Axion: Mass Dark Matter Abundance Relation

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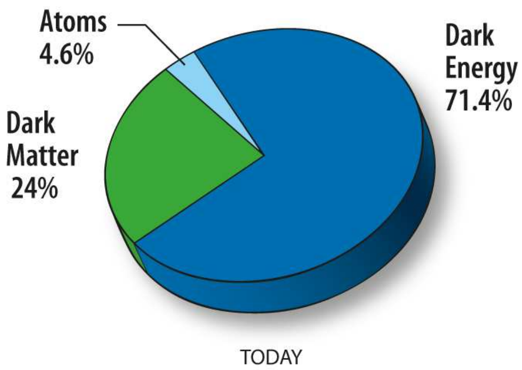
- Mystery 1: Dark Matter
- Mystery 2: T-symmetry of QED and QCD
- The Axion: a solution to both mysteries?
- Early Universe cosmology of the axion
- How to predict the axion mass if it's the dark matter

“Why move from Montréal to Darmstadt?”

Dark Matter: a Cosmic Mystery



Atoms: Standard Model.

Dark Energy: Cosmological Constant.

Strange value, but possible

Dark Matter: MYSTERY! NOT SM!

We only know 3 things about dark matter:

- It's **Matter**: gravitationally clumps.
- It's **Dark**: negligible electric charge, interactions too feeble to be detected except by gravity
- It's **Cold**: negligible pressure by redshift $z = 3000$

Another mystery: $\mathbf{T}$-symmetry in QED and QCD

$\mathbf{T}$ symmetry: “when you run a movie backwards, the *microphysics* is correct.”

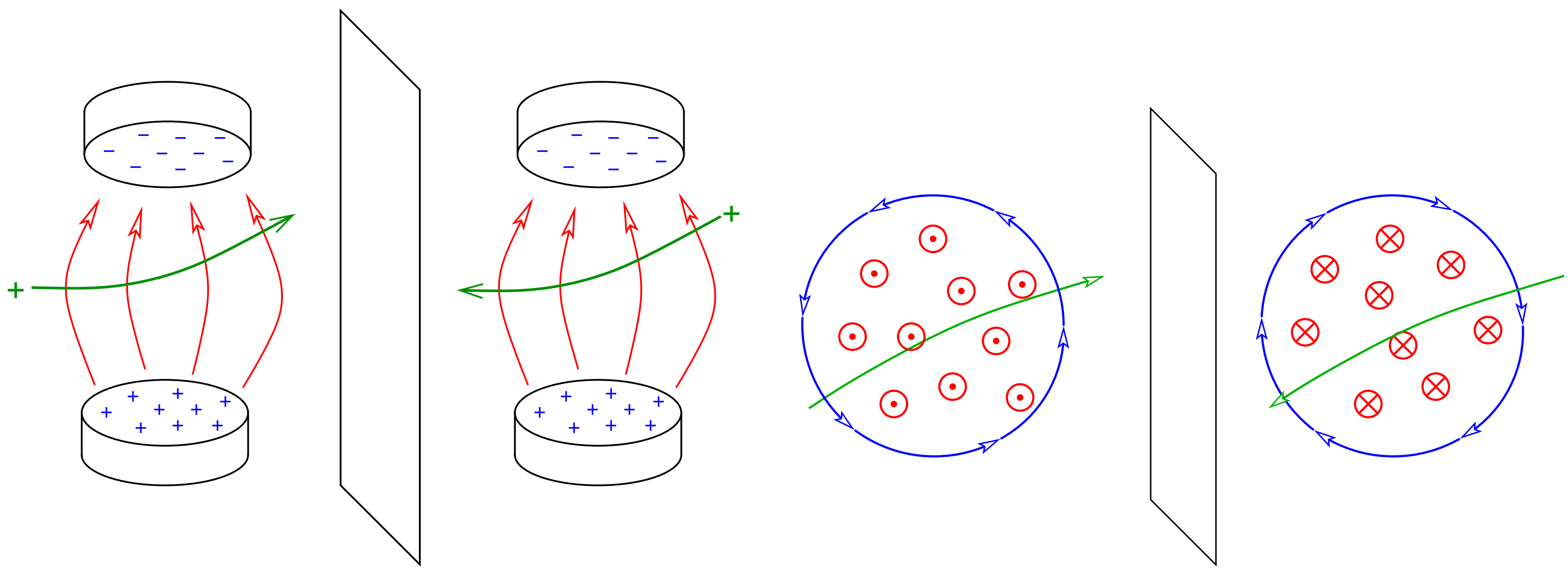
Statistical mechanics breaks $\mathbf{T}$.
But microphysics very nearly obeys it!

Weak physics breaks $\mathbf{T}$, but only through very small CKM effects. Observed in handful of experiments, all involving neutral meson oscillation.

No evidence for $\mathbf{T}$ viol in E&M or Strong interactions.

$\mathbf{T}$ in E&M

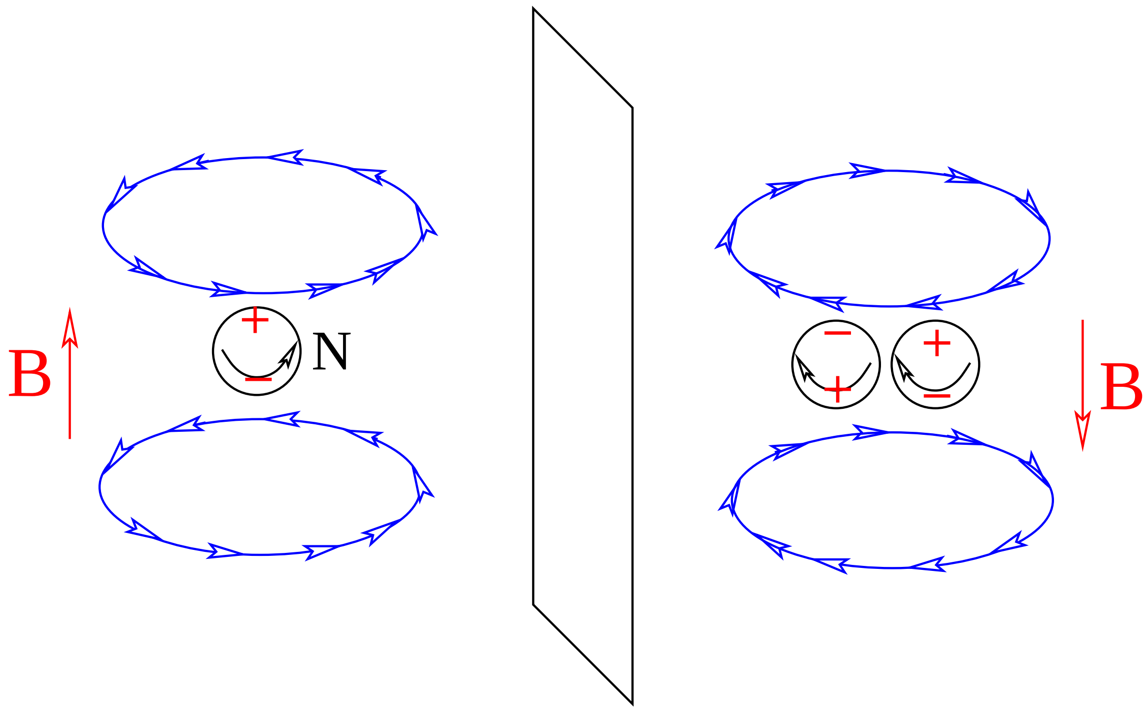
How do E, B fields change when you run movie backwards?



Q's unchanged, but J's flip. E same, but B flips.

Looking for $\mathbf{T}$: Neutron EDM

Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.



Is there an Electric Dipole Moment (EDM) aligned with spin?

If so: looks different when movie runs backwards, $\mathbf{T}$ viol!

$\mathbf{T}$ and the E&M Action

Action $S \Rightarrow$ all physics. Local field thy: $S = \int \mathcal{L}\, d^4x$.

$\mathcal{L}$ a singlet (gauge symm) and spacetime scalar (Lorentz):

$$\mathcal{L} = \frac{\vec{B}^2 - \vec{E}^2}{2e^2} + \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

$\mathbf{T}$ flip: $\vec{E} \to \vec{E}$ and $\vec{B} \to -\vec{B}$:

$(B^2 - E^2) \to (B^2 - E^2)$ **BUT** $E \cdot B \to -E \cdot B$.

$$\mathcal{L} \xrightarrow{T} \frac{\vec{B}^2 - \vec{E}^2}{2e^2} - \frac{\Theta}{4\pi^2}\vec{E}\cdot\vec{B} + \text{(electrons...)}$$

Nonvanishing Θ is a $\mathbf{T}$ violation!

E&M $\mathbf{T}$ violation is Illusory!

The $\Theta\vec{E}\cdot\vec{B}$ term has no *consequences*!

$$\vec{E}\cdot\vec{B} = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} = \partial^{\mu}K_{\mu}\,, \qquad K^{\mu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}A^{\nu}F^{\alpha\beta}$$

I can integrate $\vec{E}\cdot\vec{B}$ to a boundary term.
Vanishes if $F^{\alpha\beta}$ vanishes on boundary. Alternately, EOM:

$$0 = \partial_{\mu}\left(\frac{1}{e^2}F^{\mu\nu} + \frac{\Theta}{8\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^{\alpha}A^{\beta}\right)$$

Second term zero by antisymmetry (**if** Θ constant)

QCD and its Lagrangian

QCD is like 8 copies of E&M, but with non-linearities:

$$\text{Field strength}: \quad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu \, ,$$

g: coupling. $a = 1 \ldots 8$. f_{abc} "structure constants"

$$S = \int dt \int d^3x \left(\frac{\vec{E}_a^2 - \vec{B}_a^2}{2g^2} + \frac{\Theta}{8\pi^2} \vec{E}_a \cdot \vec{B}_a \right)$$

where $\vec{E}_a \cdot \vec{B}_a$ still a total derivative:

$$\vec{E}_a \cdot \vec{B}_a = \partial^\mu K_\mu \, , \qquad 2K_\mu = \epsilon_{\mu\nu\alpha\beta} \left(A_a^\nu F_a^{\alpha\beta} + \frac{g f_{abc}}{3} A_a^\nu A_b^\alpha A_c^\beta \right)$$

Last term *need not* vanish on boundary even if $\vec{E}_a = 0 = \vec{B}_a$ there!

It's always $8\pi^2 N_I$ with N_I integer. So Θ mod 2π has *physical consequences*

G. 't Hooft, PRL 37, 8(1976); R. Jackiw and C. Rebbi, PRL 37, 172 (1976);

Callan Dashen and Gross, Phys Lett 63B, 334 (1976)

Neutron EDM and Θ

Theory: Neutron electric dipole moment should exist,

$$d_n = -3.8 \times 10^{-16}\, e\,\mathrm{cm} \times \Theta$$

so long as Θ is not zero! Guo *et al*, arXiv:1502.02295, assumes Θ, modulo 2π, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$$

Either $|\Theta| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

Why an accident seems unlikely

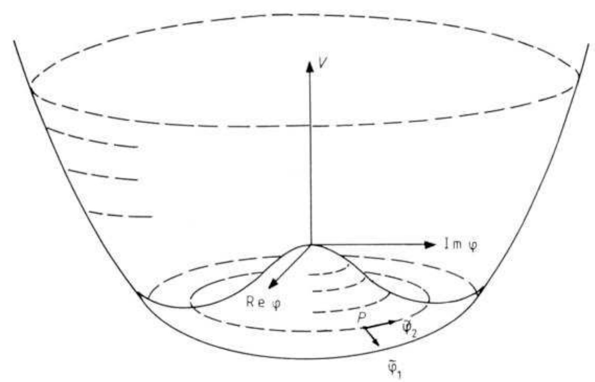
- $\mathbf{T}$ is not a fundamental symmetry. We have observed its violation (Kaon, B-meson, possibly D-meson physics)
- More $\mathbf{T}$ violation almost surely out there – otherwise, tough to explain why Universe is filled with matter!
- Renormalization: $\mathbf{T}$ viol. one place finds its way into other places, including Θ. Θ does *not get smaller* as you go from high to low scales R.G. marginal – feels $\mathbf{T}$ viol from all scales.

Axion: an explanation for $\Theta = 0$

Hypothesize an extra *complex scalar* field $\varphi = \phi e^{i\theta_A}$
Assume a *symmetry*: $\varphi \to e^{i\theta}\varphi$ $[\theta_A = \theta_A + \theta]$. Lagrangian:

$$\mathcal{L} = g^{\mu\nu}\partial_\mu\varphi^*\partial_\nu\varphi + \lambda\left(\varphi^*\varphi - f_a^2/2\right)^2 \text{ (+interactions)}$$

Spontaneous symmetry breaking with VEV f_a.
We will want $f_a \sim 10^{11}$GeV.



Peccei Quinn PRL 38, 1440 (1977);

J. E. Kim, PRL 43, 103 (1979); Shifman Vainshtein and Zakharov, NPB 166, 493 (1980)

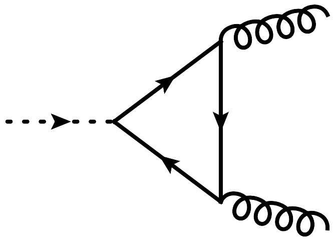
Axion: Indirect QCD coupling

φ is singlet under QCD.

But add some heavy physics which couples it indirectly

(such as heavy quarks which get mass due to $|\varphi|$)

$$\mathcal{L} = \ldots + \frac{\mathrm{Arg}\,\varphi}{8\pi^2} \vec{E}_a \cdot \vec{B}_a$$



Phase of $\varphi = f_a e^{i\theta_A}$ turns into part of $\bar{\Theta}$:

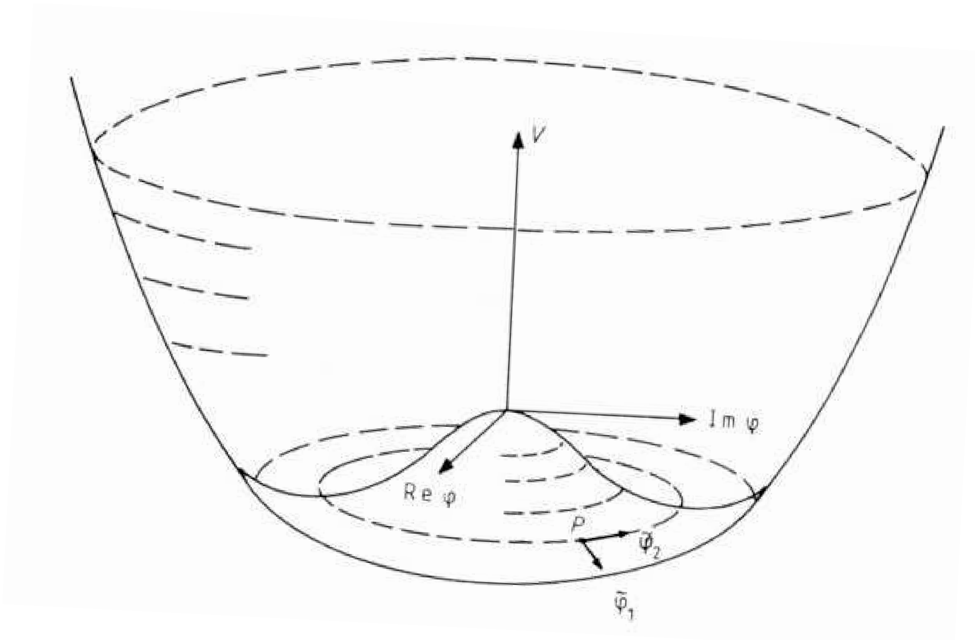
$$\mathcal{L} = \ldots + \frac{\Theta + \theta_A}{8\pi^2} \vec{E}_a \cdot \vec{B}_a$$

QCD cares about $(\Theta + \theta_A)$, not Θ alone.

Now the fun part

θ_A is a *field* and can change value.

QCD *cares about* value of $\Theta + \theta_A$. Detailed **QCD** calculation *Topological Susceptibility of* **QCD**: Potential tilted



Energy lowest where $\Theta + \theta_A = 0$, where $\mathbf{T}$ conserved.

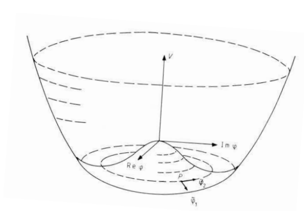
Tilt is temperature dependent, smaller in very early Universe

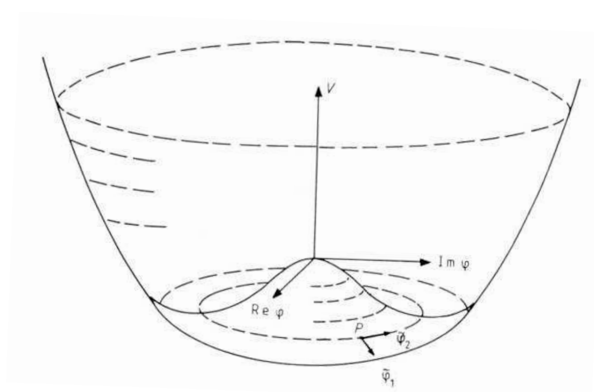
Size of tilt: nonperturbative **QCD**. Solved for temp=0. Not for high temperature!

What is predicted:

- Neutron EDM: 10^{-5} smaller than current limit
- New particle, Axion, exists, with specific interactions
- Strength of axion interactions scale with its mass (Pseudo-Nambu-Goldstone Boson)

What is not predicted: Axion Mass!





The larger ϕ, the flatter the potential, and lighter the axion. ϕ not predicted by model.

Axion in cosmology

Assume first that φ starts homogeneous [inflation]

Classical axion field!

Starts oscillating around

$t = \pi m_a^{-1}$. Damped:

- Hubble drag
- effect of dm_a/dt

Acts Like Dark Matter!

Modern axion energy density larger if field starts oscillating later: $\varepsilon_{\rm axion} \propto f_a^{0.84} \propto m_a^{-0.84}$.

Initial state of φ field?

Maybe it's the same everywhere in space.

But most likely, it's randomly different in different places!

- Inflation stretches quantum fluctuations to classical ones: $\Delta\varphi \sim H_{\text{infl.}}$. If $N_{\text{efolds}} H^2 > f_a^2$, scambles field.

  If not: need $H < 10^{-5} f_a$ to avoid excess "isocurvature" fluctuations in axion field

- Gets scrambled *after* inflation if Universe was ever really hot $T > f_a \sim 10^{11}$ GeV.

Should be able to predict DM abundance from m_a.
And that's valuable, also for the experimentalists!

L. Visinelli and P. Gondolo, PRL 103, 011802 (2014)

My Goal

Predict relation between **Dark Matter Density** and **Axion Mass** assuming space-random starting angle.

Challenges:

1. $\chi(T)$: needs Lattice Gauge Thy. Open, subdominant problem
2. Axion field dynamics: classical but with large scale hierarchy $f_a/H \sim 10^{30} \gg 1$

I will explain 2., and show how to solve it.

Solving space-inhomogeneous case

Put the Lagrangian

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \frac{\lambda}{8}(2-\varphi^*\varphi)^2 - \chi(t)\,\mathrm{Re}\,\varphi$$

as classical field thy.

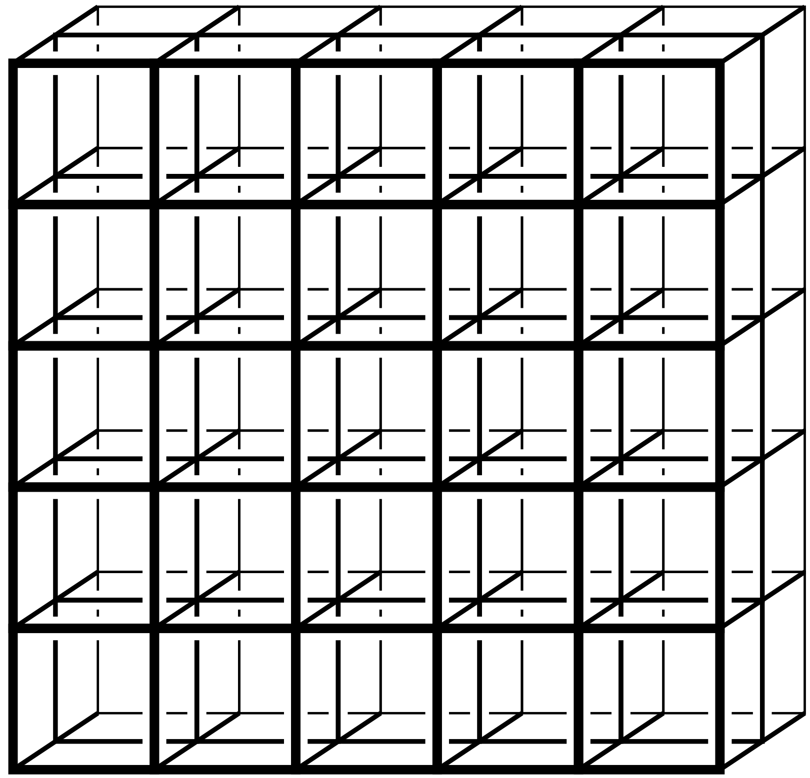
on **real-time lattice**,

with random init. condit.

Plus Hubble drag

Nonperturbative approach.

Easy enough to do!



What do you see?

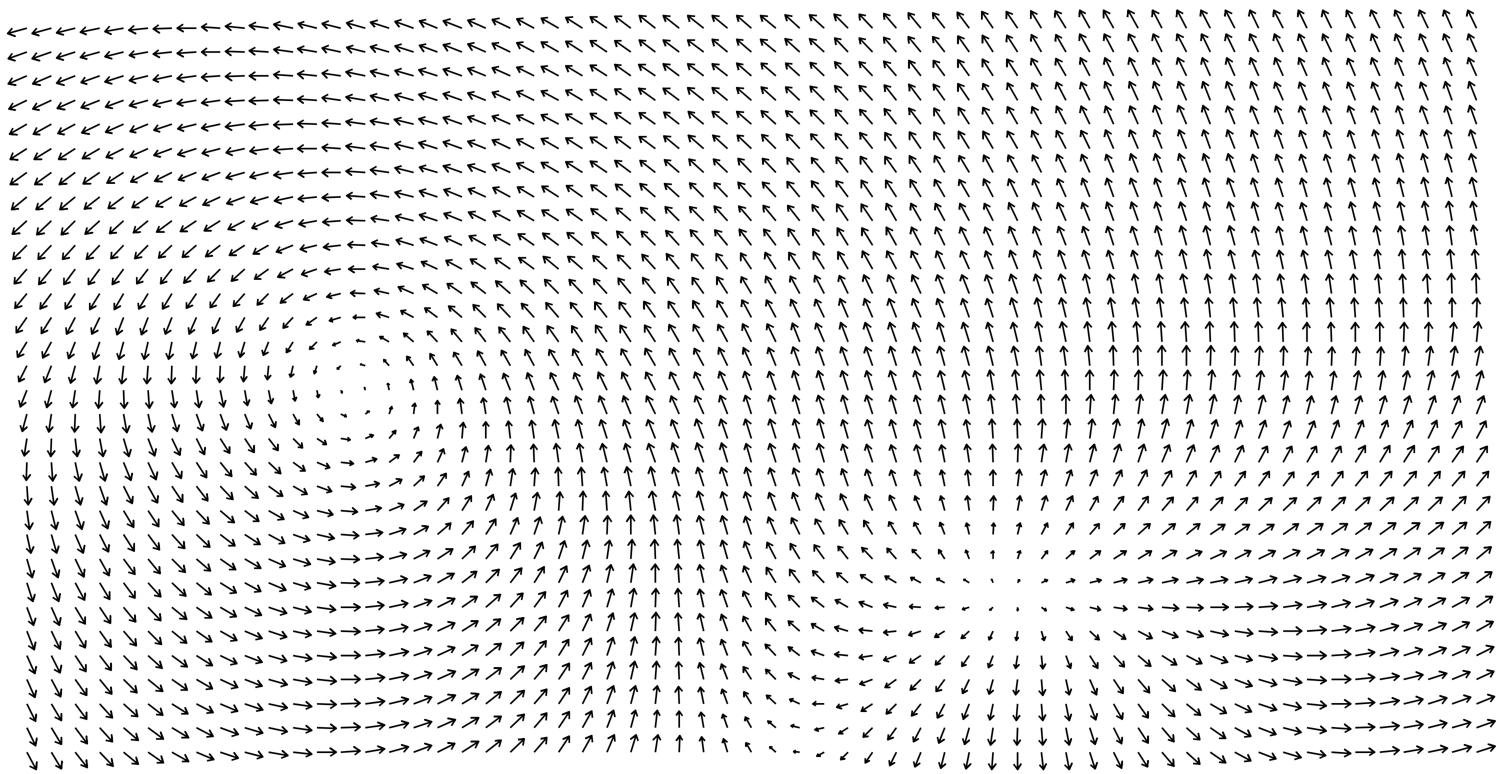
Energy density has string-like maxima, complex dynamics:

ε has stringlike structures: Annihilate when V tilts:

Axions and Topology I

φ is a complex number – plot as a 2D arrow.

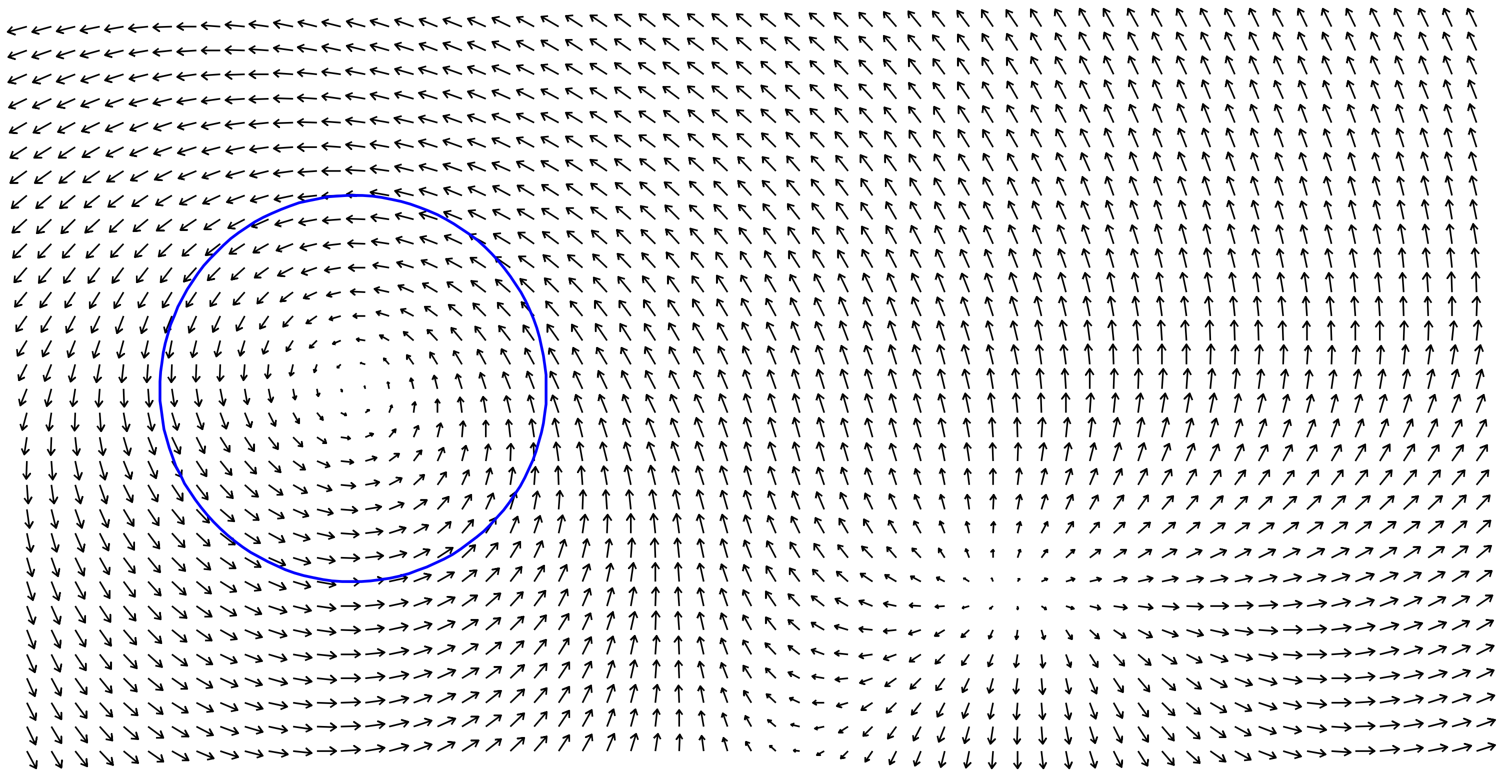
Axion field: a field of arrows. 2D slice for instance:



Strings are vortices in the field.

Axions and Topology II

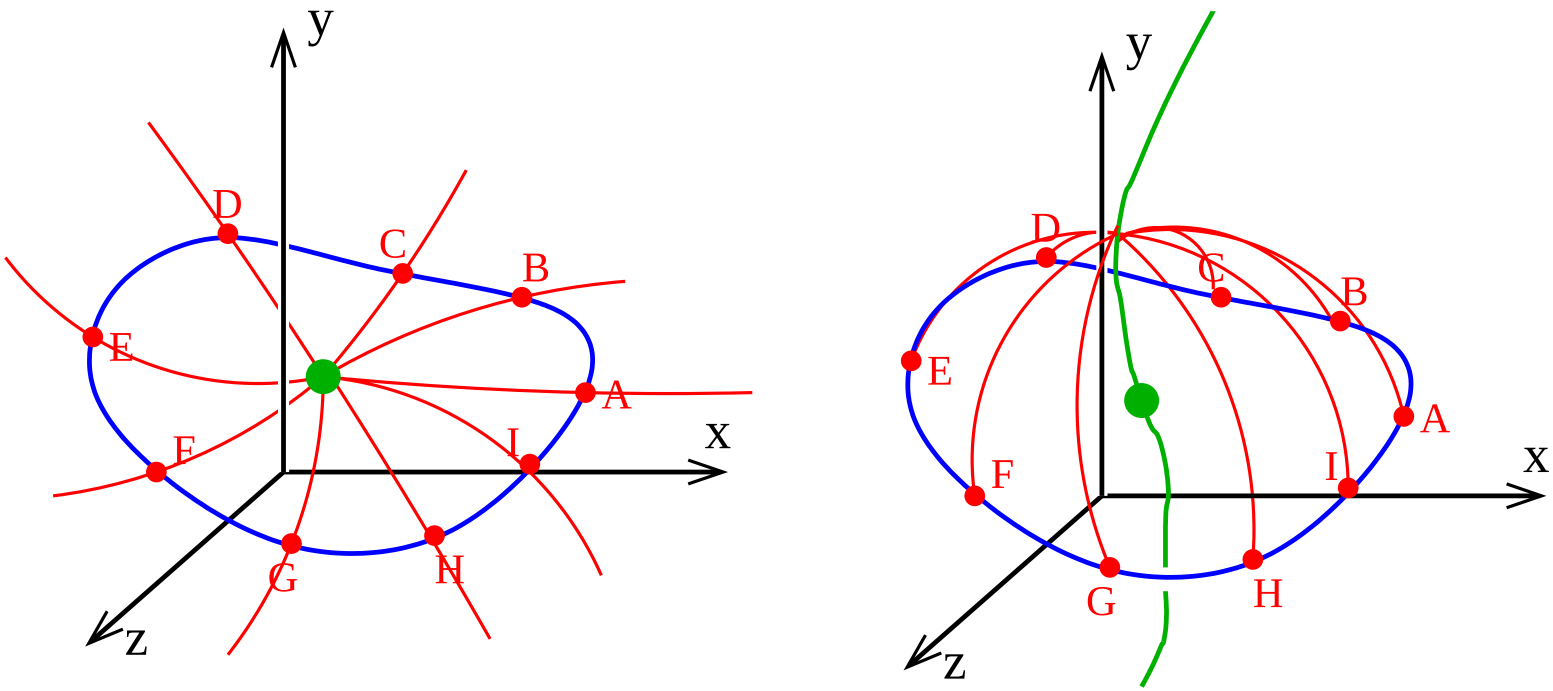
As you circle vortex, angle θ_A varies by 2π.



Continuity: angle θ_A *must* be undefined somewhere inside the circle. $\varphi = 0$ somewhere. Center of vortex.

Axions, Topology III

Singularity must exist somewhere inside such a circle



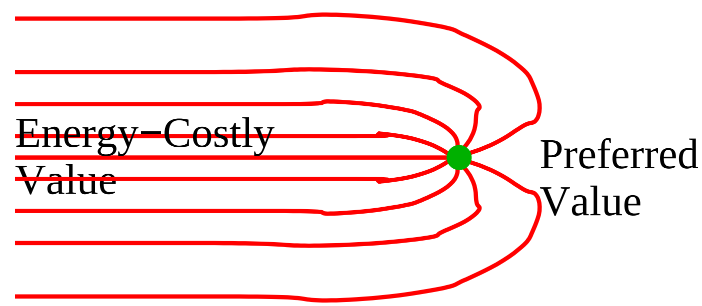
in 3D – true of *any* surface bounded by ring.

Requires a *string* of singular points – a vortex line

Strings and Walls

Random starting conditions $\Rightarrow$ network of these "strings"
Network evolves, strings straighten out, find each other.

Potential tilts: $\theta_A \to 0$ where it can. Near string: explores all values. String has "domain wall" attached.



Pulls strings towards each other, speeds breakup of network.
Complex string – domain wall network dynamics.

This dynamics produces Axions!

Conventional Wisdom

This dynamics produces axions! 3 sources:

- "misalignment" axions from $\theta_A \neq 0$ between strings
- "Wall" axions from breakup of axionic walls
- "String" axions radiated off of axionic strings

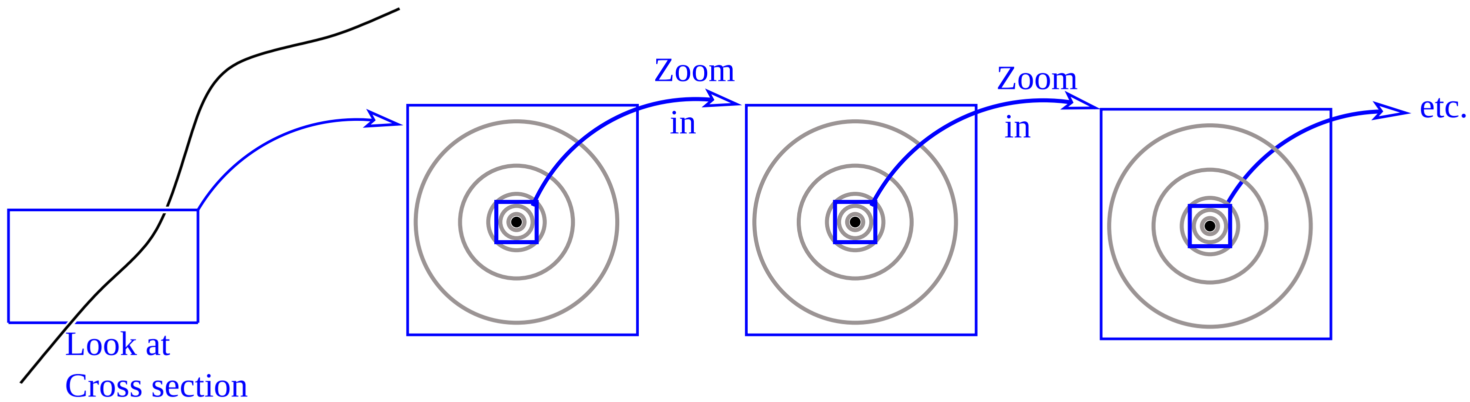
Axion production should be (much?) larger than angle-average of misalignment mechanism

Big complication

These strings are like onions (or scallions)

Layers! Innermost "core" has radius $r \sim f_a^{-1} \sim 10^{-27}$m. Outer size $\sim H^{-1} \sim 10^2$m. Energy stored logarithmically with radius $\varepsilon \propto \int \frac{rdr}{r^2}$. Name $\ln(f_a/H) \equiv \kappa \sim 70$.

Layers of String Energy



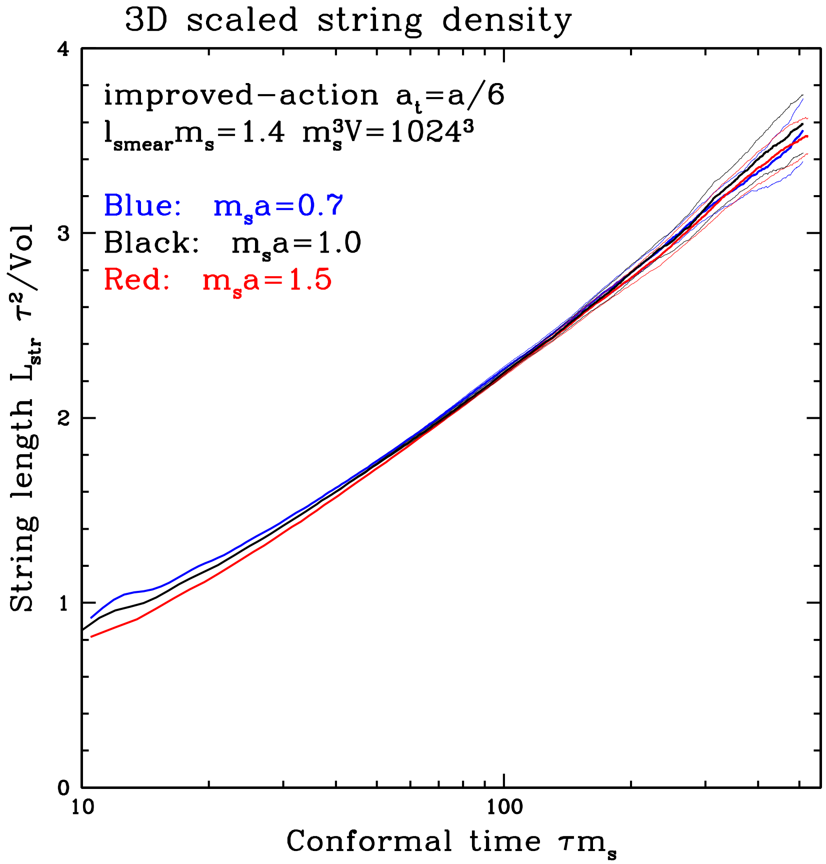
Series of "sheaths" around string:
equal energy in each $\times 2$ scale, 10^{30} scale range!

More "sheaths" $\rightarrow$ outermost layer less important:

- inter-string interactions less important
- Radiation from strings less efficient

Both "thin out" string network. Big scale separation $\rightarrow$ dense network

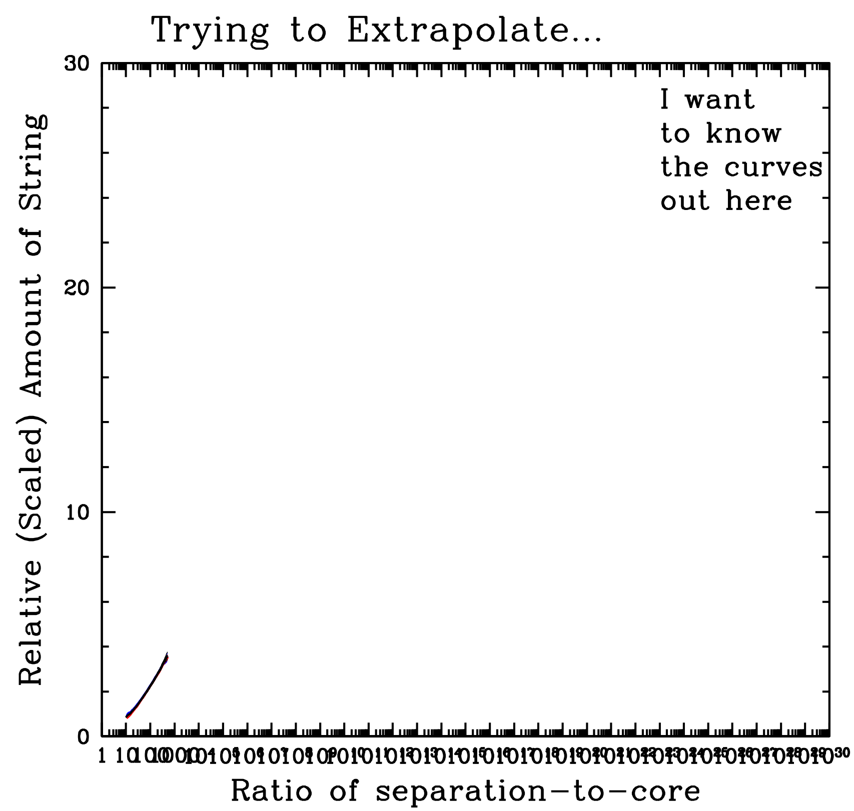
Does it matter?



String density varies strongly with # of layers. So does energy of string. Suggestive – axion production larger as more layers.

Wild extrapolation

Problem is – can I really believe an extrapolation over this large a range of scales, based on the information I have?



Need some way to capture string-core physics!

An effective description

The important axion-production physics is:

- Only long-range (light) degree of freedom is axion
- Axion strings: thin cores with high tension
  $T \simeq 70 \times \pi f_a^2$
- Interaction between axion field and string is correct

Any modified UV (high-mass) physics which does this is OK!

Find massive fields which somehow increase string tension

Global vs Local Strings

Another theory with topological string objects:
Abelian Higgs model

$$\mathcal{L}(\varphi, A_\mu) = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (D_\mu\varphi)^*(D^\mu\varphi) + \frac{\lambda}{8}\left(2\varphi^*\varphi - f_a^2\right)^2$$

with $D_\mu = \partial_\mu - ieA_\mu$ covariant derivative

String where $\mathrm{Arg}\,\varphi$ winds by 2π as before.
But now, $\vec{B}$-field flux in string core

$$A_\phi = \frac{g(r)}{er} \qquad \text{compensates so} \quad D_\mu\varphi \to 0 (r \gg 1/ef_a)$$

Abelian Higgs model

Strings carry energy in core: $\int B^2 + |D\varphi|^2 + V(\varphi)$
Energy density falls exponentially away from string core

Away from string core, only massive (heavy) fields:

$$m_\varphi^2 \sim \lambda f_a^2\,, \qquad m_A^2 = e^2 f_a^2\,.$$

Network of strings with tension $T \simeq \pi f_a^2$.

No long-range interactions between strings. “String-only”

Much ($4\times$ to $10\times$) higher string network density

Trick: global strings, local cores

Hybrid theory with A_μ and two scalars

$$
\begin{aligned}
\mathcal{L}(\varphi_1, \varphi_2, A_\mu) \;\; = \;\; & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
& + \frac{\lambda}{8}\left[(2\varphi_1^* \varphi_1 - f_a^2)^2 + (2\varphi_2^* \varphi_2 - f_a^2)^2\right] \\
& + |(\partial_\mu - i q_1 e A_\mu)\varphi_1|^2 + |(\partial_\mu - i q_2 e A_\mu)\varphi_2|^2
\end{aligned}
$$

Pick $q_1 \neq q_2$.

Two rotation symmetries, $\varphi_1 \to e^{i\theta_1}\varphi_1$, $\varphi_2 \to e^{i\theta_2}\varphi_2$

One combination gauged, one combination global (Axion)

For almost-equal charges ($q_1 = 3$, $q_2 = 4$), initial $\varphi_1 = \varphi_2$, all strings have same φ_1, φ_2 winding.

φ_1 wants $B = 2\pi/eq_1$, φ_2 wants $B = 2\pi/eq_2$.
Solution: $B = 2\pi(q_1 + q_2)/e(q_1^2 + q_2^2)$.
String has large $2\pi f_a^2$ tension of gauge-string
Plus long-range interactions $\propto (q_2 - q_1)^2/(q_2^2 + q_1^2)$

Behaves like axion-string with weak coupling to axion field.
Or, with very large string-core tension

String cores modified.
But far from cores – theory of axions only

String networks have larger tension

Strings denser, last longer, decay to more persistent loops

Results (preliminary)

- More, higher-tension, denser string network, but
- only 20% more axions than with axion-only simulation,
- **Fewer** axions than angle-averaged misalignment
- Usual argument **wrong**, involves **double-counting**
- energy in "walls" is *part* of misalignment energy
- Walls get "eaten" by strings, energy lost for axions

Conclusions

- Dark matter and $\mathbf{T}$ in QCD both mysteries
- Axion could explain both!
- Axion dynamics in early Universe: string defects
- Need auxiliary DOF to get high string tension.

Put in known dark matter density, vacuum $\chi(0)$
Recent lattice result for $\chi(T)$, these results:
Preliminary result: $m_{\rm ax} \simeq ()\ \mu{\rm eV}$

Why is $\chi(T \sim 1\text{GeV})$ Hard?

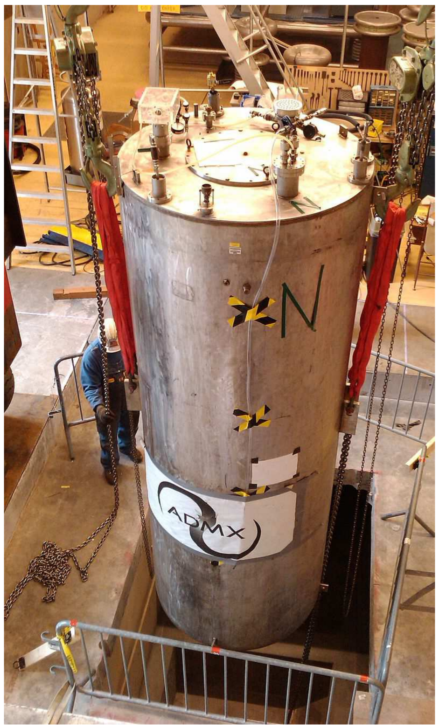
Lattice Monte-Carlo, find fraction of configs with instanton. But

- Instantons get rare fast, $\chi \sim T^{-7\ldots-8}$. Statistics??
- Slowing-down of algorithm to change N_I
- Instanton counting must have very low false-positive rate!

Chiral limit *not* a problem: $\chi \propto (m_u m_d m_s / T^3)$, if $m \ll T$.
Use $T \gg m \gg m_u$, multiply by $m_u m_d m_s / m^3$.

How do experimentalists look for Axions?

Most sensitive method: resonant cavity in magnetic field



Microwave cavity inside superconducting solenoid. $\vec{E}$ field of cavity mode aligned with $\vec{B}$ of solenoid. Cavity oscillation: oscillating $\vec{E} \cdot \vec{B}$. If cavity resonance matches $m_A/\hbar$: cavity resonance driven. Squid readout – tuneable cavity ...

What about Anthropic Principle?

Trendy Explanation for “coincidences” or “tunings”

Why is Cosmological Constant so small?
If it were 100 times bigger, matter would fly apart or collapse before life could evolve. Nature plays dice, universes with all values occur, but only universes with life get observed.

Why does **QCD** respect $\mathbf{T}$ symmetry?
If **QCD** *violated* $\mathbf{T}$*, something would go wrong with nuclear physics, which would make life impossible. Nature plays dice, only universes where life is possible get observed.*
Except that life is fine in a world where $\Theta = 10^{-2}$!