Chirally rotated Schrödinger Functional: first checks at tree-level of PT

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SFB - A4

- **Lattice regularisation:** $V = L^3 \times T$ **, spacing** a
  - inverse lattice spacing $1/a$: ultraviolet cutoff
  - observables on the lattice:

$$\langle \mathcal{O} \rangle_{\rm latt} = \langle \mathcal{O} \rangle_{\rm cont} + \text{cutoff effects}$$

  - remove cutoff $\Leftrightarrow$ continuum limit: $a \to 0$
  - reach continuum limit as fast as possible:

$$\langle \mathcal{O} \rangle_{\rm latt} = \langle \mathcal{O} \rangle_{\rm cont} + \mathrm{O}(a^2)$$

better than

$$\langle \mathcal{O} \rangle_{\rm latt} = \langle \mathcal{O} \rangle_{\rm cont} + \mathrm{O}(a)$$

- **Big effort to find a lattice regulator with leading $\mathrm{O}(a^2)$ effects**
  - add counterterms to remove the $\mathrm{O}(a)$
  - get leading $\mathrm{O}(a^2)$ automatically

- **Our lattice action: Wilson twisted mass fermions at maximal twist**
  - automatic $\mathrm{O}(a)$ improvement (R. Frezzotti and G.C. Rossi, hep-lat/0306014)

$$\langle \mathcal{O} \rangle_{\mathrm{latt}} = \langle \mathcal{O} \rangle_{\mathrm{cont}} + \mathrm{O}(a^2)$$

- **Many observables need to be renormalised**
- **Want renormalisation scheme which:**
  - is non-perturbative
  - is mass independent: massless renormalisation
  - keeps automatic $\mathrm{O}(a)$ improvement
- **Final goal: twisted mass lattice QCD simulations for $N_f = 2+1+1$**

  (dynamical up, down, strange and charm)

- **Schrödinger Functional schemes:** (M. Lüscher et al., hep-lat/9207009)
  - finite space-time volume: $V = L^3 \times T$
    - can use $\frac{1}{L}$ as renormalisation scale
    - finite size techniques: non-perturbative running with $1/L$
  - boundary conditions:
    - periodic in spatial directions
    - Dirichlet in time direction: (S. Sint, hep-lat/9312079), (M. Lüscher, hep-lat/0603029)

$$P_+\psi(x)|_{x_0=0} = 0 \qquad P_-\psi(x)|_{x_0=T} = 0$$

$$P_\pm = \frac{1}{2}\left(\mathbb{1} \pm \gamma_0\right)$$

$\Longrightarrow$ non-zero bound in the spectrum of the Dirac operator $(1/2T)$

- **Lattice regularisation and Schrödinger Functional b.c:**
  - non-zero bound in eigenvalue spectrum of the Dirac operator

    $\Longrightarrow$ allows lattice simulations at the chiral point

    $\Longrightarrow$ **non-perturbative** and **massless renormalisation** of QCD
  - General problem of SF schemes (with lattice regularisation)

    $\mathrm{O}(a)$ effects from the boundaries

    $\Longrightarrow$ add boundary counterterms

- Standard Schrödinger Functional b.c break chiral symmetry

  $\Longrightarrow$

  Wilson twisted mass fermions and standard SF b.c:

  Incompatible with bulk **automatic** $O(a)$ **improvement**

- **Let's try with a chiral rotation of the standard SF formulation**

  (S. Sint, hep-lat/0511034)

- Chiral rotation of the the quark fields (with maximal twist):

$$\psi(x) \rightarrow e^{i\frac{\alpha}{2}\gamma_5\tau^3}\,\psi(x) \qquad \bar{\psi}(x) \rightarrow \bar{\psi}(x)\,e^{i\frac{\alpha}{2}\gamma_5\tau^3} \quad ; \quad \alpha = \pi/2$$

- **Chirally rotated SF boundary conditions**

(S. Sint, hep-lat/0511034), (S. Sint, talk at Lattice 2008)

$$Q_+\psi(x)|_{x_0=0} = 0 \qquad Q_-\psi(x)|_{x_0=T} = 0$$

$$Q_\pm = \frac{1}{2}\left(\mathbb{1} \pm i\gamma_0\gamma_5\tau^3\right)$$

  - in the continuum it is a change of basis: same theory as SF
  - $\gamma_5\tau^1$–symmetry of the b.c: automatic $\mathrm{O}(a)$ improvement

- Fermion lattice action of a theory with boundaries:
  - want (massless) Wilson fermions in the bulk
  - need to define the lattice action near the desired time boundaries
  - use orbifold techniques (Y. Taniguchi, hep-lat/0412024)

- Get: desired boundary conditions at tree-level up to $\mathrm{O}(a)$ effects

(S. Sint, private comunication)

$$Q_+(1 - \frac{1}{2}a\partial_0^*)\psi(x)|_{x_0=0} = 0 \qquad Q_-(1 + \frac{1}{2}a\partial_0)\psi(x)|_{x_0=T} = 0$$

- **Relevant (d=3) boundary operator**

  - $\gamma_5\tau^1$-odd: breaking of flavour and parity symmetries
  - bulk $\mathrm{O}(a)$ effects $\Leftrightarrow$ loose bulk automatic $\mathrm{O}(a)$ improvement
  - add $d = 3$ finite boundary counterterm with coefficient $\mathcal{Z}_f$

$$\delta S = (\mathcal{Z}_f - 1)a^3 \sum_{\vec{x}} \left(\bar{\psi}\psi|_{x_0=0} + \bar{\psi}\psi|_{x_0=T}\right)$$

  - need to **tune** $\mathcal{Z}_f$ to restore the symmeries

    $\Longrightarrow$ bulk automatic $\mathrm{O}(a)$ improvement
  - **non-perturbative tuning**
    - massless scheme: $m_0 \to m_c$
    - bulk improvement: $\mathcal{Z}_f \to \mathcal{Z}_f^*$

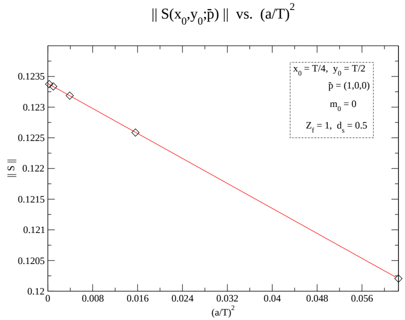
- **Irrelevant (d=4) boundary operator**
  - boundary $\mathrm{O}(a)$ effects
  - add $d = 4$ boundary counterterm with coefficient d_s

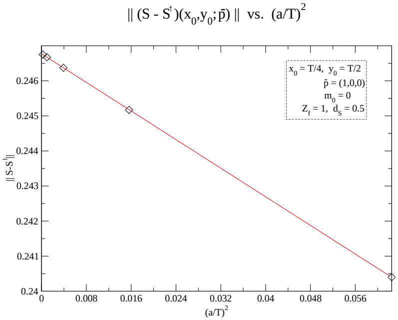
$$\delta S = a(d_s - 1)a^3 \sum_{\vec{x}} \left(\bar{\psi}\gamma_k D_k \psi|_{x_0=0} + \bar{\psi}\gamma_k D_k \psi|_{x_0=T}\right)$$

  - tune d_s to cancel the $\mathrm{O}(a)$ boundary effects
  - **perturbative tuning**
    - ★ first need to find the tree-level value: d_s^0 ($d_s = d_s^0 + d_s^1 \mathrm{O}(g_0^2)$)

- **Theory with χSF b.c. is automatic $\mathrm{O}(a)$ improved at tree-level if:**
  - $m_0 = 0$
  - $\mathcal{Z}_f = 1$
  - $d_s^0 = \frac{1}{2}$

- Example:





- Definitions:

$$g_A^{ab}(x_0)_\pm = -\langle A_0^a(x)\mathcal{Q}_\pm^b\rangle \qquad g_P^{ab}(x_0)_\pm = -\langle P^a(x)\mathcal{Q}_\pm^b\rangle$$

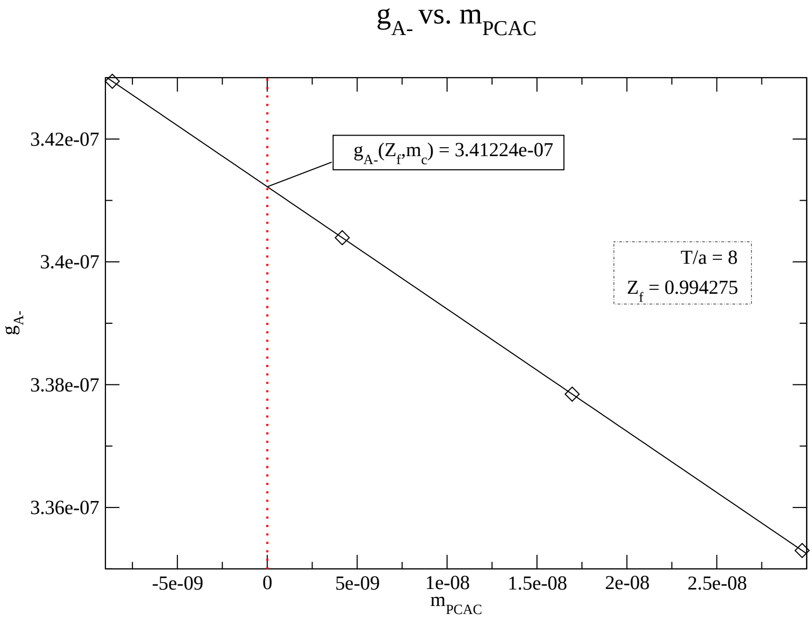
$$\mathcal{Q}_\pm^a = a^6 \sum_{\vec{y},\vec{z}} \bar{\zeta}(\vec{y})\gamma_5 \frac{1}{2}\tau^a Q_\pm \zeta(\vec{z})\, e^{i\vec{p}(\vec{y}-\vec{z})}$$

$$\zeta(\vec{x}) = \psi(x)|_{x_0=a} \qquad \bar{\zeta}(\vec{x}) = \bar{\psi}(x)|_{x_0=a}$$

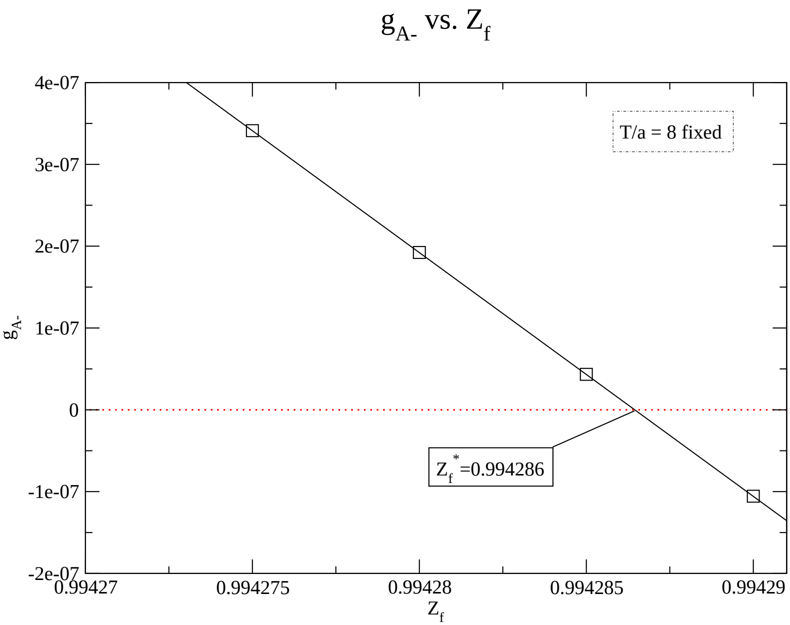
- Set: $d_s = 1/2$

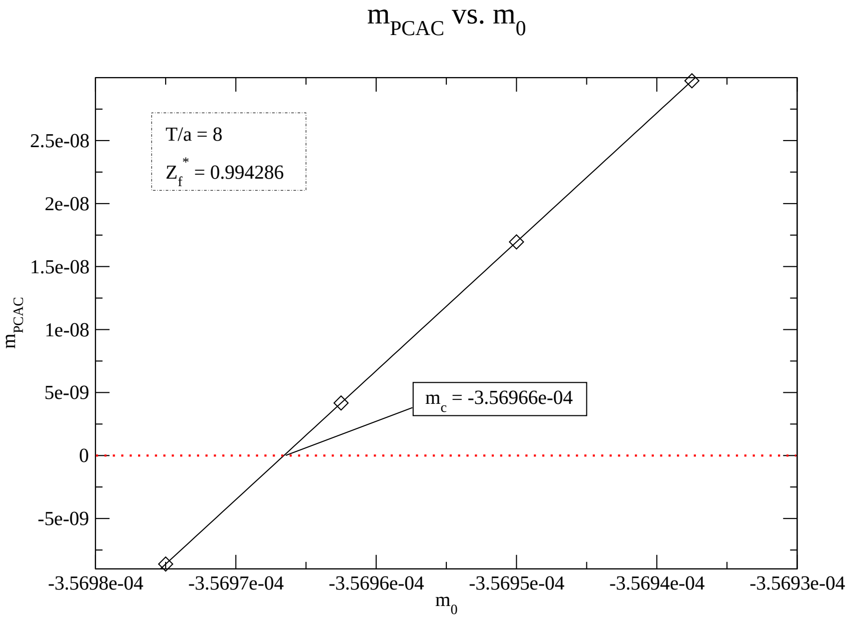
- Tuning of m_0: $m_{PCAC,-} \equiv \dfrac{\partial_0 g_A(T/2)_-}{2g_P(T/2)_-} = 0$

- Tuning of Z_f: $g_A(T/2)_- = 0$



g_{A-} vs. m_{PCAC}
$g_{A-}(Z_f,m_c) = 3.41224e-07$
T/a = 8
$Z_f = 0.994275$
3.42e-07
3.4e-07
3.38e-07
3.36e-07
g_{A-}
-5e-09
0
5e-09
1e-08
1.5e-08
2e-08
2.5e-08
m_{PCAC}


g_A- vs. Z_f
4e-07
3e-07
2e-07
1e-07
0
-1e-07
-2e-07
g_A-
T/a = 8 fixed
Z_f^* =0.994286
0.99427
0.994275
0.99428
0.994285
0.99429
Z_f


m_PCAC vs. m_0
T/a = 8
Z_f^* = 0.994286
m_c = -3.56966e-04
2.5e-08
2e-08
1.5e-08
1e-08
5e-09
0
-5e-09
m_PCAC
-3.5698e-04
-3.5697e-04
-3.5696e-04
-3.5695e-04
-3.5694e-04
-3.5693e-04
m_0

- **The use of proper tuning conditions and the precise non-perturbative determination of m_c and Z_f^* are needed for**
  - computing bulk automatic $\mathrm{O}(a)-$improved observables from our lattice twisted mass simulations
  - universality: obtain the correct theory in the continuum limit

$\Longrightarrow$

proper setup for non-perturbative renormalisation needed for many physical quantities: moments of PDFs, form factors, ...

- **At tree-level the theory is automatic** $\mathrm{O}(a)$ improved if
  - $m_0 = 0$
  - $\mathcal{Z}_f = 1$
  - $d_s = 1/2$

- **Short-time goal:**
  - first quenched studies
  - lattice perturbation theory

- **Real goal:**

  extend this to dynamical simulations with $N_f = 2 + 1 + 1$