Determining The Charm-Quark Mass and QCD Coupling from Moments of Current-Current Correlators

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Content

Outline

Motivation

- quark masses and strong coupling constant: fundamental parameters of the Standard Model, only input parameters of QCD Lagrangian
- Confinement $\Rightarrow$ free quark states not observed in Nature, experimental determination of quark masses only indirectly
- serve as boundary conditions of the renormalisation group equations
- pQCD and LQCD: calculations and precise measurements from first principles
  $\Rightarrow$ comparison with experiment = precision test of Standard Model
- LQCD provides control over non-perturbative effects of strong interaction — alternative approach to sum rules using experimental data
- possibility to choose from a variety of operators in LQCD to extract α_s and m_c

Input from continuum perturbation theory

- hadronic contributions to vacuum polarisation functions

$$q^2\,\Pi^p = i\int d^4x e^{iqx}\langle 0|\,T\{J^p(x)\,J^p(0)\}\,|0\rangle$$

$$\left(-q^2 g_{\mu\nu} + q_\mu q_\nu\right)\Pi^\delta + q_\mu q_\nu \Pi_L^\delta = i\int d^4x\, e^{iqx}\langle 0|\,T\{J_\mu^\delta(x)\,J_\nu^\delta(0)\}|0\rangle\,,$$

with $\delta = v, a$ $J^p = \bar\psi\gamma_5\psi$, $J_\mu^v = \bar\psi\gamma_\mu\psi$ and $J_\mu^a = \bar\psi\gamma_\mu\gamma_5\psi$

- low momentum region: expansion of $\Pi^{p,\,\delta}$ in $z = \frac{q^2}{4m_c^2(\mu)}$ in $\overline{MS}$ scheme

$$\Pi^{p,\delta}(q^2) = \frac{3}{16\pi^2}\sum_{k\geq -1}\bar C_k^{p,\delta} z^k\,,\qquad \bar C_k = \sum_{m\geq 0}\left(\frac{\alpha_s}{\pi}\right)^m \bar C_k^{(m)}\left(\log\left(\frac{m_c^2(\mu)}{\mu^2}\right)\right)$$

- coefficients for vector, axial vector and pseudoscalar correlator available up to third order in α_s (recent publications 0805:3358 [hep-ph], 0806:3405 [hep-ph])

Input from lattice QCD

- moments of renormalised correlators from charmed currents at zero spatial momentum:

$$C^{p,\delta}(t) = a^6 \sum_{\vec{x}} \langle J_c^{p,\delta}(\vec{x},t) J_c^{p,\delta}(\vec{0},0)\rangle$$

$$G_n^{p,\delta} = \sum_{t/a=-N_t/2+1}^{N_t/2-1} \left(\frac{t}{a}\right)^n C^{p,\delta}(t)\,,$$

with $J_c^{p,\delta} = \bar{\psi}_c \Gamma^{p,\delta} \psi_c$, $\Gamma^p = \gamma_5$, $\Gamma^v = \sum_i \gamma_i$, $\Gamma^a = \gamma_0\gamma_5$

- cut-off independence and dimensional analysis imply

$$G_2^{p,\delta} = g_2^{p,\delta}(\alpha_s(\mu), m_c(\mu)/\mu) + \mathcal{O}((a\,m_c)^m)$$

$$G_n^{p,\delta} = \frac{g_n^{p,\delta}(\alpha_s(\mu), m_c(\mu)/\mu)}{(am_c(\mu))^{n-2}} + \mathcal{O}((a\,m_c)^m)\,,$$

with $n > 2$, exponent m depends on lattice discretization scheme

Analysis using data from lattice QCD (cont.)

Aim:

1. extrapolate non-perturbatively measured, renormalized lattice moments to zero lattice spacing and zero light quark mass
2. match with perturbative expansion of continuum moments renormalised in $\overline{MS}$ scheme
3. $\Rightarrow$ extract $m_c(\mu)$ and $\alpha_s(\mu)$

- quenched analysis by Bochkarev and de Forcrand (hep-lat/9505025)
- analysis with dynamical u, d, s quark and Highly Improved Staggered Quark action by HPQCD (0805.2999v2 [hep-lat]) $\Rightarrow m_c$ and α_s with uncertainty of $\mathcal{O}(1\%)$ (combined statistical and systematic)

Outline

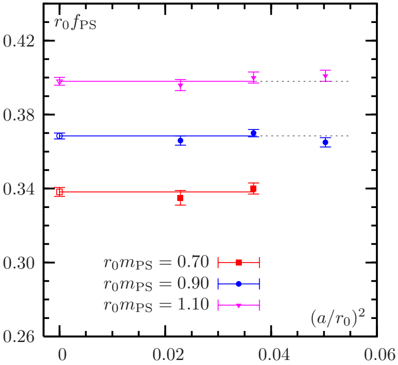
Twisted Mass Lattice QCD (JHEP 0108:058,2001)

- Wilson-type fermion discretisation for $n_f = 2$ mass degenerate quark flavours *up*, *down*:

$$\mathcal{S}_{tm} = a^4 \sum_x \bar{\psi}(x) \left[D_W + m_0 + i\mu_f \gamma_5 \tau^3 \right] \psi(x) \, ;$$

- m_0 bare (untwisted) quark mass, μ_f twisted mass parameter
- automatic $\mathcal{O}(a)$ improvement of physical observables, if $m_0 \to m_{cr}$ $\Leftrightarrow$ "maximal twist" (JHEP 0108:058,2001)
- bare quark mass given by μ_f alone
- no charm in the sea, but heavy charm doublet added in valence sector $\Rightarrow$ partially quenched analysis

Twisted Mass Lattice QCD — $\mathcal{O}(a)$ improvement



- numerically well established in the light sector (e.g. scaling of f_{PS} PoS LAT2007:022,2007)
- in our case, e.g. $G_2^{p,\delta}(a, \mu_l, \mu_c) = g_2^{p,\delta}(\alpha_s(\mu), m_c(\mu)/\mu) + c_2\,(am_c)^2$
- possibly severe $\mathcal{O}(a^2)$ effects due to $a \sim 0.33 - 0.51\,\text{GeV}^{-1}$ but $m_c \sim 1\,\text{GeV}$

Twisted Mass Lattice QCD — Ensembles

- twisted mass configurations with $n_f = 2$ at maximal twist available via LDG
- three lattice spacings $a/\text{fm} = 0.0995(7),\ 0.0855(5),\ 0.0667(5)$
- analysis for fourth lattice spacing $a \approx 0.055\,\text{fm}$ in preparation
- spatial lattice size $L \gtrsim 2\,\text{fm}$
- for each lattice spacing several values of μ_l s.t.
  $m_{PS} = 300 \sim 600\,\text{MeV}$
  $\Rightarrow \mu_l$ dependence of lattice moments
- for each pair (a, μ_l) several μ_c covering physical point in $m_{\eta_c},\ m_{J/\psi}$
  $\Rightarrow$ interpolation at physical point
- ≈ 240 and ≈ 150 independent gauge configurations on $24^3 \times 48$ and $32^2 \times 64$, repsectively

Twisted Mass Lattice QCD — Renormalisation

- study $P-P$, $A_0 - A_0$ (η_c meson) and $V_i - V_i$ (J/ψ meson) correlators
- local currents $\bar{\psi}(x)\Gamma^{p,\delta}\psi(x)$ not conserved on the lattice

$\Rightarrow$ either need (non-perturbative) renormalisation factors Z_A, Z_V, Z_P/Z_S

$\Rightarrow$ or consider RG invariant combinations of correlation functions, e.g. ratios of moments $G_n^{p,\delta}/G_m^{p,\delta}$ of same Lorentz structure

- renormalisation factors Z_A, Z_V, Z_P/Z_S by ETMC from LQCD
- $\Rightarrow$ large class of operators to extract α_s, m_c in twisted mass LQCD

Outline

Reducing finite a errors

- start with $G_n = \sum_{t/a} a^6 C(t, \vec{p} = \vec{o}) \, (t/a)^n$ s.t.

$$G_n = \frac{g_n(\alpha_s(\mu), m_c(\mu)/\mu))}{(am_c(\mu)^{n-2}} + \alpha_s^0 \left((am_c)^m + \ldots\right) + \alpha_s^1 \left((am_c)^m + \ldots\right)$$

- devide by lowest order lattice perturbation theory contribution:

$$G_n/G_n^{(0)} = \frac{g_n}{g_n^{(0)}} \left(\frac{m_{pole\ c}^{(0)}}{m_c(\mu)} \right)^{n-2} + \alpha_s^1 \left((am_c)^m + \ldots\right)$$

$\rightarrow$ cancellation of explicit factors of the lattice spacing
$\rightarrow$ cancellation of finite a corrections to all orders in a and α_s^0

Suppressing errors in the bare charm quark mass

- multiplication of $G_n/G_n^{(0)}$ by $(am_{\eta_c}/2am^{(0)}_{pole\ c})^{n-2}$ (hep-lat/9404012)
- modifies leading $m_c(\mu)$ dependence:

$$\frac{G_n}{G_n^{(0)}}\left(\frac{am_{\eta_c}}{2a\,m^{(0)}_{pole\ c}}\right)^{n-2} = \frac{g_n}{g_n^{(0)}}\left(\frac{m_{\eta_c}}{2m_c(\mu)}\right)^{n-2} + \mathcal{O}((am_c)^m\alpha_s)\,;$$

  l.-h. s.: am_{η_c} measured in LQCD; r.-h. s.: $m_{\eta_c} \to m^{exp}_{\eta_c}$ (same for $m_{J/\psi}$)
- reduced sensitivity on (small) shifts in $a\mu_c$ due to correlation of am_{η_c} and $a\mu_c$

Definition of reduced moments R_n

$$R_n = \begin{cases} G_2/G_2^{(0)} & \text{for } n = 2 \\ \left(G_n/G_n^{(0)}\right)^{1/(n-2)} \left(\frac{am_{\eta_c}}{2am^{(0)}_{pole\ c}}\right) & \text{for } n \geq 4 \end{cases}$$

implying the relation to continuum quantities in the $\overline{MS}$ scheme

$$R_n = \begin{cases} r_2(\alpha_s, m_c/\mu) + \mathcal{O}((am_c)^m \alpha_s) & \text{for } n = 2 \\ r_n(\alpha_s, m_c/\mu) \frac{m_{\eta_c}}{2\, m_c(\mu)} + \mathcal{O}((am_c)^m \alpha_s) & \text{for } n \geq 4 \end{cases}$$

r_n are related to coefficients of the polarisation functions $\Pi(q^2)$ via

$$r_{2k+2} = \left(\frac{\bar{C}_k}{\bar{C}_k^{(0)}}\right)^{\frac{1}{2k}}$$ as 3rd degree polynomial in α_s

Final formulae for m_c and α_s

- obtain $m_c(\mu)$ as solution of equations:

$$m_c(\mu) = \frac{m_{\eta_c}^{exp}}{2} \frac{r_n(\alpha_s(\mu), m_c(\mu)/\mu)}{R_n} \quad \text{for } n \geq 4$$

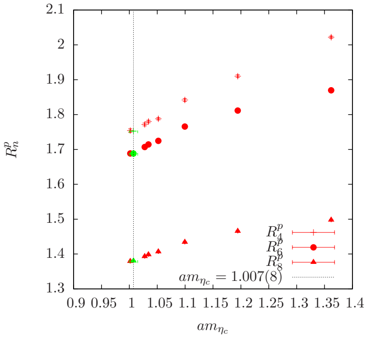
  with α_s fixed
- defining equations for α_s as solution of:

$$R_2 = r_2(\alpha_s(\mu), m_c(\mu)/\mu)$$

  for fixed $m_c(\mu)$
- scale μ corresponds to meson masses m_{η_c}, $m_{J/\psi} \approx 3\,\text{GeV}$
- analogoue formulae for α_s from R_n/R_{n+2} and m_c from $R_n \sim \sqrt{G_n/G_{n-2}}$

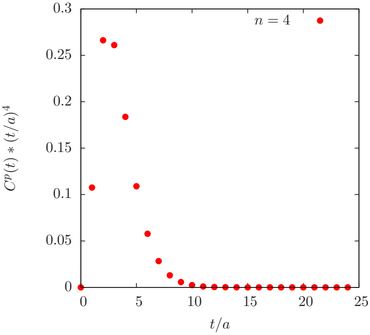
Outline

Interpolation of reduced moments



- am_{η_c} −dependence of R_n^p, $n = 4, 6, 8$ at $a = 0.0667\,\mathrm{fm}$, $a\mu_{sea} = 0.003$, interpolation point marked by dashed line
- uncertainty of interpolation point induced by $a \lesssim 0.7\%$
- despite suppression of tuning errors significant slope in the neighbourhood of the interpolation point
- higher moments $R_n \sim am_{\eta_c}/a\mu_c$ dominant behaviour

Finite T cutoff — restriction on high moments



- for large t $C^p(t)(t/a)^n \sim \mathrm{e}^{-m_{\eta_c} t}\, t^n$
- dominant support moves to larger t with growing n $\longrightarrow$ growing truncation effects in $t-$sum
- with $am_{\eta_c} \gtrsim 1$ and $N_t = 48$ negligable effects up to 10th, 12th moment

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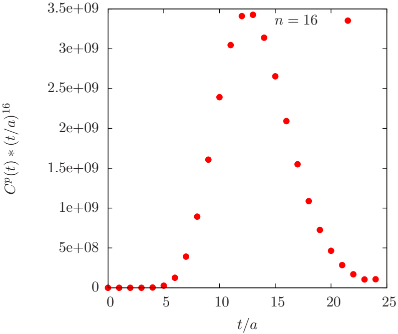

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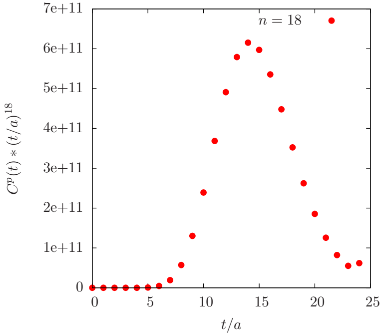

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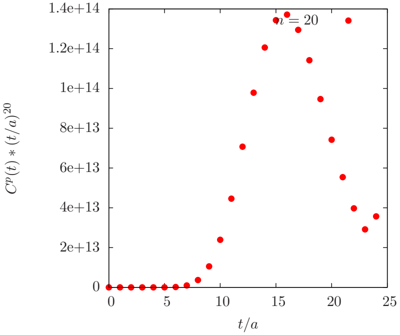

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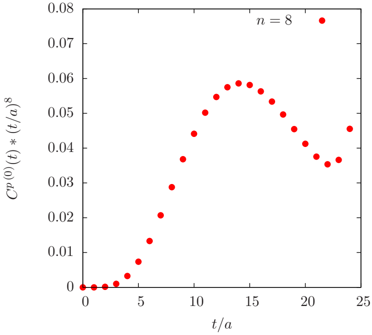
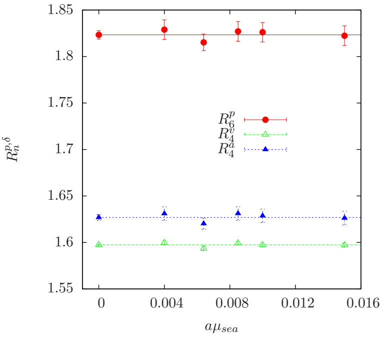
Finite T cutoff at tree-level



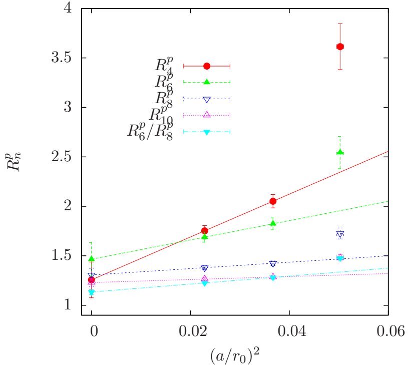
- η_c meson much lighter than in dynamical case $am_{\eta_c} \approx 2a\mu_c$
- severe cutoff already for moment no. 8
- $\Rightarrow$ moments calculated on $N_s^3 \times \mathbb{Z}$ lattice (Nucl.Phys.B800:94-108,2008)

Sea quark mass dependence — $\lim_{m_{u,d} \to 0}$



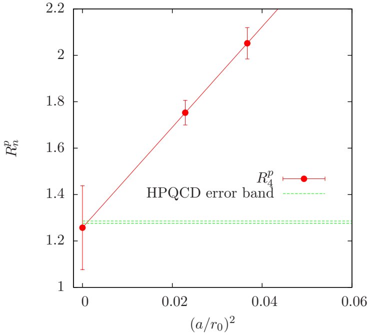
- at present level of accuracy no significant systematic dependence on $a\mu_{sea}$ observable
- fitted to a constant for all moments from all correlators (cf. HPQCD ansatz $R_n(a) \sim Rn(0)(1 + f_{n,1}(2m_{u/d} + m_s)/m_c + \ldots)$ motivated by χPT

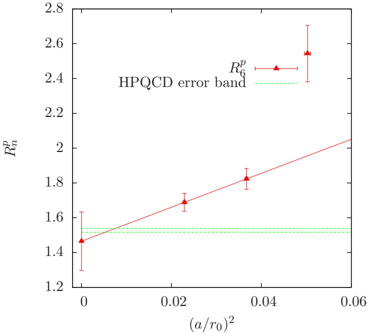
Continuum extrapolation — $\lim_{a \to 0}$



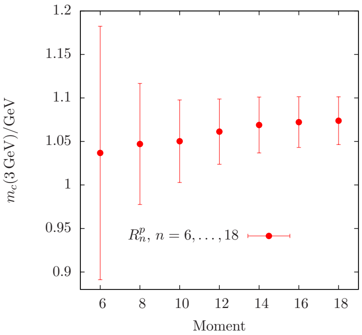
- reproduction of HPQCDs Fig. 1 of 0805:2999 [hep-lat];
- low moments inherit larger uncertainties from Z factor
- if $\mathcal{O}(a)$ term negligable, then apparently huge $\mathcal{O}(a^2)$ terms (slope of straight line)
- requires additional lattice spacing: $a = 0.055\,\mathrm{fm}$ already provided by ETMC, analysis in progress

Continuum extrapolation — comparison with HPQCD results for $R^p_{4,\,6}$



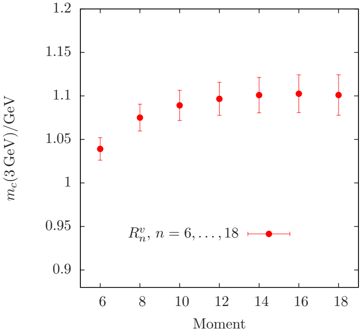


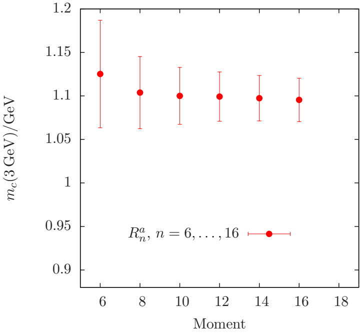
Preliminary results for the charm quark mass — R^p



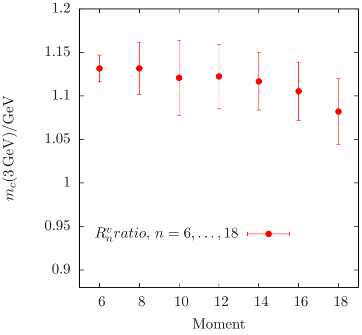
- fig. shows $m_c(\mu = 3\,\mathrm{GeV})$ obtained from matching R_n^p to pQCD expansion for $n = 6, \ldots, 18$
- strong coupling set to $\alpha_{\overline{MS}}(n_f = 4, \mu = 3\,\mathrm{GeV}) = 0.252(10)$ to extract m_c
- central values approx. 10% above HPQCD results
- truncation error from perturbative series not included

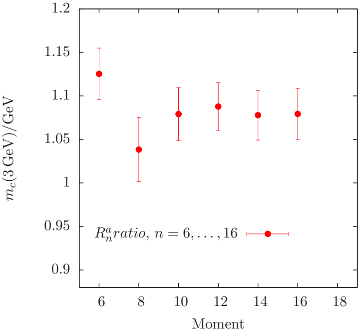
Preliminary results for the charm quark mass — R^v, R^a





Preliminary results for the charm quark mass — for R^v, R^a ratios





Conclusions and Outlook

- calculated $m_c(\mu)$ from current-current correlators by matching LQCD to pQCD
- (reduced) moments from pseudoscalar, vector and axial vector currents as well as ratios of these give compatible results, but at present $\approx 10\%$ larger values
- check of continuum extrapolation with additional lattice spacing
- use advanced χPT motivated combined extrapolation $(a,\, m_{u,\,d}) \rightarrow (0,\, 0)$
- neglect vacuum polarisation effects due to *strange* and *charm* quark ($n_f = 2$, partially quenched); possibly use ETMC's $n_f = 2 + 1 + 1$ configurations
- large uncertainties of lower moments from renormalisation constants (yet results agree with those from RGI moments), maybe look for additional RGI combinations of lattice operators in the charm sector
- analysis for α_s (requires m_c as input)

Thank you very much for your attention.
Farewell, till our next meeting.