Where in F-theory is the supersymmetric standard model?

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Written in collaboration with various subsets of:
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Research program goal:

Global view of $\mathcal{V}_d = \{$ string vacua $\}$ w/SUSY in d-dimensions

- Constraints on $\mathcal{G}_d = \{$ d-dim SUGRA (+ YM, matter) $\}$

  (string constraints/new quantum consistency conditions)

- What do "typical" d-dimensional vacua look like?
  Is the SSM natural?

Much progress in last 8 years.

10D: $\mathcal{V}_{10} = \mathcal{G}_{10}$ ($\mathcal{N} = 1$)

- $U(1)^{496}, U(1)^{248} \times E_8$ inconsistent [Adams/DeWolfe/WT]

6D: $\mathcal{V}_6 \cong \mathcal{G}_6$

- F-theory closely matches { 6D SUGRA }
- Systematic global picture of 6D F-theory models
- Many interesting boundary cases – much current research
- Almost all vacua: na gauge group G, matter everywhere in moduli space
  e.g. $G_2 \times SU(2)$ with (7, 2) matter

4D: F-theory $\subset \mathcal{G}_4$, but big set

- F-theory geometry fairly well understood, outline of global picture
- Superpotential, fluxes, brane dynamics complicate picture
- Most geometries $\rightarrow G$, matter
- $E_8, SU(3) \times SU(2), \ldots$ typical, $SU(5)$ requires tuning moduli
- Several ways to get SSM: each a little "unnatural"

F-theory:

A nonperturbative approach to constructing 8D, 6D, 4D string vacua

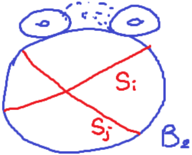
— IIB w/ varying axiodilaton on complex space $\mathbb{P}^1, B_2, B_3$, generally not CY

Power of holomorphy: algebraic geometry encodes global space of vacua

Basic picture:
Given an *elliptically fibered* K3, CY3 or CY4,

$\pi : X_2 \to \mathbb{P}^1$, $\pi : X_3 \to B_2$ or $\pi : X_4 \to B_3$

Geometry $\longrightarrow$ 8D, 6D, 4D supergravity.



Physics encoded in Weierstrass model

$$y^2 = x^3 + fx + g, \qquad f \in \Gamma(\mathcal{O}(-4K_B)), g \in \Gamma(\mathcal{O}(-6K_B))$$

Codimension 1 (Kodaira) singularities (S_i): gauge group, Codim. 2: matter

Massless spectrum of 6D SUGRA $\Rightarrow$ F-theory geometry

Classifying elliptic Calabi-Yau threefolds/6D F-theory models

(Gross: finite number of topological types; desire explicit construction)

1. Classify bases B_2: complex surfaces that support elliptic CY3
2. Given B_2: consider all "tunings" of generic Weierstrass model

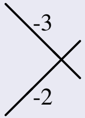
Classifying B_2's (restrict to smooth bases)

- Grassi: All B_2 blow-ups of minimal surfaces $\mathbb{P}^2, \mathbb{F}_m (m \leq 12)$, Enriques.
- Non-Higgsable clusters: $C \cdot C \leq -3 \Rightarrow$ forces na G [Morrison/WT]

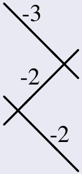
$\overline{\quad -m \quad}$

$(m = 3, 4, 5, 6, 7, 8, 12)$

$\mathfrak{su}(3), \mathfrak{so}(8), \mathfrak{f}_4$
$\mathfrak{e}_6, \mathfrak{e}_7, \mathfrak{e}_8$



$\mathfrak{g}_2 \oplus \mathfrak{su}(2)$



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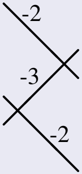
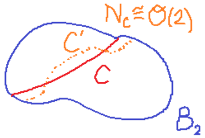


$\mathfrak{su}(2) \oplus \mathfrak{so}(7) \oplus \mathfrak{su}(2)$

Geometry of non-Higgsable groups



The base B_2 is a complex surface.

Contains homology classes of complex curves C_i

For $C \cong \mathbb{P}^1 \cong S^2$, local geometry encoded by *normal bundle* $\mathcal{O}(m)$

$C \cdot C = m$; $e.g., N_C \cong \mathcal{O}(2) \cong TC$: deformation has 2 zeros, $C \cdot C = +2$

If $N_C \cong \mathcal{O}(-n), n > 0$, C is *rigid* (no deformations)

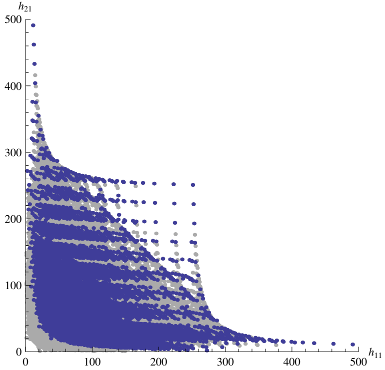
For $\mathcal{O}(-n), n > 2$, base space is so curved that singularities must pile up to preserve Calabi-Yau structure on total space $\Rightarrow$ non-Higgsable gauge group

Familiar heterotic dual example: K3 with 24 + 0 instantons $\Rightarrow E_8$

Classification of base surfaces B_2: start with $\mathbb{P}^2, \mathbb{F}_m$, blow up to get all bases B_2

Finite number of possibilities: non-Higgsable clusters bound complexity

- 61,539 toric bases (some not strictly toric: -9, -10, -11 curves) [Morrison/WT]



- Beyond toric: 162,404 "semi-toric" bases w/ 1 $\mathbb{C}^*$-structure [Martini/WT]
- All bases for EF CY threefolds w/ $h^{2,1}(X) \geq 150$ [WT/Wang]

Global picture of 6D F-theory models

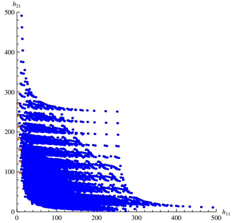
— Systematic approach to constructing B_2's, tunings $\Rightarrow$ global picture
(Many interesting ongoing technical issues:
smaller $h^{2,1}(X)$, U(1)'s, discrete G, exotic matter; residual swampland ...)

— Toric bases surprisingly effective for classifying CY3's, particularly for large Hodge numbers; most known CY3's elliptic (cf. [Candelas/Constantin/Skarke])

— Proven upper bound $h^{2,1} \leq 491$ rigorously

— "Most" bases B_2 have non-Higgsable G_{NA}

(all but weak Fano = gdP)

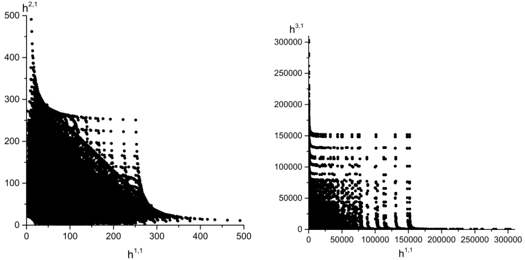


— Typical gauge groups products of $E_8, G_2 \times SU(2), \ldots$
no generic $SU(3), SU(5)$

4D F-theory compactifications: Story parallel in many ways

– Compactify on elliptic Calabi-Yau fourfold, base B_3 = complex threefold
Most known CY4's elliptic (cf. [Gray/Haupt/Lukas, Anderson/Gao/Gray/Lee])

– Empirical data suggest similar structure (though less complete for CY4's)



4D theories significantly more subtle:
- Minimal models (Mori theory) more subtle
- F-theory $\subset \mathcal{V}_4$ (e.g. heterotic on quintic)
- Fluxes, superpotential, seven-brane dynamics not completely understood

But evidence so far: moduli space of CY4 geometries parallel to CY3 story

4D F-theory geometry

Analog of minimal bases:
Fano threefolds, B_2 bundles over $\mathbb{P}^1$, $\mathbb{P}^1$ bundles over B_2.

Similar non-Higgsable clusters: divisors (surfaces) w/ negative normal bundles
[Anderson/WT, Grassi/Halverson/Shaneson/WT, Morrison/WT]

Single group clusters: $SU(2), SU(3), G_2, SO(7), SO(8), F_4, E_6, E_7, E_8$

(cannot have: non-Higgsable $SU(5), SO(10)$)

the only connected (w/ matter) 2-factor products that can appear are:

$$G_2 \times SU(2), \quad SO(7) \times SU(2), \quad SU(2) \times SU(2), \quad SU(3) \times SU(2), \quad SU(3) \times SU(3)$$

4D clusters can have chains, loops, branching ...

6D: protected by D-terms; 4D, F terms also relevant.

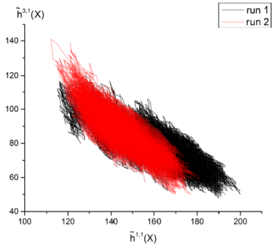
Exploring threefold bases for EF CY4's

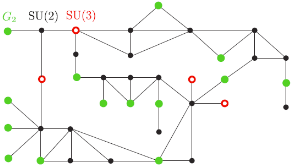
Some large classes of bases explored:

[Anderson/WT, Halverson/WT, WT/Wang, Morrison/WT (ta), Halverson/Tian].

Monte Carlo on toric threefold bases (w/ Yinan Wang)

Explore connected toric threefold bases from $\mathbb{P}^3$ by blow-up, -down transitions





Estimate number of connected toric threefold bases $\sim 10^{48\pm 2}$

Non-Higgsable G: $\sim 14\times$ SU(2), $\sim 10 \times G_2$, $\sim 3 \times F_4$, $\sim 2\times$ SU(3), $\sim 1\times$ SO(8); $\sim 10\%$ of products are SU(3) $\times$ SU(2).

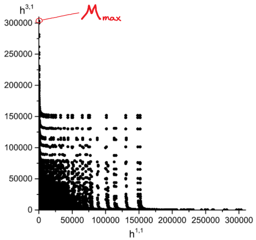
Physics: F-theory flux vacua (w/ Y. Wang)

Can we identify the F-theory geometry with most flux vacua?

Conventional wisdom (Ashok-Denef-Douglas): $\Rightarrow$ in regime $h^{1,1} \ll h^{3,1}$

$$\#\text{vacua } N(X) \sim 10^{0.9\ h^{3,1}(X)}$$

vacua $N(X) \sim 10^{0.9\ h^{3,1}(X)}$



$\mathcal{M}_{\max}$ is elliptically fibered; B_2 over $\mathbb{P}^1$. Dominates set of flux vacua?

$N(\mathcal{M}_{\max}) \sim 10^{272,000}$ non-Higgsable $G_{\max} = E_8^9 \times F_4^8 \times (G_2 \times SU(2))^8$

Circumstantial evidence: $\sum_{X \neq \mathcal{M}_{\max}} N(X) < 10^{-3000} N(\mathcal{M}_{\max})$

Physics: realizing the standard model in F-theory

We have some sense of a global picture of the space of elliptic CY4's.

Ignoring the outstanding issues of G-flux and seven-brane DOF
What are the options for realizing $G = SU(3) \times SU(2) \times U(1)$ in F-theory?

1. Tune the whole thing — but not on divisors with NHC's
2. Tune part of G, get part from NHC; *e.g.* NH $SU(3)$, tune $SU(2) \times U(1)$
3. Get all of G from non-Higgsable structure

Unification

- $SU(5), SO(10)$ cannot appear as NHC's. Can't enhance NHC $\rightarrow SU(5)$ so e.g. SU(5) only from tuning (or forced by superpotential)
- $E_6, \ldots$ possible for NHC's, could break *e.g.* from fluxes on branes.

Where in F-theory is the supersymmetric standard model?

Consider "naturalness" of alternatives

Tuning through SU(5) GUT (*a la* [Beasley/Heckman/Vafa, Donagi/Wijnholt])
(or just SU(3) $\times$ SU(2) $\times$ U(1) (*e.g.*, [Lin/Weigand]))

- Can't be done on $\mathcal{M}_{\text{max}}$
- Requires tuning moduli [Braun/Watari, Halverson/Tian]
- For SU(5), flux breaking needs special (*e.g.* non-toric) divisors: Yukawas, … [Heckman/Morrison/Vafa, Marsano/Saulina/Schafer-Nameki]

Non-Higgsable SU(3) $\times$ SU(2) [Grassi/Halverson/Shaneson/WT]

- NHC's fairly natural ($\sim$ 10% of products, $\sim$ 2/base from MC w/Wang)
- Can't be done on $\mathcal{M}_{\text{max}}$
- Tuning the U(1) may be expensive ($\sim$ SU(2) on $-K + X_{\text{eff}}$, breaking adjoint)
- Some non-Higgsable U(1) cases but rare [Martini/WT, Morrison/Park/WT, Wang]

GUT breaking through 7-brane flux on non-Higgsable E_6, E_7, E_8

- Doesn't seem to work on $\mathcal{M}_{\text{max}}$
  (E_8's all on $D = \mathbb{F}_m$, no Yukawas [Beasley/Heckman/Vafa])
- Need (suppressed?) exotic (*e.g.* non-toric) local structure.

Summary of situation for realizing supersymmetric standard model

Conventional wisdom: $\mathcal{M}_{\text{max}}$ dominates flux vacua
but no clear way to get SSM.

Statistics of bases B_3 (from toric Monte Carlo):
$SU(3) \times SU(2)$ non-Higgsable is natural.
Standard model matter multiplets naturally arise.
U(1) seems to require tuning. Anthropic?

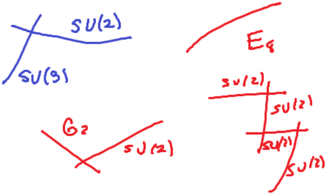
Tuning SU(5): requires many moduli on small $h^{1,1}$ bases ($\sim \mathbb{P}^3$)
Maybe fewer moduli on surfaces close to NHC's, larger $h^{1,1}$
Tradeoff between tuning and NHC obstruction

Upshot: 4D F-theory seems to generically predict G, matter.
But the SSM does not seem particularly natural.

Research ongoing ...

Another application: NHC hints for dark matter candidates: Two possibilities:

I) "hidden sector" dark matter, e.g. from a disconnected cluster



II) WIMP dark matter (from $SU(2) \times G$, $G = SU(2), SU(3), SO(7), SO(8)$)

For $\mathcal{M}_{\max}$, get disconnected $E_8, F_4, G_2 \times SU(2)$ (and subgroup) sectors

Conclusions

- We have a good handle on the classification of elliptic Calabi-Yau threefolds
- A plausible "bird's-eye" picture of the global space of elliptic CY4's (with much work remaining to be done)
- Significant questions regarding the connection of geometry and physics in 4D
- Several ways of realizing SU(3) $\times$ SU(2) $\times$ U(1) in F-theory
- Some sense of what may be more or less natural in 4D F-theory landscape
- No overwhelmingly clear sign of where the SUSY standard model is favored