Introduction to Mathematica and FORM

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http://wwwth.mpp.mpg.de/members/hahn/mmaform.pdf

Computer Algebra Systems

- Commercial systems: Mathematica, Maple, Matlab/MuPAD, MathCad, Reduce, Derive . . .
- Free systems: FORM, GiNaC, Maxima, Axiom, Cadabra, Fermat, GAP, Singular, Sage . . .
- Generic systems: Mathematica, Maple, Matlab/MuPAD, Maxima, MathCad, Reduce, Axiom, Sage, GiNaC . . .
- Specialized systems: Cadabra, Singular, Magma, CoCoA, GAP . . .
- Many more . . .

Mathematica vs. FORM

Mathematica

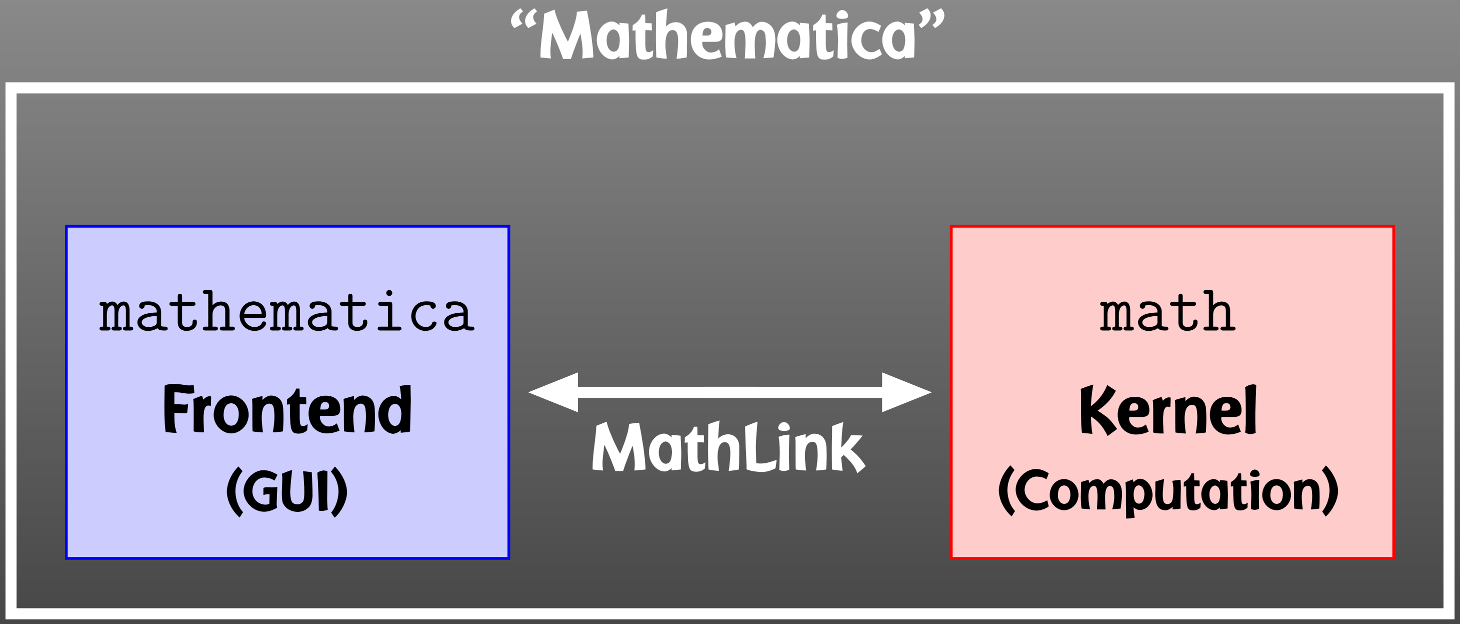
- Much built-in knowledge,
- ‘Big and slow’ (esp. on large problems),
- Very general,
- GUI, add-on packages...

FORM

- Limited mathematical knowledge,
- ‘Small and fast’ (also on large problems),
- Optimized for certain classes of problems,
- Batch program (edit-run cycle).

Mathematica

Mathematica Components



`http://wwwth.mpp.mpg.de/members/hahn/intro_math.pdf`

<rant>Why I hate the Frontend</rant>

Frontend:

- ☺ Nice formatting
- ☺ Documentation
- ☺ Ease of use
- ☹ No obvious relation between screen and definitions
- ☹ Always interactive
- ☹ Slow startup

Kernel:

- ☹ Text interface
- ☹ No pretty-printing
- ☺ 1-to-1 relation to definitions
- ☺ Interactive and non-interactive
- ☺ Scriptable
- ☺ Fast startup

Expert Systems

In technical terms, Mathematica is an **Expert System.**
Knowledge is added in form of **Transformation Rules.**
An expression is transformed until no more rules apply.

Example:

```
myAbs[x_] := x /; NonNegative[x]
myAbs[x_] := -x /; Negative[x]
```

We get:

```
myAbs[3] ☞ 3
myAbs[-5] ☞ 5
myAbs[2 + 3 I] ☞ myAbs[2 + 3 I]
```

— no rule for complex arguments so far

```
myAbs[x] ☞ myAbs[x]
```

— no match either

Immediate and Delayed Assignment

Transformations can either be

- **added "permanently" in form of Definitions,**

```
norm[vec_] := Sqrt[vec . vec]
norm[{1, 0, 2}] ☞ Sqrt[5]
```

- **applied once using Rules:**

```
a + b + c /. a -> 2 c ☞ b + 3 c
```

Transformations can be **Immediate** or **Delayed**. Consider:

```
{r, r} /. r -> Random[] ☞ {0.823919, 0.823919}
{r, r} /. r :> Random[] ☞ {0.356028, 0.100983}
```

Mathematica is one of those programs, like TEX, where you wish you'd gotten a US keyboard for all those braces and brackets.

Almost everything is a List

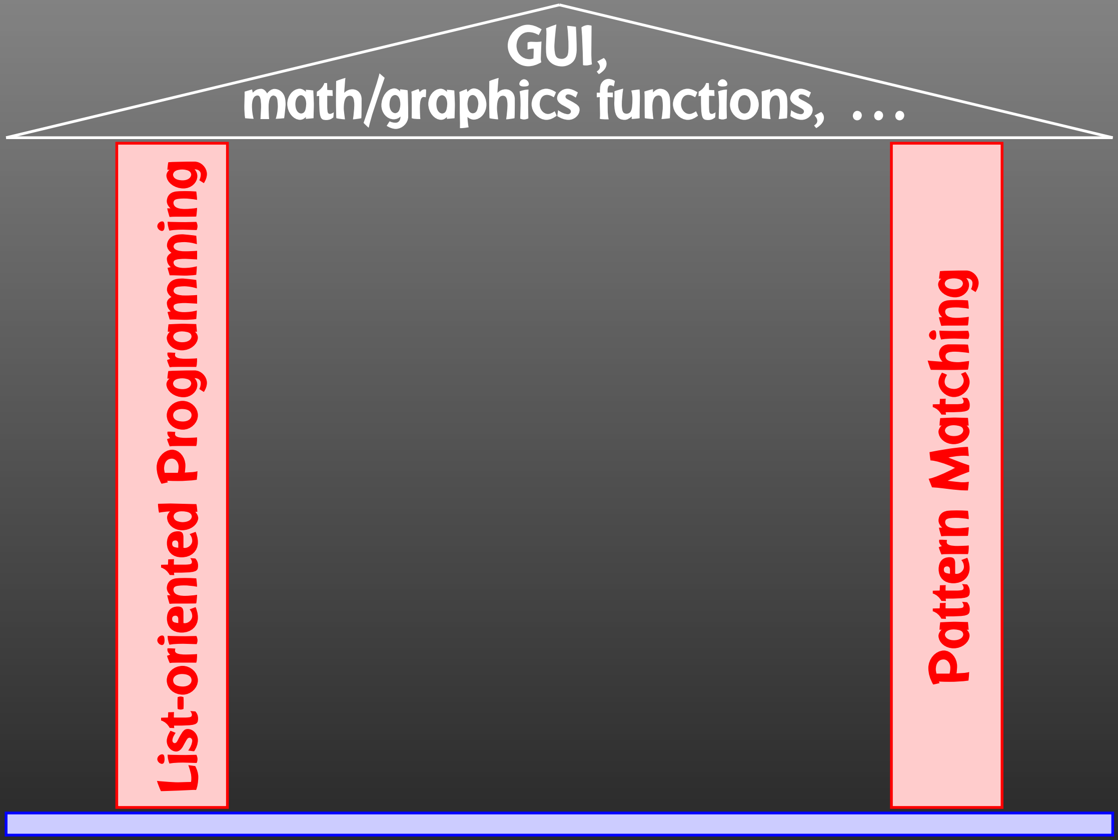
All Mathematica objects are either Atomic, e.g.

```
Head[133] ☞ Integer
Head[a] ☞ Symbol
```

or (generalized) Lists with a Head and Elements:

```
expr = a + b
FullForm[expr] ☞ Plus[a, b]
Head[expr] ☞ Plus
expr[[0]] ☞ Plus       — same as Head[expr]
expr[[1]] ☞ a
expr[[2]] ☞ b
```

The Pillars of Mathematica



List-oriented Programming

Using Mathematica's list-oriented commands is almost always of advantage in both speed and elegance.

Consider:

```
tab = Table[Random[], {10^7}];

test1 := Block[ {sum = 0},
  Do[ sum += tab[[i]], {i, Length[tab]} ];
  sum ]

test2 := Apply[Plus, tab]
```

Here are the timings:

```
Timing[test1][[1]] ☞ 8.29 Second
Timing[test2][[1]] ☞ 1.75 Second
```

Map, Apply, and Pure Functions

Map applies a function to all elements of a list:

```
Map[f, {a, b, c}] ☞ {f[a], f[b], f[c]}
f /@ {a, b, c} ☞ {f[a], f[b], f[c]}     — short form
```

Apply exchanges the head of a list:

```
Apply[Plus, {a, b, c}] ☞ a + b + c
Plus @@ {a, b, c} ☞ a + b + c     — short form
```

Pure Functions are a concept from formal logic. A pure function is defined 'on the fly':

```
(# + 1)& /@ {4, 8} ☞ {5, 9}
```

The # (same as #1) represents the first argument, and the & defines everything to its left as the pure function.

List Operations

Flatten removes all sub-lists:

```
Flatten[f[x, f[y], f[f[z]]]] ☞ f[x, y, z]
```

Sort and **Union** sort a list. **Union** also removes duplicates:

```
Sort[{3, 10, 1, 8}] ☞ {1, 3, 8, 10}
Union[{c, c, a, b, a}] ☞ {a, b, c}
```

Prepend and **Append** add elements at the front or back:

```
Prepend[r[a, b], c] ☞ r[c, a, b]
Append[r[a, b], c] ☞ r[a, b, c]
```

Insert and **Delete** insert and delete elements:

```
Insert[h[a, b, c], x, {2}] ☞ h[a, x, b, c]
Delete[h[a, b, c], {2}] ☞ h[a, c]
```

More Speed Bumps

Consider:

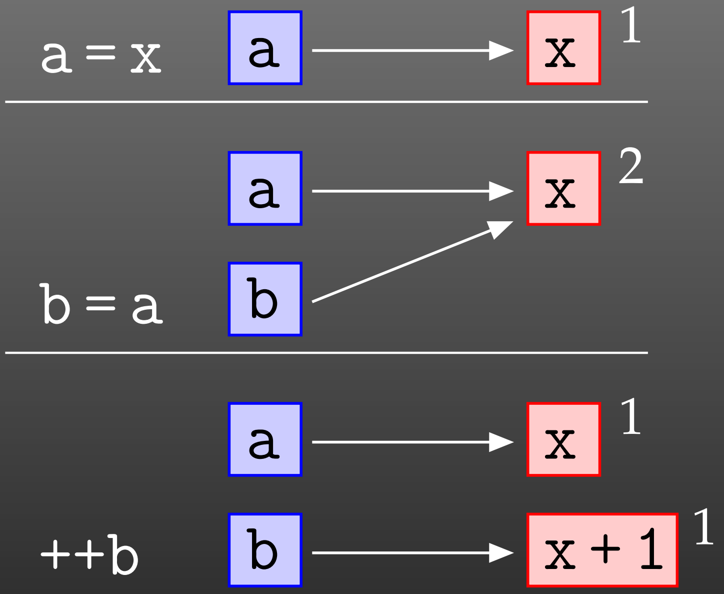
```
tab = Table[Random[], {10^5}];

test1 := Block[ {res = {}},
  Do[ AppendTo[res, tab[[i]]], {i, Length[tab]} ];
  res ]

test2 := Block[ {res = {}},
  Do[ res = {res, tab[[i]]}, {i, Length[tab]} ];
  Flatten[res] ]
```

The timings:

```
Timing[test1][[1]] ☞ 19.47 Second
Timing[test2][[1]] ☞  0.11 Second
```

Reference Count

Assignments that don't change the content make no copy but just increase the Reference Count.



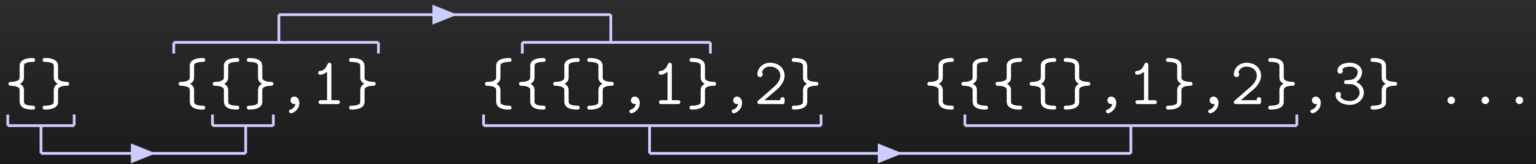
Reference Count and Speed

```
test1 := ...
  ... AppendTo[res, tab[[i]]] ...
  res

test2 :=
  ... res = {res, tab[[i]]} ...
  Flatten[res]
```

`test1` **has to re-write the list every time an element is added:**

```
{}   {1}      {1,2}       {1,2,3}       ...
```

`test2` **does that only once at the end with** `Flatten`**:**



```
{}   {{},1}   {{{},1},2}   {{{{},1},2},3} ...
```

Patterns

One of the most useful features is **Pattern Matching:**

`_`	— matches one object
`__`	— matches one or more objects
`___`	— matches zero or more objects
`x_`	— named pattern (for use on the r.h.s.)
`x_h`	— pattern with head **h**
`x_:1`	— default value
`x_?NumberQ`	— conditional pattern
`x_ /; x > 0`	— conditional pattern

Patterns take function overloading to the limit, i.e. functions behave differently depending on *details* of their arguments:

```
Attributes[Pair] = {Orderless}
Pair[p_Plus, j_] := Pair[#, j]& /@ p
Pair[n_?NumberQ i_, j_] := n Pair[i, j]
```

Attributes

Attributes characterize a function's behavior before and while it is subjected to pattern matching. For example,

```
Attributes[f] = {Listable}
f[l_List] := g[l]
f[{1, 2}] ☞ {f[1], f[2]}     — definition is never seen
```

Important attributes: `Flat`, `Orderless`, `Listable`, `HoldAll`, `HoldFirst`, `HoldRest`.

The `Hold...` **attributes are needed to pass variables by reference:**

```
Attributes[listadd] = {HoldFirst}
listadd[x_, other__] := x = Flatten[{x, other}]
```

This would not work if `x` **were expanded before invoking** `listadd`, **i.e. passed by value.**

Memorizing Values

For longer computations, it may be desirable to ‘remember’ values once computed. For example:

```
fib[1] = fib[2] = 1
fib[i_] := fib[i] = fib[i - 2] + fib[i - 1]
```

```
fib[4] ☞ 3
```

```
?fib ☞ Global`fib
        fib[1] = 1
        fib[2] = 1
        fib[3] = 2
        fib[4] = 3
        fib[i_] := fib[i] = fib[i - 2] + fib[i - 1]
```

Note that Mathematica places more specific definitions before more generic ones.

Decisions

Mathematica's **If Statement** has three entries: for True, for False, but also for Undecidable. For example:

```
If[8 > 9, yes, no] ☞ no
If[a > b, yes, no] ☞ If[a > b, yes, no]
If[a > b, yes, no, dunno] ☞ dunno
```

Property-testing Functions end in `Q`: `EvenQ`, `PrimeQ`, `NumberQ`, `MatchQ`, `OrderedQ`, ... These functions have no undecided state: in case of doubt they return `False`.

Conditional Patterns are usually faster:

```
good[a_, b_] := If[TrueQ[a > b], 1, 2]
```

— `TrueQ` removes ambiguity

```
better[a_, b_] := 1 /; a > b
better[a_, b_] = 2
```

Equality

Just as with decisions, there are several types of equality, decidable and undecidable:

```
a == b ☞ a == b
a === b ☞ False
a == a ☞ True
a === a ☞ True
```

The full name of '===' is `SameQ` and works as the `Q` indicates: in case of doubt, it gives `False`. It tests for Structural Equality.

Of course, equations to be solved are stated with '==':

```
Solve[x^2 == 1, x] ☞ {{x -> -1}, {x -> 1}}
```

Needless to add, '=' is a definition and quite different:

```
x = 3        — assign 3 to x
```

Selecting Elements

Select selects elements fulfilling a criterium:

```
Select[{1, 2, 3, 4, 5}, # > 3 &] ☞ {4, 5}
```

Cases selects elements matching a pattern:

```
Cases[{1, a, f[x]}, _Symbol] ☞ {a}
```

Using **Levels** is generally a very fast way to extract parts:

```
list = {f[x], 4, {g[y], h}}
Depth[list] ☞ 4
```
— `list` is 4 *levels* deep (0, 1, 2, 3)

```
Level[list, {1}] ☞ {f[x], 4, {g[y], h}}
Level[list, {2}] ☞ {x, g[y], h}
Level[list, {3}] ☞ {y}
Level[list, {-1}] ☞ {x, 4, y, h}

Cases[expr, _Symbol, {-1}]//Union
```
— find *all variables* in `expr`

Mathematical Functions

Mathematica is equipped with a large set of mathematical functions, both for symbolic and numeric operations.

Some examples:

`Integrate[x^2, {x,3,5}]`	— integral
`D[f[x], x]`	— derivative
`Sum[i, {i,50}]`	— sum
`Series[Sin[x], {x,1,5}]`	— series expansion
`Simplify[(x^2 - x y)/x]`	— simplify
`Together[1/x + 1/y]`	— put on common denominator
`Inverse[mat]`	— matrix inverse
`Eigenvalues[mat]`	— eigenvalues
`PolyLog[2, 1/3]`	— polylogarithm
`LegendreP[11, x]`	— Legendre polynomial
`Gamma[.567]`	— Gamma function

Graphics

Mathematica has formidable graphics capabilities:

```
Plot[ArcTan[x], {x, 0, 2.5}]
ParametricPlot[{Sin[x], 2 Cos[x]}, {x, 0, 2 Pi}]
Plot3D[1/(x^2 + y^2), {x, -1, 1}, {y, -1, 1}]
ContourPlot[x y, {x, 0, 10}, {y, 0, 10}]
```

Output can be saved to a file with `Export`:

```
plot = Plot[Abs[Zeta[1/2 + x I]], {x, 0, 50}]
Export["zeta.eps", plot, "EPS"]
```

[?] Hint: To get a high-quality plot with proper LaTeX labels, don't waste your time fiddling with the `Plot` options. Use the `psfrag` LaTeX package.

Numerics

Mathematica can express **Exact Numbers**, e.g.

`Sqrt[2]`, `Pi`, $\frac{27}{4}$

It can also do **Arbitrary-precision Arithmetic**, e.g.

```
N[Erf[28/33], 25] ☞ 0.7698368826185349656257148
```

But: Exact or arbitrary-precision arithmetic is fairly slow!
Mathematica uses **Machine-precision Reals** for fast arithmetic.

```
N[Erf[28/33]] ☞ 0.769836882618535
```

Arrays of machine-precision reals are internally stored as **Packed Arrays** (this is invisible to the user) and in this form attain speeds close to compiled languages on certain operations, e.g. eigenvalues of a large matrix.

Compiled Functions

Mathematica can ‘compile’ certain functions for efficiency.

This is not compilation into assembler language, but rather a **strong typing** of an expression such that intermediate data types do not have to be determined dynamically.

```
fun[x_] := Exp[-((x - 3)^2/5)]
cfun = Compile[{x}, Exp[-((x - 3)^2/5)]]

time[f_] := Timing[Table[f[1.2], {10^5}]][[1]]

time[fun]  ☞  2.4 Second
time[cfun]  ☞  0.43 Second
```

`Compile` is implicit in many numerical functions, e.g. in `Plot`.

In a similar manner, `Dispatch` hashes long lists of rules beforehand, to make the actual substitution faster.

Blocks and Modules

`Block` implements Dynamical Scoping

A local variable is known everywhere, but only for as long as the block executes (“temporal localization”).

`Module` implements Lexical Scoping

A local variable is known only in the block it is defined in (“spatial localization”). This is how scoping works in most high-level languages.

```
printa := Print[a]
a = 7
btest := Block[{a = 5}, printa]
mtest := Module[{a = 5}, printa]
```

```
btest ☞ 5
mtest ☞ 7
```

In C: Only Modules

Most languages have only Lexical Scoping:

```
#include <stdio.h>

static int a = 7;

static void printa() {
  printf("%d\n", a);
}

int main() {
  int a = 5;
  printa();
  return 0;
}
```

DownValues and UpValues

Definitions are usually assigned to the symbol being defined: this is called **DownValue**.

For seldomly used definitions, it is better to assign the definition to the next lower level: this is an **UpValue**.

```
x/: Plus[x, y] = z
?x ☞ Global`x
     x /: x + y = z
```

This is better than assigning to `Plus` directly, because `Plus` is a very common operation.

In other words, Mathematica **“looks” one level inside each object** when working off transformations.

Output Forms

Mathematica knows some functions to be **Output Forms.** These are used to format output, but don't "stick" to the result:

`{{1, 2}, {3, 4}}//MatrixForm` ☞ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

`Head[%]` ☞ `List` — not `MatrixForm`

Some important output forms:

`InputForm`, `FullForm`, `Shallow`, `MatrixForm`, `TableForm`, `TeXForm`, `CForm`, `FortranForm`.

```
TeXForm[alpha/(4 Pi)] ☞ \frac{\alpha}{4\pi}
CForm[alpha/(4 Pi)] ☞ alpha/(4.*Pi)
FullForm[alpha/(4 Pi)]
  ☞ Times[Rational[1, 4], alpha, Power[Pi, -1]]
```

MathLink

The **MathLink API** connects Mathematica with external C/C++ programs (and vice versa). **J/Link** does the same for Java.

```
:Begin:
:Function:      copysign
:Pattern:       CopySign[x_?NumberQ, s_?NumberQ]
:Arguments:     {N[x], N[s]}
:ArgumentTypes: {Real, Real}
:ReturnType:    Real
:End:

#include "mathlink.h"

double copysign(double x, double s) {
  return (s < 0) ? -fabs(x) : fabs(x);
}

int main(int argc, char **argv) {
  return MLMain(argc, argv);
}
```

For more details see arXiv:1107.4379.

Scripting Mathematica

Efficient batch processing with Mathematica:

Put everything into a script, using sh's Here documents:

```
#! /bin/sh ................. Shell Magic
math << \_EOF_ ........... start Here document (note the \)
  AppendTo[$Echo, "stdout"];
  << FeynArts`
  top = CreateTopologies[...];
  ...
_EOF_ .................... end Here document
```

Everything between "<< \*tag*" and "*tag*" goes to Mathematica as if it were typed from the keyboard.

Note the "\\" before *tag*, it makes the shell pass everything literally to Mathematica, without shell substitutions.

Scripting Mathematica

- Everything contained in **one compact shell script,** even if it involves several Mathematica sessions.
- Can combine with arbitrary shell programming, e.g. can use **command-line arguments** efficiently:

```
#! /bin/sh
math -run "arg1=$1" -run "arg2=$2" ... << \END
  ...
END
```

- Can easily be **run in the background,** or combined with utilities such as **make.**

Debugging hint: `-x` **flag** makes shell echo every statement,

```
#! /bin/sh -x
```

Commercial Software?

Mathematica licenses cost money ($\sim$ 5 k€/license).

While your Mathematica program runs, it blocks one license, so don't 'just' leave your Mathematica session open.

- Parallelize
- Script, Distribute, Automate
- Crunch numbers outside Mathematica

But: don't overdo it.
If your calculation takes 5 min in total, don't waste time improving.

Parallel Kernels

Mathematica has built-in support for parallel Kernels:

```
LaunchKernels[];
ParallelNeeds["mypackage`"];

data = << mydata;
ParallelMap[myfunc, data];
```

Parallel Kernels count toward Sublicenses.
# Sublicenses = 8 $\times$ # interactive Licenses.

Parallel Functions

- More functions:

```
ParallelArray   ParallelEvaluate   ParallelNeeds
ParallelSum     ParallelCombine    ParallelTable
ParallelDo      ParallelProduct    ParallelTry
ParallelMap     ParallelSubmit
DistributeDefinitions   DistributeContexts
```

- Automatic parallelization (so-so success):
  `Parallelize[`*expr*`]`
- 'Intrinsic' functions (e.g. `Simplify`) not parallelizable.
- Multithreaded computation partially automatic (OMP) for some numerical functions, e.g. `Eigensystem`.
- Take care of side-effects of functions.
- Usual concurrency stuff (write to same file, etc).

Crunch Numbers outside Mathematica

- **Conversion** of Mathematica expression to Fortran/C **painless.**
- Optimized output can **easily run faster** than in Mathematica.
- Showstopper: Functions not available in Fortran/C, e.g. `NDSolve`, `Zeta`. Maybe 3rd-party substitute (GSL, Netlib).
- Mathematica has built-in C-code generator, e.g.

  ```
  myfunc = Compile[{{x}}, x^2 + Sin[x^2]];
  Export["myfunc.c", myfunc, "C"]
  ```

  But no standalone code: shared object for use with Mathematica (i.e. also needs license).
- FormCalc's code-generation functions produce optimized standalone code.

Code-generation Functions

FormCalc's code-generation functions are public and disentangled from the rest of the code. They can be used to **write out an arbitrary Mathematica expression as optimized Fortran or C code:**

- `*handle* = OpenCode["*file.F*"]`
  opens *file.F* as a Fortran file for writing,
- `WriteExpr[*handle*, {*var* -> *expr*, ...}]`
  writes out Fortran code which calculates *expr* and stores the result in *var*,
- `Close[*handle*]`
  closes the file again.

Code generation

Traditionally: Output in Fortran.
Code generator is meanwhile rather sophisticated, e.g.

- **Expressions too large** for Fortran are split into parts, as in

  ```
  var = part1
  var = var + part2
  ...
  ```

- **High level of optimization,** e.g. common subexpressions are pulled out and computed in temporary variables.
- **Many ancillary functions** make code generation versatile and highly automatable, such that the resulting code needs few or no changes by hand:
  `VarDecl`, `ToDoLoops`, `IndexIf`, `FileSplit`, ...

C Output

- Output in C99 makes integration into C/C++ codes easier:

  ```
  SetLanguage["C"]
  ```

Code structured by e.g.

- Loops and tests handled through macros, e.g. `LOOP(var,1,10,1)...ENDLOOP(var)`
- Introduced data types `RealType` and `ComplexType` for better abstraction, can e.g. be changed to different precision.

Mathematica $\leftrightarrow$ Fortran

Mathematica $\rightarrow$ Fortran:

- Get FormCalc from http://feynarts.de/formcalc
- Write out arbitrary Mathematica expression:

```
h = OpenCode["file"]
WriteExpr[h, {var -> expr, ...}]
Close[h]
```

Fortran $\rightarrow$ Mathematica:

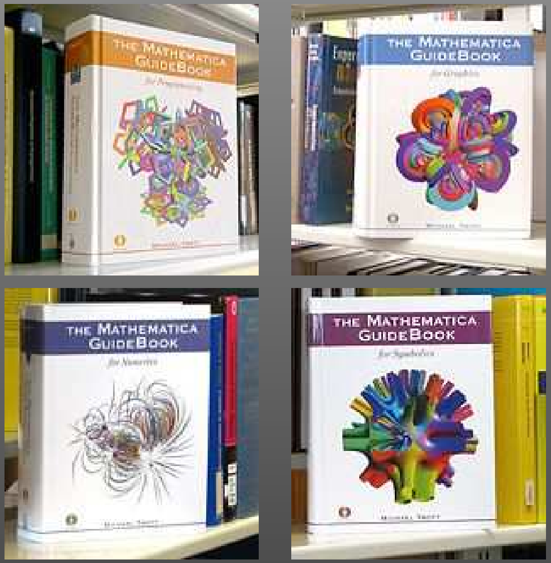
- Get http://feynarts.de/formcalc/FortranGet.tm
- Compile: `mcc -o FortranGet FortranGet.tm`
- Load in Mathematica: `Install["FortranGet"]`
- Read Fortran code: `FortranGet["file.F"]`

Mathematica Summary

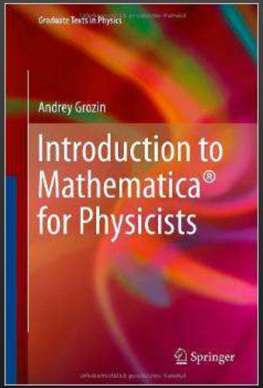
- Mathematica makes it wonderfully easy, even for fairly unskilled users, to manipulate expressions.
- Most functions you will ever need are already built in. Many third-party packages are available at MathSource, http://library.wolfram.com/infocenter/MathSource.
- When using its capabilities (in particular list-oriented programming and pattern matching) right, Mathematica can be very efficient.
  Wrong: `FullSimplify[veryLongExpression]`.
- Mathematica is a general-purpose system, i.e. convenient to use, but not ideal for everything.
  For example, in numerical functions, Mathematica usually selects the algorithm automatically, which may or may not be a good thing.

Books

- Michael Trott
  The Mathematica Guidebook
  for $\{$Programming, Graphics, Numerics, Symbolics$\}$ (4 vol)
  Springer, 2004-2006.



- Andrei Grozin
  Introduction to Mathematica for Physicists
  Springer, 2013.


Graduate Texts in Physics
Andrey Grozin
Introduction to Mathematica® for Physicists
Springer

FORM

FORM Essentials

- A FORM program is divided into **Modules.**
  Simplification happens only at the end of a module.
- FORM is **strongly typed** –
  all variables have to be declared:
  Symbols, Vectors, Indices, (N)Tensors, (C)Functions.
- FORM works **on one term at a time:**
  Can do “`Expand[(a + b)^2]`” (**local** operation) but
  not “`Factor[a^2 + 2 a b + b^2]`” (**global** operation).
- FORM is mainly strong on **polynomial expressions.**
- FORM program + documentation + course available from
  http://nikhef.nl/∼form.

A Simple Example in FORM

```
Symbols a, b, c, d;
Local expr = (a + b)^2;
id b = c - d;
print;
.end
```

Running this program gives:

```
FORM by J.Vermaseren,version 4.0(Mar 1 2013) Run at: Tue May  8 10:14:12 2013
    Symbols a, b, c, d;
    Local expr = (a + b)^2;
    id b = c - d;
    print;
    .end

Time =       0.00 sec    Generated terms =          6
             expr        Terms in output =          6
                         Bytes used      =        104

   expr =
      d^2 - 2*c*d + c^2 - 2*a*d + 2*a*c + a^2;

  0.00 sec out of 0.00 sec
```

Module Structure

A FORM program consists of **Modules.** A Module is terminated by a "dot" statement (`.sort`, `.store`, `.end`, ...)

- **Generation Phase** ("normal" statements)
  During the execution of "normal" statements terms are only generated. This is a purely **local operation** - only one term at a time needs to be looked at.
- **Sorting Phase** ("dot" statements):
  At the end of the module all terms are inspected and similar terms collected. This is the only 'global' operation which requires FORM to look at all terms 'simultaneously.'

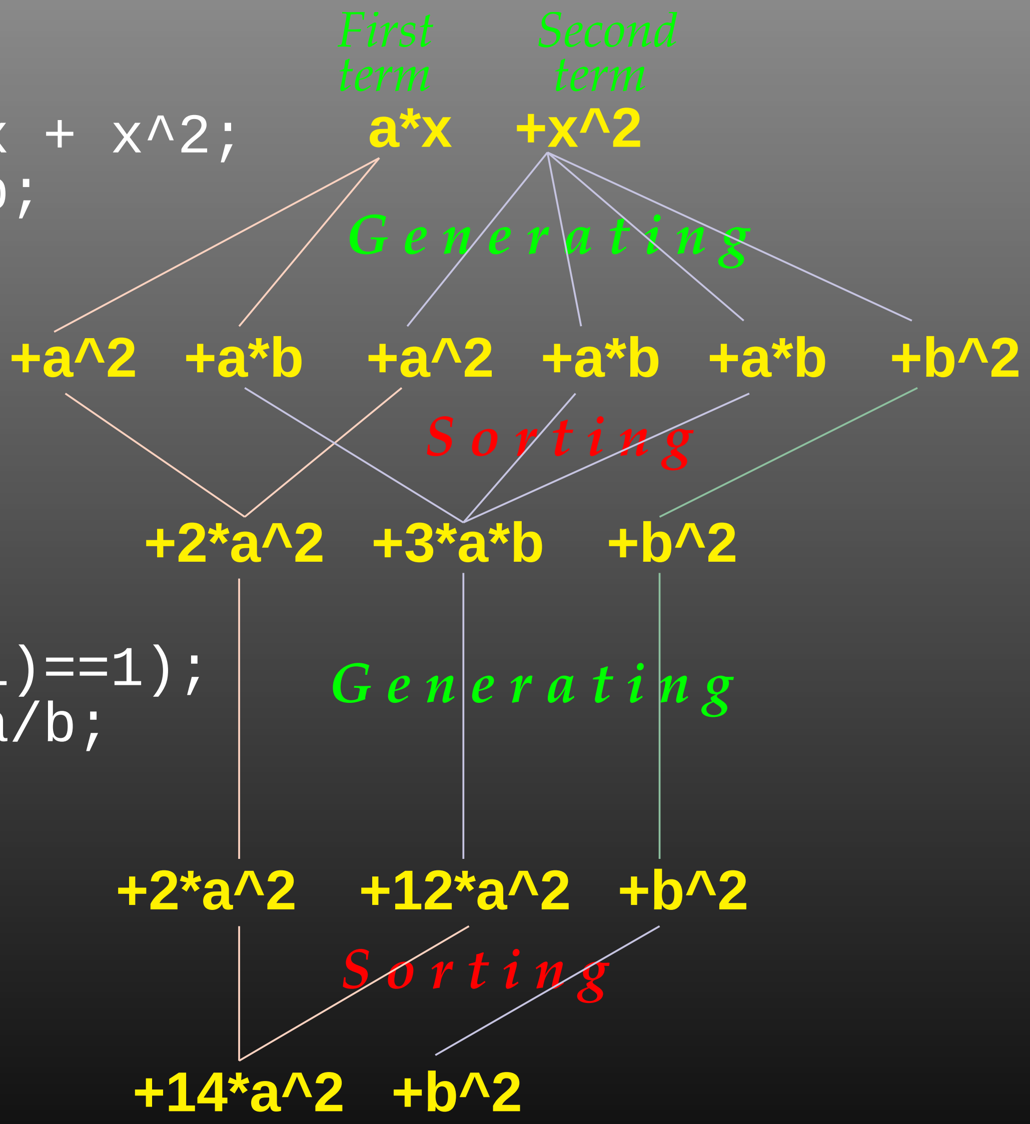
Sorting and Generating

First module

```
l expr = a*x + x^2;
id x = a + b;



.sort
```

Second module

```
if(count(b,1)==1);
multiply 4*a/b;
endif;



print;
.end
```



Id-Statement

The central statement in FORM is the `id`**-Statement:**

```
a^3*b^2*c
id a*b = d;  ☞  a*c*d^2           — multiple match
once a*b = d;  ☞  a^2*b*c*d       — single match
only a*b = d;  ☞  a^3*b^2*c       — no exact match possible
```

`id` **does not, by default, match negative powers:**

```
x + 1/x
id x = y;  ☞  x^-1 + y
id x^n? = y^n;  ☞  y^-1 + y       — wildcard exponent
```

Patterns

Patterns are possible, too:

```
f(a, b, c) + f(1, 2, 3)
id f(a, b, c) = 1; ☞ 1 + f(1, 2, 3)
```

— explicit match

```
id f(a?, b?, c?) = 1; ☞ 2
```

— wildcard match

```
id f(?a) = g(?a); ☞ g(a, b, c) + g(1, 2, 3)
```

— group-wildcard match

```
id f(a?int_, ?a) = a; ☞ 1 + f(a, b, c)
```

— constrained wildcard

```
id f(a?{a,b}, ?a) = a; ☞ a + f(1, 2, 3)
```

— alternatives

Bracketing, Collecting

`bracket` puts specified items outside the bracket.
`antibracket` puts specified items inside the bracket.
`collect` moves the bracket contents to a function.

```
Symbols a, b, c, d;
Local expr = (a + b)*(c + d);
print;
.sort
     expr = a*c + a*d + b*c + b*d;

bracket a, b;
print;
.sort
     expr = + a * ( c + d )
            + b * ( c + d );

CFunction f;
collect f;
bracket f;
print;
.end
     expr = + f(c + d) * ( a + b );
```

Preprocessor

FORM has a Preprocessor which operates before the compiler.

Many constructs are familiar from C, but the FORM preprocessor can do more:

- `#define`, `#undefine`, `#redefine`,
- `#if{,def,ndef}` ... `#else` ... `#endif`,
- `#switch` ... `#endswitch`,
- `#procedure` ... `#endprocedure`, `#call`,
- `#do` ... `#enddo`,
- `#write`, `#message`, `#system`.

The preprocessor works across modules, e.g. a `do`-loop can contain a `.sort` statement.

Dollar Variables

- Not strongly typed, can contain ‘everything.’
- Preserved across module boundaries.
- Can be operated on during preprocessing (`#$X = ...`) and normal operation (`$X = ...`).
- Can receive matched pattern: `once f(x?$var) = ...`
- No arrays.

```
s a, b;
L F = (a + b)^6;
#$n = 0;
$n = $n + 1;
print "term %$ is %t", $n;
.end
```

☞

```
term 1 is  + a^6
term 2 is  + 6*a^5*b
term 3 is  + 15*a^4*b^2
term 4 is  + 20*a^3*b^3
term 5 is  + 15*a^2*b^4
term 6 is  + 6*a*b^5
term 7 is  + b^6
```

Special Commands for High-Energy Physics

- **Gamma matrices:** `g_`, `g5_`, `g6_`, `g7_`.
- **Fermion traces:** `trace4`, `tracen`, `chisholm`.
- **Levi-Civita tensors:** `e_`, `contract`.
- **Index properties:** `{,anti,cycle}symmetrize`.
- **Dummy indices:** `sum`, `replaceloop`.
  (e.g. $\sum_i a_i b_i + \sum_j a_j b_j = 2 \sum_i a_i b_i$)
- **Very efficient combinatorics:** `dd_`, `distrib_`.

FORM Summary

- FORM is a freely available Computer Algebra System with some specialization on High Energy Physics.
- Programming in FORM takes more ‘getting used to’ than in Mathematica. Also, FORM has no GUI or other programming aids.
- FORM programs are module oriented with global (= costly) operations occurring only at the end of module. A strategical choice of these points optimizes performance.
- FORM is much faster than Mathematica on polynomial expressions and can handle in particular huge (GB) expressions.

FORM $\leftrightarrow$ Mathematica

Mathematica $\longrightarrow$ FORM:

- Get FormCalc from http://feynarts.de/formcalc
- After compilation the `ToForm` utility should be in the executables directory (e.g. Linux-x86-64):

```
ToForm < file.m > file.frm
```

FORM $\longrightarrow$ Mathematica:

- Get http://feynarts.de/formcalc/FormGet.tm
- Compile it with `mcc -o FormGet FormGet.tm`
- Load it in Mathematica with `Install["FormGet"]`
- Read a FORM output file: `FormGet["file.out"]`
  Pipe output from FORM: `FormGet["!form file.frm"]`

Exercise

Write a Mathematica function that works somewhat similar to `Select`. It should, however, not just give back the list of items for which the test function is true, but

{ list of items for which test is `True`,
list of items for which test is `False` }

Try to find a version in which the test function is not evaluated more than once per item!

`http://wwwth.mpp.mpg.de/members/hahn/mmaform.pdf`
`http://wwwth.mpp.mpg.de/members/hahn/intro_math.pdf`
`http://wwwth.mpp.mpg.de/members/hahn/mmaform.tar.gz`