LECTURES ON PARTON SHOWER MATCHING AT NLO

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LEADING ORDER

- For many of the theory predictions needed in the searches for new physics as well as measuring properties of the SM, leading order predictions are used
- ✦ The reasons for this are clear:
 - In many regions of phase-space they do a decent job, in particular for shapes of distributions
 - Parton showers and hadronizations models are tuned to data
 - Many flexible lowest order (LO) tools are readily available
- Unfortunately LO predictions describe total rates rather poorly

NEED FOR NLO

- ✦ If we would have the same flexible tools available at NLO, the experimental analyses will benefit a various ways:
 - NLO predictions predict rates much more precisely
 - Reduced theoretical uncertainties due to meaningful scale dependence
 - Shapes are better described
 - Correct estimates for PDF uncertainties
 - Even data-driven analyses might benefit: smaller uncertainty due to interpolation from control region to signal region
- These accurate theoretical predictions are particularly needed for
 - O searches of signal events in large backgrounds samples and
 - precise extraction of parameters (couplings etc.) when new physics signals have been found

QUANTITATIVE PREDICTIONS



For precise, quantitative comparisons between theory and data, (at least) Next-to-Leading-Order corrections are a must

IMPROVING MC's

- Parton shower MC programs are only correct in the soft-collinear region. Hard radiation cannot be described correctly
- There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:
 - NLO+PS matching: Include full NLO corrections to the matrix elements to reduce theoretical uncertainties in the matrix elements. The real-emission matrix elements will describe the hard radiation
 - ME+PS merging: Include matrix elements with more final state partons to describe hard, well-separated radiation better

NLO+PS MATCHING

LIMITATIONS OF FIXED ORDER CALCULATIONS

In the small transverse momentum region, this calculation breaks down (it's even negative in the first bin!), and anywhere else it is purely a LO calculation for V+1j



AT NLO



- ✦ We have to integrate the real emission over the complete phasespace of the one particle that can go soft or collinear to obtain the infra-red poles that will cancel against the virtual corrections
- We can NOT use the same merging procedure as used at LO (MLM or CKKW): requiring that all partons should produce separate jets is not infrared safe
- We have to invent a new procedure to match NLO matrix elements with parton showers



- ◆ In a fixed order calculation we have contributions with *m* final state particles and with *m*+1 final state particles $\sigma^{\text{NLO}} \sim \int d^4 \Phi_m B(\Phi_m) + \int d^4 \Phi_m \int_{\text{Loop}} d^d l \, V(\Phi_m) + \int d^d \Phi_{m+1} \, R(\Phi_{m+1})$
 - ♦ We could try to shower them independently
 - ★ Let I^(k)_{MC}(O) be the parton shower spectrum for an observable O, showering from a k-body initial condition
 - ✦ We can then try to shower the *m* and *m*+1 final states independently

$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m (B + \int_{\rm loop} V) \right] I_{\rm MC}^{(m)}(O) + \left[d\Phi_{m+1} R \right] I_{\rm MC}^{(m+1)}(O)$$

NAIVE (WRONG) APPROACH

★ In a fixed order calculation we have contributions with *m* final state particles and with *m*+1 final state particles $\int \sqrt{44} = D(4) + \sqrt{44} = \frac{1}{44} + \frac{1}{44}$

$$\sigma^{\rm NLO} \sim \int d^4 \Phi_m \, B(\Phi_m) + \int d^4 \Phi_m \int_{\rm loop} d^d l \, V(\Phi_m) + \int d^d \Phi_{m+1} \, R(\Phi_{m+1})$$

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DOUBLE COUNTING

$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m (B + \int_{\rm loop}^{V}) \right] I_{\rm MC}^{(m)}(O) + \left[d\Phi_{m+1} R \right] I_{\rm MC}^{(m+1)}(O)$$

- ✦ But this is wrong!
- If you expand this equation out up to NLO, there are more terms then there should be and the total rate does not come out correctly
- Schematically $I_{MC}^{(k)}(O)$ for 0 and 1 emission is given by

$$\sum_{MC}^{(k)}(O) \sim \Delta_a(Q^2, Q_0^2)$$

$$+ \Delta_a(Q^2, t) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s(t)}{2\pi} P_{a \to bc}(z)$$

• And Δ is the Sudakov factor

$$\Delta_a(Q^2, t) = \exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s(t')}{2\pi} P_{a \to bc}\right]$$











- There is double counting between the real emission matrix elements and the parton shower: the extra radiation can come from the matrix elements or the parton shower
- There is also an overlap between the virtual corrections and the Sudakov suppression in the zero-emission probability

DOUBLE COUNTING IN VIRTUAL/ SUDAKOV

- ★ The Sudakov factor ∆ (which is responsible for the resummation of all the radiation in the shower) is the no-emission probability
- It's defined to be Δ = 1 P, where P is the probability for a branching to occur
- By using this conservation of probability in this way, Δ contains contributions from the virtual corrections implicitly
- Because at NLO the virtual corrections are already included via explicit matrix elements, Δ is double counting with the virtual corrections
- In fact, because the shower is unitary, what we are double counting in the real emission corrections is exactly equal to what we are double counting in the virtual corrections (but with opposite sign)!

AVOIDING DOUBLE COUNTING

- There are a couple of methods to circumvent this double counting
 MC@NLO (Frixione & Webber)
 POWHEG (Nason)
 - KRKNLO (Cracow group), Vincia (Skands et al.), ...

MC@NLO PROCEDURE

Frixione & Webber (2002)

To remove the double counting, we can add and subtract the same term to the *m* and *m*+1 body configurations

$$\frac{d\sigma_{\mathrm{MC@NLO}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

Where the MC are defined to be the contribution of the parton shower to get from the m body Born final state to the m+1 body real emission final state

MC@NLO PROCEDURE



 Double counting is explicitly removed by including the "shower subtraction terms"

MC@NLO PROPERTIES

- ✦ Good features of including the subtraction counter terms
 - 1. **Double counting avoided**: The rate expanded at NLO coincides with the total NLO cross section
 - 2. **Smooth matching**: MC@NLO coincides (in shape) with the parton shower in the soft/collinear region, while it agrees with the NLO in the hard region
 - Stability: weights associated to different multiplicities are separately finite. The *MC* term has the same infrared behavior as the real emission (there is a subtlety for the soft divergence)
- Not so nice feature (for the developer):
 - 4. **Parton shower dependence**: the form of the *MC* terms depends on what the parton shower does exactly. Need special subtraction terms for each parton shower to which we want to match

DOUBLE COUNTING AVOIDED

$$\frac{d\sigma_{\mathrm{MC@NLO}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

✦ Expanded at NLO

$$I_{\rm MC}^{(m)}(O)dO = 1 - \int d\Phi_1 \frac{MC}{B} + d\Phi_1 \frac{MC}{B} + \dots$$
$$d\sigma_{\rm NLOwPS} = \left[d\Phi_m (B + \int_{\rm loop} V + \int d\Phi_1 MC) \right] I_{\rm MC}^{(m)}(O)dO$$
$$+ \left[d\Phi_{m+1}(R - MC) \right]$$
$$\simeq d\Phi_m (B + \int_{\rm loop} V) + d\Phi_{m+1}R = d\sigma_{\rm NLO}$$

SMOOTH MATCHING

$$\frac{d\sigma_{\mathrm{MC@NLO}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

✦ Smooth matching:

• Soft/collinear region: $R \simeq MC \Rightarrow d\sigma_{MC@NLO} \sim I_{MC}^{(m)}(O)dO$ • Hard region (shower effects suppressed), ie. $MC \simeq 0 \quad I_{MC}^{(m)}(O) \simeq 0 \quad I_{MC}^{(m+1)}(O) \simeq 1$ $\Rightarrow d\sigma_{MC@NLO} \sim d\Phi_{m+1}R$

STABILITY & UNWEIGHTING

$$\frac{d\sigma_{\mathrm{MC@NLO}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- The MC subtraction terms are defined to be what the shower does to get from the m to the m+1 body matrix elements. Therefore the cancellation of singularities is exact in the (R - MC) term*: there is no mapping of the phase-space in going from events to counter events as we have in the CS-dipoles/FKS subtraction
- The integral is bounded all over phase-space; we can therefore generate unweighted events!

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• "S-events" (which have m body kinematics)
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• "H-events" (which have *m*+1 body kinematics)

* up to a subtlety that I'll mention later

FKS SUBTRACTION

$$\frac{d\sigma_{\mathrm{MC@NLO}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- ✦ The MC counter terms render the real emission finite
- So, do we still need the CS-dipoles/FKS subtraction terms?

FKS SUBTRACTION

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YES!

NLO SUBTRACTION

$$\frac{d\sigma_{\mathrm{MC}@\mathrm{NLO}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

★ We cannot do the one-particle integral over the MC terms analytically: we do not get the explicit poles in 1/€ and 1/€² to cancel the poles in the virtual corrections. So we need to extract them using a subtraction method G

$$\frac{d\sigma_{\mathrm{MC@NLO}}}{dO} = \left[d\Phi_m (B + (\int_{\mathrm{loop}} V + \int d\Phi_1 G) + \int d\Phi_1 (MC - G) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

NEGATIVE WEIGHTS

$$\frac{d\sigma_{\mathrm{MC@NLO}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1} (R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

- We generate events for the two terms between the square brackets (S- and H-events) separately
- There is no guarantee that these contributions are separately positive (even though predictions for infra-red safe observables should always be positive!)
- Therefore, when we do event unweighting we can only unweight the events up to a sign. These signs should be taken into account when doing a physics analysis (i.e. making plots etc.)
- ✦ The events are only physical when they are showered

POSSIBLE ISSUES WITH THE MC@NLO METHOD

- MC subtraction terms need to be defined over the full phase-space, even though the shower has a cut-off.
 - Can be considered a power corrections to the parton shower and is therefore beyond expected accuracy
- Value of the scale entering α_s in the MC subtraction terms
 - Can be considered a higher order difference and is therefore beyond expected accuracy
- Shower does, in general, not reproduce exactly the IR singularities in the soft limit (for subleading terms in colour)
 - Can be considered a power corrections and is therefore beyond expected accuracy
 - Other solution would be to change the shower to include complete colour dependence (at least for a single emission)
- ✦ Fraction of negative weights can be large (30% negative weights is not rare)
 - Requires larger samples of unweighted events to obtain the same statistical precision

POWHEG

Nason (2004)

 Consider the probability of the first emission of a leg (inclusive over later emissions)

$$d\sigma = d\Phi_m B \left[\Delta(Q^2, Q_0^2) + \Delta(Q^2, t) d\Phi_{(+1)} \frac{MC}{B} \right]$$

 One could try to get NLO accuracy by replacing B with the NLO rate (integrated over the extra phase-space)

$$B \to B + V + \int d\Phi_{(+1)} R$$

This naive definition is not correct: the radiation is still described only at leading logarithmic accuracy, which is not correct for hard emissions.

POWHEG

★ This is double counting.
To see this, expand the equation up to the first emission $d\Phi_m \left[B + V + \int d\Phi_1 R \right] \left[1 - \int \Phi_1 \frac{MC}{B} + d\Phi_1 \frac{MC}{B} \right]$ which is not equal to the NLO

In order to avoid double counting, one should replace the definition of the Sudakov form factor with the following:

$$\Delta(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} d\Phi_1 \frac{MC}{B}\right] \to \tilde{\Delta}(Q^2, Q_0^2) = \exp\left[-\int_{Q_0^2}^{Q^2} d\Phi_1 \frac{R}{B}\right]$$

corresponding to a modified differential branching probability

 $d\tilde{p} = d\Phi_{(+1)}R/B$

★ Therefore we find for the POWHEG differential cross section $d\sigma_{\text{powheg}} = d\Phi_m \left[B + V + \int d\Phi_1 R \right] \left[\Delta(Q^2, Q_0^2) + \Delta(Q^2, t) d\Phi_1 \frac{R}{B} \right]$

PROPERTIES

$$d\sigma_{\text{powheg}} = d\Phi_m \left[B + V + \int d\Phi_1 R \right] \left[\Delta(Q^2, Q_0^2) + \Delta(Q^2, t) d\Phi_1 \frac{R}{B} \right]$$

- The term in the square brackets integrates to one (integrated over the extra parton phase-space between scales Q₀² and Q²) (this can also be understood as unitarity of the shower below scale t) POWHEG cross section is normalised to the NLO
- Expand up to the first-emission level:

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left[B + V + \int d\Phi_{(+1)}R \right] \left[1 - \int d\Phi_{(+1)}\frac{R}{B} + d\Phi_{(+1)}\frac{R}{B} \right] = d\sigma_{\text{NLO}}$$

so double counting is avoided

 Its structure is identical an ordinary shower, with normalisation rescaled by a local K-factor and a different Sudakov for the first emission: no negative weights are involved.

POSSIBLE ISSUES WITH POWHEG METHOD



- NLO-factor multiples the complete first emission Sudakov terms: Large, arbitrary NNLO terms are included
 - scale dependence looks like NLO (i.e., is relatively small), even though distribution is only LO accurate in the tail
 - Can be ameliorated (see next slide)
- Order/evolution variable used in POWHEG and shower are not the same: formally needs a truncated, vetoed parton shower

POWHEG: IMPROVED

In POWHEG, only singular part of real emission needs to be put in Sudakov:

$$d\sigma_{\rm NLO+PS} = d\Phi_B \bar{B}^s(\Phi_B) \left[\Delta^s(p_\perp^{\rm min}) + d\Phi_{R|B} \frac{R^s(\Phi_R)}{B(\Phi_B)} \Delta^s(p_T(\Phi)) \right] + d\Phi_R R^f(\Phi_R)$$

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where

$$\bar{B}^s(\Phi_B) = B(\Phi_B) + \left| V(\Phi_B) + \int d\Phi_{R|B} R^s(\Phi_{R|B}) \right|$$

and we have split the Real emission matrix elements in a singular and finite part:

 $R(\Phi_R) = R^s(\Phi_R) + R^f(\Phi_R)$

POWHEG: $R^{s}(\Phi) = F R(\Phi)$, $R^{f}(\Phi) = (1 - F)R(\Phi)$ Original is F = 1: exponentiate the full real; it can be damped by hand

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POWHEG looks now similar to MC@NLO. MC@NLO has the real matrix elements split according to:

MC@NLO:
$$R^{s}(\Phi) = P(\Phi_{R|B})B(\Phi_{B}) = MC$$

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Need exact mapping $(\Phi_{R,}\Phi_{B}) \Rightarrow \Phi$ in MC subtraction term R^s

DAMPED POWHEG



- ✦ Inclusion of NNLO terms can be varied by changing F
- Should this be considered an uncertainty or a tuning parameter?
FOUR-LEPTON PRODUCTION

Plot from RF, Frixione, Hirschi, Maltoni, Pittau & Torrielli (2011)



♦ 4-lepton invariant mass is almost insensitive to parton shower effects. 4-lepton transverse moment is extremely sensitive

FOUR-LEPTON PRODUCTION



 Differences between Herwig (black) and Pythia (blue) showers large in the Sudakov suppressed region (much larger than the scale uncertainties)

✦ Contributions from gg initial state (formally NNLO) are of 5-10%
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HIGGS BOSON PRODUCTION



QUANTITATIVE PREDICTIONS



For precise, quantitative comparisons between theory and data, (at least) Next-to-Leading-Order corrections are a must





1302.1415



- ✦ Effectively the scale for which a 1-jet event becomes a 0-jet event (left) or 2-jet event becomes a 1-jet event (based on k_T-algorithm)
- NLO+PS work well at low scales, but not so much at large scales: easily explained by only having LO (left) or PS (right) accuracy

SUMMARY

 We want to match NLO computations to parton showers to keep the good features of both approximations

• In the MC@NLO method:

by including the shower subtraction terms in our process we avoid double counting between NLO processes and parton showers

• In the POWHEG method:

apply an NLO-factor, and modify the (Sudakov of the) first emission to fill the hard region of phase-space according to the real-emission matrix elements

MATRIX ELEMENTS — PARTON SHOWER MERGING AT LO

IMPROVING MC's

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- There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:
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 - ME+PS merging: Include matrix elements with more final state partons to describe hard, well-separated radiation better



- 1. Fixed order calculation
- 2. Computationally expensive
- 3. Limited number of particles
- 4. Valid when partons are hard and well separated
- 5. Quantum interference correct
- 6. Needed for multi-jet description



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Shower MC

- 1. Resums logs to all orders
- 2. Computationally cheap
- 3. No limit on particle multiplicity
- 4. Valid when partons are collinear and/or soft
- 5. Partial interference through angular ordering
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Approaches are complementary: merge them!



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Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

PS ALONE VS. MATCHED SAMPLE

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result \Rightarrow Large variation in results (small prediction power)



PS ALONE VS. MATCHED SAMPLE

In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.



GOAL FOR ME-PS MERGING/ MATCHING

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions



2nd QCD radiation jet in top pair production at the LHC

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Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space



Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space

- ★ So double counting no problem, but what about getting smooth distributions that are independent of the precise value of Q^c?
- Below cutoff, distribution is given by PS
 need to make ME look like PS near cutoff
- ✦ Let's take another look at the PS!



- How does the PS generate the configuration above (i.e. starting from e⁺e⁻ -> qqbar events)?
- ✦ Probability for the splitting at t₁ is given by

$$(\Delta_q(Q^2, t_1))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(Q^2, t_{\rm cut}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\rm cut}))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$



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- How does the PS generate the configuration above (i.e. starting from e⁺e⁻ -> qqbar events)?
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$$(\Delta_q(Q^2, t_1))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$\frac{(\Delta_q(Q^2, t_{\rm cut}))^2}{2\pi} \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\rm cut}))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$



$$(\Delta_q(Q^2, t_{\rm cut}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\rm cut}))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$



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Leading Logarithmic approximation of the matrix element BUT with α_s evaluated at the scale of each splitting



$$(\Delta_q(Q^2, t_{\rm cut}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\rm cut}))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Leading Logarithmic approximation of the matrix element BUT with α_s evaluated at the scale of each splitting

Sudakov suppression due to disallowing additional radiation above the scale t_{cut}

 $\mathcal{M}|^2(\hat{s},p_3,p_4,...)$

To get an equivalent treatment of the corresponding matrix element, do as follows:

- Cluster the event using some clustering algorithm
 this gives us a corresponding "parton shower history"
- 2. Reweight α_s in each clustering vertex with the clustering scale $|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(Q^2)} \frac{\alpha_s(t_2)}{\alpha_s(Q^2)}$
- 3. Use some algorithm to apply the equivalent Sudakov suppression $(\Delta_q(Q^2, t_{\text{cut}}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\text{cut}}))^2$

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Rikkert Frederix

 $\mathcal{M}|^2(\hat{s},p_3,p_4,...)$

MLM MATCHING

[M.L. Mangano, 2002, 2006] [J. Alwall et al 2007, 2008]

The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t₀!



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The simplest way to do the Sudakov suppression is to run the shower on the event, starting from t₀!



- ✦ If hardest shower emission scale $k_{T1} > t_{cut}$, throw the event away, if all $k_{T1,2,3} < t_{cut}$, keep the event
- ◆ The suppression for this is (∆_q(Q², t_{cut}))⁴so the internal structure of the shower history is ignored. In practice, this approximation is still pretty good

✦ Allows matching with any shower, without modifications! Rikkert Frederix

CKKW MATCHING



 Once the 'most-likely parton shower history' has been found, one can also reweight the matrix element with the Sudakov factors that give that history

$$(\Delta_q(Q^2, t_{\rm cut}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\rm cut}))^2$$

 To do this correctly, must use same variable to cluster and define this sudakov as the one used as evolution parameter in the parton shower. Parton shower can start at t_{cut}

CKKW MATCHING



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50

- ♦ W+jets production: diff. jet rate for 0→1 transition (~ pT of hardest jet)
- ◆ Small dependence on the merging scale for small values, ~10%
 - When taken too large, the parton shower cannot fill the region all the way up to the merging scale anymore, leading to large deficits

MERGING ME+PS AT NLO ACCURACY

FOUR-LEPTON PRODUCTION



FOUR-LEPTON PRODUCTION



In the tail of the pT spectrum, there are large theoretical uncertainties. This is no surprise! Here the NLO calculation has actually only LO accuracy, because there must be a hard parton/jet recoiling against the 4lepton system.

FOUR-LEPTON PRODUCTION



In the tail of the pT spectrum, there are large theoretical uncertainties. This is no surprise! Here the NLO calculation has actually only LO accuracy, because there must be a hard parton/jet recoiling against the 4lepton system.

Can we include the NLO corrections to 4 leptons + 1 (hard) jet here?

LIMITATIONS

There are more observables very sensitive to theory uncertainties -- all related to **hard emissions** in the real-emission matrix elements and even stronger if they are emitted by the shower.

Even though our NLO computation is "inclusive in all extra radiation" (which is made explicit by the parton shower), the shower is only correct in the strict collinear approximation. It cannot generate hard extra jets correctly (i.e. jets beyond the first, which is included in the real emission corrections of the NLO computation and therefore already has a large uncertainty associated with it)



LIMITATIONS

There are more observables very sensitive to theory uncertainties -- all related to **hard emissions** in the real-emission matrix elements and even stronger if they are emitted by the shower.

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CKKW (2004) and MLM (2004)

- At LO this has been solved ~10 years ago: use tree-level matrix elements of various multiplicities to generate hard radiation, and the parton shower for the collinear and soft
- Double counting no problem: we simply throw events away when the matrixelement partons are too soft, or when the parton shower generates too hard radiation
- Applying the matrix-element cut is easy: during phase-space integration, we only generate events with partons above the matching scale



- ✦ For the cut on the shower, there are two methods. Throwing events away after showering is not very efficient, although it is working ("MLM method")
- Instead we can also multiply the Born matrix elements by suitable product of Sudakov factors (i.e. the no-emission probabilities) Δ(Q^{max}, Q^c) and start the shower at the scale Q^c ("CKKW method").
- For a given multiplicity we have $\sigma_{n,\text{excl}}^{\text{LO}} = B_n \Theta(k_{T,n} Q^c) \Delta_n(Q_{\max}, Q^c)$ Rikkert Frederix

WHAT TO DO AT NLO

- ✦ Let's start very simple and see what to do…
- ✦ Let's consider
 - a very simple process: production of a single EW vector boson (or Higgs boson)
 - O an observable most-sensitive to QCD radiation: k_T-jet resolution variable (with R=1), √y ~ p_T(j) [y₀₁ ~ p_T²(j₁); y₁₂ ~ p_T²(j₂); etc]

LEADING ORDER V



- Simplest prediction of all
- Just gives a delta-function at zero p_T due to energymomentum conservation
- Cannot be used to make reliable predictions for this observable

Physical curve	No
Tail	N/A
Integral	LO
Extendible to multi-jet	Yes

√y01

do/d/y01

LEADING ORDER V+1 JET





- Non-trivial distribution that is LO accurate
- Need a generation cut, otherwise the integral over the pT spectrum diverges
- Cannot be used to make reliable predictions at low pT

Physical curve	Only at high-p _T
Tail	LO
Integral	ω
Extendible to multi-jet	Yes



NEXT-TO-LEADING ORDER V

 $\sqrt{y_{01}}$

$$\left| \sum_{n} \left| 2 + 2 \right| \right|^{2} + 2 \operatorname{Re} \left[\sum_{n} \left| 2 + 2 \right| \right|^{2} + 2 \operatorname{Re} \left[\sum_{n} \left| 2 + 2 \right| \right|^{2} \right]$$



- Curve is non-physical at low pT: divergent real-emission corrections are compensated for by divergent virtual corrections
- Including higher order corrections (NNLO, etc), does not fix the nonphysical behaviour at small pT

Physical curve	Only at high-p _T
Tail	LO
Integral	NLO
Extendible to multi-jet	Yes

do/dvy01

NLO+PS V



- To get a physical shape at low pT need to resum radiation at all orders
- Can either be done analytically, or with a parton shower
- Parton shower also includes hadronisation and other nonfactorisable corrections
- Most used methods are MC@NLO and POWHEG

Physical curve	Yes
Tail	LO
Integral	NLO
Extendible to multi-jet	Yes

NLO(+PS) V+1 JET



- ✦ Distribution diverges at small pT
- ✦ Have to put a generation cut
- Parton shower can easily be added, but this does not solve the low-p_T problem

Physical curve	Only at high-p _T
Tail	NLO
Integral	ω
Extendible to multi-jet	Yes

MINLO V+1JET



- Include suitable Sudakov Form factors in the NLO V+1j predictions
- ✦ Distributions is NLO accurate
- Integral is not NLO accurate: the difference starts at O(α_s^{3/2})
- Parton shower can easily be attached

Physical curve	Yes
Tail	NLO
Integral	LO+
Extendible to multi-jet	Yes

Minlo

- The Minlo approach can be summarised as follows:
 - Renormalisation and factorisation scale setting, a la CKKW
 - **O** Together with matching to the Sudakov form factor, $\exp\left[-R\left(v\right)\right]$
 - Matching requires to subtract the O(alpha_s) expansion of the Sudakov form factor times the Born to prevent double counting with the NLO corrections
 - NLO accuracy of V+1j observables is not hampered by the scale setting and inclusion of the form factor: differences are beyond NLO



MINLO

[Hamilton, Nason, Zanderighi (2012)]

✦ Start from a NLO calculation with one extra jet

1. Set μ_R everywhere it occurs and likewise for all μ_F set $\mu_F \to \mu_F \sqrt{v}$:

$$d\sigma \to d\sigma' = d\sigma \ (\mu_R = K_R \max(Q_{\mathcal{B}}, Q_{\mathcal{B}J}), \ \mu_F \to K_F \sqrt{y}) \ . \tag{2.22}$$

2. Replace the additional power of $\bar{\alpha}_s$ that accompanies the NLO corrections according to

$$d\sigma' \to d\sigma'' = d\sigma' \left(\bar{\alpha}_{\rm s}^{\rm NLO} \left(\mu_{\rm R}^2 \right) \to \bar{\alpha}_{\rm s} \left(K_{\rm R}^2 y \right) \right) \,. \tag{2.23}$$

3. Multiply the LO component by the $\mathcal{O}(\bar{\alpha}_s)$ expansion of the inverse of the Sudakov form factor times $\bar{\alpha}_s \left(K_R^2 y\right) / \bar{\alpha}_s \left(\mu_R^2\right)$:

$$d\sigma'' \to d\sigma''' = d\sigma'' - d\sigma''|_{\rm LO} \,\bar{\alpha}_{\rm S} \left(K_{\rm R}^2 \, y \right) \left(G_{12} L^2 + \left(G_{11} + 2S_1 + \bar{\beta}_0 \right) L + 2\bar{\beta}_0 \ln \frac{\mu_{\rm R}}{K_{\rm R} \, Q} \right) \, (2.24)$$

4. Multiply by the Sudakov form factor times $\bar{\alpha}_s \left(K_R^2 y \right) / \bar{\alpha}_s \left(\mu_R^2 \right)$:

$$d\sigma^{\prime\prime\prime} \to d\sigma_{\mathcal{M}} = \exp\left[-R\left(v\right)\right] \frac{\bar{\alpha}_{s}\left(K_{R}^{2} y\right)}{\bar{\alpha}_{s}\left(\mu_{R}^{2}\right)} d\sigma^{\prime\prime\prime}.$$
(2.25)

MINLO DECOMPOSED



RESUMMED CROSS SECTION

 $d\sigma_{\mathcal{M}} = d\sigma_{\mathcal{R}} + d\sigma_{\mathcal{M}\mathcal{R}} + d\sigma_{\mathcal{F}}$

 $L = \log(1/v) = \log(Q^2/y)$

RESUMMED CROSS SECTION



Well-known formula; used e.g. in the Caesar approach

ert Frederix

Rik

- Sudakov form factor exp[R] not identical to what's (originally) used in Minlo. But Minlo approached can be improved to incorporate these terms (not relevant when colour is trivial)
- Written as **total derivative**: straight-forward to show that this is NLO correct in phase-space Φ up to $dσ_F$ after integration over *L* and expanding in $α_S$
- However, not NLO correct in the $d\Phi dL$ phase space (i.e., tail is not NLO correct)

ACCURACY OF MINLO

 $d\sigma_{\mathcal{M}} = d\sigma_{\mathcal{R}} + d\sigma_{\mathcal{M}\mathcal{R}} + d\sigma_{\mathcal{F}}$

$$\int_{\Lambda^2}^{Q^2} \frac{\mathrm{d}q_{\mathrm{T}}^2}{q_{\mathrm{T}}^2} \log^m \frac{Q^2}{q_{\mathrm{T}}^2} \alpha_{\mathrm{S}}^n \left(q_{\mathrm{T}}^2\right) \exp \mathcal{S}\left(Q, q_{\mathrm{T}}\right) \approx \left[\alpha_{\mathrm{S}}\left(Q^2\right)\right]^{n - \frac{m+1}{2}}$$

coefficient

Explicit derivation, using the general form of the differential NLO V+1j cross sections in the small y limit,

$$\frac{d\sigma_{S}}{d\Phi dL} = \frac{d\sigma_{0}}{d\Phi} \sum_{n=1}^{2} \sum_{m=0}^{2n-1} H_{nm} \bar{\alpha}_{S}^{n} \left(\mu_{R}^{2}\right) L^{m}$$

gives

$$\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_{0}}{d\Phi} \exp\left[-R\left(v\right)\right] \prod_{\ell=1}^{n_{i}} \frac{q^{\left(\ell\right)}\left(x_{\ell}, \mu_{F}^{2}v\right)}{q^{\left(\ell\right)}\left(x_{\ell}, \mu_{F}^{2}\right)} \left[\bar{\alpha}_{s}^{2}\left(K_{R}^{2}y\right)\left[\tilde{R}_{21}L + \tilde{R}_{20}\right] + \bar{\alpha}_{s}^{3}\left(K_{R}^{2}y\right)L^{2}\tilde{R}_{32}\right]$$
Only non-zero when exp[R] and Minlo
Sudakov exponent are different, or
when exp[R] is not NNLL_{\sigma} accurate.
Therefore, assume that it is not known

Known

MINLO ACCURACY FOR (INCLUSIVE) O-JET OBSERVABLES

$$\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi} \exp\left[-R\left(v\right)\right] \prod_{\ell=1}^{n_i} \frac{q^{(\ell)}\left(x_\ell, \mu_F^2 v\right)}{q^{(\ell)}\left(x_\ell, \mu_F^2\right)} \left[\bar{\alpha}_s^2\left(K_R^2 y\right) \left[\tilde{R}_{21} L + \tilde{R}_{20}\right] + \bar{\alpha}_s^3\left(K_R^2 y\right) L^2 \tilde{R}_{32}\right]$$

✦ After integration over the logarithm L (taking R₂₁=0, which is okay for the processes considered here) this results into terms of

$$\int dL' \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL'} = -\frac{d\sigma_0}{d\Phi} \left[\widetilde{R}_{20} - \bar{\beta}_0 \mathcal{H}_1 \left(\mu_R^2 \right) \right] \sqrt{\frac{\pi}{2}} \frac{1}{\left| 2G_{12} \right|^{1/2}} \bar{\alpha}_{\mathrm{S}}^{3/2} \left(1 + \mathcal{O} \left(\sqrt{\bar{\alpha}_{\mathrm{S}}} \right) \right)$$

Hence, diff. NLO-0jet cross section not correct with NLO-1jet Minlo

[Hamilton, Nason, Oleari, Zanderighi (2012); RF, Hamilton (2015)]
MINLO V+1JET



- Include suitable Sudakov Form factors in the NLO V+1j predictions
- ✦ Distributions is NLO accurate
- Integral is not NLO accurate: the difference starts at O(α_s^{3/2})
- Parton shower can easily be attached

Physical curve	Yes
Tail	NLO
Integral	LO+
Extendible to multi-jet	Yes

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FxFx / Meps@nlo: V & V+1J merging



MEPS@NLO: [Hoeche, Krauss, Schonherr, Siegert; +Gehrmann (2012)]

- Merge NLO+PS for V with Minlo for V+1j, at "merging scale" Q
- ✦ Above Q the tail is NLO accurate
- For not-too-small Q, integral is NLO accurate
- Used by ATLAS & CMS for LHC run II analyses

Physical curve	"Yes"		
Tail	NLO		
Integral	"NLO" (depending on Q)		
Extendible to multi-jet	Yes		

DIFFERENCES BETWEEN FXFX & MEPS@NLO

- Both FxFx and MEPS@NLO merging are based on making MC@NLO calculation for jet-multiplicities *exclusive* in more jets
 - Veto additional radiation; resum dependence on the veto scale (=merging scale)
- ✦ Major difference is in the way this exclusivity is applied
 - O CKKW-L approach (i.e. Sudakov rejection based on shower kernels)
 - Used in Sherpa's "MEPS@NLO"
 - Using shower kernels prevents for a direct link with Minlo approach (and comparison to analytic resummation and accuracy), but prevents issues with mismatch in k_T and shower ordering values
 - Minlo (CKKW) from hard scale down to the scale of the softest jet not affected by veto; MLM-type rejection from there down to merging scale
 - Used in MadGraph5_aMC@NLO w/ Pythia/Herwig: "FxFx merging"
 - Direct link with Minlo, but MLM-type rejection prevents mismatches in ordering values

FXFX MERGING: HIGGS BOSON PRODUCTION



- Transverse momentum of the Higgs and of the 1st jet.
- Agreement with H+0j at MC@NLO and H+1j at MC@NLO in their respective regions of phase-space; Smooth matching in between; Small dependence on matching scale
- Alpgen (LO matching) shows larger kinks

FXFX MERGING: HIGGS BOSON PRODUCTION

RF & Frixione, 2012



✦ Differential jet rates for 1->0 and 2->1

FXFX MERGING: HIGGS BOSON PRODUCTION

RF & Frixione, 2012



- ✦ Differential jet rates
- ✦ Matching up to 2 jets at NLO
- ◆ Results very much consistent with matching up to 1 jet at NLO

FOUR-LEPTON PRODUCTION



In the tail of the pT spectrum, there are large theoretical uncertainties. This is no surprise! Here the NLO calculation has actually only LO accuracy, because there must be a hard parton/jet recoiling against the 4lepton system.

Can we include the NLO corrections to 4 leptons + 1 (hard) jet here?

FOUR-LEPTON PRODUCTION



MERGING SCALE DEPENDENCE

Next-to-leading order electroweak corrections

Electroweak corrections in particle-level event generation

Conclusions

Merging systematics: $pp \rightarrow \ell^- \bar{\nu} + jets$



 \Rightarrow dead zones in incl. obs. if Q_{cut} too high

 $\mathcal{O} \mathcal{Q} \mathcal{O}$

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Kallweit, Lindert, Maierhöfer, Pozzorini, MS JHEP04(2016)021



← For pT(W) > 1 TeV, Sudakov peak for d_{12} is around 50 GeV



Kallweit, Lindert, Maierhöfer, Pozzorini, MS JHEP04(2016)021

$$\int dL' \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL'} = -\frac{d\sigma_0}{d\Phi} \left[\widetilde{R}_{20} - \overline{\beta}_0 \mathcal{H}_1 \left(\mu_R^2 \right) \right] \sqrt{\frac{\pi}{2}} \frac{1}{\left| 2G_{12} \right|^{1/2}} \,\overline{\alpha}_{\mathrm{s}}^{3/2} \left(1 + \mathcal{O} \left(\sqrt{\overline{\alpha}_{\mathrm{s}}} \right) \right)$$

 Even for a factor 1000 or more in ratio of hard scale over merging scale: no sign of lack of NLO for inclusive observables seems to a problem for this process and observable

FxFx / Meps@nlo: V & V+1J merging



MEPS@NLO: [Hoeche, Krauss, Schonherr, Siegert; +Gehrmann (2012)]

- Merge NLO+PS for V with Minlo for V+1j, at "merging scale" Q
- ✦ Above Q the tail is NLO accurate
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- Used by ATLAS & CMS for LHC run II analyses

Physical curve	"Yes"		
Tail	NLO		
Integral	"NLO" (depending on Q)		
Extendible to multi-jet	Yes		

UNLOPS: V & V+1J MERGING



- "Unitarise" FxFx/MEPS@NLO predictions
- Difference between NLO+PS V and FxFx/MEPS@NLO computed numerically and subtracted below Q
- Almost NLO accurate integral (not exactly due to incorrect phase-space mappings/kinematics below Q)
- ✦ Allows for smaller Q
- ✦ Available in Pythia8

Physical curve	"Yes"
Tail	NLO
Integral	"NLO"
Extendible to multi-jet	Yes

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Integral	"NLO"
Extendible to multi-jet	Yes

"SUBTRACT WHAT YOU ADD" PHILOSOPHY

 The difference between the inclusive NLO 0-jet observables and the inclusive NLO 1-jet Minlo calculations is subtracted at a (projected) 0-jet Born-like phase-space point



- This (potentially) removes the NLO ambiguities of taking the merging scale to small values
- ✦ Idea is very interesting, but current implementation/subtleties need improvement:
 - Shape of contribution below merging scale is strictly generated by shower: not even LO contributions there, i.e. MC@NLO or POWHEG matching there. Problematic for large merging scales
 - There is a non-trivial dependence on the mapping used to got from 1-jet to 0-jet kinematics (i.e. for the real emission corrections to the NLO 1-jet)
 - Rather poor efficiency for small merging scales as adding/subtraction is done at the level of event files

GENEVA



- Start from NNLO for V, add NNLL' analytic resummation
- High-enough orders in resummation accuracy circumvents the need of merging scale: already includes NLO for the complete pT(j) spectrum
- Non-trivial to attach parton shower
- Only available for W-boson production: rather difficult to extend, even though in principle possible

Physical curve	Yes
Tail	NLO
Integral	NNLO
Extendible to multi-jet	Tricky

GENEVA

- Not based on MC@NLO or POWHEG for event generation. Rather, just like UNLOPS, use projections to underlying kinematics to allow for event generation
 - No real issues with inefficiencies here: can put this cut to very small value ~1 GeV; similar to a shower cut-off or phase-space slicing parameter in NNLO computations
 - Projections done very carefully. No issues with mismatches
 - First steps to N-jettiness subtraction instead of slicing for NNLO?
- Split phase-space according to variable that is easy to resum: N-jettiness
 - It is known how to resum N-jettiness up to NNLL' accuracy
 - NNLO corrections naturally included in NNLL' resummation
 - N-jettiness and shower evolution are very different: need some gymnastics to attach a parton shower: recent study on underlying event studies shows that this seems to be under control [Alioli, Bauer, Guns, Tackmann (2015)]
- ✦ Very powerful approach
 Rikkert Frederix

MINLO-REVISITED V+1J



[Hamilton, Nason, Oleari, Zanderighi (2012); Hamilton, Nason, Re, Zanderighi (2013); RF, Hamilton (2015)]

- ✦ Much simpler as Geneva
- Like Minlo V+1j, include Sudakov form factors to make distribution physical at low pT
- Modify the Sudakov form factors with subleading, process dependent terms such that total integral becomes NLO accurate
- ✦ Can include NNLO corrections for V

Physical curve	Yes
Tail	NLO
Integral	(N)NLO
Extendible to multi-jet	Yes

An explicit comparison between the diff.-jet-rate-resummation formula (which integrates to the correct NLO 0-jet diff. cross section) and Minlo shows that [Banfi, Salam, Zanderighi (2005); [Banfi, Salam, Zanderighi (2005); Dokshitzer, Diakonov, Troian (1980)]

$$\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi} \exp\left[-R\left(v\right)\right] \prod_{\ell=1}^{n_i} \frac{q^{(\ell)}\left(x_\ell, \mu_F^2 v\right)}{q^{(\ell)}\left(x_\ell, \mu_F^2\right)} \left[\bar{\alpha}_{\mathrm{S}}^2\left(K_R^2 y\right) \left[\tilde{R}_{21} L + \tilde{R}_{20}\right] + \bar{\alpha}_s^3\left(K_R^2 y\right) L^2 \tilde{R}_{32}\right]$$

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 RF, Hamilton (2015)]

An explicit comparison between the diff.-jet-rate-resummation formula (which integrates to the correct NLO 0-jet diff. cross section) and Minlo shows that [Banfi, Salam, Zanderighi (2005); [Banfi, Salam, Zanderighi (2005); Dokshitzer, Diakonov, Troian (1980)]

$$\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi} \exp\left[-R\left(v\right)\right] \prod_{\ell=1}^{n_i} \frac{q^{(\ell)}\left(x_\ell, \mu_F^2 v\right)}{q^{(\ell)}\left(x_\ell, \mu_F^2\right)} \left[\bar{\alpha}_{\mathrm{S}}^2\left(K_R^2 y\right) \left[\tilde{R}_{21} L + \tilde{R}_{20}\right] + \bar{\alpha}_s^3\left(K_R^2 y\right) L^2 \tilde{R}_{32}\right]$$

✦ After integration over the logarithm L (taking R₂₁=0, which is okay for the processes considered here) this results into terms of

$$\int dL' \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL'} = -\frac{d\sigma_0}{d\Phi} \left[\widetilde{R}_{20} - \overline{\beta}_0 \mathcal{H}_1 \left(\mu_R^2 \right) \right] \sqrt{\frac{\pi}{2}} \frac{1}{\left| 2G_{12} \right|^{1/2}} \,\overline{\alpha}_{\mathrm{s}}^{3/2} \left(1 + \mathcal{O} \left(\sqrt{\overline{\alpha}_{\mathrm{s}}} \right) \right)$$

Hence, diff. NLO-0jet cross section not correct with NLO-1jet Minlo
 [Hamilton, Nason, Oleari, Zanderighi (2012);
 RF, Hamilton (2015)]

Explicitly compute and remove that term in the Minlo calculation such that the integral $\int_{dL} \frac{d\sigma_{MR}}{d\Phi dL}$ is zero up to NLO is process dependent and not a constant in phase-space is that

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$$\int dL' \, \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL'} = -\frac{d\sigma_0}{d\Phi} \, \left[\widetilde{R}_{20} - \bar{\beta}_0 \mathcal{H}_1 \right] \, dL'$$

Can either be done analytically μ_R^2 or numerically by enforcing)

+ Hence, diff. NLO-0jet cross section not correct with NLO 1jet Minlo

[Hamilton, Nason, Oleari, Zanderighi (2012); RF, Hamilton (2015)]

RAPIDITY OF THE HIGGS BOSON



- Only observable truly NNLO correct
- ◆ Extended Minlo' method (HJJ★) agrees with NNLOPS by construction
- Normal HJJ Minlo shows larger uncertainty bands and different central value: it's only LO accurate for this observable

TRANSVERSE MOMENTUM OF THE LEADING JET



- ◆ Extended Minlo' method (HJJ★) agrees with NNLOPS by construction.
 - O apart from $p_T < 5$ GeV region: grid-granularity to compute δ not fine enough
 - O Also region $60 < p_T < 80$ GeV shows 3-5% deviations: pT derivative of the numerator of δ changes very rapidly
- Normal HJJ Minlo shows unphysical uncertainty band. Formally only LO for this observable

TRANSVERSE MOMENTUM OF THE SECOND JET



- ✦ Extended Minlo' method HJJ★ agrees with Minlo HJJ, as expected
 - O apart close to the Sudakov peak: the difference between HJJ* and HJJ is beyond LL/NNLL_o accuracy, which is important close to the Sudakov peak
- NNLOPS only LO accurate for this observable: uncertainty band is too small (this is due to the POWHEG method)

Y₁₂ RESOLUTION PARAMETER



- Similar picture as for p_T(j₂), but low p_T region easier to see due to logarithmic x-axis
- First observable where we see some non-zero dependence on the freezing parameter ρ (red solid). Well below the Sudakov peak where higher-logarithmic corrections are large as well as nonperturbative corrections

$\begin{array}{l} Y_{12} \ RESOLUTION \ PARAMETER \\ WITH \ \hline Y_{01} > 200 \ GeV \end{array}$



- ★ At very large y₁₂, all scales are large and of the same order —> the Minlo method switches off: HJJ★ agrees with HJJ
- When y₁₂ « y₀₁, large logarithms build up, and the extended Minlo' method brings the HJJ* to the NNLOPS

HIGGS BOSON PT IN EVENTS WITH EXACTLY 2 JETS



- At small p_T, all scales are of the same order. The Minlo method does not do much: HJJ* agrees with HJJ
- At large p_T, HJJ[★] agrees with NNLOPS dominated by events with one hard jet (p_T(j₁) ~ p_T(H)) and one soft jet: a 30 GeV jet comes basically for free
 - The pT(H) spectrum with N_{jets}=2 becomes essentially N_{jets}≥1 pT(H) distribution

	NLO rate?	NLO tail?	physical?	comment
NLO V+0j	\checkmark	×	×	
NLO+PS V+0j	\checkmark	×	\checkmark	fully automated
Minlo V+1j	×	\checkmark	\checkmark	
FxFx/ MEPS@NLO V+0,1j	√ *	√ %	\checkmark	Combines NLO+PS with Minlo
UNLOPS	√ *	√ %	\checkmark	Unitarity on incl. X-sect. is imposed
Geneva	\checkmark	\checkmark	\checkmark	allows for NNLO
Minlo' V+1j	\checkmark	\checkmark	\checkmark	allows for NNLO

- Comparison to data
- ✦ Z+jets
- Exclusive jet multiplicity and hardest and 3rd hardest jet pT spectra
- Uncertainty band contains ren. & fac. scale, PDF & merging scale dependence
- Rather good agreement between data and theory



- ✦ Agreement between FxFx merged results, matched to Herwig++ and Pythia8, and Atlas and CMS data is rather good
- Where data and theory differ, also differences between the results matched to HW++ and PY8 differ


NLO+PS MATCHING INCLUDING EW CORRECTIONS

- In POWHEG, two independent implementations of QCD+EW corrections to W-boson production exist [Bernaciak & Wackeroth (2012); Barzè et al. (2012)]
- MG5_aMC and Sherpa working towards automation. Some first results with Sherpa+OpenLoops have been presented, although they include only EW corrections of virtual origin [Kallweit et al. (2015)]







 In the last couple of years the accuracy of event generation has greatly improved, and full automation has been achieved at NLO accuracy

• First results for matching NNLO ME to PS

- ✦ A lot of freedom in tuning has been replaced by accurate theory descriptions:
 - More predictive power
 - Better control on uncertainties
 - Greater trust in the measurements
- Latest developments include the merging matrix elements of various multiplicities and matching those to the parton shower, including some EW corrections