The story of a simpler universe

Enrico Pajer

Universiteit Utrecht

Delta Institute for Theoretical Physics

Outline

- Introduction: how the universe just got simpler
- The conformal limit of inflation
- Conclusion

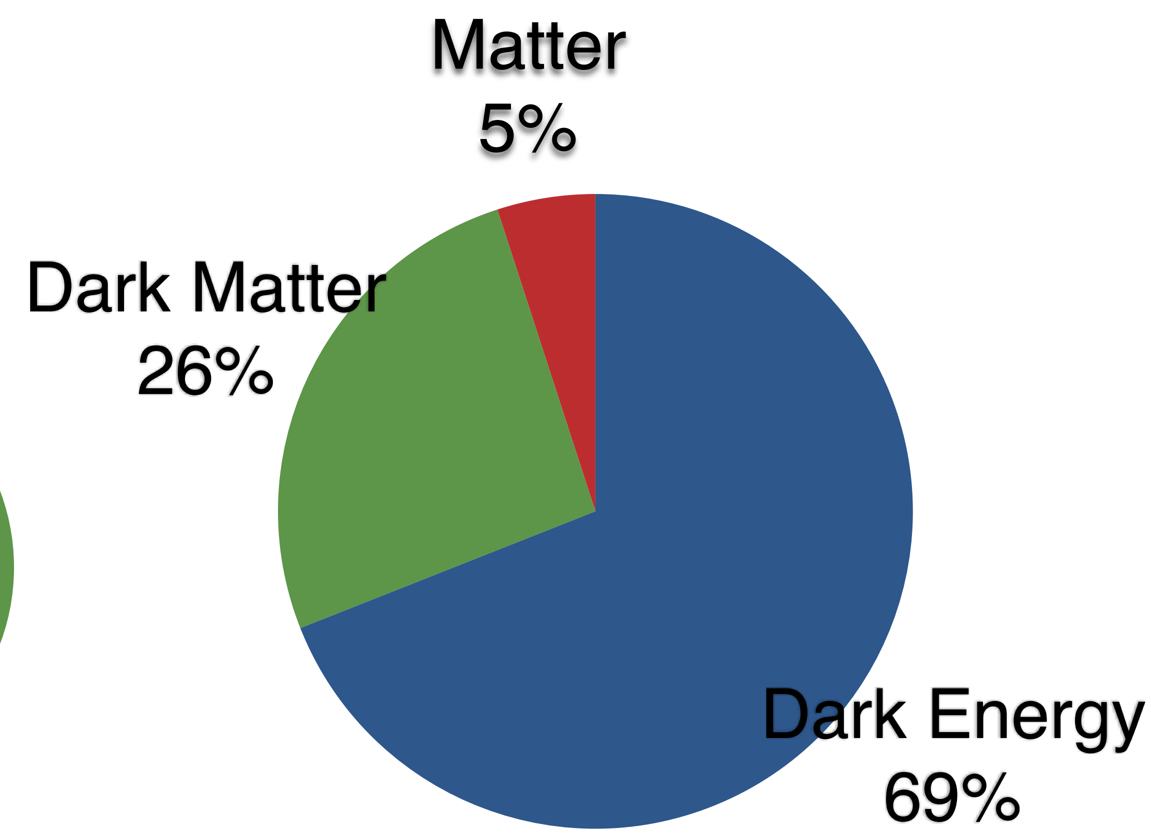
Introduction

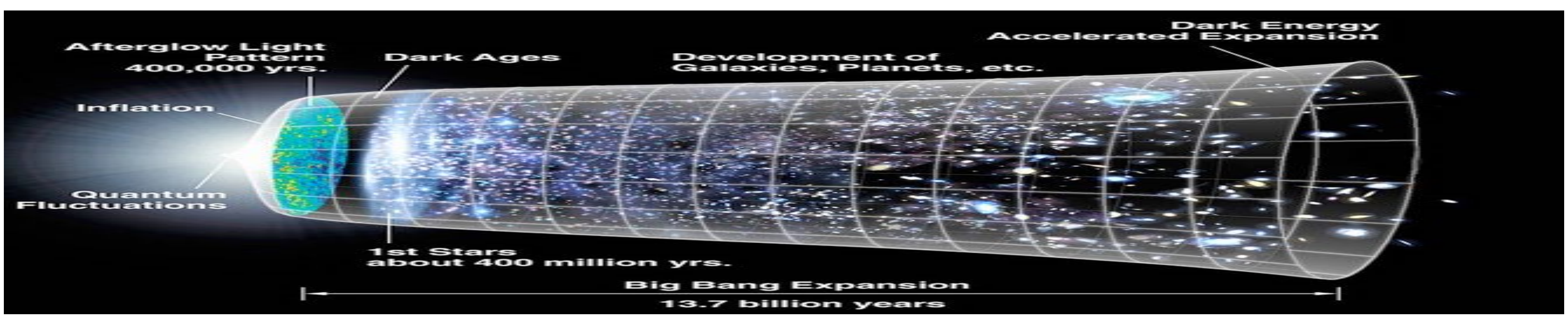
- Cosmic pies
- Cosmic pies recipe book
- No news is interesting news

Cosmic pies

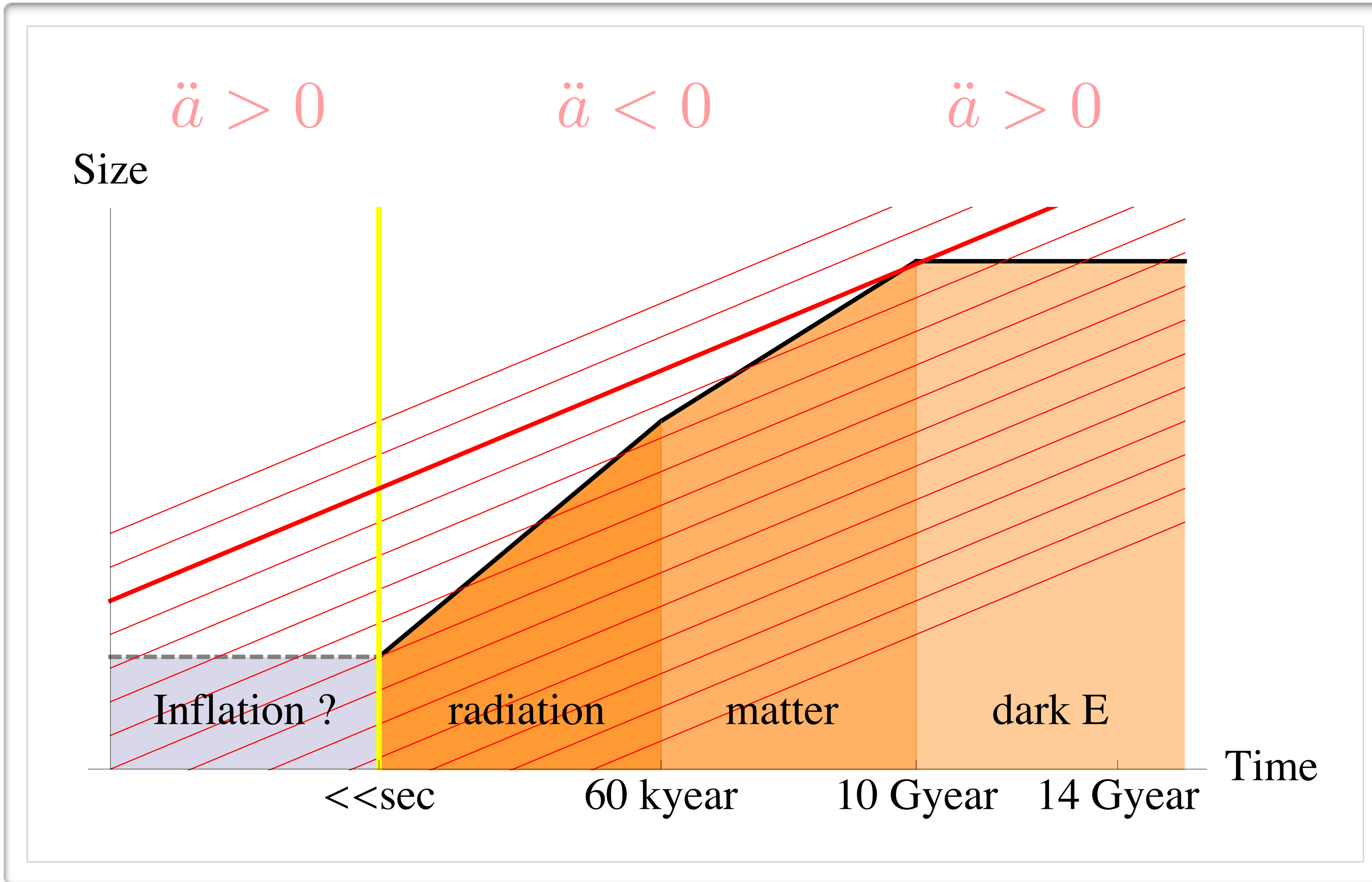
Radiation
100%

Matter
100%

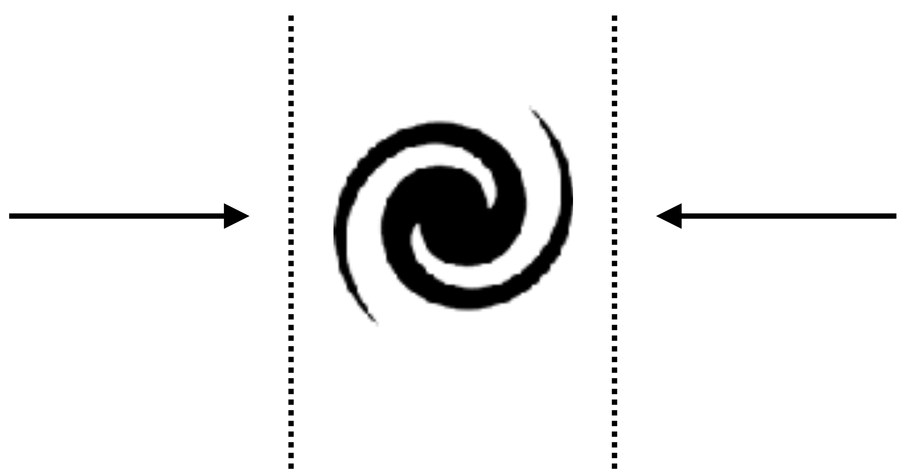




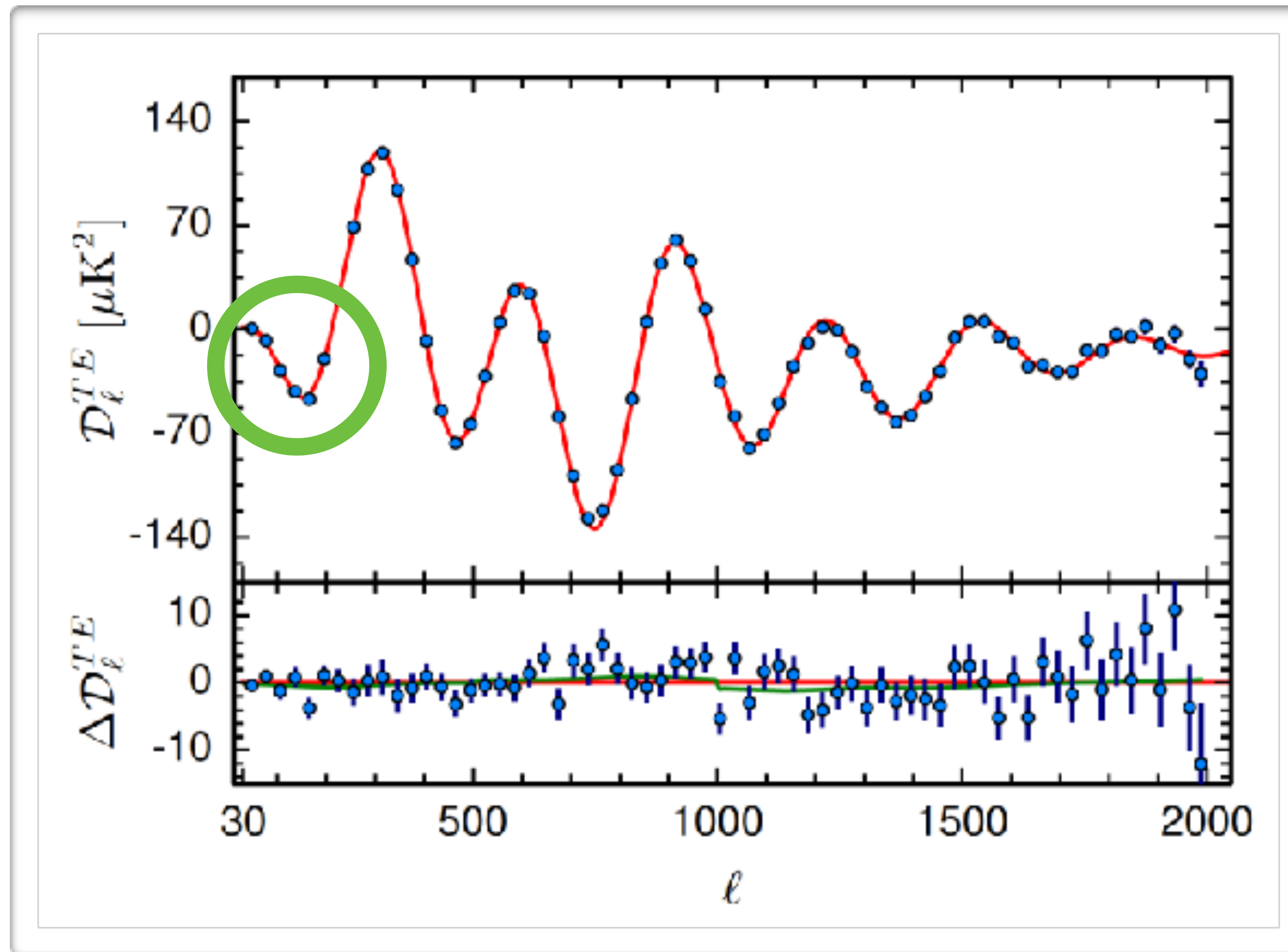
Cosmic (pie) evolution



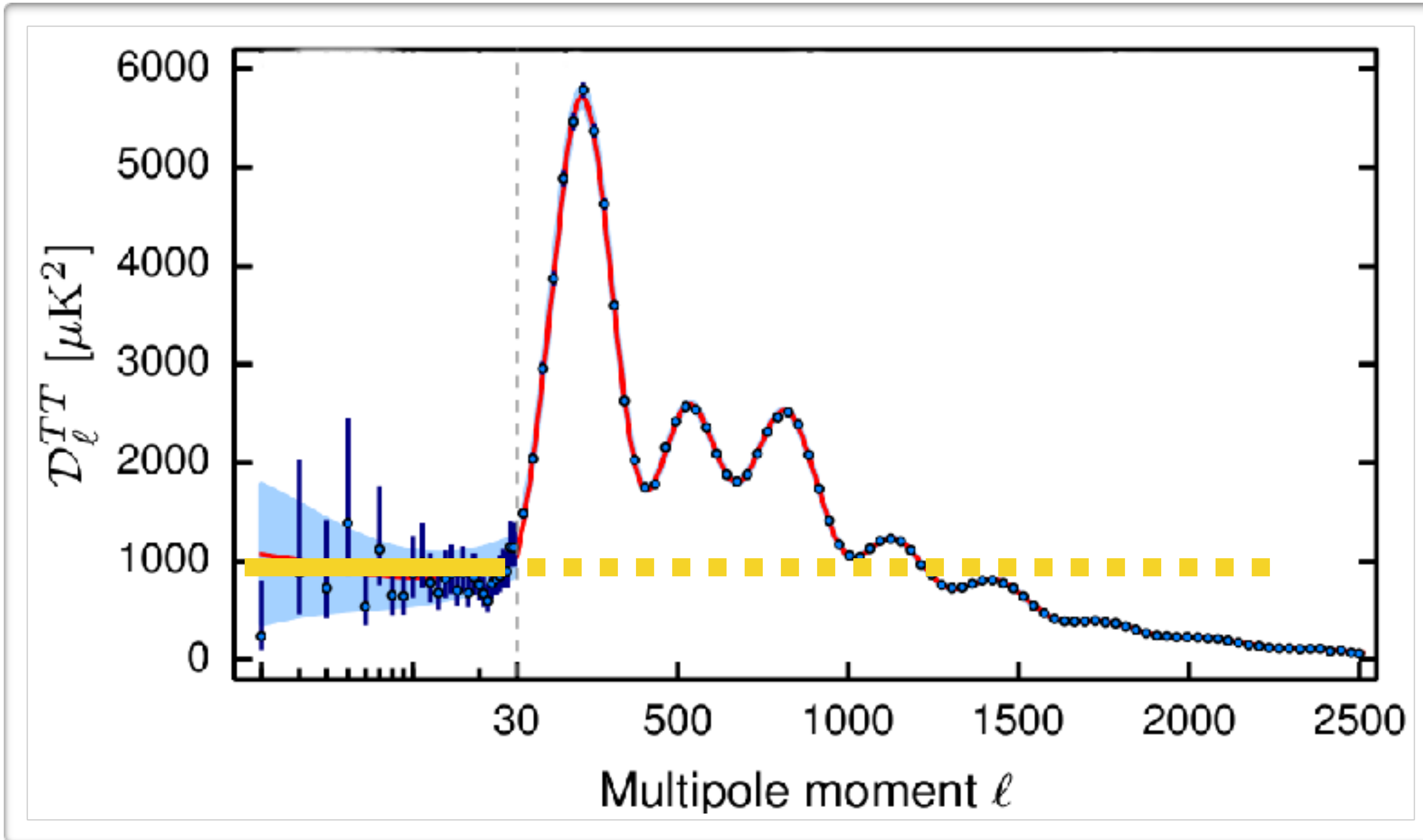
Cosmic recipe book, or how we learned to love inflation



Out-of-phase, coherent waves. Primordial explanation!



Scale invariance



Scale invariance emerges naturally from *de Sitter isometries*

Inflation

- A phase of quasi-de Sitter (accelerated expansion) in the first fraction of a second

- At least one (new) field, the *inflaton*

- At least three *new* (independent?) scales

    - Hubble Rate of expansion: H
    - Rate of deceleration: $\epsilon \equiv \frac{\dot{H}}{H^2}$
    - Rate of deceleration or inflaton mass: $\eta \equiv \frac{\dot{\epsilon}}{\epsilon H}$

Primordial power spectra

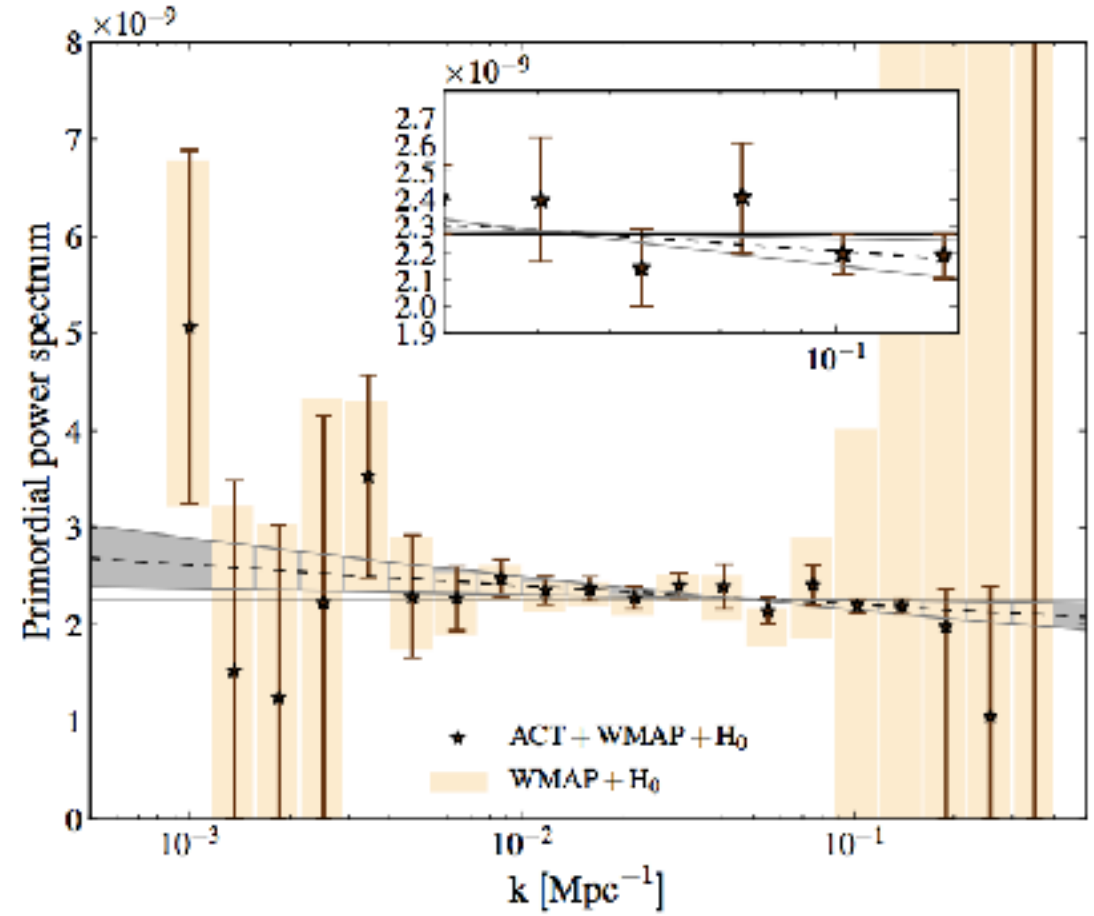
- Describe the free theory on time-dep background
- Scalars, a.k.a. *curvature* perturbation

$$\langle g_{ii}(k) g_{ii}(k) \rangle \sim \frac{H^2}{\epsilon M_{Pl}^2} \frac{1}{k^{3+(1-n_s)}}$$

- Tensors: aka *gravitational waves*

$$\langle g_{ij}^{TT}(k) g_{ij}^{TT}(k) \rangle \sim \frac{H^2}{M_{Pl}^2} \frac{1}{k^{3+8\epsilon}}$$

Observations

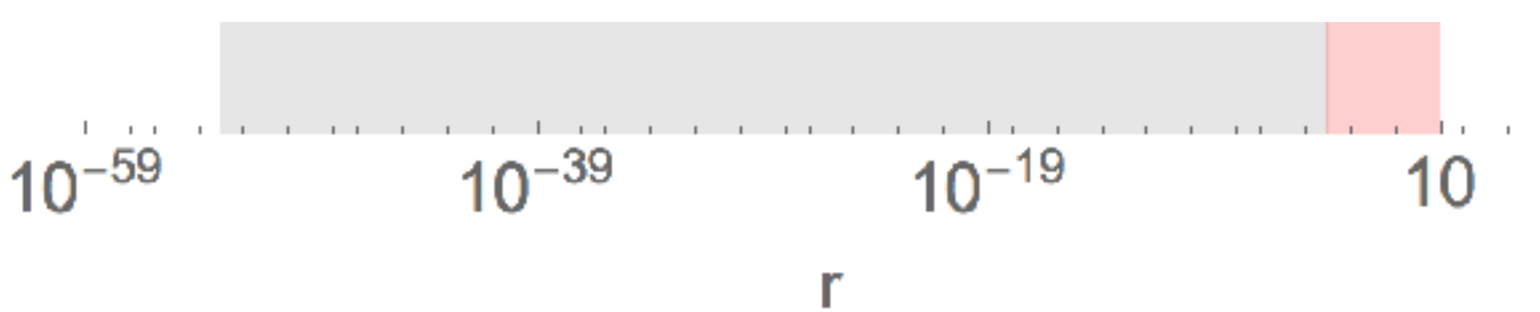


Observed:

- Scalar amplitude
- scalar tilt: $1 - n_s = 2\epsilon + \eta = 0.035$

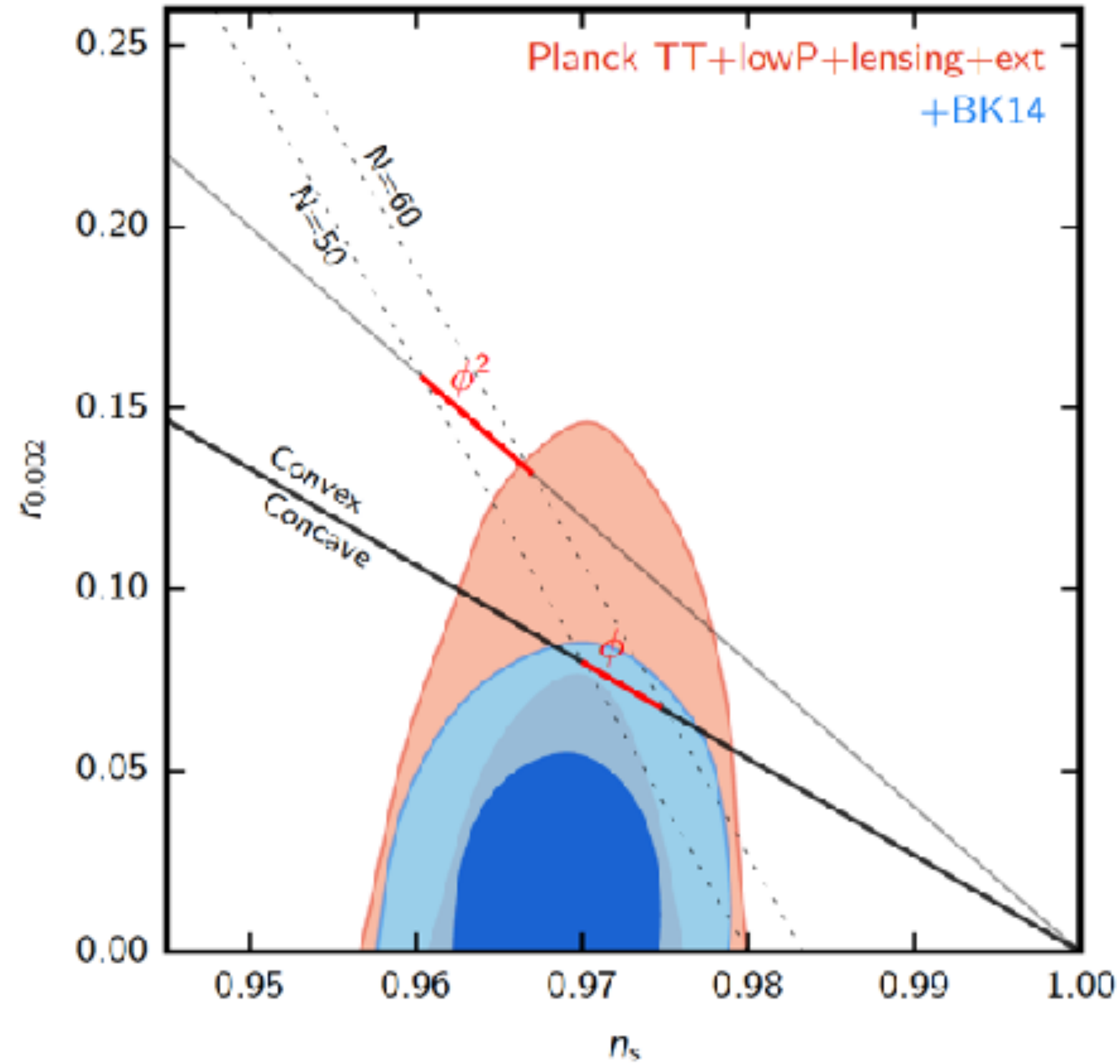
Might be observed in the future

- running of the scalar tilt: $\alpha_s \sim (1 - n_s)^2$
- tensor amplitude: $r \equiv \dfrac{(g_{ij}^{TT})^2}{(g_{ii})^2} = 16\epsilon$ tensor tilt: $n_T = 8\epsilon$



No news is interesting news

- Improved constraints from CMB B-mode polarization lead to r<0.07 [Planck, Bicep2/Keck 15]
- The vanilla quadratic potential is out. Linear is next.
- A *new hierarchy* (of scales)



$$\left.\begin{aligned} 1-n_s = 2\epsilon+\eta = 0.035 \\ r = 16\epsilon < 0.07 \end{aligned}\right\} \Rightarrow \epsilon \ll \eta$$

Symmetries

- Little primordial data, many models. How to proceed?
- Understand the symmetries of gravity plus scalars around (quasi) dS space, a.k.a. cosmological inflation
- Spontaneous breaking of (gauged) time translations: EFT of inflation and soft theorems
- dS isometries? This talk.
- Shift-symmetry constraints on correlators?

The conformal limit of inflation [EP, Pimentel & van Wijck 2016]

- New hierarchy of scales and slow-roll parameters
- de Sitter isometries and Conformal symmetry
- All correlators are fixed by Conformal symmetry
- Consistency relations + Conformal symmetry
- Wave functional of the universe

New slow-roll hierarchy

- To describe *our universe* the traditional slow-roll hierarchy needs to be updated for η>>ε

- *Decoupling limit* with fixed power spectrum

$$\epsilon \to 0 \quad \& \quad M_{\rm Pl} \to \infty \quad \text{with} \quad \frac{H^2}{\epsilon M_{\rm Pl}^2} = 10^{-7}$$

- *Gravitational interaction are turned off*. Gravity is only background

- *Conformal limit* = Decoupling + no breaking of dS isometries (e.g. no speed of sound cs)

- Simple class of models

$$\epsilon \sim \frac{1}{N^\beta} \quad \Rightarrow \quad \eta \sim \xi_n \sim \frac{1}{N} \gg \epsilon \qquad \left(\xi_{n+1} \equiv \frac{\dot{\xi}_n}{\xi_n H}\right)$$

dS as CFT

- Conformal limit: inflation is just a *scalar field in dS* (not true at O(ϵ))

$$ds^2 = \frac{-d\tau^2 + dx^2}{\tau^2 H^2}$$

- accelerated expansion: perturbations leave the horizon

- mass determines time dependence outside the horizon

$$\phi(x,\tau) \sim \sum_{\Delta} \tau^{\Delta} O_{\Delta}(x) \qquad \Delta = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$

- *3+1 dS isometries = SO(4,1) = Euclidean 3d conformal group*

- dS time translation = dilation, dS boost = special conformal transf

$$D: \; -\tau\partial_\tau - x\partial_x \quad \Rightarrow \quad -\Delta - x\partial_x$$

Two point function

- Dilations fix immediately the two point function

$$\langle \phi(k)\phi(k)\rangle' \stackrel{!}{=} \frac{C(\tau_*)}{k^{3-2\Delta}}$$

- Convert to zeta all modes at the same time

$$\zeta = \phi/\sqrt{2\epsilon(\tau_*)}$$

- this reproduces the right tilt for ϵ=0

$$1 - n_s = 2\epsilon + \eta \simeq \eta = -2\Delta$$

Interactions

- Higher n-point functions probe interactions (non-Gaussianities in the CMB and galaxies)
- Interactions in the scalar sector and from gravity
- Maldacena computed

$$\langle\zeta^3\rangle' = (n_s - 1)\left[\frac{1}{k_1^3 k_2^3} + 2\,\mathrm{perms}\right] + \epsilon \times \mathrm{equi.} + (\eta^2, \epsilon^2)$$
$$\simeq -\eta\left[\frac{1}{k_1^3 k_2^3} + 2\,\mathrm{perms}\right]$$

- *The leading bispectrum follows from symmetry!*

Soft Theorems

- In standard single field inflation, the squeezed limit of any n-point function is related to (n-1)-point functions
- This is the Ward identity for non-linearly realized t-translation invariance. It is a manifestation of the equivalence principle
- A similar (simpler) argument holds without gravity

$$\begin{aligned}\langle \phi_L \phi_S \phi_S \rangle &= \langle \phi_L \langle \phi_S \phi_S \rangle_{\phi_L} \rangle \\ &= \langle \phi_L \phi_L \rangle \partial_{\phi_L} \langle \phi_S \phi_S \rangle\end{aligned}$$

Bispectrum

- Conformal invariance (triple K-integral) + the squeezed limit fully fixes the full bispectrum to leading order.

- We recover the bispectrum and deriver new terms that are higher order in slow-roll

$$\langle \zeta^3 \rangle' = -(\eta + \eta\xi)\left[\frac{1}{k_1^3 k_2^3} + 2\,\mathrm{perms}\right] + \eta\xi \times \mathrm{equi.} + (\eta^3)$$

- non-Gaussianity is fixed by the *running of the spectral tilt* (observable) ηξ

- This an (observable) *consistency condition for single field* inflation in the scalar sector

- Gauge fixing the graviton breaks conformal invariance. Correlators with a graviton propagator are invariant only up to a gauge transformation (e.g. 4-point function). But gravity decouples…

Wave functional of the universe

- Quantum field theory in the Schroedinger picture (common in gauge-gravity duality)
- "The wave function of the Universe is like a diamond. From every angle it looks a different color. Yet it has no color." [M. Mirbabayi]

$$\langle O(\phi)\rangle = \int [d\phi(x)]\, |\psi|^2 O(\phi), \qquad \psi = \int [d\phi(x,\tau)]\, e^{iS[\phi]}$$

Weyl invariance

- The wave function of the universe in any FLRW is invariant under (3d) spatial diff's (Ward identities for squeezed limit correlators)

- In dS the wave function is also Weyl invariant

$$g_{ij} \to e^{2\lambda} g_{ij}, \quad \phi \to e^{\Delta\lambda}\phi, \quad \zeta \to \zeta + \lambda$$

- interactions are then constrained

$$\psi \sim \exp\left\{ \int_{x,y} P_\phi(x-y)^{-1} \phi(x)\phi(y) \left[1 - \Delta\zeta(x)\right] \right\}$$

- reproduces the *local part of the bispectrum*

Conformal limit

Hard to test experimentally, but interesting theoretically:

- All n-point correlators are *dS invariant* (leading order)
- new "scalar" consistency relation *away from the squeezed limit*
- new (physical!) relation: $f_{NL}^{eq} \sim \alpha_s$
- Perhaps relevant for flows away from CFT

Conclusions

- Observations dictate a new hierarchy: $\varepsilon << \eta$
  - Primordial correlators are fixed by conformal symmetry
  - *New consistency condition:* $f_{NL}^{eq} \sim \alpha_s$ (physical)
  - Shift-symmetry implications?