# Cosmological singularities in holography

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New Ideas in String Phenomenology
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Based on work with Adam Bzowski and Thomas Hertog: 1512:05761 + ongoing

# Cosmological singularities

- It appears that our universe began with a big bang singularity, *i.e.* our inflating FLRW cosmology evolved out of a phase with Planckian curvature and energy density, beyond the purview of any (semi)classical theory of gravity
  - I will neglect the possibility of a bouncing universe

- Primary goal of quantum gravity: provide an understanding of singularities. This is necessary for cosmology in order to describe the initial conditions of the universe.

- Holography provides a tool for studying singularities. Crunching AdS spacetimes have been constructed from M-theory compacted on $S^7$. These are $\mathcal{N} = 8$, D = 4 = d+1 bulk theories with a boundary dual that is the (deformed) CFT on a stack of M2 branes.

Hertog and Horowitz: 0406134 & 0503071
Aharoney, Bergman, Jafferis, Maldacena: 0806:1218

- Further exploration and other models: Craps, Hertog & Turok: 0711.1824 & 0905.0709, Barbon & Rabinovici: 1102.3015, Smolkin & Turok: 1211.1322, Craps, Rajaraman & Sethi: hep-th/0601062, Das, Michelson, Nrayan & Trivedi: hep-th/0602107, Awad, Das, Nampuri, Narayan & Trivedi: 0807:1517, Engelhardt, Hertog & Horowitz 1404:2309 &1503.08838

- Recently: Kumar & Vaganov: 1510.03281, Brandenberger, Cai, Das, Ferreira, Morrison & Wang: 1601:00231, Ferreira& Brandenberger: 1602:08152

# Holography $\longleftrightarrow$ cosmology

- What is the signature of the bulk singularity in the dual field theory?

- The form of the answers

  - Correlation functions

  - Properties of the state and stability of the boundary theory

- The relation to cosmology

  - Learn something about properties of singularities in quantum gravity (initial conditions)

  - Learn something about cosmologies (time dependent spacetimes) with strong gravity

# Holographic setup

- We study a specific consistent truncation to a theory with gravity and a single scalar field with $m^2 = -2$:

$$S_{\rm bulk} = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 + 2 + \cosh(\sqrt{2}\phi)\right]$$

Duff & Liu: 9901149

  - The gauge coupling has been chosen such that the pure AdS solution with $\phi = 0$ has $R_{\rm AdS} = 1$
  - The tachyonic scalar is within the Breitenlohner-Freedman bound. With appropriate asymptotic boundary conditions, the theory is non-perturbatively stable at $\phi = 0$.

- For non-trivial scalar fields, the boundary CFT is deformed by an operator of weight $\Delta$

$$m^2 = \Delta(\Delta - d)$$

$$S_{\rm boundary} = S_{\rm CFT} + \int d^3x\sqrt{-\gamma}\ \mathcal{O}_\Delta$$

# Bulk geometry

- Non-standard asymptotic boundary conditions to allow for non-trivial instanton solutions

$$S_E = \int d^4x \sqrt{g_E} \left( -\frac{1}{2}R + \frac{1}{2}(\partial\phi)^2 - 2 - \cosh(\sqrt{2}\phi) \right)$$

$$ds_E^2 = d\rho^2 + A^2(\rho) \left( d\theta^2 + \sin^2(\theta) d\Omega_2^2 \right)$$

<u>Lorentzian Continuations</u>

- Outside the light cone

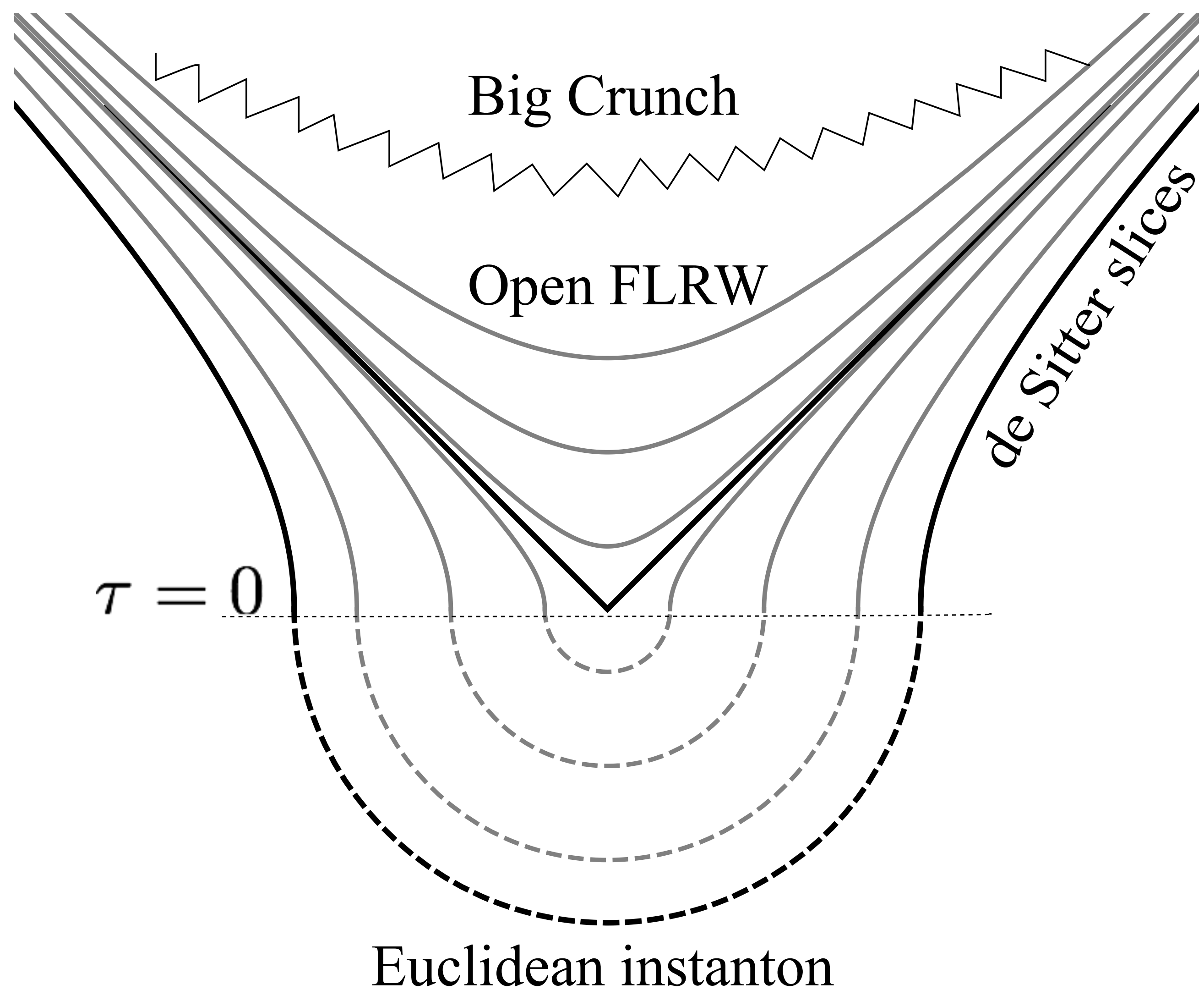
$\theta \to i\tau + \pi/2$

$$ds^2 = d\rho^2 + a_{\text{out}}(\rho)^2 \left( -d\tau^2 + \cosh^2(\tau) d\Omega^2 \right)$$

- Inside the light cone

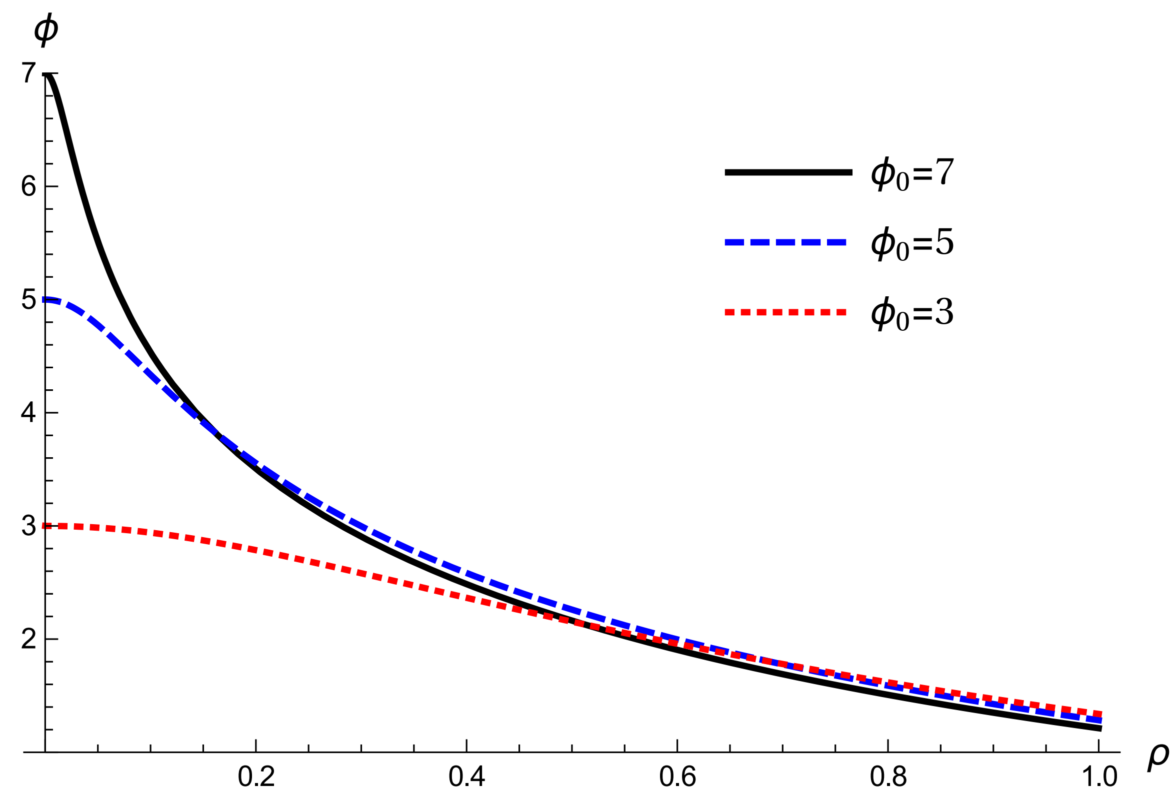
$\rho \to it$ and $\theta \to i\chi$

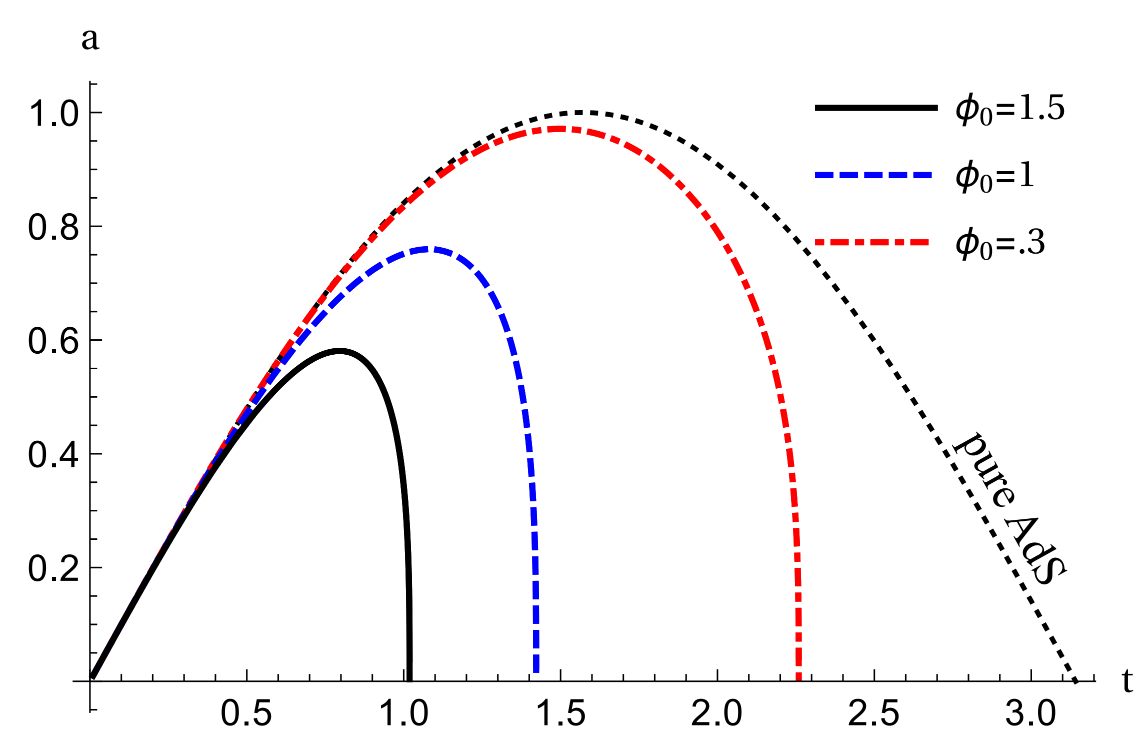
$$ds^2 = -dt^2 + a_{\text{in}}(t)^2 \left( d\chi^2 + \sinh^2(\chi) d\Omega^2 \right)$$



# Bulk evolution

- The set of regular Euclidean instantons is a one parameter family of solutions that can be characterised by the value of the scalar field at the origin $\phi_0$
- Outside the light cone the scalar field dies off and the space is asymptotically AdS
- Larger values of $\phi_0$ correspond to crunches that happen at earlier and earlier FLRW times. As $\phi_0 \to \infty$ the big crunch singularity lies along the light cone.



# Holographic setup: boundary conditions

- The Lorentzian continuation picks a natural slicing of AdS such that the boundary is 3-dimensional de Sitter space:

$$ds^2 = d\rho^2 + a(\rho)^2 \left(-d\tau^2 + \cosh^2(\tau) d\Omega_2^2\right)$$

- Then the asymptotic behaviour of the scalar field is given by:

$$\phi = \phi_- \mathrm{e}^{-\Delta_- \rho} + \phi_+ \mathrm{e}^{-\Delta_+ \rho} + \cdots$$

  - where $\Delta_- / \Delta_+$ are the smaller/larger roots of $m^2 = -\Delta(d - \Delta)$

- For our case of $m^2 = -2$

$$\phi = \alpha\, \mathrm{e}^{-\rho} + \beta\, \mathrm{e}^{-2\rho} + \cdots$$

  - We impose Neuman boundary conditions $\quad \beta = \text{fixed (source)}, \quad \Delta = 1$
    $\alpha = \langle \mathcal{O}_1 \rangle \text{ (VEV)}$

Maldacena: 1012:0274

# Boundary theory: massive scalar in de Sitter

- The choice of boundary condition $\beta = \mathrm{const.}$ corresponds to a boundary theory where a conformally coupled scalar on de Sitter space receives a massive deformation.

$$S = -\int d^3x \sqrt{-\gamma_{dS}} \left( \frac{1}{2}(\partial\varphi)^2 + (\frac{3}{8} + \beta)\varphi^2 \right)$$

- The bulk singularity does not hit the boundary at finite boundary time - the boundary theory is globally well defined.

- Although the boundary theory is strongly coupled, we gain intuition from the free theory: expect a critical deformation, $\beta_c^{fr} = -3/8$, where the boundary theory is that of a minimally coupled massless scalar.

  - We lose the existence of the Euclidean vacuum

  - The vacuum does not respect the full de Sitter symmetry group/ the zero mode of the correlation has to be removed/ only derivatives of the correlators can be considered

    Allen, 1985

  - In the massless limit, two-point functions do not die off in the IR

    Kirsten & Garriga: 9305013
    Tolley & Turok: 0108119

# Mass deformation determined by the regular instantons

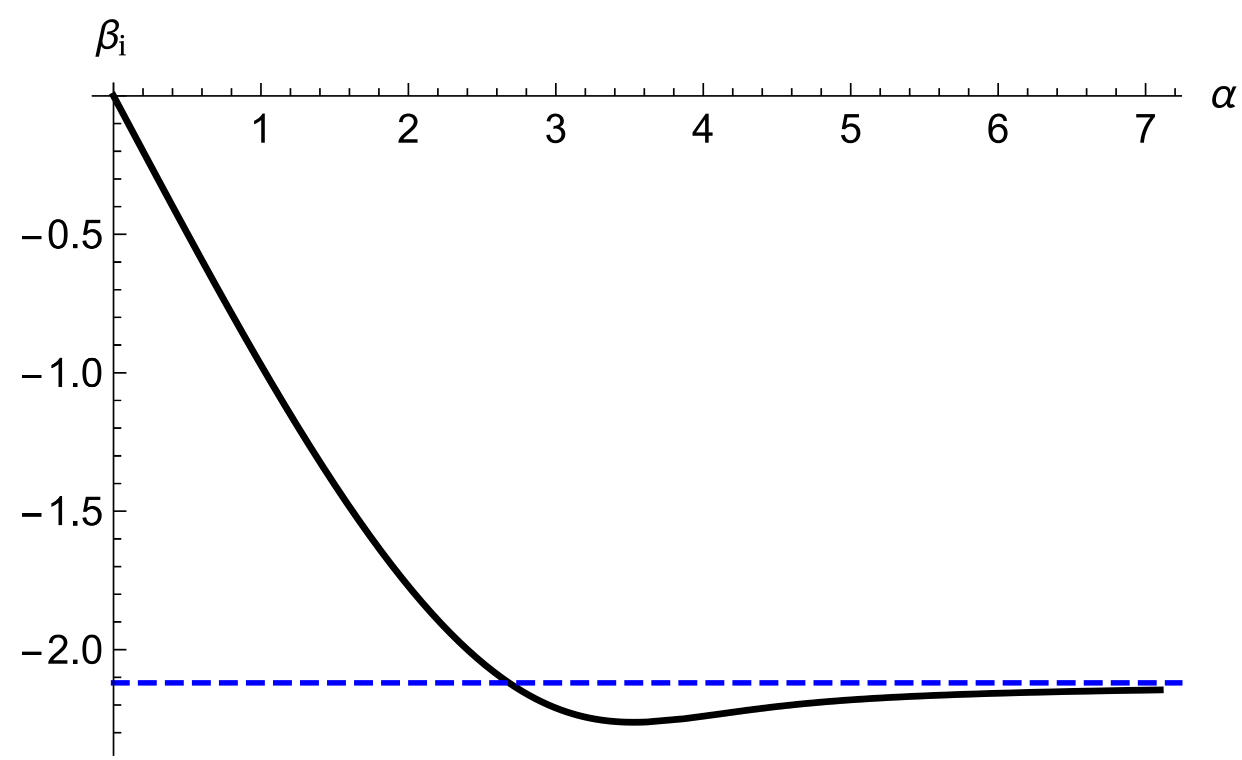
- The one parameter family of regular instantons defines a curve in the $\alpha$-$\beta$ plane.

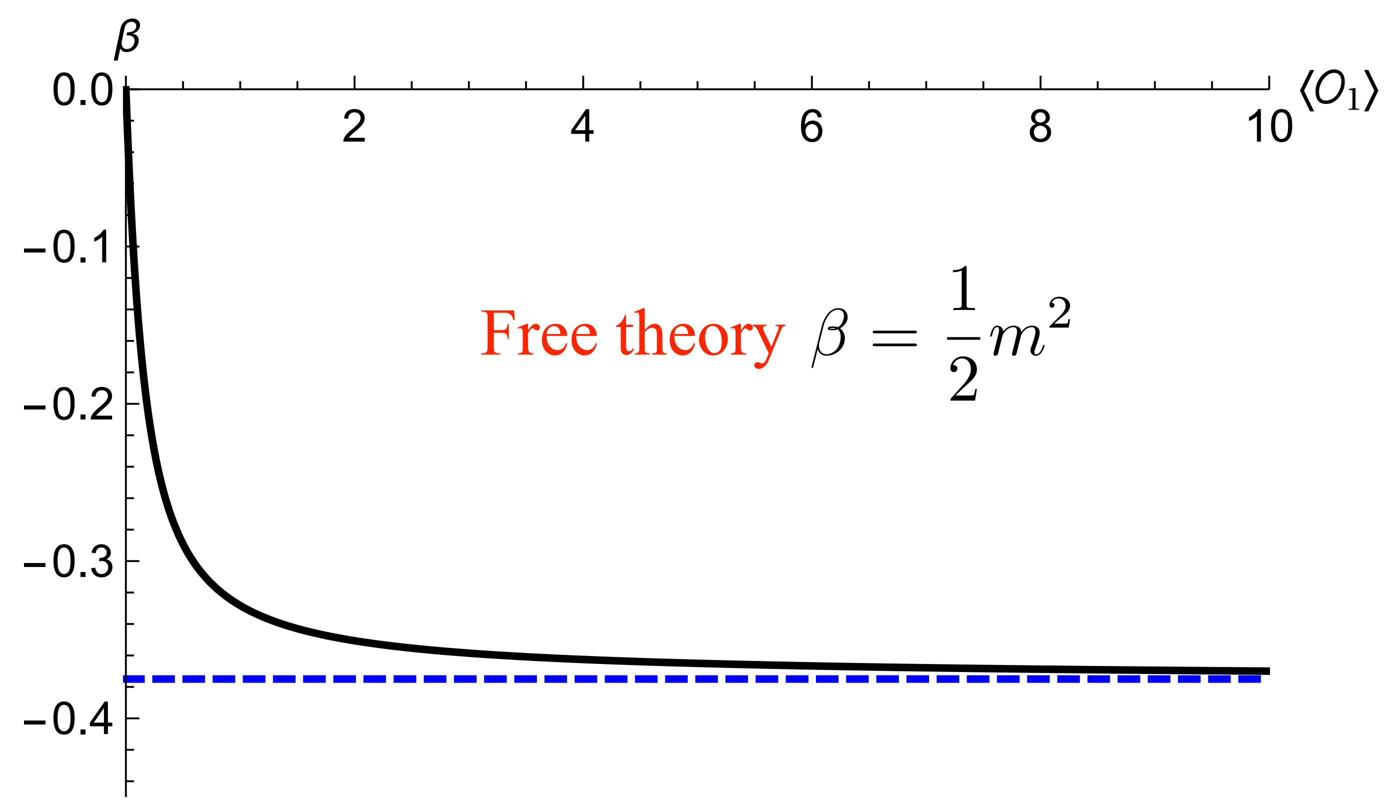
- A choice of boundary conditions does not guarantee either existence or uniqueness of the instanton solution

- There is a critical $\beta \approx -2.12$ at which a singular instanton enters, and below which two instanton geometries correspond to a given boundary theory

  - This corresponds to a second saddle point entering the computation for boundary correlators

  - not a feature of other consistent truncations

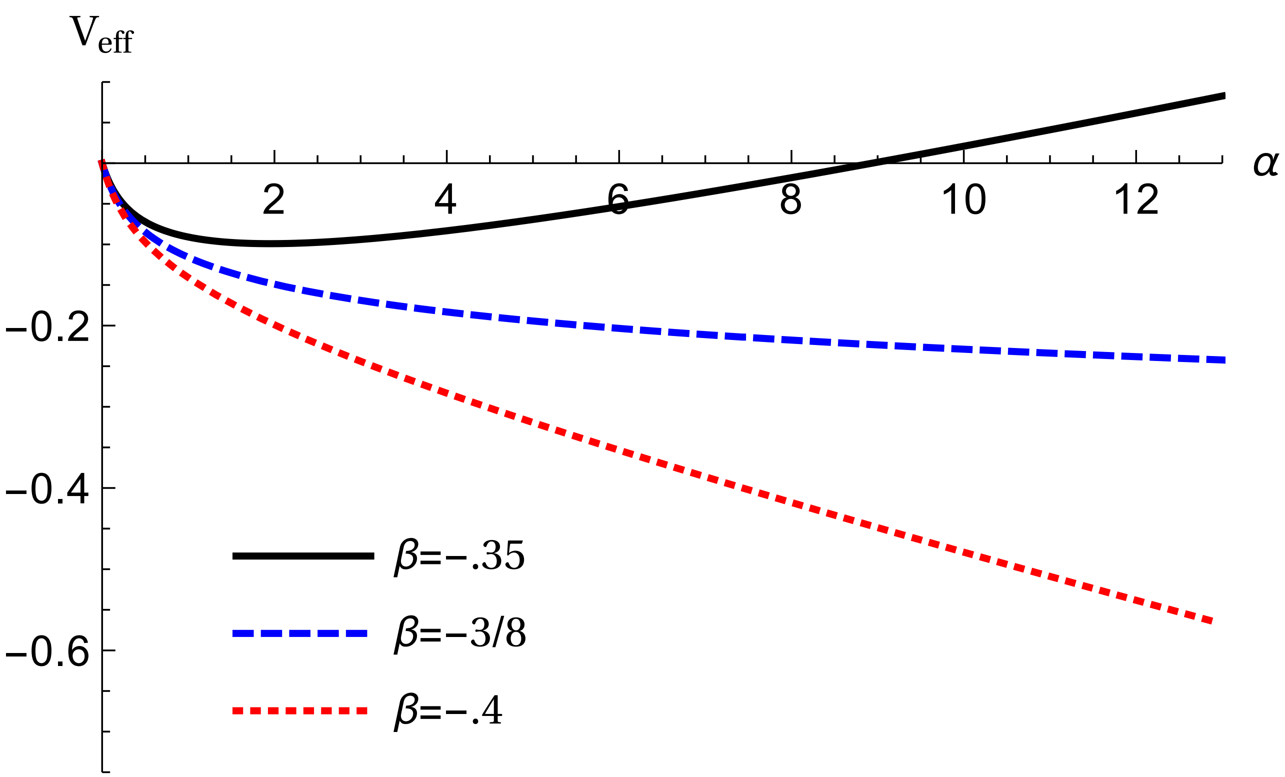


# Effective Potential: scalar on de Sitter

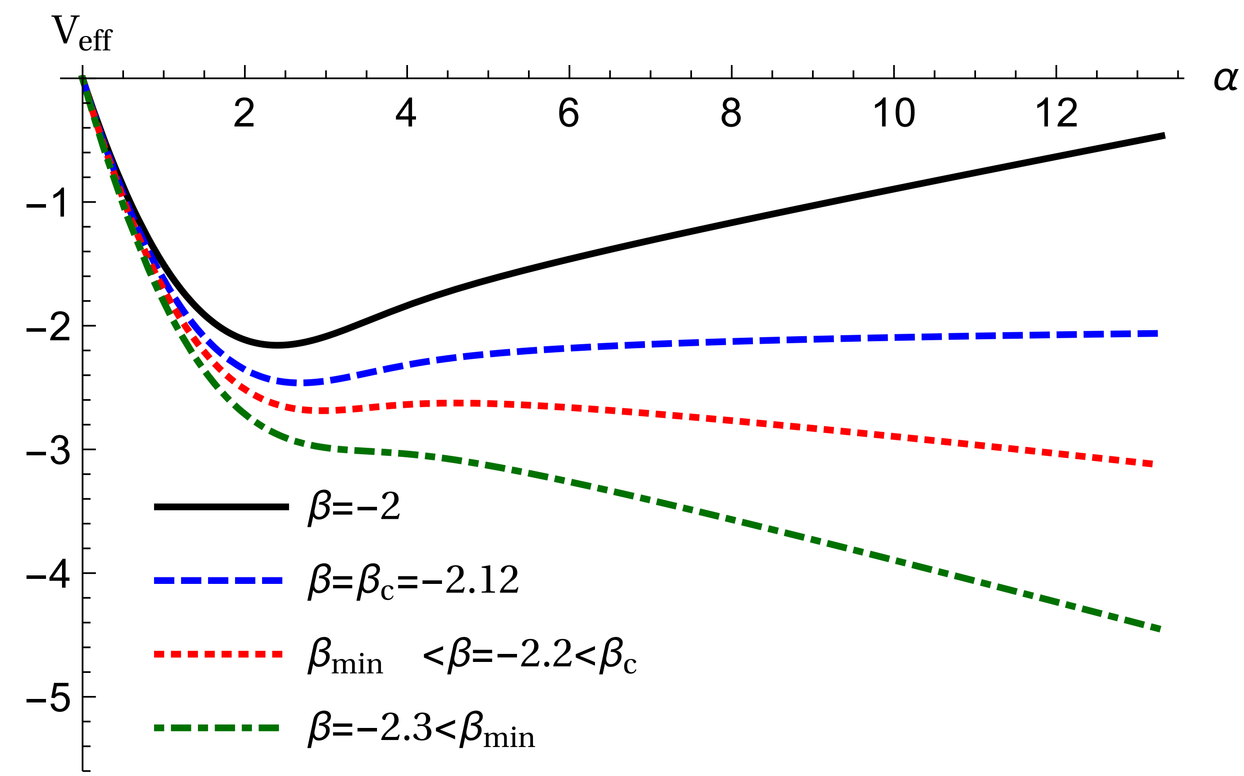
$$\Gamma_\beta(\alpha) = \Gamma_0(\alpha) - \int d^3x \sqrt{-\gamma_{\mathrm{dS}}} \int_0^{\alpha(x)} \beta_{\mathrm{BC}}(\alpha) \mathrm{d}\alpha \qquad \Gamma_\beta(\alpha) = -Vol_{ds} V_{\mathrm{eff}}(\alpha)$$

## Free theory




- Stable for $\beta > \beta_c^{fr} = -3/8$
- No stable vacua for tachyonic scalars

## Strong coupling

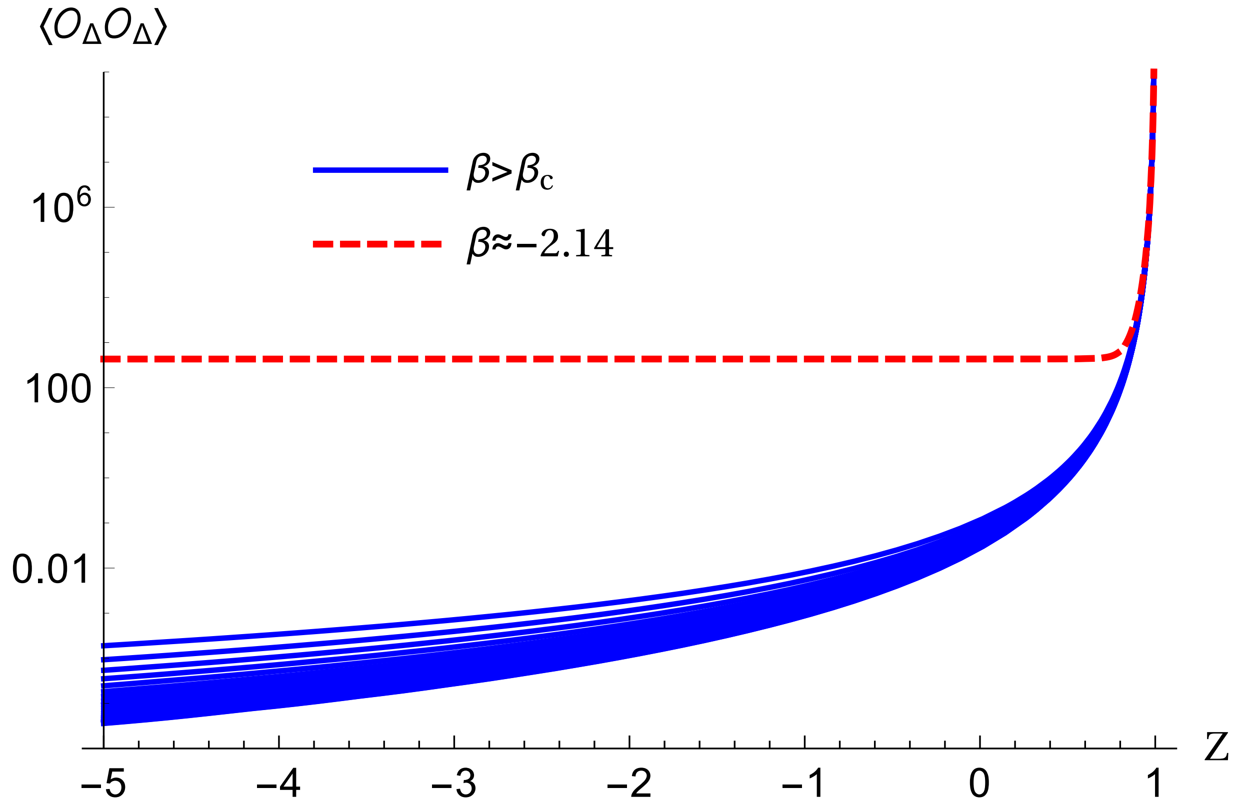



- Stable for $\beta > \beta_c \approx -2.12$
- Regime of with a metastable minimum corresponding to the first saddle point, and an unstable maxima corresponding to the second saddle point and no overall stable vacuum.
- No (meta)stable vacua for large deformations

# Correlation functions: geodesic method

$$\langle\psi|\mathcal{O}_\Delta(x)\mathcal{O}_\Delta(x')|\psi\rangle = \sum_i w_i \mathrm{e}^{-\Delta\mathcal{L}^{\mathrm{i}}_{\mathrm{reg}}(\mathrm{x},\mathrm{x}')}$$

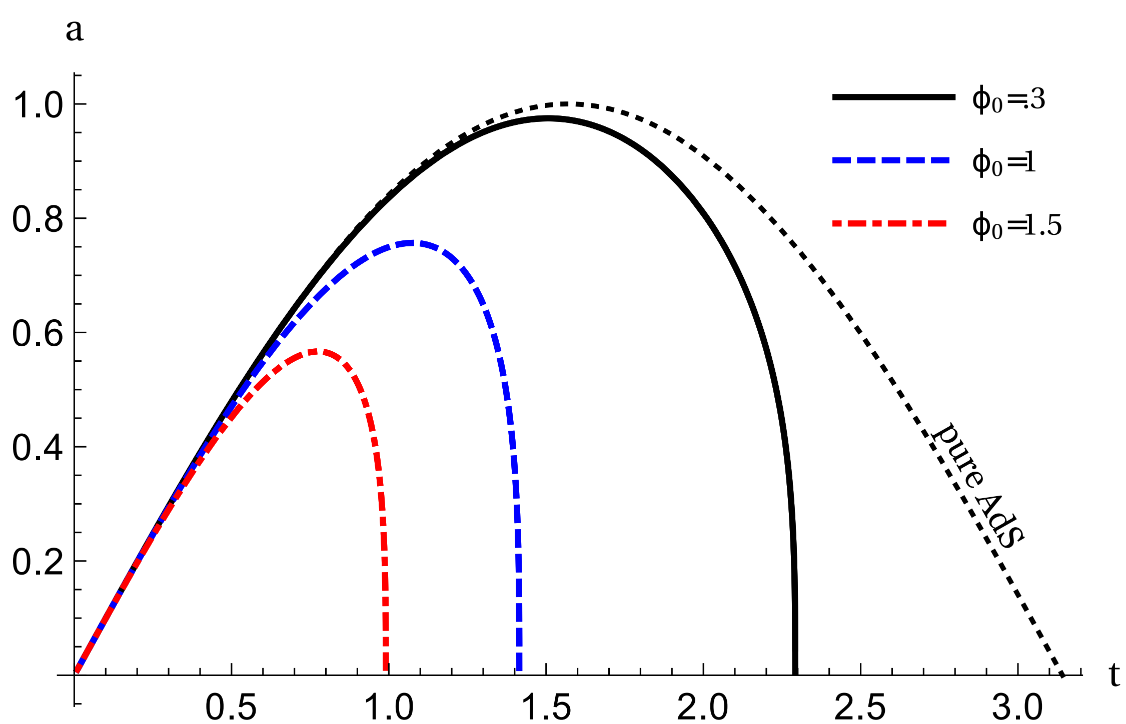
- $\langle\psi|$ is the state of the boundary theory
- $\Delta \gg 1$
- $w_i$ is a weighting, given by the Euclidean action, relevant in the case of multiple saddle points
- $\mathcal{L}_{\mathrm{reg}}$ is the regularised length of a geodesic connecting boundary points x and x'

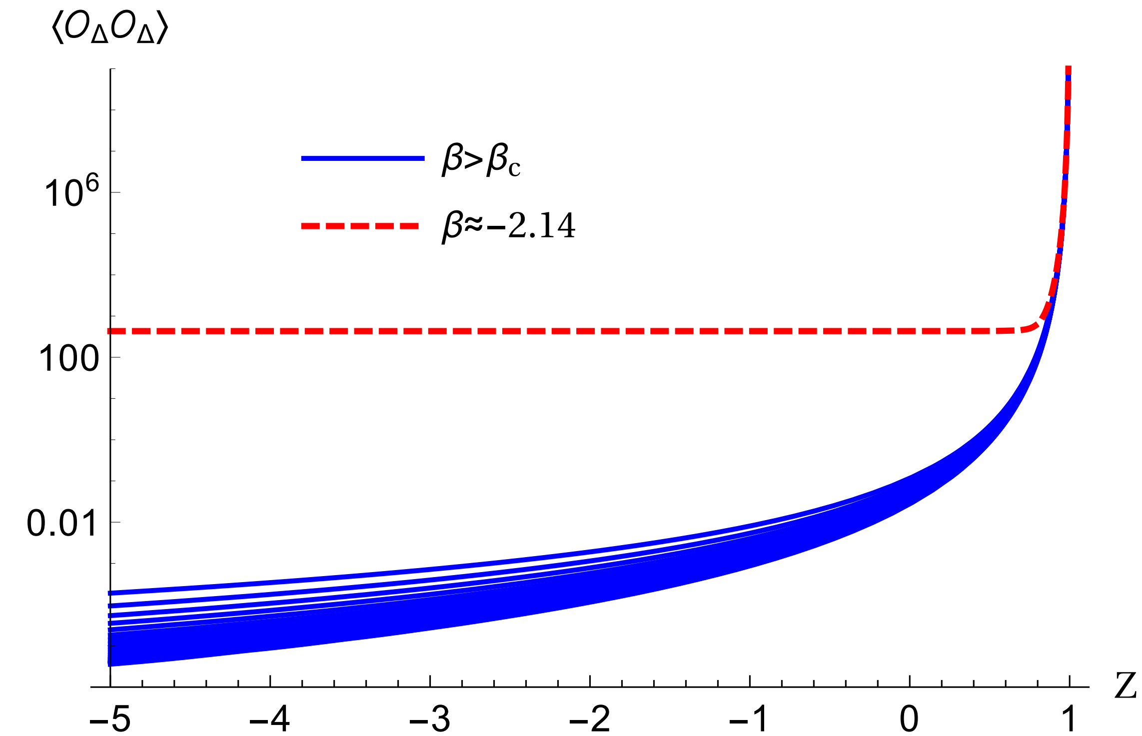


# The IR behavior

- The strong IR tail is produced by the second saddle point, where $a_{\max} \to 0$

$$\langle \mathcal{O}_\Delta(x)\mathcal{O}_\Delta(x')\rangle \approx \sum_i w_i \left( (a_1^i)^2 \mathrm{e}^{\pi a^i_{\max}} (-2Z)^{-a^i_{\max}} \right)^\Delta$$




- The proper time between the closest approach of the geodesic $t_{\max}$, and the crunch shrinks as we increase $\phi_0$. Therefore the IR tail near the critical deformation comes from geodesics which come *close* to the singularity.

# Comparison with free theory: $\beta \to \beta_c$

## Free theory

- To compare to the geodesic approximation we need to compute the two-point function for a heavy operator.

$$\mathcal{O}_{N/2} =: \varphi^N :$$

$$\langle \mathcal{O}_{N/2}(x)\mathcal{O}_{N/2}(x')\rangle = N!G^N(Z)$$

- Expanding the free theory around the massless limit, one finds:

$$\langle \mathcal{O}_\Delta \mathcal{O}_\Delta \rangle_{\text{free}} \propto \frac{1}{(\beta_c^{fr}-\beta)^{2\Delta}}\left[1+2\Delta(\beta_c^{fr}-\beta)\log(-2Z) + \mathcal{O}(\beta_c^{fr}-\beta)^2\right] + \text{ subleading}$$

## Strong coupling

- In the limit of the critical deformation we need numerical fits to describe the geometry

$$a_1 \approx \frac{A}{\sqrt{\beta_c-\beta}}$$

$$\log(a_{\max}) \approx B_1 - \frac{B_2}{\beta_c-\beta}$$

- Using these we find the two point function in the limit $\beta \to \beta_c$ is:

$$\langle \mathcal{O}_\Delta \mathcal{O}_\Delta \rangle_{\text{s.c.}} \propto \frac{1}{(\beta_c-\beta)^{\Delta}}\left[1-\Delta \mathrm{e}^{\mathrm{B}_1}\mathrm{e}^{-\frac{\mathrm{B}_2}{\beta_c-\beta}}\log(-2\mathrm{Z}) + \mathcal{O}\left(\mathrm{e}^{-\frac{2\mathrm{B}_2}{\beta_c-\beta}}\right)\right] + \text{ subleading}$$

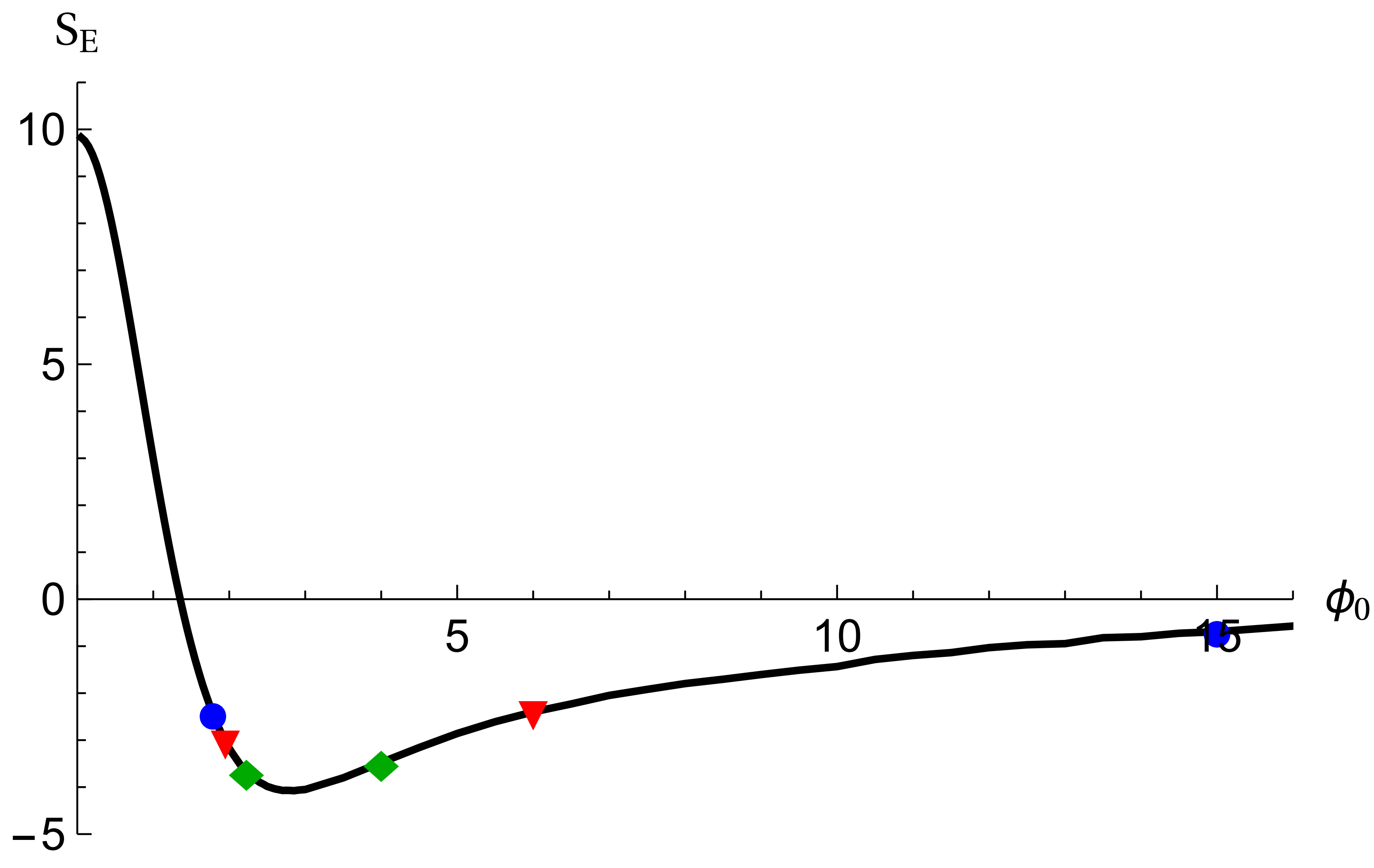
- The Z-dependence matches the free theory

- The expansion in $\mathrm{e}^{-1/(\beta_c-\beta)}$ is reminiscent of an instanton expansion - especially in view of the metastable effective potential.

# Euclidean Action

- The Euclidean action is not single single valued, signature of a first order phase transition.

- Also indicated from purely field theory calculation by Nosaka, Shimizu and Terashima (1512.00249)



# Conclusions

- The boundary theory on de Sitter is stable (for some range of parameters), whereas this is not the case for a Minkowski boundary.

- Signatures of the bulk singularity are reminiscent of the massless minimally coupled scalar in de Sitter

  - The dependence on boundary separation of correlation functions in the IR matches that of the free theory in the massless limit

  - The IR tail of the two point function comes from geodesics which come arbitrarily close to the singularity

  - Speculation: cosmological singularities are linked to strong correlations on large length scales (also one of the main virtues of inflation)

- We find indications of an instability for a branch of instanton solutions

  - Instability in the boundary theory indicated by $V_{eff.}(\varphi^2)$

  - Correlation functions dual to heavy bulk fields show hints of an instanton expansion in the limit of singular geometry

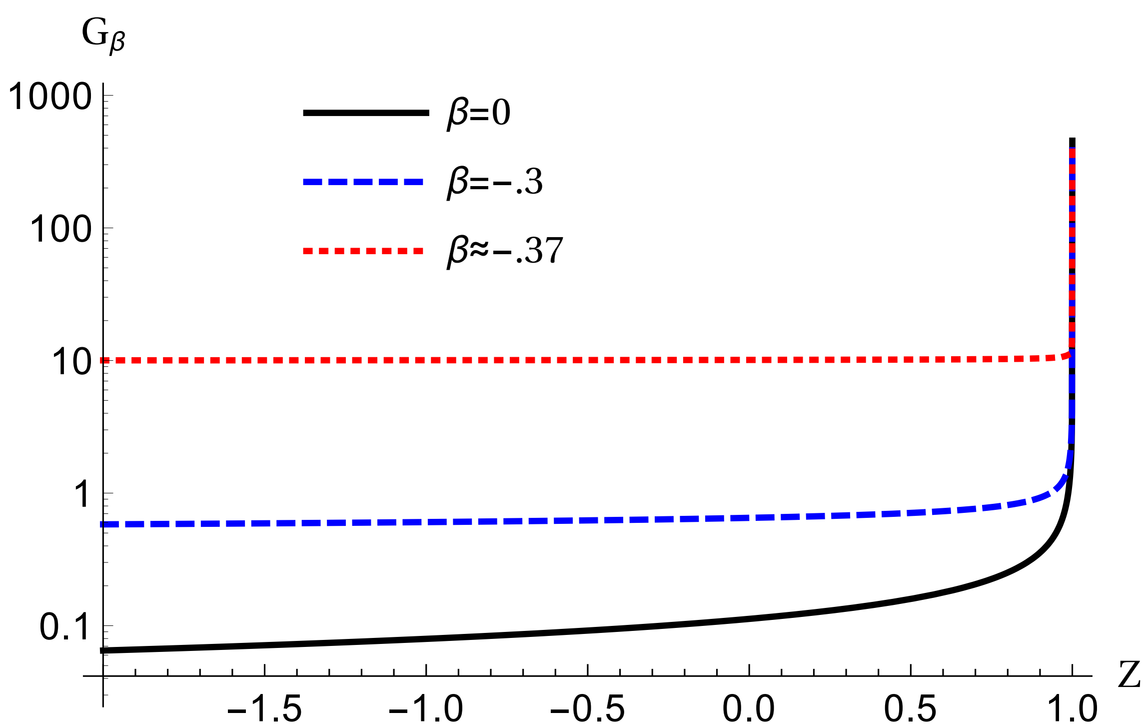
  - Double valued Euclidean action indicates first order phase transition

Thank you for your attention

# Free scalar on $dS_3$

- All correlators only depend on the de Sitter invariant distance: $Z = \eta_{\mu\nu}\Delta X^\mu \Delta X^\nu$

$$Z = \begin{cases} \cos D & \text{for } D \leq \pi\,, \\ -\cosh(D-\pi) & \text{for } D > \pi\,. \end{cases}$$



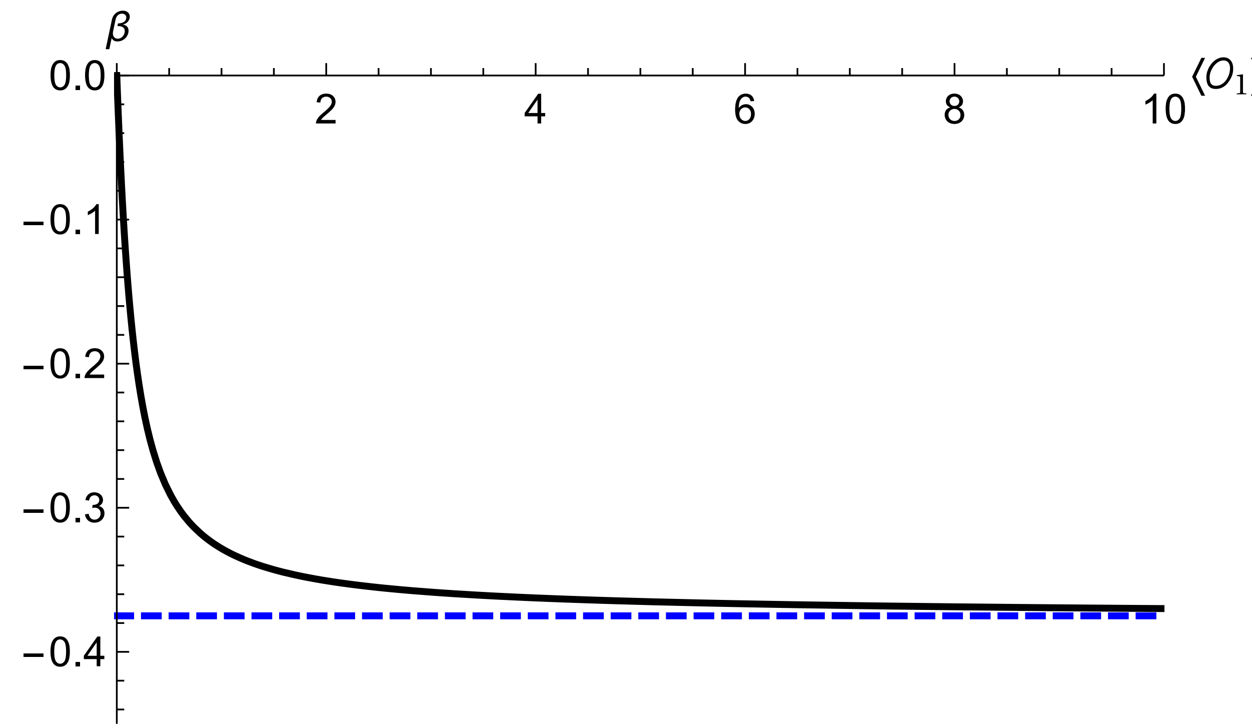

- Two-point function

$$G_\beta(Z) = \frac{\sin\left(\sqrt{1-8\beta}\,\arcsin\sqrt{\frac{1+Z}{2}}\right)}{2\pi\sin\left(\frac{\pi}{2}\sqrt{1-8\beta}\right)\sqrt{1-Z^2}}$$

- One-point function

$$\varphi^2(x) = \lim_{x'\to x}\left[\varphi(x)\varphi(x') - \langle\varphi(x)\varphi(x')\rangle_0 \times 1\right]$$

$$\alpha = \langle\varphi^2\rangle = -\frac{\sqrt{1-8\beta}}{4\pi}\cot\left(\frac{\pi}{2}\sqrt{1-8\beta}\right)$$




Anninos, Denef, Harlow: 1207:5517

# Quantum effective action

- The effective potential for $\langle\varphi^2\rangle$, (*i.e.* $\alpha$ ) can be computed from the standard quantum effective action.

$$\frac{\delta\Gamma_0}{\delta\alpha(x)} = \sqrt{-\gamma_{\mathrm{dS}}}\beta_i(x)$$

- We are interested in a theory with fixed von Neumann boundary conditions. We deform the theory by some fixed source $\beta_{\mathrm{BC}}$, resulting in a shift of the quantum effective action:

$$\Gamma_\beta(\alpha) = \Gamma_0(\alpha) - \int d^3x\sqrt{-\gamma_{\mathrm{dS}}}\int_0^{\alpha(x)} \beta_{\mathrm{BC}}(\alpha)\mathrm{d}\alpha$$

- The effective potential for $\alpha$ is obtained from the quantum effective action via the definition: $\Gamma_\beta(\alpha) = -Vol_{ds}V_{\mathrm{eff}}(\alpha)$, resulting in:

$$V_{\mathrm{eff}}(\alpha) = -\int_0^\alpha \beta_i(\alpha)d\alpha + \int_0^\alpha \beta_{\mathrm{BC}}(\alpha)\mathrm{d}\alpha$$

Hertog & Horowitz: 0412169

- The extrema of this potential are in one-to-one correspondence with the regular solutions (either in the free theory or the regular instanton solutions) that obey the von Neumann boundary conditions.

# Geodesics in crunching AdS

Kumar & Vaganov: 1510.03281

- Geodesics do not enter the region $t > t_{\rm max}$, where $t_{\rm max}$ is defined by $a(t_{\rm max}) = a_{\rm max}$. As the boundary separation of the endpoints is taken to infinity, the geodesic lies along the surface $t = t_{\rm max}$

  - This has been used to argue that boundary correlators do not encode information about the singularity because geodesics don't get *close* to it. We do not find this to be the case.

- Geodesic length can only depend on the de Sitter invariant distance (Z) between the boundary points

- Geodesics with small boundary separations only probe the near boundary region - asymptotically AdS region

$$\mathcal{L}(Z \to 1^-) = 2\log a(\rho_{cut}) + \log\left(2(1-Z)\right) - \frac{\alpha^2}{12}(1-Z) + \mathcal{O}(1-Z)^2$$

- Regularisation removes the universal volume divergence

$$\mathcal{L}_{\rm reg} = \lim_{\rho_{\rm cut}\to\infty}(\mathcal{L} - 2\rho_{\rm cut}) - \log(a_1^2) \quad \text{with} \quad a = a_1 e^{\rho} + a_{-1}e^{-\rho} + \mathcal{O}(e^{-2\rho})$$

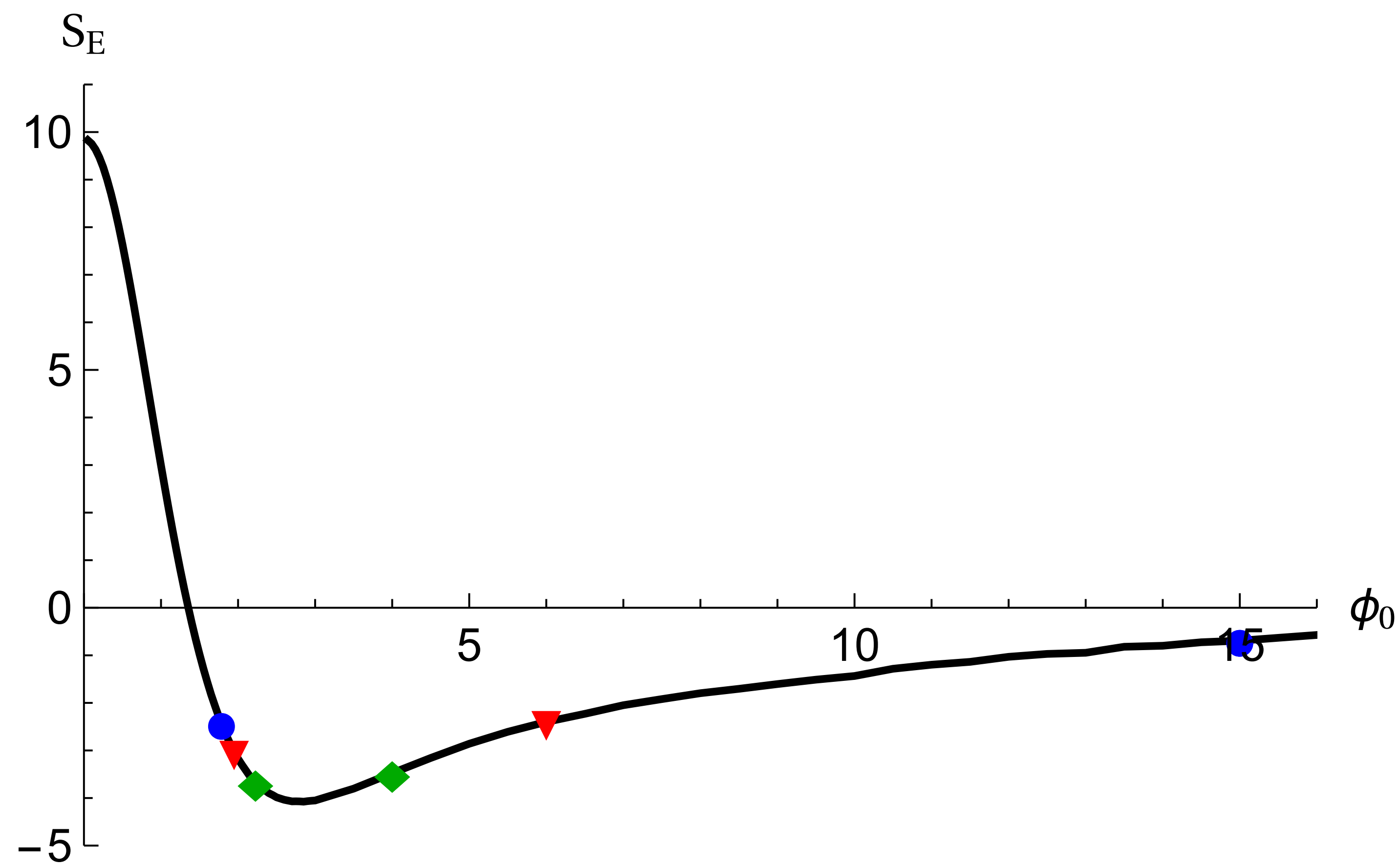
# Weight saddle points via the Euclidean action

- Weights are given by: $w_i = \begin{cases} 1 & \text{for } \beta > \beta_c\,, \\ \frac{\mathrm{e}^{-\mathrm{S_{E\,i}}}}{\sum_{\mathrm{j}} \mathrm{e}^{-\mathrm{S_{E\,j}}}} & \text{for } \beta_{\min} \leq \beta \leq \beta_c\,, \end{cases}$

- When the second saddle point enters, it has a less negative Euclidean action, and is therefore subdominant

  - reminiscent of Maldacena's eternal BH in AdS story

    hep-th/0106112



# Comparison with free theory: $\phi_0 \ll 1$

## Free theory

- To compare to the geodesic approximation we need to compute the two-point function for a heavy operator.

$$\mathcal{O}_{N/2} =: \varphi^N :$$

$$\langle \mathcal{O}_{N/2}(x)\mathcal{O}_{N/2}(x')\rangle = N! G^N(Z)$$

- Focusing on the IR, we expand the two point function around $Z \to \infty$. Additionally, the $\phi_0 \ll 1$ limit corresponds to small $\beta$:

$$\langle \mathcal{O}_\Delta \mathcal{O}_\Delta \rangle_{\text{free}} \propto \left[(-2Z)^{-\Delta} - 4\Delta(-2Z)^{-\Delta}\log(-2Z)\beta\right] + \mathcal{O}(\beta^2) + \text{ subleading}$$

## Strong coupling

- In the near AdS limit we can solve the bulk perturbatively in $\beta$, we find:

$$a_1 = \frac{1}{2} + \frac{2\beta^2}{9} + \mathcal{O}(\beta^3)$$
$$a_{\max} = 1 - \frac{4\beta^2}{3} + \mathcal{O}(\beta^3)$$

- Plugging into the IR expansion of the holographic two point function we find

$$\langle \mathcal{O}_\Delta \mathcal{O}_\Delta \rangle_{\text{sc}} \propto \left[(-2Z)^{-\Delta} + \frac{\Delta}{12}(-2Z)^{-\Delta}\left(\log(-2Z) + \pi - 2/3\right)\beta^2\right] + \mathcal{O}(\beta^3) + \text{subleading}$$

- The Z-dependence matches the free theory, however, the subleading dependence enters at a different order in $\beta$

# Holographic Renormalization

$$S_{\text{div}} = -2a_1^3 e^{3\rho_{\text{cut}}} + \frac{a_1}{4} e^{\rho_{\text{cut}}} (\alpha^2 - 6) + \text{finite terms}$$

$$S_{\text{ct}} = \int_{\rho=\rho_{\text{cut}}} \mathrm{d}^3\mathrm{x}\sqrt{\gamma^\rho}\left(2 + \frac{1}{2}\mathrm{R}[\gamma^\rho] + \frac{1}{2}\phi^2\right)$$

$$S_{-} = -\int_{\rho=\rho_{\text{cut}}} \mathrm{d}^3\mathrm{x}\sqrt{\gamma^\rho}\,\phi\,\pi_{\mathrm{r}}$$

$$\pi_r = \frac{1}{\sqrt{\gamma^\rho}}\frac{\delta(S_E + S_{\text{ct}})}{\delta\phi} = \partial_\rho\phi + \phi$$

$$S_{\text{ren}} = \lim_{\rho_{\text{cut}}\to\infty}\left[-\int \mathrm{d}^4\mathrm{x}\sqrt{\mathrm{g_E}}\mathrm{V}(\phi) + \int_{\rho=\rho_{\text{cut}}} \mathrm{d}^3\mathrm{x}\sqrt{\gamma^\rho}\left(2 + \frac{1}{2}\mathrm{R}[\gamma^\rho] - \mathrm{K} - \frac{1}{2}\phi^2 - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\rho}\phi^2\right)\right]$$

# Some numerical fits