One-loop Pfaffians and large-field inflation

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New Ideas in String Phenomenology
16.02.2017

Based on [1702.00420] with C. Wieck

Motivation – Moduli stabilization and inflation

Moduli Stabilization

- Moduli stabilization in string theory
  - KKLT [Kachru,Kallosh, Linde, Trivedi '03]
  - Racetrack [Dorey '99; Dijkgraaf, Vafa '02; Escoda, Gomez-Reino, Quevedo '03]
  - LARGE Volume [Balasubramanian, Berglund, Conlon, Quevedo '05]
- Moduli have no perturbative potential $\Rightarrow$ stabilize via fluxes / non-perturbative (exponential) term
- Prefactors of exponential often field-dependent Pfaffians

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Inflation

- Want to study large field inflation [talks by Blumenhagen,McAllister,Cicoli,. . .]
  - Aligned inflation [Kim, Nilles, Peloso '04]
  - N-flation [Dimopoulos, Kachru, McGreevy, Wacker '05]
  - Axion monodromy inflation [Silverstein, Westphal '08]
  - . . .

Motivation – Questions we want to address

Starting point

- Large field inflation
- (Kähler) modulus in exponential
- Inflaton in Pfaffian prefactor

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Questions

1. Can the exponential still stabilize the modulus?
2. Is large field inflation still possible?
3. What about
   - Slow-roll?
   - Values of r and n_s?
   - Single field inflation?

Outline

1. Setup
2. Inflaton-dependent Pfaffians
   - Polynomial Pfaffians
   - Exponential Pfaffians
   - Periodic/Theta-function Pfaffians
3. Conclusion

Setup

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Disclaimer

- Study inflation in low energy EFT
- Use ingredients from string theory

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- Want: Minimize potential over entire moduli space, but:
  - Some ingredients (e.g. Kähler potential) unknown
  - Computation power for $\mathcal{O}(100)$ moduli beyond what is currently feasible

Setup

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- Study inflation in low energy EFT
- Use ingredients from string theory
- Want: Minimize potential over entire moduli space, but:
  - Some ingredients (e.g. Kähler potential) unknown
  - Computation power for $\mathcal{O}(100)$ moduli beyond what is currently feasible
- Study generic toy model rather than concrete realization
  - One Kähler modulus T and one inflaton field Φ
  - Assume other moduli have been stabilized by other mechanisms at higher scale
  - Fine-tuned uplift to deSitter (not address the CC problem here)

Setup – Kähler and superpotential

The Model

$$K = K(T + \overline{T}, \Phi + \overline{\Phi}) \,, \qquad W = W_0 + \mu\Phi^2 + A(\Phi)e^{-\alpha T}$$

Possible origin

- IIB flux compactification with mobile D7 branes
  [Ibanez, Marchesano, Valenzuela '14; Bielleman, Ibanez, Pedro, Valenzuela, Wieck '16]
- D7 brane chaotic inflation [Hebecker, Kraus, Witkowski '14; Arends, Hebecker, Heimpel, Kraus, Lust, Mayrhofer, Schick, Weigand '14]
  - T is 4-cycle volume in CY3
  - Φ is position modulus of mobile D7 brane
    $\Rightarrow$ large field excursion from unwinding
  - W_0 and μ generated from G_3-flux (Need to be careful with tuning of μ [Hebecker, Mangat, Rompineve '14])
  - α given by gauge-kin function (GC) or GW invariants (instantons)
  - $A(\Phi)$ can be polynomial / exponential / periodic (depending on origin of NP term)

Inflaton-dependent Pfaffians

Polynomial Pfaffian

The model

$$K = -3\log(T+\overline{T}) + \frac{1}{2}(\Phi+\overline{\Phi})^2\,,$$

$$W = W_0 + \mu\Phi^2 + A_0\left(\sum_{m=0}^{n}\delta_m\Phi^m\right)^p e^{-\alpha T}$$

Polynomial Pfaffian

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Polynomial Pfaffian [Donagi, Wijnholt '10; Cvetic, Donagi, Halverson, Marsano '12; Curio '09 '10]

- M-Theory: From Euclidean M5 branes wrapping a vertical elliptical divisor
- F-Theory/Type IIB: From Euclidean D3 (ED3) branes on internal 4-cycle
- NP term from intersection of ED3 with GUT divisor
- Φ are ED3 and D7 brane moduli
- Heterotic: From WS instanton on a curve w/ volume T
- Φ are CS or bundle moduli

Polynomial Pfaffian

Parameter examples

- Use notation $T|_{\theta=0} = t + i\sigma, \quad \Phi|_{\theta=0} = \chi + i\varphi$
- Use params $W_0 = 4 \times 10^{-3}, \quad \mu = 10^{-5}, \quad \alpha = 2\pi/5$
- That means for $t_0 = 10$: $A_0 \sim -\mathcal{O}(100), \quad \Delta_{\text{up}} \sim \mathcal{O}(W_0)$

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Simplifications

- δ_i are VEVs of heavy fields, $\delta_m > \delta_{m+1}$ (sub-Planckian VEVs)
- Polynomial factors to some power $p \leftrightarrow \#$ zero modes
  $\Rightarrow \delta_m = 0$ unless m multiple of p
- Ansatz $A(\Phi) = A_0(1 + \delta\Phi^m)$

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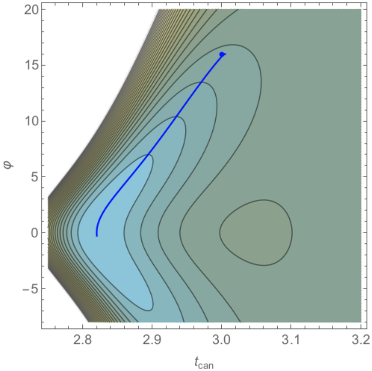
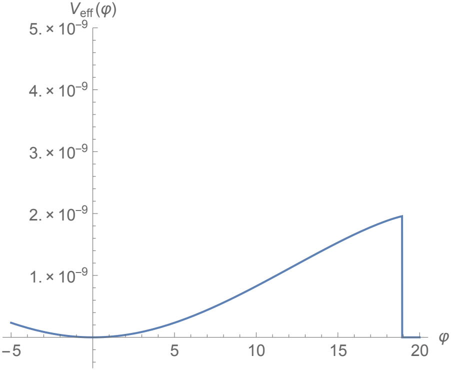
Observations

- χ stabilized at zero for realistic parameter ranges
- σ displaced for large δ (integrate out dynamically)
- Results depend on m mod 4

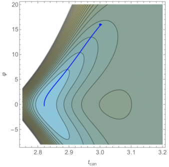
Polynomial Pfaffian for $m = 1 \bmod 4$ or $m = 3 \bmod 4$

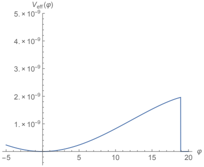




Inflaton trajectory for $m = 1$ and $\delta = 1/3$

Polynomial Pfaffian for $m = 1 \bmod 4$ or $m = 3 \bmod 4$

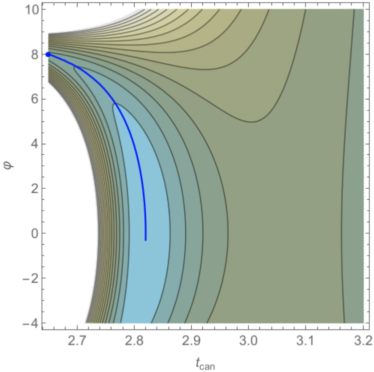


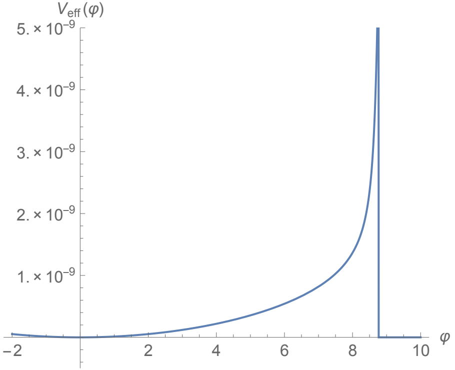


Parameter examples

- δ can be quite large ($\delta \lesssim \mathcal{O}(1)$ for $m = 1$)
- More generally: If $\delta\varphi_\star^m \gtrsim 10^m \to 2^{\text{nd}}$ (false) vacuum
- Destabilization point of t depends on W/μ_0 (as for $A = \text{const}$ [Buchmuller, Dudas, Heurtier, Westphal, Wieck, Winkler '15])
- Valley flat $\Rightarrow$ 60 e-folds of slow-roll inflation possible (runaway even flattened further)
- Orthogonal direction steep $\Rightarrow$ single field inflation despite curved trajectory
- After 60 e-folds: $n_s \approx 0.964$, $r \approx 0.078$

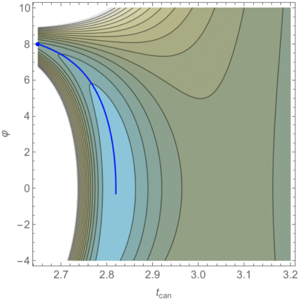
Polynomial Pfaffian for $m = 2 \bmod 4$

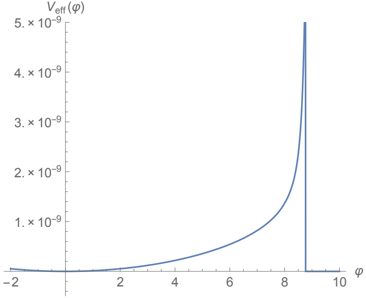




Inflaton trajectory for $m = 2$ and $\delta = 1/40$

Polynomial Pfaffian for $m = 2 \bmod 4$

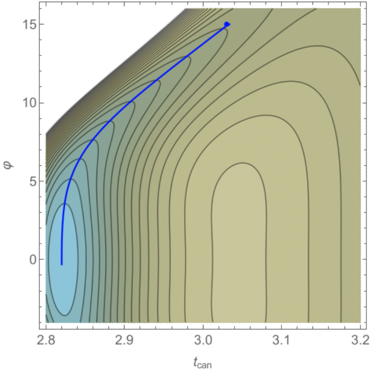


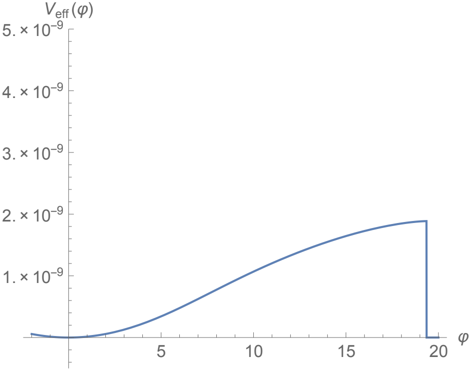


Behavior

- Inflaton trajectory curved towards smaller t (V exp. steep)
- δ needs to be much smaller ($\delta\varphi^m \ll 1$)
- 60 e-folds of slow-roll inflation might be possible
- But n_s and r are ruled out (too large)

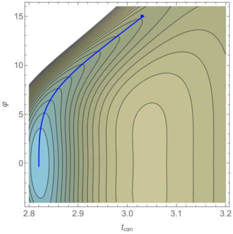
Polynomial Pfaffian for $m = 0 \bmod 4$

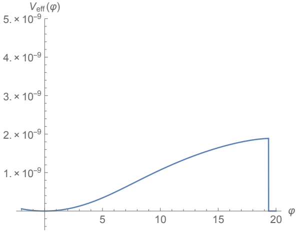




Inflaton trajectory for $m = 4$ and $\delta = 1/2500$

Polynomial Pfaffian for $m = 0 \bmod 4$





Behavior

- Interestingly most explicitly computed Pfaffians are of this form [Buchbinder, Donagi, Ovrut '02; Curio '09; Cvetic, Donagi, Halverson, Marsano '12]
- Inflaton trajectory curved towards larger t
- V flattend even further (as for $m = 1, 3 \bmod 4$)
- δ can be larger: False vacuum for $\delta\varphi^m \gtrsim 5^m$
- Orthogonal direction again steep
- After 60 e-folds of slow-roll inflation: $n_s \approx 0.976$, $r \approx 0.048$

Exponential Pfaffian

The model

$$K = -3\log(T+\overline{T}) + \frac{1}{2}(\Phi+\overline{\Phi})^2\,,$$
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Exponential Pfaffian

- Relaxion Monodromy [McAllister, Schwaller, Servant, Stout, Westphal '16]
  - Wrap NS5 brane on 2-cycle Σ_2
  - $N \gg 1$ NS5 brane flux $\to$ $N \gg 1$ D3 brane flux
  - Large field range from axion $b \sim \int_{\Sigma_2} B$
  - Strong backreaction on 4-cycle Σ_4 with NP effect $\Rightarrow$ $A \sim e^{-N}$

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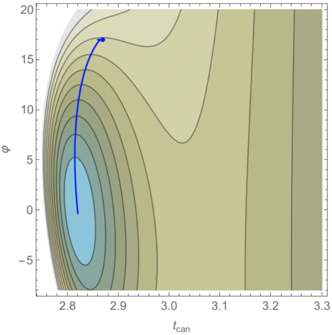
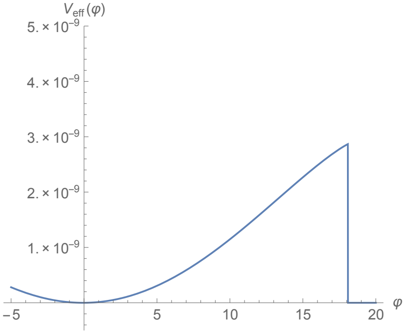
  - Strong backreaction on 4-cycle Σ_4 with NP effect $\Rightarrow A \sim e^{-N}$
- Higgsotic Inflation [Ibanez, Marchesano, Valenzuela '14]
  - Wrap D7 on 2-cycle Σ_2
  - Large field range from unwinding
  - G_3 flux leads to accumulated D3 brane charge for each winding
  - Backreaction on 4-cycle Σ_4 with NP effect $\Rightarrow A \sim e^{-\delta\Phi}$
    δ depends on warping, distance between Σ_2 and Σ_4, ...

Exponential Pfaffian for $\delta = 1/(4\pi^2)$

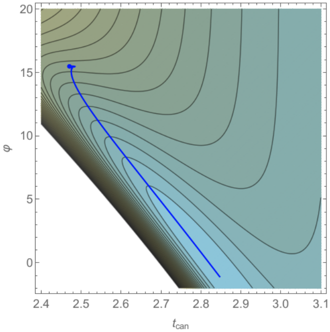


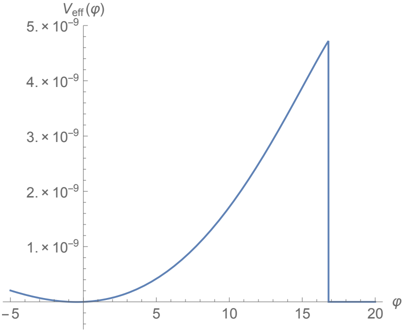


Behavior

- Potential tilted towards steep flank
- χ, σ stabilized at 0
- After 60 *e*-folds of slow-roll inflation: $n_s \approx 0.96$, $r \approx 0.09$

Exponential Pfaffian for $\delta = 1/3$





Behavior

- Potential strongly tilted towards steep flank
- At most ~ 50 e-folds of slow-roll inflation: $n_s \approx 0.94$, $r \approx 0.18$ (ruled out)

Periodic/Theta-function Pfaffian

The model

$$K = -3\log(T+\overline{T}) + \frac{1}{2}(\Phi+\overline{\Phi})^2\,,$$
$$W = W_0 + \mu\Phi^2 + A_0\,\vartheta_j(i\Phi,q)^\delta e^{-\alpha T}$$

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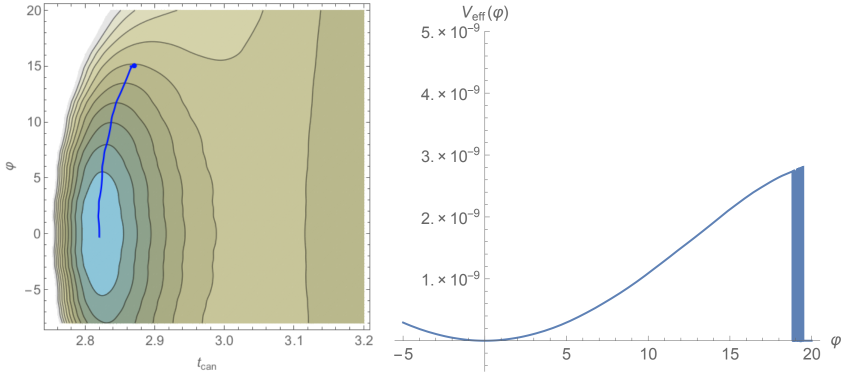
Theta function Pfaffian

- Gauge-kinetic function receives 1-loop correction: $f = T + \frac{1}{4\pi^2}\log[\vartheta_j\ldots]$
- $A \sim \vartheta_j$ in setups with toroidal compactifications [Berg,Haack,Koers '04] No pileup of charge
- More generally (from periodicity of intermediate Jacobian parameterizing Wilson line moduli) [Corvilain, Grimm, Regalado '16]
- Can be dominant if previous exp. dependence suppressed:
  - Need $\overline{\text{D3}}$ / $\overline{\text{D7}}$ branes for tadpole cancellation
  - Σ_4 far away only sees dipole with net charge $N_{\text{D3}} - N_{\overline{\text{D3}}}$

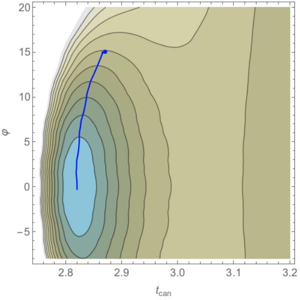
[Flauger, McAllister, Pajer, Westphal, Xu '09; Flauger, McAllister, Silverstein, Westphal '14]

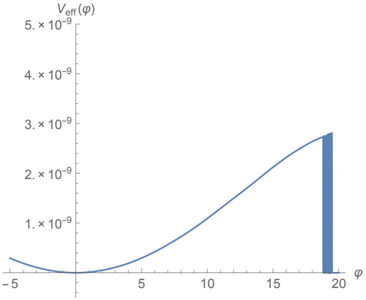
Periodic Pfaffian $\vartheta_3(i\Phi, q)$



$$\vartheta_3(\varphi, q) = 1 + 2\sum_{n=1}^{\infty} q^{n^2}\cos(2n\varphi), \quad q = e^{i\pi\langle Z\rangle} = 0.4 - 0.3i, \quad \delta = \frac{1}{4\pi^2}$$

Periodic Pfaffian for $\vartheta_3(i\Phi, q)$

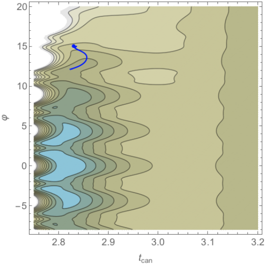


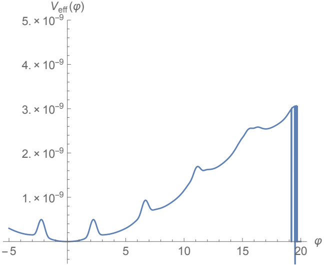


Behavior

- Periodic modulations imprinted on potential and consequently inflaton trajectory
- Similar to other periodic modulations [Kappl, Nilles, Winkler '15]
- 60 e-folds of slow-roll inflation w/ realistic n_s, r possible
- Running of n_s substantial and δ-dependent

Periodic Pfaffian for $\vartheta_3(i\Phi, q)$





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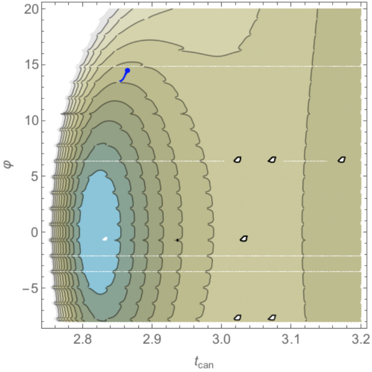
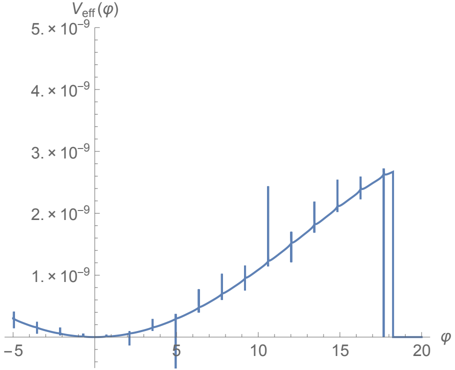
Behavior

- Periodic modulations imprinted on potential and consequently inflaton trajectory
- Inflaton trapped in false vacuum
- no successful inflation

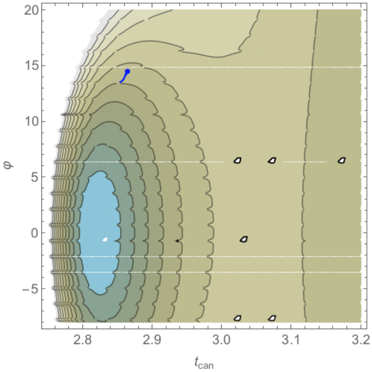
Periodic Pfaffian $\vartheta_2(i\Phi, q)$

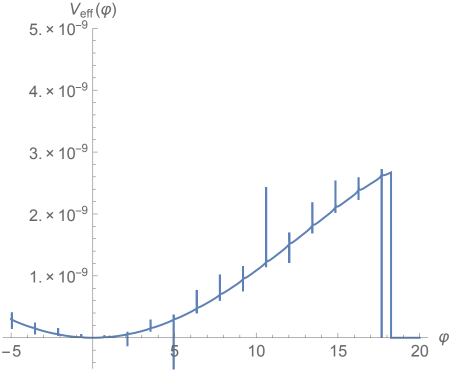




$$\vartheta_2(\varphi, q) = 2q^{1/4} \sum_{n=0}^{\infty} q^{n(n+1)} \cos((2n+1)\varphi)$$
$$q = e^{i\pi\langle Z\rangle} = 0.4 - 0.3i, \quad \delta = \frac{1}{4\pi^2}$$

Periodic Pfaffian for $\vartheta_2(i\Phi, q)$





Behavior

- $\vartheta_2(\varphi, q)^\delta$ has branch cuts for $\varphi > \pi/2$
- Approximation breaks down

Conclusion

Conclusion

Polynomial Pfaffian

- $m = 0, 1, 3 \bmod 4$: Inflaton trajectory curves right $(t > t_0)$: V flat, further flattened by Pfaffian, δ can be large
- $m = 2 \bmod 4$: Inflaton trajectory curves left $(t < t_0)$: V exponentially steep, δ needs to be small

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  V exponentially steep, δ needs to be small

Exponential Pfaffian

- Inflaton trajectory tilts to the right ($t < t_0$):
  V exponentially steep

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- $m = 2 \mod 4$: Inflaton trajectory curves left ($t < t_0$): V exponentially steep, δ needs to be small

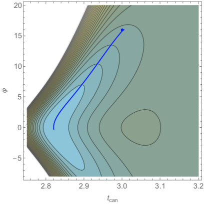
Exponential Pfaffian

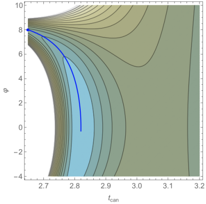
- Inflaton trajectory tilts to the right ($t < t_0$): V exponentially steep

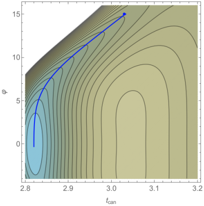
Periodic/Theta function Pfaffian

- Periodic function imprinted onto inflaton trajectory
- Large modulations trap inflaton in false vacuum, running substantial

Conclusion







Thank you for your attention