}}\right)}{s'_{12}v_{15}v_{45}} \\
&-\frac{(-s_{23})^{2\epsilon}(-v_{45})^{-2\epsilon-1}\epsilon\ln\left(1-\frac{s_{23}v_{15}}{m^2v_{45}}\right)\ln^2\left(-\frac{m^2v_{45}}{s_{23}v_{15}}\right)}{s'_{12}v_{15}} - \frac{\pi^2\epsilon\ln\left(-\frac{s_{23}}{m^2}\right)}{s'_{12}v_{15}v_{45}} \\
&-\frac{\pi^2(-s_{23})^{2\epsilon}(-v_{45})^{-2\epsilon-1}\epsilon\ln\left(1-\frac{s_{23}v_{15}}{m^2v_{45}}\right)}{s'_{12}v_{15}} - \frac{8(-s_{23})^{2\epsilon}(-v_{45})^{-2\epsilon-1}\epsilon\sin^{-1}\left(\frac{\sqrt{s_{23}}}{2m}\right)^2\ln\left(\frac{v_{15}}{v_{45}}\right)}{s'_{12}v_{15}} \\
&+\frac{2(-s_{23})^{2\epsilon}(-v_{45})^{-2\epsilon-1}\epsilon\ln\left(-\frac{m^2v_{45}}{s_{23}v_{15}}\right)\text{Li}_2\left(\frac{s_{23}v_{15}}{m^2v_{45}}\right)}{s'_{12}v_{15}} + \frac{2(-s_{23})^{2\epsilon}(-v_{45})^{-2\epsilon-1}\epsilon\text{Li}_3\left(\frac{s_{23}v_{15}}{m^2v_{45}}\right)}{s'_{12}v_{15}}
\end{aligned}
$$

Conclusions

- Mellin-Barnes method is useful for extraction of IR pieces of 1-loop massive n-point functions.
- High level of automatization: *AMBRE*, *MB.m*, *HypExp2*.
- There is still room for improvement (e.g. better continuation procedure, automatic derivation of residua sums, etc).
- Applied to calculation of IR-parts of some LHC-relevant 5- and 6-point functions.
- Extension to two loops is straightforward.

Thank you for your attention!