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What Is The Magnetic Weak Gravity Conjecture For Axions?

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Motivation

Is it possible to have an **axionic field** ϕ with a **superplanckian** period / field range?

$$\phi \to \phi + 2\pi f \quad \text{with} \quad f > M_{pl} \ ?$$

Superplanckian axions are important for

1.) models of **Large field inflation** and

2.) for dynamically solving the EW hierarchy problem **(Cosmological Relaxation)**

Motivation

Is it possible to have an **axionic field** ϕ with a **superplanckian** period / field range?

$\phi \to \phi + 2\pi f$ with $f > M_{pl}$?

Do theories of superplanckian axions reside in the swampland?

Motivation

Charting the swampland:

The question can be attacked from 2 sides:

1.) **Top-down:** study low energy EFTs of QG directly.
E.g. try to construct models with superplanckian axions in string theory

2.) **Bottom-up:** search for inconsistencies in the EFT in the presence of gravity.
Formulate swampland criteria ("folk theorems").

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One example: **Weak Gravity Conjecture**

[Arkani-Hamed, Motl, Nicolis, Vafa 2006]

Motivation

The Weak Gravity Conjecture: [Arkani-Hamed, Motl, Nicolis, Vafa 2006]
a bottom up swampland criterion for U(1) theories with charged particles and monopoles coupled to gravity.

1.) **Electric form:**

Require the existence of superextremal charged particles:

$$m \lesssim q M_{pl}$$

2.) **Magnetic form:**

Gives a lower bond on the cutoff scale Λ of the EFT:

$$\Lambda \lesssim e M_{pl}$$

Motivation

The Weak Gravity Conjecture for axions:

1.) **Electric form:**

Consider an axion θ with a potential due to instantons

$$V \sim \sum_n e^{-nS} \cos\left(\frac{n\theta}{f}\right)$$

Translating the electric WGC bound to an axion theory:

$$S \lesssim \frac{M_{pl}}{f} \, .$$

Constrains axion inflation models with 'instantonic' potentials.

Motivation

Generalise the **magnetic WGC** to **(p+1)-form** theories with electric p-branes and **magnetic (d-p-4)-branes**.

WGC: "The minimally charged magnetic brane should not be a black brane"

$$\Lambda \lesssim \left(g_{el}^2 M_{pl}^{d-2}\right)^{\frac{1}{2(p+1)}} \ .$$

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Generalise the **magnetic WGC** to **(p+1)-form** theories with electric p-branes and **magnetic (d-p-4)-branes**.

WGC: "The minimally charged magnetic brane should not be a black brane"

For an **axion** theory (i.e. $p = -1$) in 4 dimensions find:

$$g_{el} = \frac{1}{f}\,, \qquad \Lambda \lesssim \left(\frac{M_{pl}^2}{f^2}\right)^{\frac{1}{2(p+1)}}\,.$$

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For $f > M_{pl}$ **this implies** $\Lambda \to 0 \,.$

$f > M_{pl}$

?

Motivation

What is the **magnetic WGC** for **axions**?

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What is the **magnetic WGC** for **axions**?

WGC for **(p+1)-form theories:**
"The minimally charged magnetic brane should not be a black brane."

For an axion theory the magnetic object is a (cosmic) string.

Strategy: study the properties of string solutions in axion theories and look for possible pathologies.

Outline

What is the magnetic WGC for axions?

1.) Study explicit cosmic string solutions for $f < M_{pl}$ & $f > M_{pl}$.

- **Static solutions**
- **Time-dependent solutions**

2.) Strings in theories with $f_{eff} > M_{pl}$.

3.) Summary and Outlook

Explicit String Solutions

1.) Static solution: **Cohen-Kaplan** string:

Both the metric and field configuration are static.

2.) Non-static solutions.

- **Gregory**'s solution:

  The metric is time-dependent, but the field configuration is static.

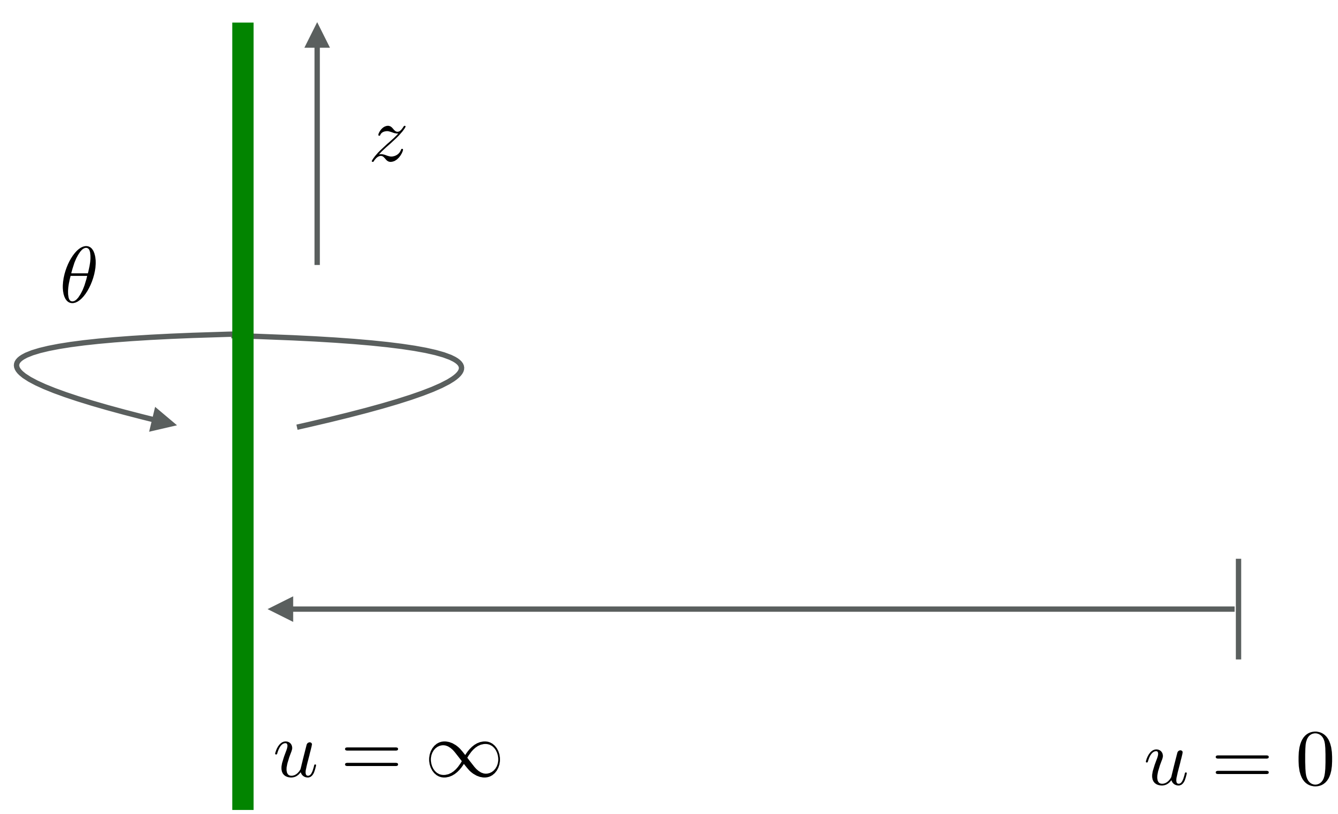
- Both metric and the field configuration become time-dependent. No analytical solution, but we will argue that this gives rise to **topological inflation**.

Explicit String Solutions

1.) Static solution: **Cohen-Kaplan** string: [Cohen, Kaplan 1988]

$$ds^2 = \frac{u}{u_0}(-dt^2 + dz^2) + \gamma^2 \left(\frac{u_0}{u}\right)^{1/2} \exp\left(\frac{u_0^2 - u^2}{u_0}\right)(du^2 + d\theta^2)$$

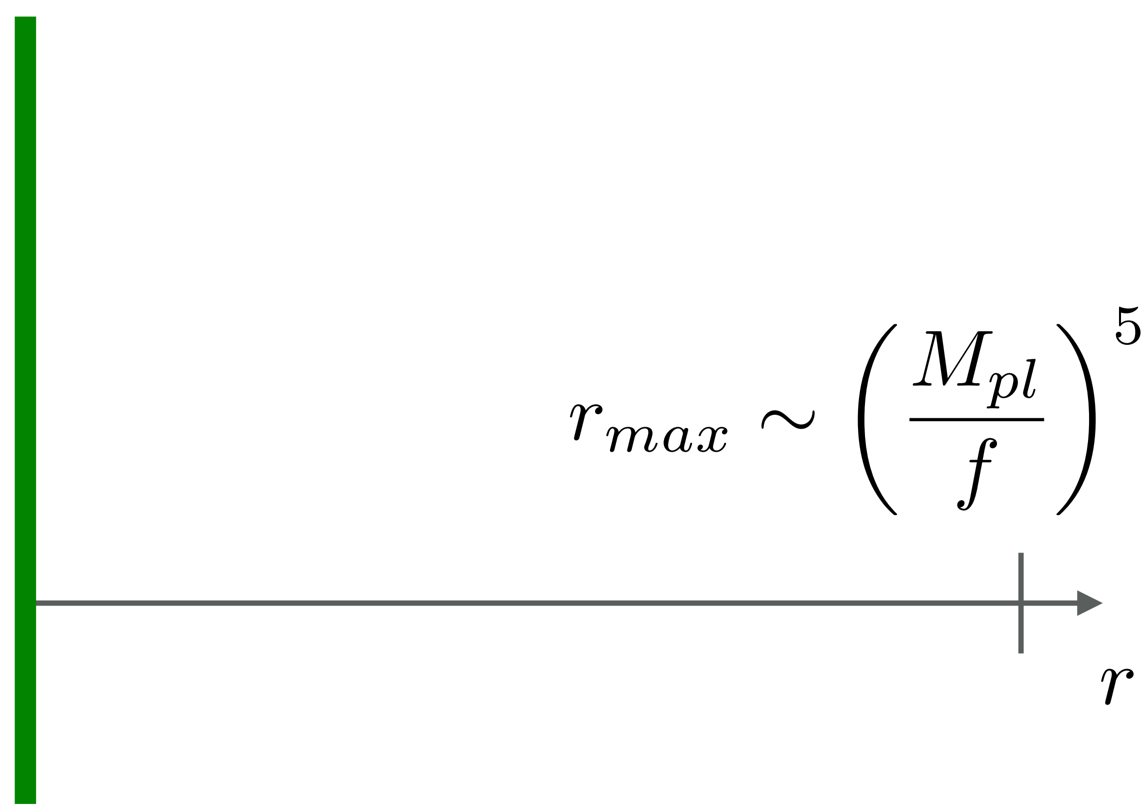
$$u_0 \equiv \frac{2M_{pl}^2}{f^2}$$



Explicit String Solutions

1.) Static solution: **Cohen-Kaplan** string: **2 problems**

- There is a physical **singularity** at at a finite proper distance from the string core.



$$r_{max} \sim \left(\frac{M_{pl}}{f}\right)^{5/4} e^{M_{pl}^2/f^2}$$

Explicit String Solutions

1.) Static solution: **Cohen-Kaplan** string: **2 problems**

- There is a physical **singularity** at at a finite proper distance from the string core.
- For $f > \sqrt{2}M_{pl}$ the **deficit angle** around the string is **negative** everywhere.

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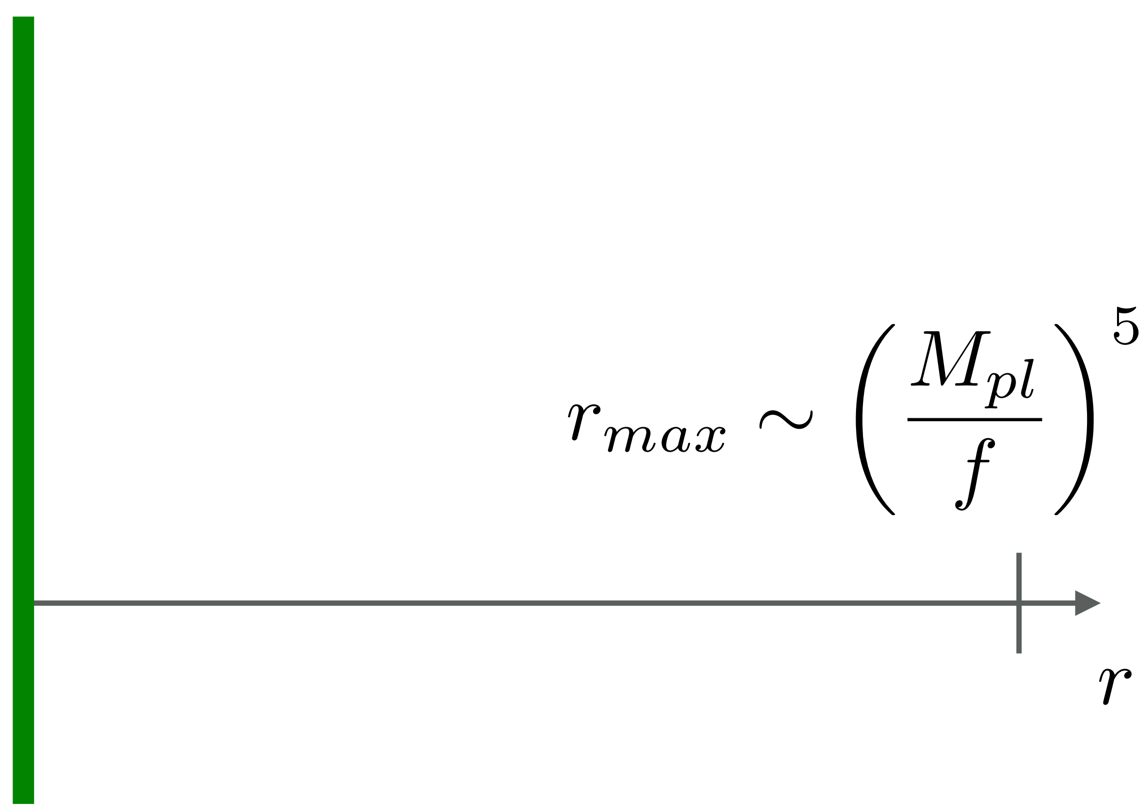


Non-Static String Solutions

2.) Consider the non-static case:

- **Gregory**'s solution: [Gregory 1996]

  The metric is time-dependent, but the field configuration is static.

  Spacetime expands along the string direction, while the width of the string remains constant.

  The singularity at finite proper distance from the string core is absent!

  However, this solution only exists for $f \lesssim M_{pl}$.
  [Gregory, Santos 2002]

Non-Static String Solutions

2.) Consider the non-static case:

- Let both the metric and the field configuration become time-dependent.

  This is what is needed if one hopes to find a non-singular string solution for the superplanckian axion.

  No analytical solution available.

  We will find that superplanckian axion will lead to **topological inflation**.
  [Linde 1994; Vilenkin 1994]

Non-Static String Solutions

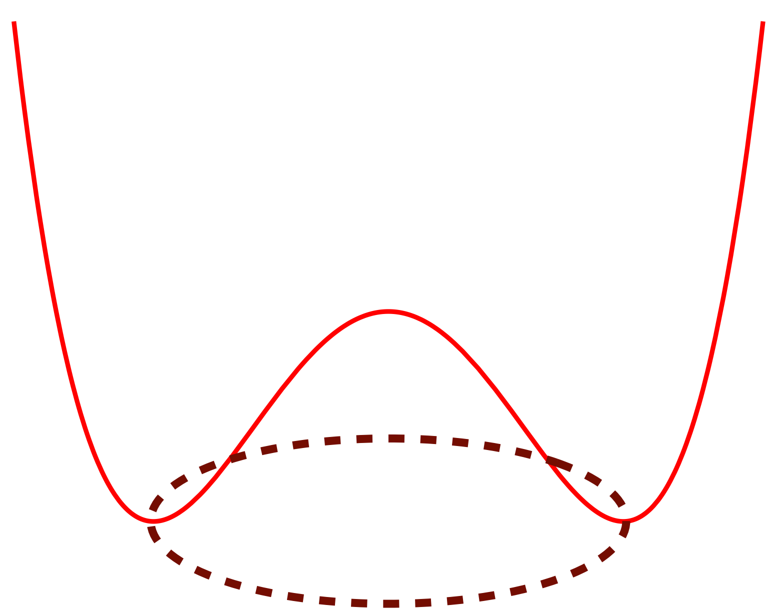
Topological inflation from the superplanckian axion.
[Linde 1994; Vilenkin 1994]

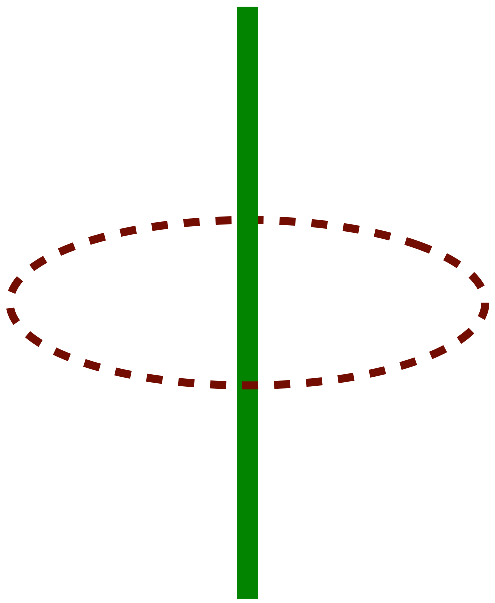
Consider the following UV completion:

$$\mathcal{L} = \frac{1}{2}|\partial\Phi| - \frac{\lambda}{4}\left(|\Phi| - f^2\right)^2$$

Axion = phase of Φ .

Vacuum manifold is S^1 and allows for **string solutions**.





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Core radius R: $\quad \frac{f^2}{R^2} \sim \lambda f^4 \quad \Rightarrow \quad R \sim \frac{1}{\sqrt{\lambda} f}$

Hubble radius H_0^{-1}: $\quad H_0^2 M_{pl}^2 \sim \lambda f^4 \quad \Rightarrow \quad H_0^{-1} \sim \frac{M_{pl}}{\sqrt{\lambda} f^2}$

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Topological inflation occurs if the Hubble radius due to the energy density in the string core is smaller than the core radius.

Core radius: $R \sim \frac{1}{\sqrt{\lambda} f}$ Hubble radius: $H_0^{-1} \sim \frac{M_{pl}}{\sqrt{\lambda} f^2}$

$$\boxed{H_0^{-1} < R \qquad \Leftrightarrow \qquad f > M_{pl}}$$

Summary: String Solutions

- $f < M_{pl}$:

  Non-singular solutions with time-dependent metric but static field configuration exist.

- $f > M_{pl}$:

  Expect non-singular solution should exhibit topological inflation.

  [Also numerical evidence by Cho 1988; Dolan et. al. 2017]

Summary: String Solutions

- $f < M_{pl}$:

  Non-singular solutions with time-dependent metric but static field configuration exist. ✓

- $f > M_{pl}$: ?

  Expect non-singular solution should exhibit topological inflation.

  [Also numerical evidence by Cho 1988; Dolan et. al. 2017]

Suggests candidate **magnetic WGC** for **axions**:
"The minimally charged string solutions should exist as a field theoretic object."

Strings In Theories With $f_{eff} > M_{pl}$

Superplanckian axions do not seem to exist when working top-down.

Typically, construct superplanckian axions from a more fundamental theory with two or more subplanckian axions.

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Typically, construct superplanckian axions from a more fundamental theory with two or more subplanckian axions.

E.g., take two axions φ_1, φ_2 with $f_1, f_2 < M_{pl}$.

Idea: make one combination of axions massive, such that the remaining axion is forced on a long winding trajectory.

This can e.g. be engineered by instanton alignment.

[Kim, Nilles, Peloso 2004] [For other approaches see Berg, Pajer, Sjors 2009; Bachlechner, Dias, Frazer, McAllister 2014; Shiu, Staessens, Ye 2015; Hebecker, Mangat, Rompineve, LW 2015]

Here we will take a different route.

Strings In Theories With $f_{eff} > M_{pl}$

Here: make one axion combination massive by coupling it to a 3-form/(-1)-form theory following Kaloper and Sorbo.

[Kaloper, Sorbo 2008; Kaloper, Lawrence, Sorbo 2011]

Consider a (-1)-form field with corresponding field strength F_0.

$$\mathcal{L}_{KS} = -\frac{1}{2g^2}|F_0|^2$$

F_0 is quantized in units of 2π. We can gauge shifts of F_0 by coupling it to an axion χ:

$$\mathcal{L}_{KS} \to -\frac{1}{2g^2}|F_0 + \chi|^2$$

Then a shift $F_0 \to F_0 + 2\pi$ is compensated by $\chi \to \chi - 2\pi$.

Strings In Theories With $f_{eff} > M_{pl}$

Here: take two axions φ_1, φ_2 with $f_1, f_2 < M_{pl}$ and with UV completion

$$\mathcal{L} = \frac{1}{2}|\partial\Phi_1|^2 - \frac{\lambda}{4}(|\Phi_1| - f_1^2)^2 + \frac{1}{2}|\partial\Phi_2|^2 - \frac{\lambda}{4}(|\Phi_2| - f_2^2)^2$$

There will also be two species of strings associated with Axions 1 and 2, respectively.

Strings In Theories With $f_{eff} > M_{pl}$

Here: take two axions φ_1, φ_2 with $f_1, f_2 < M_{pl}$ and take $f_1 = f_2$ for simplicity.

Make the combination $\chi = \varphi_1 - N\varphi_2$ massive while the orthogonal field ψ remains massless.

Gauge a 0-from field strength by χ :

$$\mathcal{L} = -\frac{f^2}{2}(\partial\varphi_1)^2 - \frac{f^2}{2}(\partial\varphi_2)^2 - \frac{1}{2g^2}|F_0 + \varphi_1 - N\varphi_2|^2 + \cdots .$$

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Explicit realisation in type IIB compactification given in [Hebecker, Mangat, Rompineve, LW 2015]

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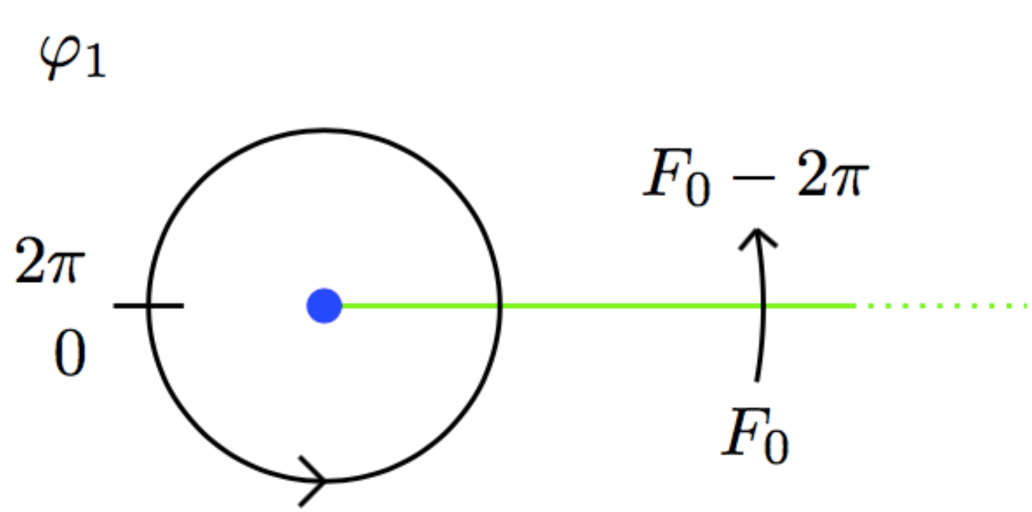
$$\mathcal{L} = -\frac{f^2}{2}(\partial\varphi_1)^2 - \frac{f^2}{2}(\partial\varphi_2)^2 - \frac{1}{2g^2}|F_0 + \varphi_1 - N\varphi_2|^2 + \cdots .$$

Now consider a string associated with the axion 1:



φ_1 shifts by 2π who going around the string.

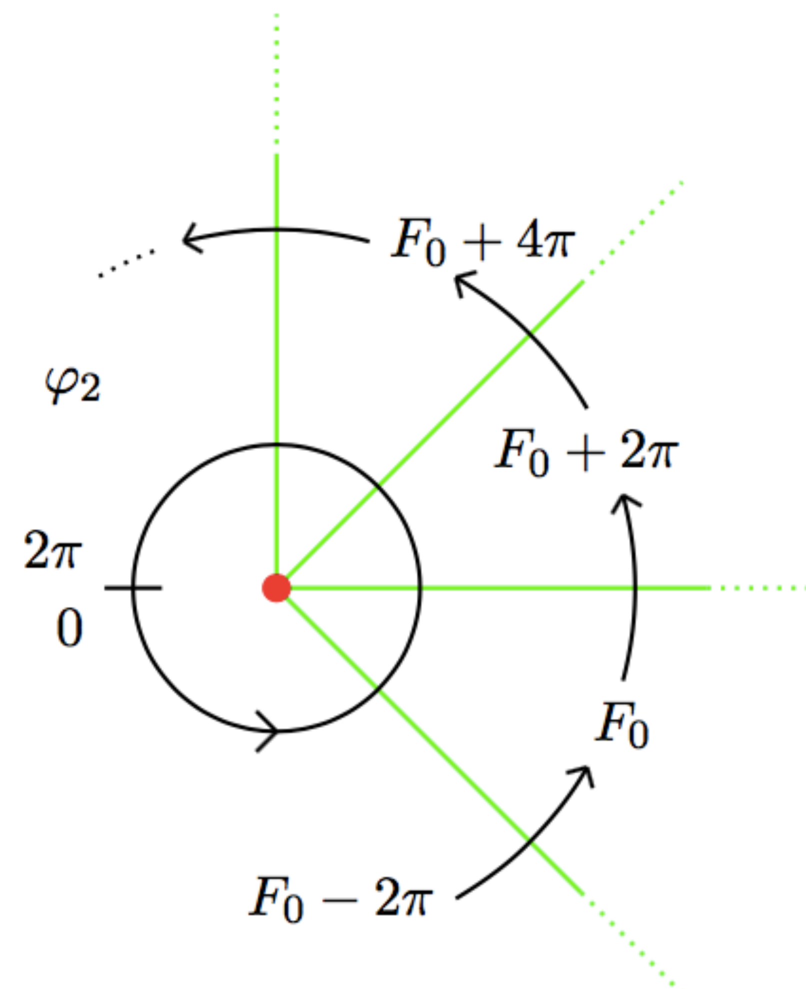
This is now associated with a shift by -2π of F_0 .

There must be a domain wall attached to every string of type 1.

Strings In Theories With $f_{eff} > M_{pl}$

$$\mathcal{L} = -\frac{f^2}{2}(\partial\varphi_1)^2 - \frac{f^2}{2}(\partial\varphi_2)^2 - \frac{1}{2g^2}|F_0 + \varphi_1 - N\varphi_2|^2 + \cdots .$$

Now consider a string associated with the axion 2:



There must be N domain walls attached to every string of type 2.

Strings In Theories With $f_{eff} > M_{pl}$

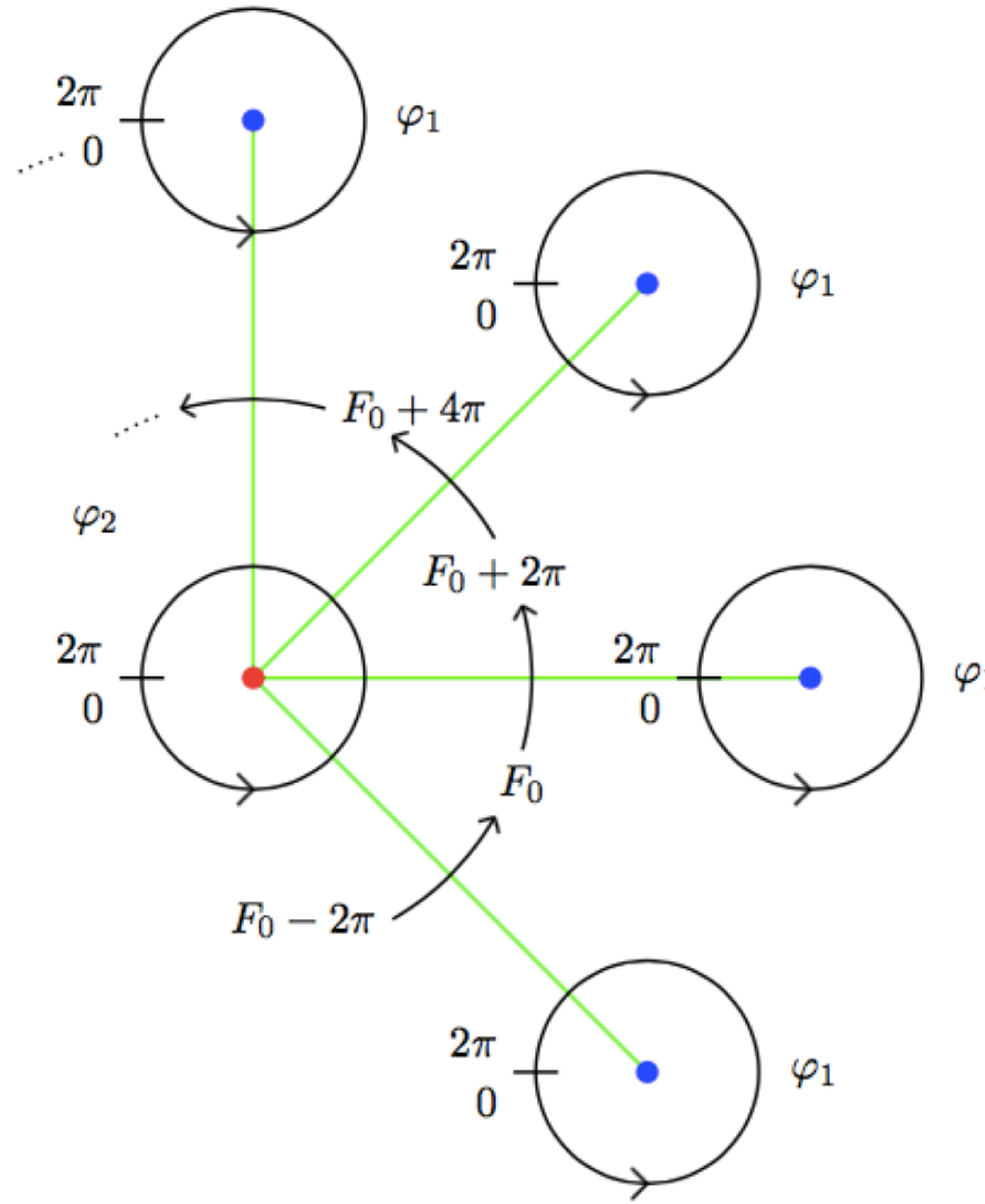
Can now build an effective string for the axion ψ with $f_{eff} = \sqrt{N^2+1} f$.

Going around this composite string find:

$$\psi \to \psi + 2\pi$$

$$\chi \to \chi \,,$$

$$F_0 \to F_0 \,.$$



[See also Higaki et.al. 2016]

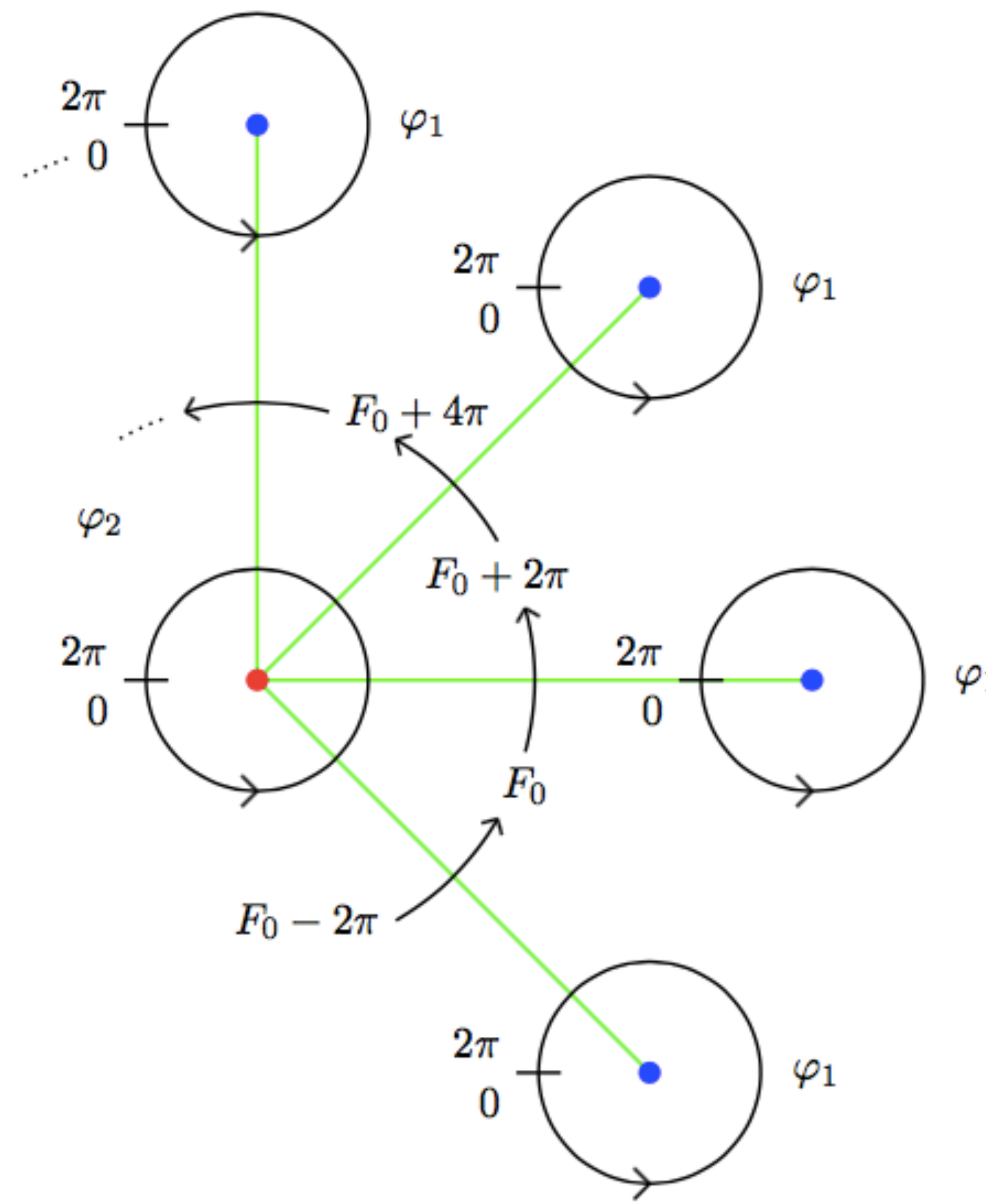
[See Saraswat 2016 for an analogous construction in 1-form theories]

Strings In Theories With $f_{eff} > M_{pl}$

Study this object for possible pathologies.

From previous analysis would expect that this solution should not be static.

Calculate string tension to determine when gravity will become important.



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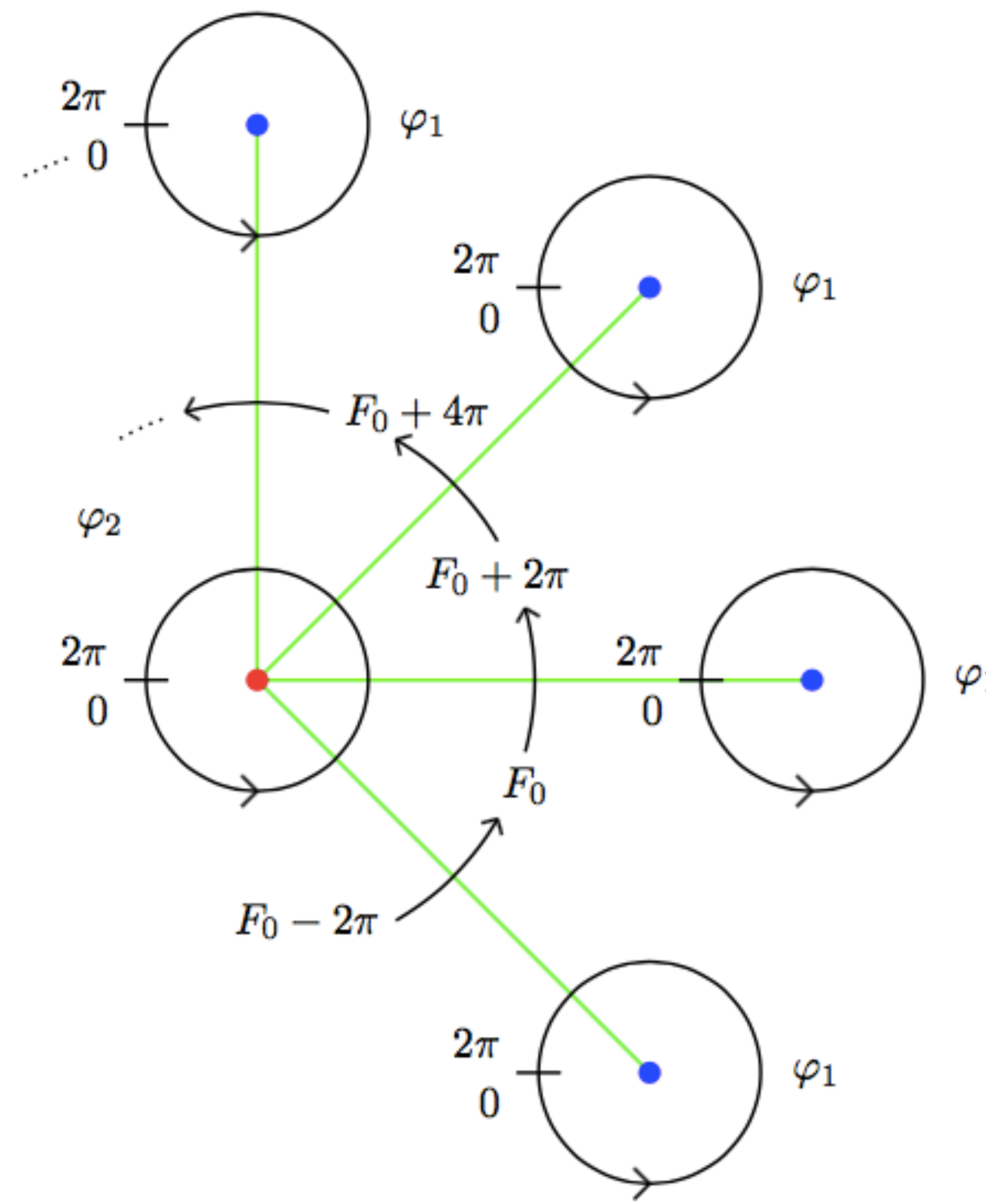
$$T \sim f_{eff}^2$$



Conclusions

- Studied string solutions as the magnetic objects in axion theories.
- For $f < M_{pl}$ such strings can arise from **static** field configurations.
- For $f > M_{pl}$ both spacetime and the field configuration will become **time-dependent**. Expect **topological inflation**.
- Provided a UV completion for a string with $f_{eff} > M_{pl}$ built out of the individual strings of a theory of two subplanckian axions. Expect this solution **not to be static**, potentially even singular.
- Candidate **magnetic WGC** for **axions**:
  "The minimally charged string solutions should exist as a field theoretic object."

Many

Thanks!