Real Radiation and Sudakov Logarithms

electroweak corrections at high energies

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in collaboration with

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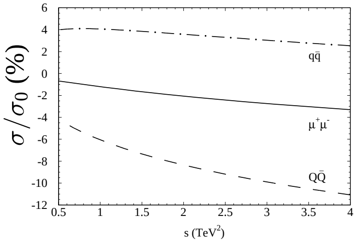
SFB Meeting 23 - 25 March 2009

At TeV-energies electroweak corrections become large, because they are enhanced by Sudakov Logarithms

$$\frac{\alpha}{4\pi \sin^2 \theta_W} \log^2 \left(\frac{M_{W/Z}^2}{s} \right) \sim 7\% \quad \text{at 1 TeV}$$

Mainly the virtual corrections are calculated (no need for real corrections)



$e^+e^- \longrightarrow f\bar{f}$:
Jantzen, Kühn, Penin, Smirnov '05

motivation

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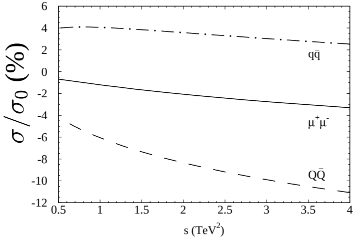


$e^+e^- \rightarrow f\bar{f}$:

Jantzen, Kühn, Penin, Smirnov '05

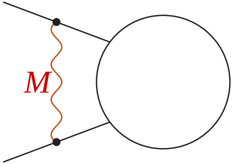
There are some open questions about the relevance of real corrections

Ciafaloni, Comelli '99 - '06
Baur '07

Sudakov Logarithms

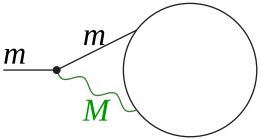
At high energies electroweak corrections are dominated by mass singularities.

soft singularities



divergent if $M = 0$

collinear singularities



divergent if $m = 0$ and $M = 0$

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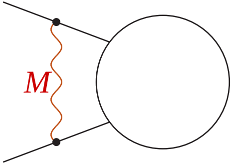

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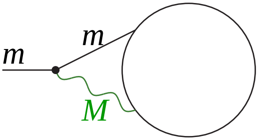
collinear singularities



divergent if $m = 0$ and $M = 0$

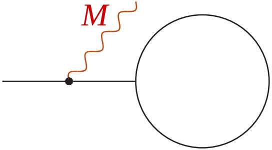
If $M \neq 0$, but $M^2 \ll s, |t|, |u|$: Divergences $\rightarrow$ Sudakov Logarithms

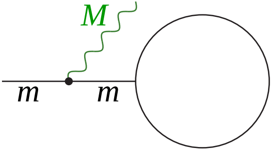
$$\frac{\alpha}{4\pi}\left[c_{sc}^{(V)} \underbrace{\log^2\left(\frac{M^2}{s}\right)}_{\text{soft + collinear}} + c_c^{(V)} \underbrace{\log\left(\frac{M^2}{s}\right)}_{\text{collinear}} + c_s^{(V)} \underbrace{\log\left(\frac{M^2}{s}\right)}_{\text{soft}} + c_0^{(V)} + \mathcal{O}\left(\frac{M^2}{s}\right)\right]\sigma_0(s)$$

Sudakov Logarithms

Similar for real emission:

soft singularities | **collinear singularities**





divergent if $M = 0$ | divergent if $m = 0$ and $M = 0$

$$\frac{\alpha}{4\pi}\left[c_{sc}^{(R)} \underbrace{\log^2\left(\frac{M^2}{s}\right)}_{\text{soft + collinear}} + c_c^{(R)} \underbrace{\log\left(\frac{M^2}{s}\right)}_{\text{collinear}} + c_s^{(R)} \underbrace{\log\left(\frac{M^2}{s}\right)}_{\text{soft}} + c_0^{(R)} + \mathcal{O}\left(\frac{M^2}{s}\right)\right]\sigma_0(s)$$

$$\underbrace{+ \frac{\alpha}{4\pi}\int \mathrm{d}z \left[\ldots\right]_+ \sigma_0(zs)}_{\text{collinear}}$$

Cancellation

$$\sigma^{(V)} = \frac{\alpha}{4\pi}\left[c_{sc}^{(V)}\log^2 + c_c^{(V)}\log + c_s^{(V)}\log + c_0^{(V)}\right]\sigma_0(s)$$

$$\sigma^{(R)} = \frac{\alpha}{4\pi}\left[c_{sc}^{(R)}\log^2 + c_c^{(R)}\log + c_s^{(R)}\log + c_0^{(R)}\right]\sigma_0(s) + \frac{\alpha}{4\pi}\int \mathrm{dz}[\dots]_+\sigma_0(zs)$$

In QED or QCD (sum over *colors*)

$c^{(V)} = -c^{(R)}$ (KLN-Theorem)

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In QED or QCD (sum over *colors*)

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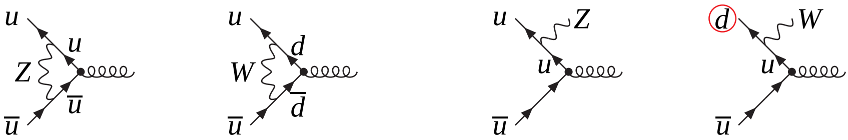
In the EW theory we must **NOT** sum over *isospin*
$\rightarrow$ Logarithms no longer cancel (Bloch-Nordsieck-Violation)



Questions

When can't we see real radiation

- *collinear* radiation into the beam pipe
- *collinear* radiation into a jet
- *soft* gauge boson decays into particles similar to the background

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Two Questions

how does the

- the group structure (BN-Violations)
- the restriction on the phase space

influence the sum of virtual and real corrections

Simplified model

- no mixing between the gauge groups (SU(2)-theory)
  $\rightarrow \quad M_W = M_Z$
- massless fermions, vectorlike coupling

We study an explicit process: $f\bar{f} \quad \rightarrow \quad f'\bar{f}'$

Full inclusive cross section at $\mathcal{O}(\alpha)$

- neutral initial state: $\sigma_{uu}: \quad u\bar{u} \quad \rightarrow \quad \sum f'\bar{f}' \; (V)$
- charged initial state: $\sigma_{du}: \quad d\bar{u} \quad \rightarrow \quad \sum f'\bar{f}' \; (V)$

Structure of Sudakov Logarithms

Correction to neutral initial state

$$\sigma_{uu}^{(V)} = \frac{\alpha}{4\pi}\left[-3\left[\log^2 + 3\log\right] - \frac{26}{3}\log\right]\sigma_{uu}^{0}(s)$$

$$\sigma_{uu}^{(R)} = \frac{\alpha}{4\pi}\left[+4\left[\log^2 + 3\log\right] + \frac{26}{3}\log\right]\sigma_{uu}^{0}(s) + \int \mathrm{dz}\left[\ldots\right]_{+}\sigma_{uu}^{0}(zs)$$

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Correction to charged initial state

$$\sigma_{du}^{(V)} = \frac{\alpha}{4\pi}\left[-3\left[\log^2 + 3\log\right] - \frac{26}{3}\log\right]\sigma_{du}^0(s)$$

$$\sigma_{du}^{(R)} = \frac{\alpha}{4\pi}\left[+\frac{5}{2}\left[\log^2 + 3\log\right] + \frac{26}{3}\log\right]\sigma_{du}^0(s) + \int dz\left[\ldots\right]_+ \sigma_{uu}^0(zs)$$

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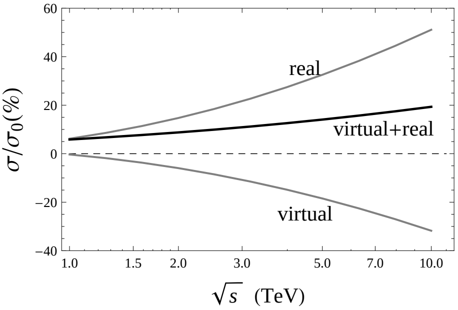
Sum of virtual and real contribution

$$\sigma_{uu} = \sigma_{uu}^{(V)} + \sigma_{uu}^{(R)} = +1\,\frac{\alpha}{4\pi}\left[\log^2 + 3\log\right]\sigma_{uu}^{0} + \int \mathrm{dz}\left[\ldots\right]_+ \sigma_{uu}^{0}(zs)$$

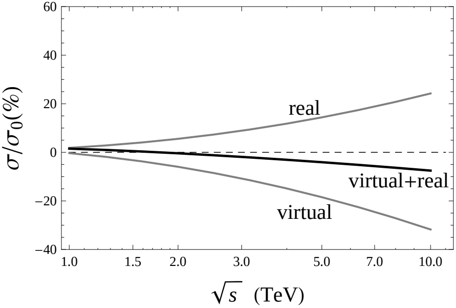
$$\sigma_{du} = \sigma_{du}^{(V)} + \sigma_{du}^{(R)} = -\frac{1}{2}\,\frac{\alpha}{4\pi}\left[\log^2 + 3\log\right]\sigma_{du}^{0} + \int \mathrm{dz}\left[\ldots\right]_+ \sigma_{uu}^{0}(zs)$$

Real and virtual corrections

neutral initial state



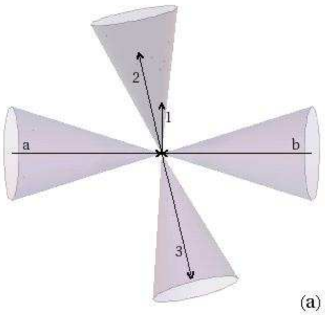
charged initial state

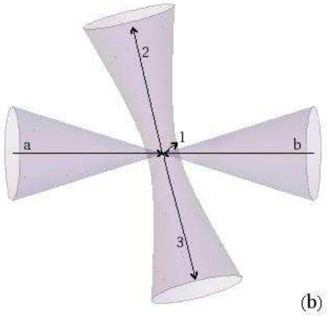


- neutral initial state: compensation possible
- charged initial state: only weakening possible

Scenario (a):
only collinear emission

Scenario (b):
collinear **and** soft emission



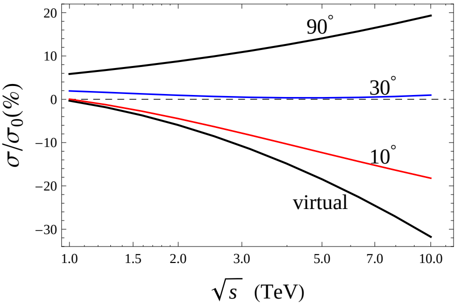


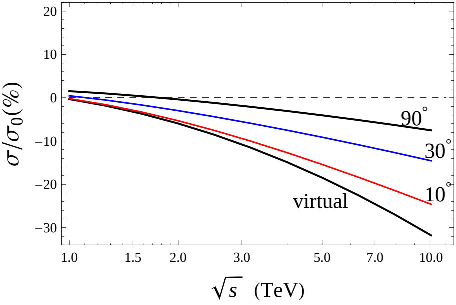
$\theta_a, \theta_b, \theta_2, \theta_3 < \theta_{cut}$

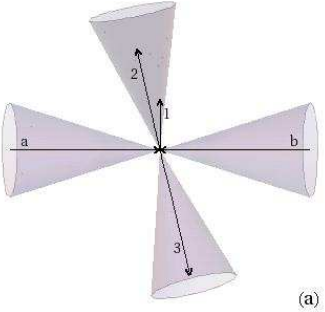
$\theta_a, \theta_b < 5^\circ$ and $\theta_{23} > 180^\circ - \theta_f$

$$E_1 < \sqrt{s}/2$$

Scenario (a): collinear radiation

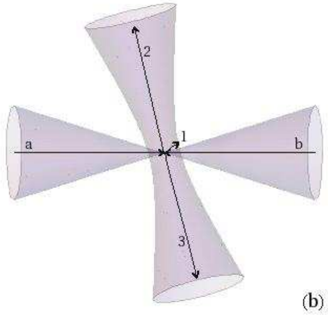




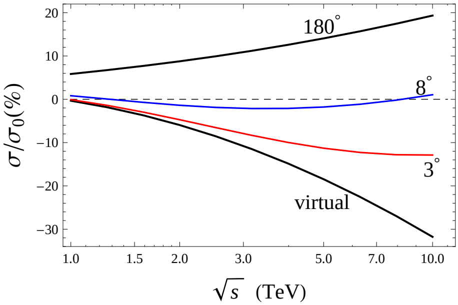


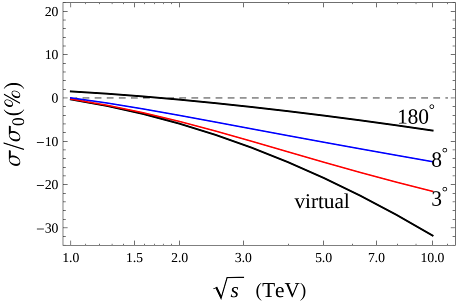
$$\theta_a, \theta_b, \theta_2, \theta_3 < \theta_{cut}$$

Scenario (b): collinear and soft radiation

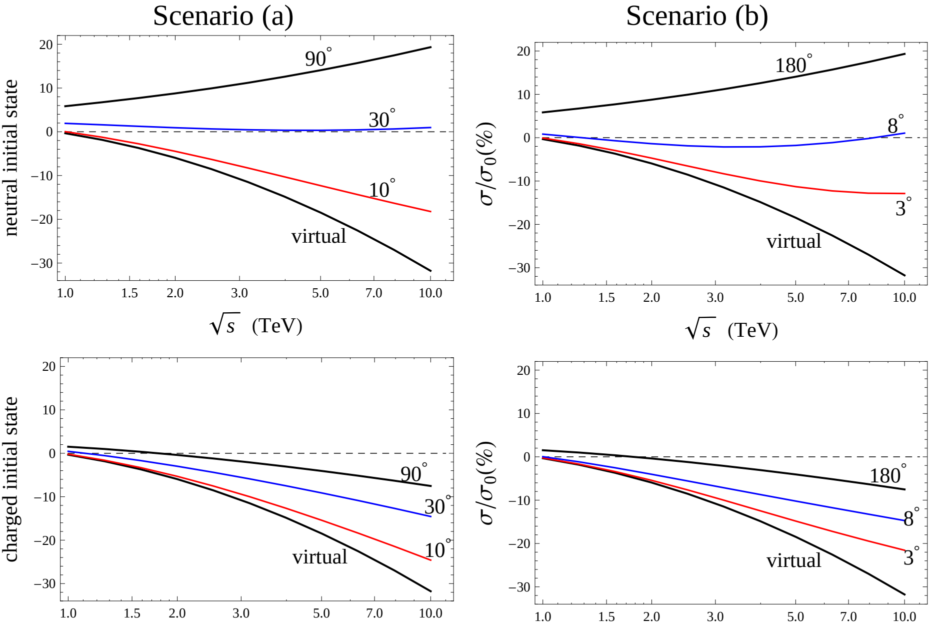


$$\theta_a, \theta_b < 5^\circ \quad \text{and} \quad \theta_{23} > 180^\circ - \theta_f$$





Scenario (a) and (b)



summary and outlook

summary

- we studied the structure of the Sudakov Logarithms in the four-fermion process
- two different restrictions on the phase space (collinear vs collinear+soft)
- combination of both gives us an idea of importance of real radiation

outlook

- calculating the four-fermion process in the standard model