Theory uncertainties and resummation

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Outline

- Renormalisation scale uncertainties
- Factorisation scale uncertainties
- Two-scale problems: the need for resummation
- Principles of NLL resummation
- Resummation uncertainties

Renormalisation and factorisation scales

- Any fixed-order cross section in hadron collisions depends on two unphysical parameters, the renormalisation and factorisation scales

$$d\sigma_{pp\to X} \sim \sum_{i,j} f_{i/p}(\mu_F) \otimes f_{j/p}(\mu_F) \otimes d\hat{\sigma}_{ij\to X}\left(\alpha_s(\mu_R), \frac{\mu_F}{\mu_R}, \ldots\right)$$

factorisation scale

renormalisation scale

- Varying these scales produces higher order contributions, e.g

$$\alpha_s^n(x\mu_R) = \alpha_s^n + (n\,\beta_0 \ln x)\,\alpha_s^{n+1}(\mu_R)$$

Scale uncertainties

- Varying renormalisation and factorisation scales is a natural way to estimate theoretical uncertainties

$$d\sigma_{pp\to X}(x\mu_R, y\mu_F) = \underbrace{d\sigma_{pp\to X}(\mu_R, \mu_F)}_{\sim\alpha_s^n} + \mathcal{O}(\alpha_s^{n+1})$$

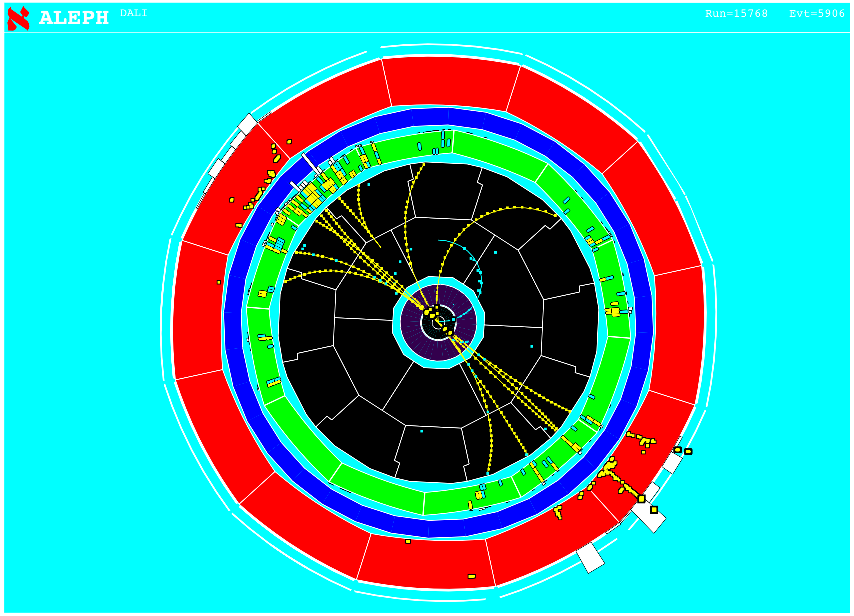
- Relevant questions are:
  - how to choose the default (central) value of μ_R and μ_F?
  - over what range should we vary these scales?
  - how should we add uncertainties?

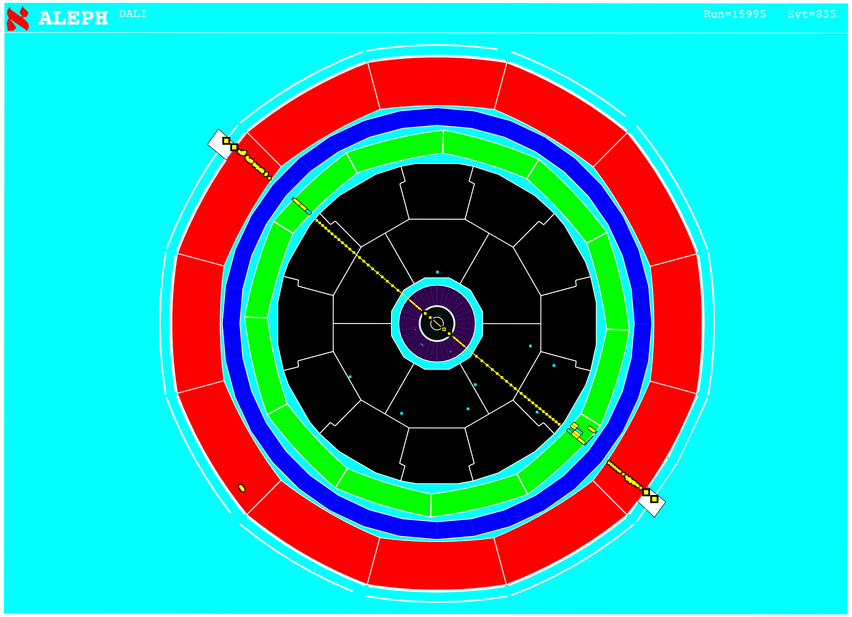
Although is no theoretically sound answer to any of these questions, we will try to reflect on how to reasonably assess theory uncertainties

Short-distance observables

- Consider a simple counting observable in e^+e^- annihilation

$$R = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



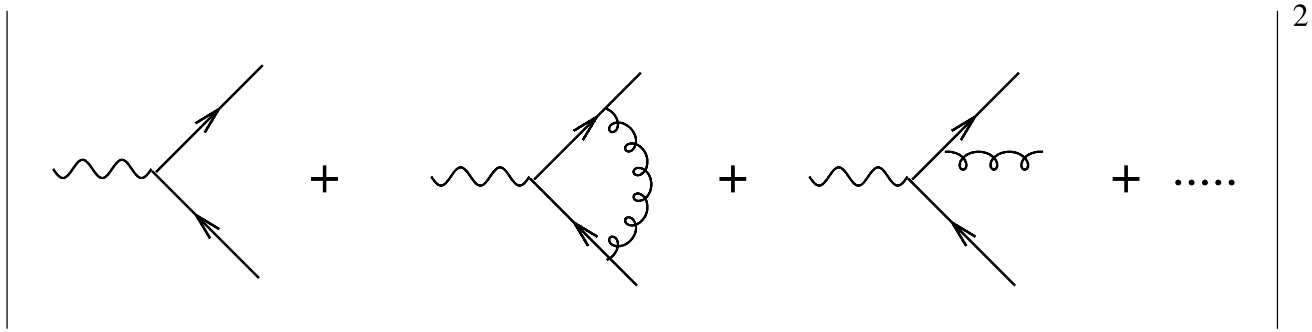


The ratio R in QCD

- Since the ratio *R* is infrared and collinear safe, it admits a massless limit

$$R\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{m_q^2}{\mu_R^2}\right) = \hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) + \mathcal{O}\left(\left(\frac{m_q^2}{Q^2}\right)^p\right)$$

$$\hat{R} = \frac{\sigma(e^+e^- \to \text{partons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Renormalisation group analysis

- The massless limit $\hat{R}$ does not depend on the renormalisation scale μ_R

$$\left(\mu_R^2 \frac{\partial}{\partial \mu_R^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) \hat{R}\left(\alpha_s(\mu^2), \frac{Q^2}{\mu_R^2}\right) = 0 \quad \Rightarrow \quad Q^2 \frac{\partial \hat{R}}{\partial Q^2} = \beta(\alpha_s) \frac{\partial \hat{R}}{\partial \alpha_s}$$

- The formal solution of the above renormalisation group equation is

$$\hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) = \hat{R}\left(\alpha_s(Q^2), 1\right)$$

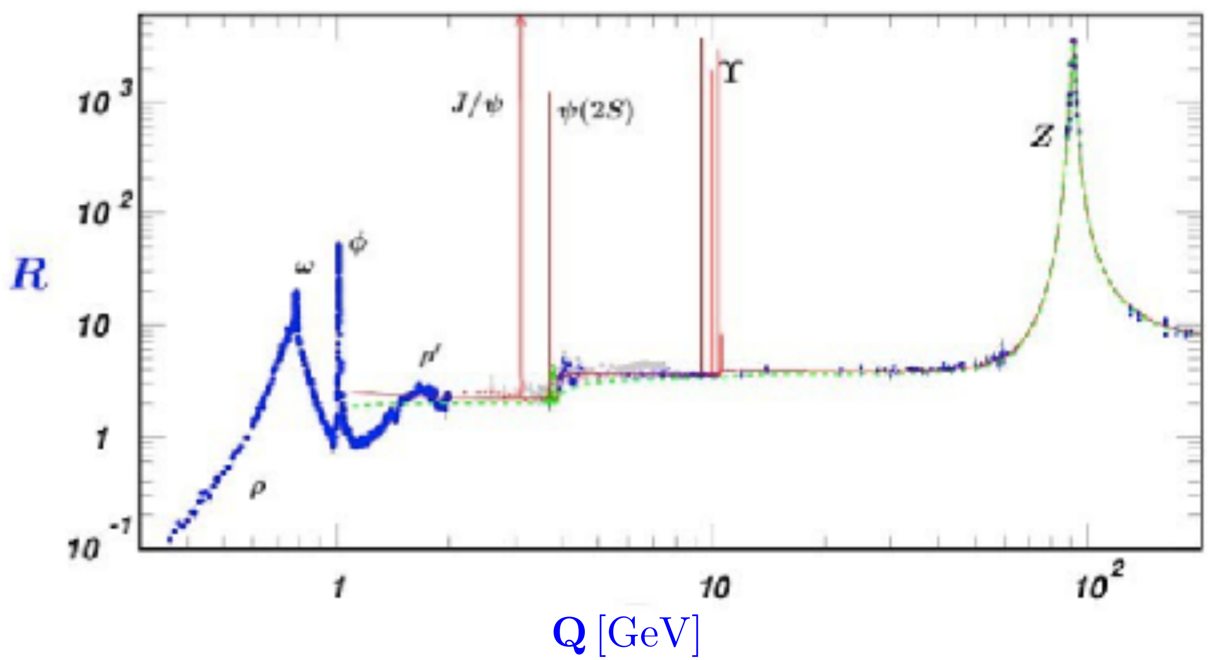
- Renormalisation group, and the fact that $\hat{R}$ depends on a single scale give enough conditions to determine the central value of μ_R

Setting the central scale as a resummation

- For a one-scale observable like $\hat{R}$, the dependence on μ_R appears in the following form

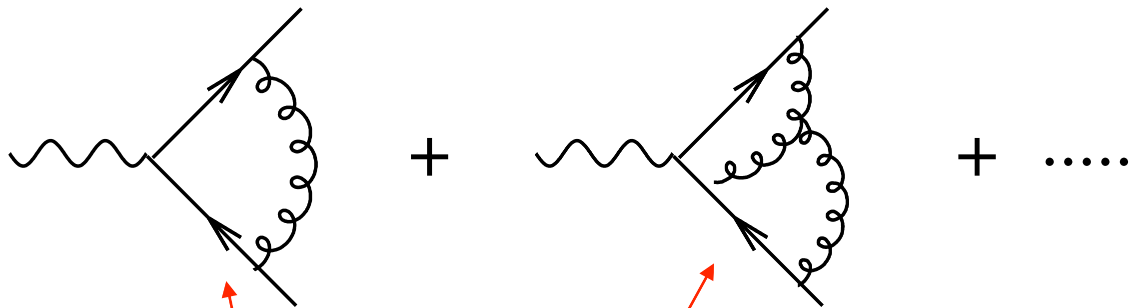
$$\hat{R}\left(\alpha_s(\mu_R), \frac{Q^2}{\mu_R^2}\right) = R_0 + R_1\alpha_s(\mu_R^2) + \left(R_1\beta_0 \ln\frac{Q^2}{\mu_R^2} + R_2\right)\alpha_s^2(\mu_R^2) + \mathcal{O}(\alpha_s^3)$$

- Choosing $\mu_R^2 = Q^2$ resums terms $\ln(\mu_R^2/Q^2)$ at all orders in $\alpha_s(Q^2)$



Meaning of renormalisation scale

- The renormalisation scale is the price to pay for the renormalisation of UV divergences



$$\underbrace{\alpha_s(\Lambda) \times \left(1 + c_1 \alpha_s(\Lambda) \ln \frac{\Lambda^2}{\mu_R^2}\right)}_{\equiv \alpha_s(\mu_R)} + \ldots$$

- The renormalised coupling $\alpha_s(\mu_R)$ contains all quantum fluctuations with virtuality larger than μ_R
- The ratio *R* is fully inclusive, so it is not able to probe quantum fluctuations above the com energy Q

Renormalisation scale: central value

- In hadronic collisions, especially in multi-jet events, observables depend on multiple scales

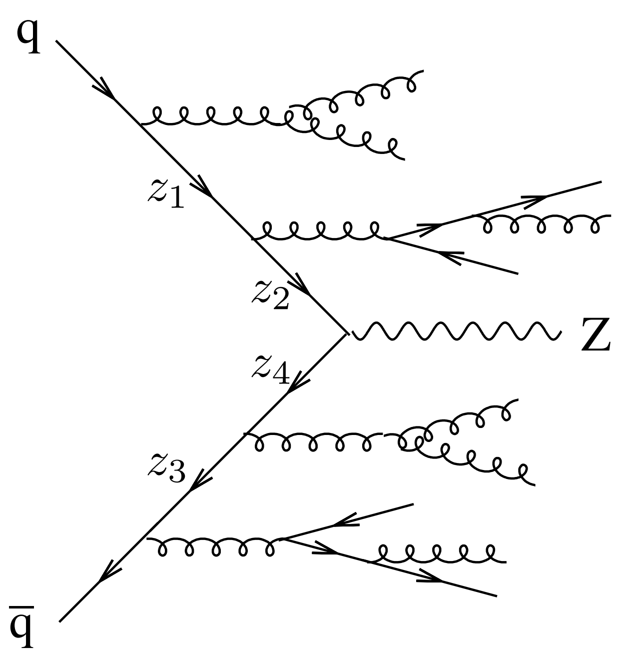
$$\sigma\left(\alpha_s(\mu_R^2), \frac{s_1}{\mu_R^2}, \ldots, \frac{s_1}{\mu_R^2}\right) = \underbrace{\sigma_0}_{\sim\alpha_s^n} + \underbrace{\alpha_s\beta_0 \ln\frac{s_1\ldots s_n}{\mu_R^{2n}}}_{\sim\alpha_s^{n+1}+\sigma_1} + \mathcal{O}(\alpha_s^{n+2})$$

- One possibility to set a default scale is to cancel the logarithm of μ_R that appears at one loop $\Rightarrow$ $\mu_R^2 = (s_1\, s_2 \ldots s_n)^{1/n}$

- Possible caveats with such choice
  - The similarity with *R* can be deceiving, since this log-enhanced term has the same kinematics as tree-level, and does not account for the physics of extra-gluon radiation $\Rightarrow$ physical meaning of renormalisation scale?
  - In multi-scale observables, these are not the only logs around, so you can have extra logs of ratios of scales $\Rightarrow$ soft-collinear resummation

Renormalisation scale: central value

- In inclusive production of a particle of mass *M* (e.g. Higgs), we have two scales, the mass of the particle and the partonic com energy square $\hat{s}$
- Without extra gluons, $\hat{s} = M$, so we're back to the one-scale problem
- With additional gluons, $\hat{s}$ and *M* can be different

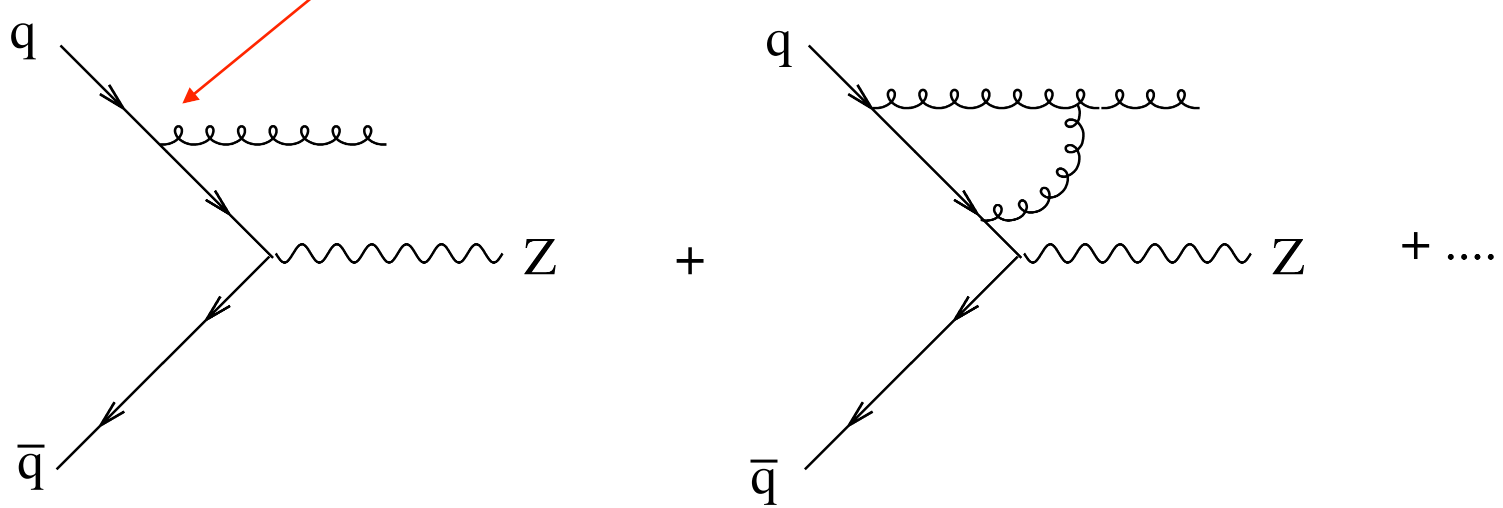


$$\hat{s} = \frac{M}{z_1\, z_2 \ldots z_n}$$

- The partonic com energy is sensitive to the typical energy scale of extra gluon radiation

Renormalisation scale: central value

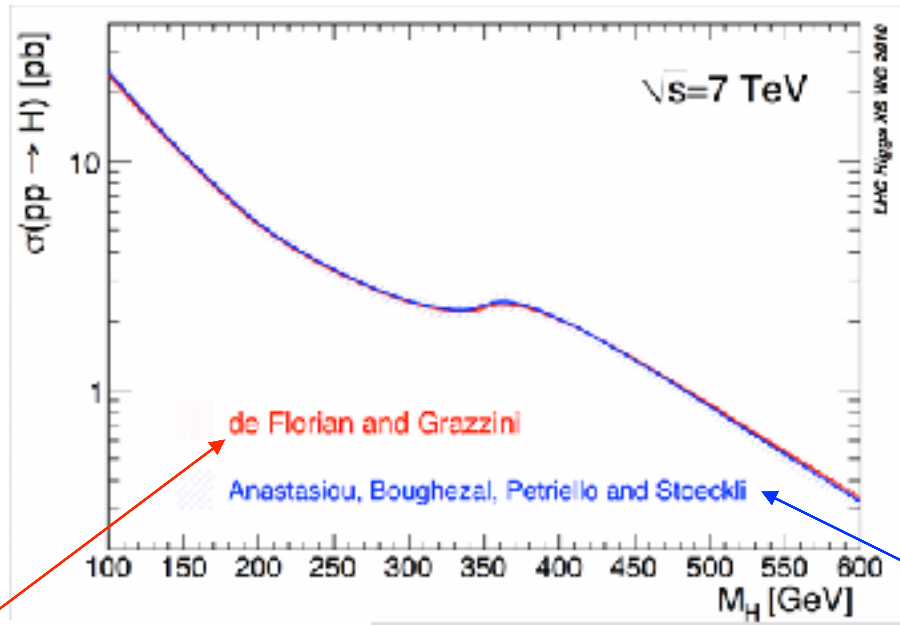
- For one emission at fixed transverse momentum k_t (one-scale problem), one can show that $\alpha_s = \alpha_s(k_t)$ resums $\ln(\mu_R^2/k_t^2)$ at all orders



- Physical meaning: integrate out of all quantum fluctuations from the cutoff to the scale k_t into the running coupling $\alpha_s(k_t)$

Renormalisation scale: central value

- Strategy: with QCD emissions, set the renormalisation scale close to the upper bound of transverse momentum of emitted gluons
- For instance, for the Higgs total cross section, an “optimal” scale can be obtained by imposing that the perturbative series is well convergent



NNLL threshold resummation
central scale M_H

NNLO fixed-order
central scale $M_H/2$

- This choice in general depends on the observable, and on the accuracy of the calculation $\Rightarrow$ threshold resummations of $\ln(M/\hat{s})$ help

Renormalisation scale variation

- Is varying renormalisation a good probe of theory uncertainties?

$$\sigma_0(x\mu_R,\dots) = \underbrace{\sigma_0(\mu_R,\dots)}_{\sim\alpha_s^n} + \underbrace{(n\beta_0 \ln x)\,\alpha_s(\mu_R)\,\sigma_0(\mu_R,\dots) + \sigma_1(\mu_R,\dots)}_{\sim\alpha_s^{n+1}} + \dots$$

- Varying *x* by a factor of two is sensible only after one has identified a suitable central scale, otherwise σ_1 will contain large logs of μ_R

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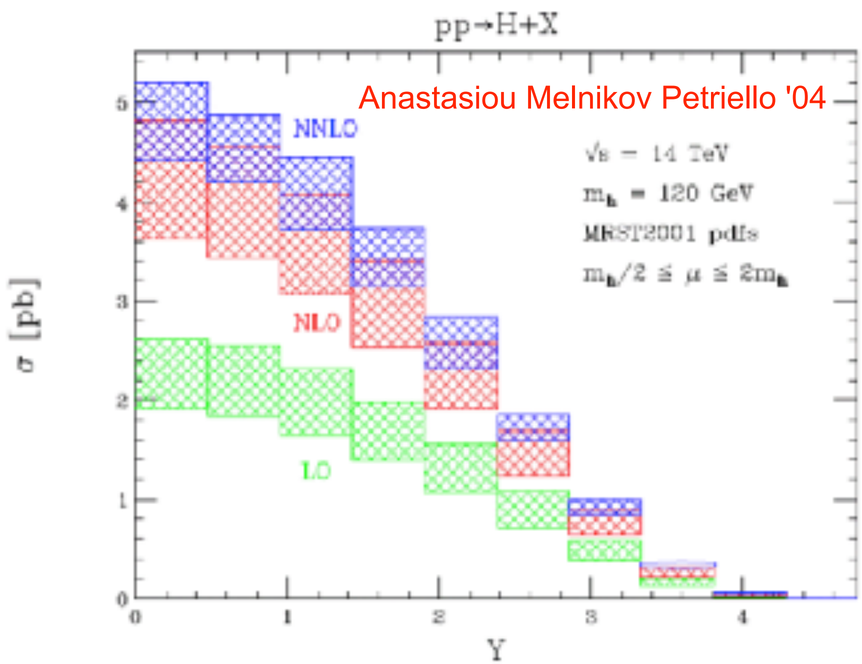
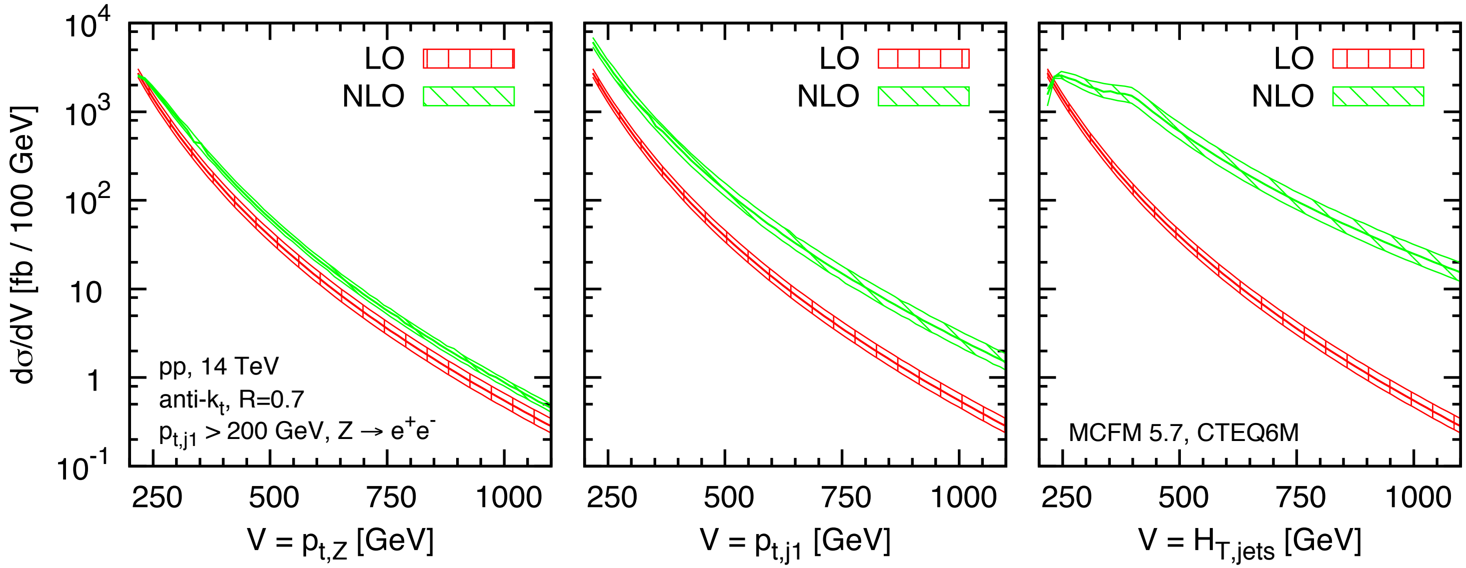


- The procedures gives an estimate of missing higher orders as long as K-factors (e.g. σ_1/σ_0) are not too large

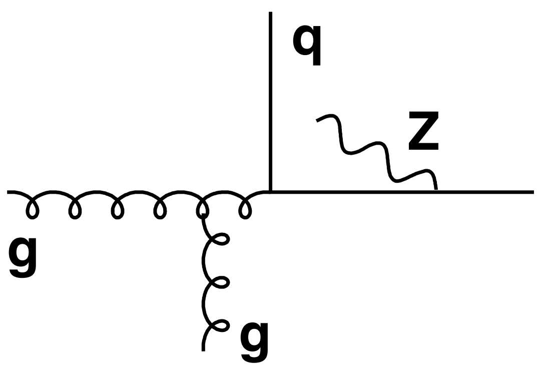
Renormalisation scale variation

- Large K-factors can have different origins than a bad choice of default renormalisation scale



- It is nowadays possible to perform approximate higher-order calculations that account for the opening of new channels

Salam Sapeta '10

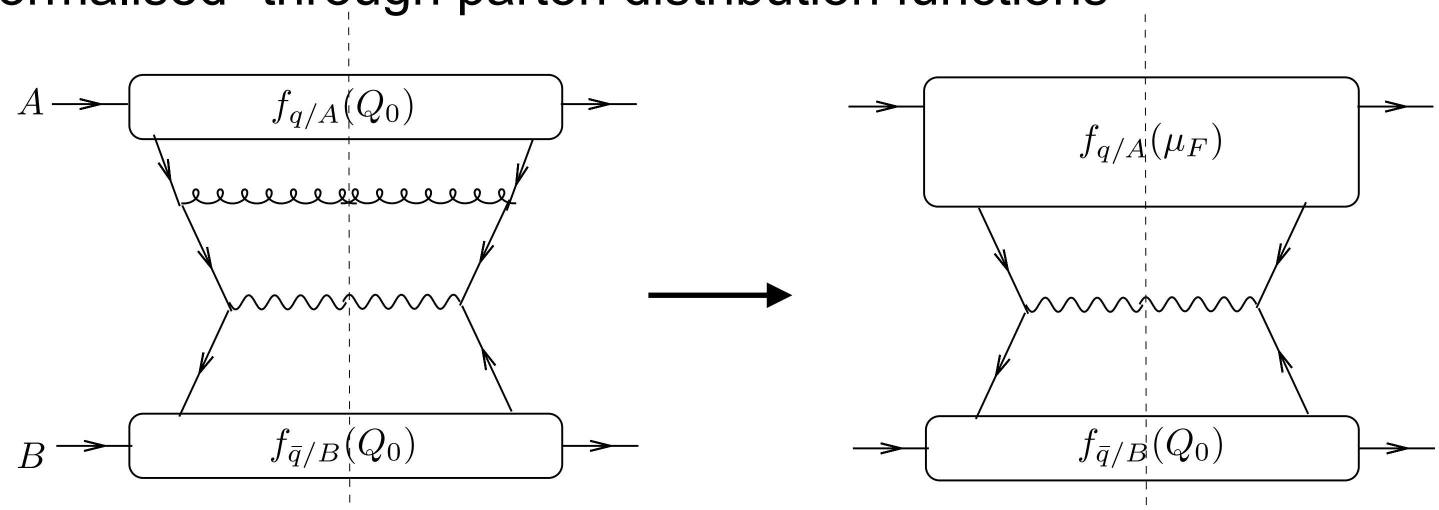


Factorisation scale: central value

- Consider a cross section in hadron collisions

$$d\sigma_{pp\to X} \sim \sum_{i,j} f_{i/p}(\mu_F) \otimes f_{j/p}(\mu_F) \otimes d\hat{\sigma}_{ij\to X}\left(\alpha_s(\mu_R), \frac{\mu_F}{\mu_R}, \dots\right)$$

- The partonic cross section $d\hat{\sigma}_{ij}$ contains collinear singularities that are "renormalised" through parton distribution functions

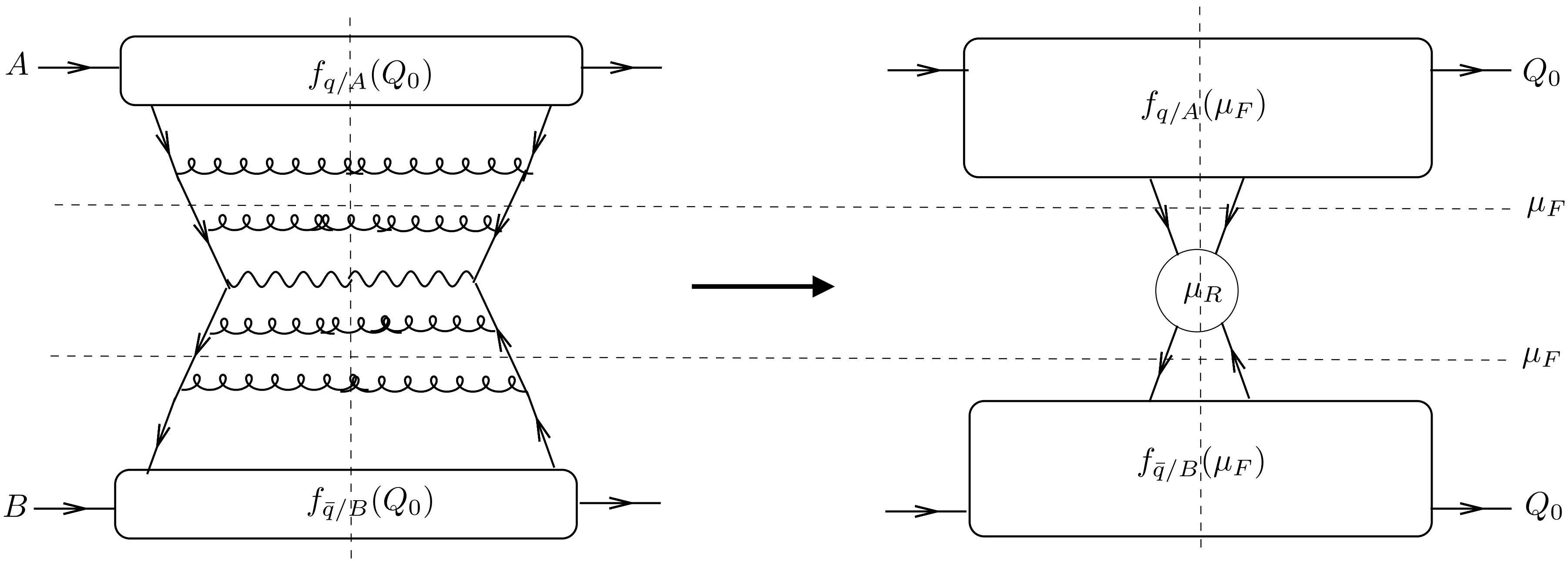


- The price to pay for this renormalisation is the introduction of the factorisation scale μ_F

- Choosing $\mu_F = \mu_R$ will make $\ln(\mu_F/\mu_R)$ to disappear formally, but logarithms of collinear origin might appear as logs of other scales

Factorisation scale: central value

- The parton distribution function $f_{i/A}(\mu_F)$ inclusively resums the contribution of multiple emissions collinear to hadron *A*
- Collinear emissions with transverse momenta up to μ_F inside $f_{i/A}(\mu_F)$ are not observed (unresolved)



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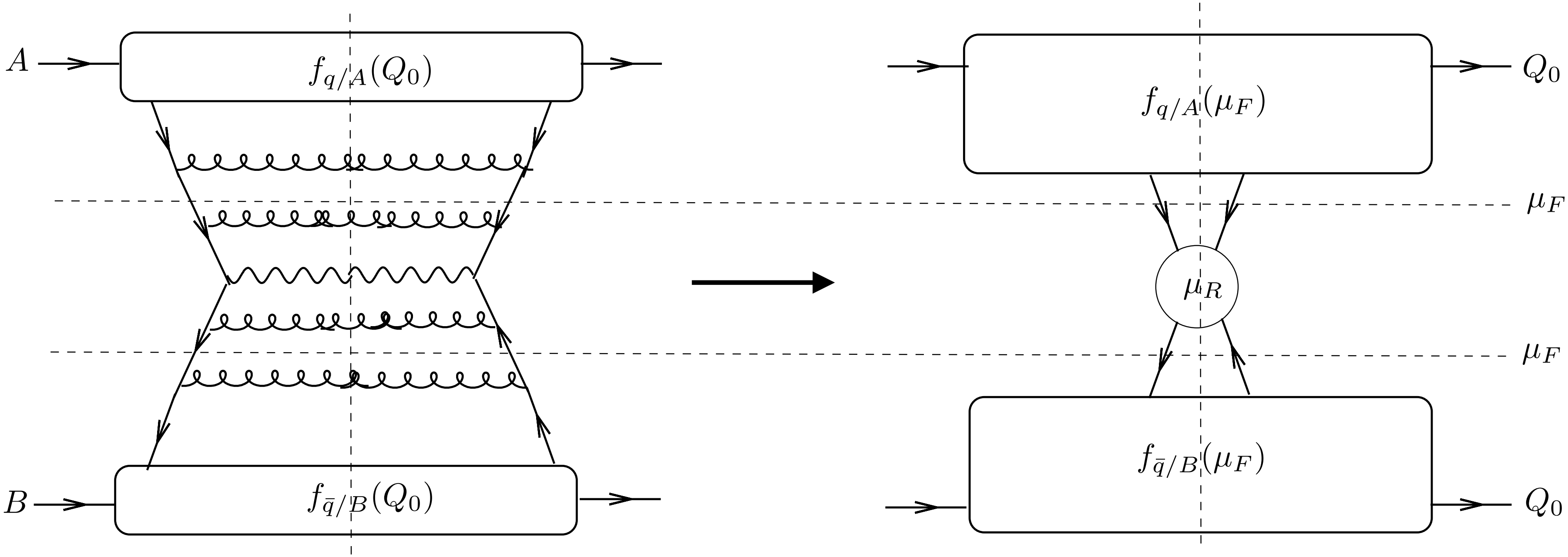
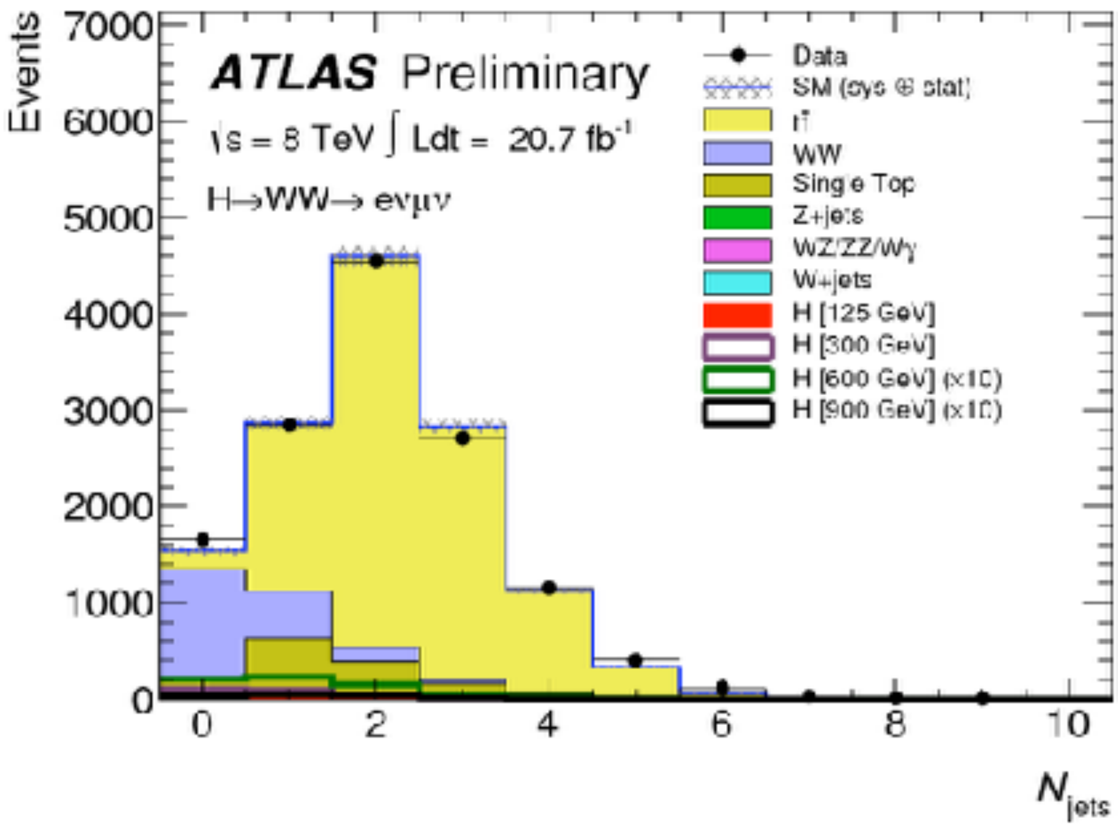


- Strategy: set the central value of μ_F close to the minimum transverse momentum of resolved partons, and allow it to vary independently of μ_R, since it accounts for a different physical effect

Higgs production with a jet veto

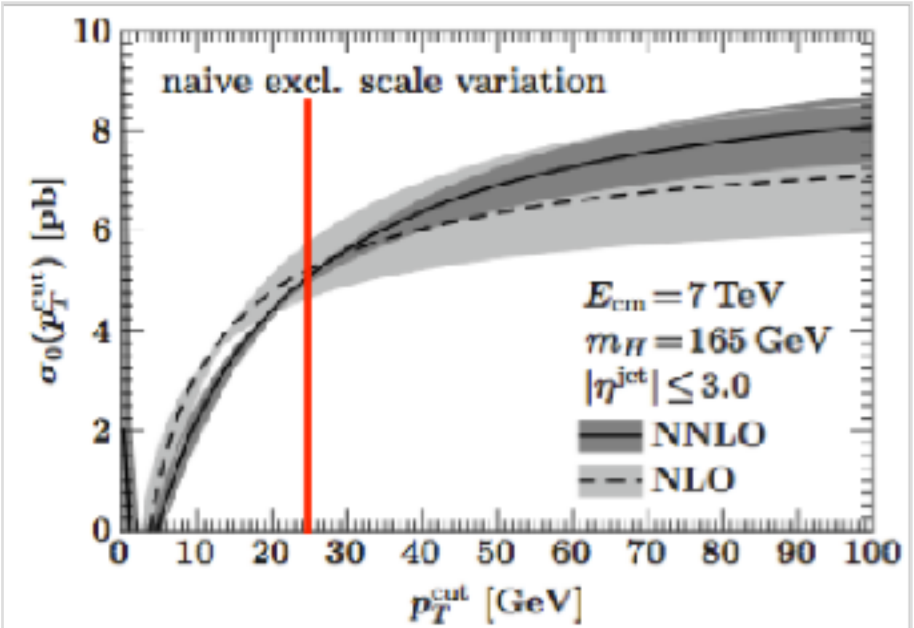
- In order to suppress the large top-antitop background to $H \to WW$ we require that all jets have a transverse momentum below $p_{\mathrm{t,veto}}$



- This works: the zero-jet cross section $\sigma_{0-\mathrm{jet}}$ is least contaminated by the huge (yellow) top-antitop background

Higgs plus zero jets: infrared sensitivity

- Scale variations sometimes highlight the pathological behaviour of some cross sections, for instance their infrared sensitivity



$$\sigma_{0-\mathrm{jet}} = \sigma_{\mathrm{tot}} - \sigma_{\geq 1-\mathrm{jet}}$$

large K-factor

large logs

$\ln(m_H/p_T^{\mathrm{cut}})$

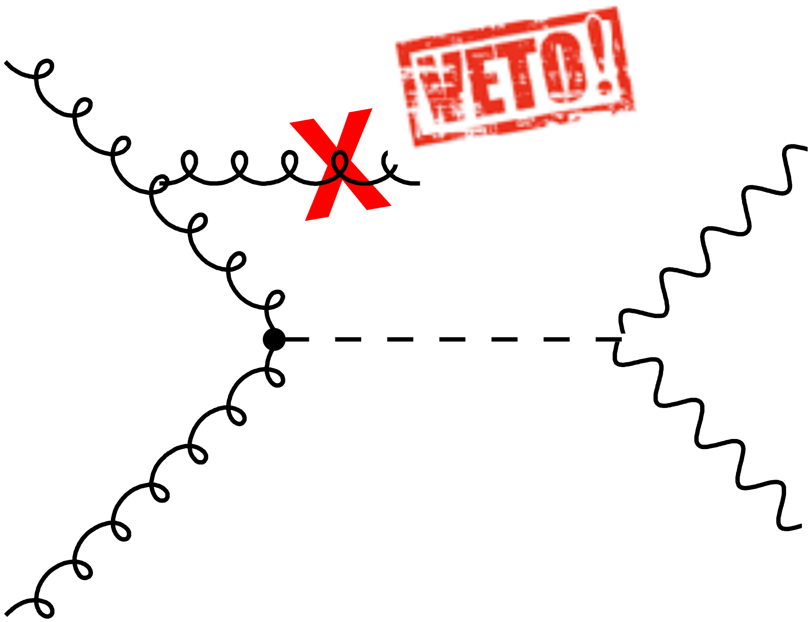
- The cancellation of two large effects gives a spurious vanishing of scale at low values of the jet-veto resolution p_T^{cut}

Stewart Tackmann ‘12

- Zero-jet cross sections are the typical example of a two-scale problem, that cannot be solved by simply adjusting μ_R and μ_F

Higgs plus zero jets as a two-scale problem

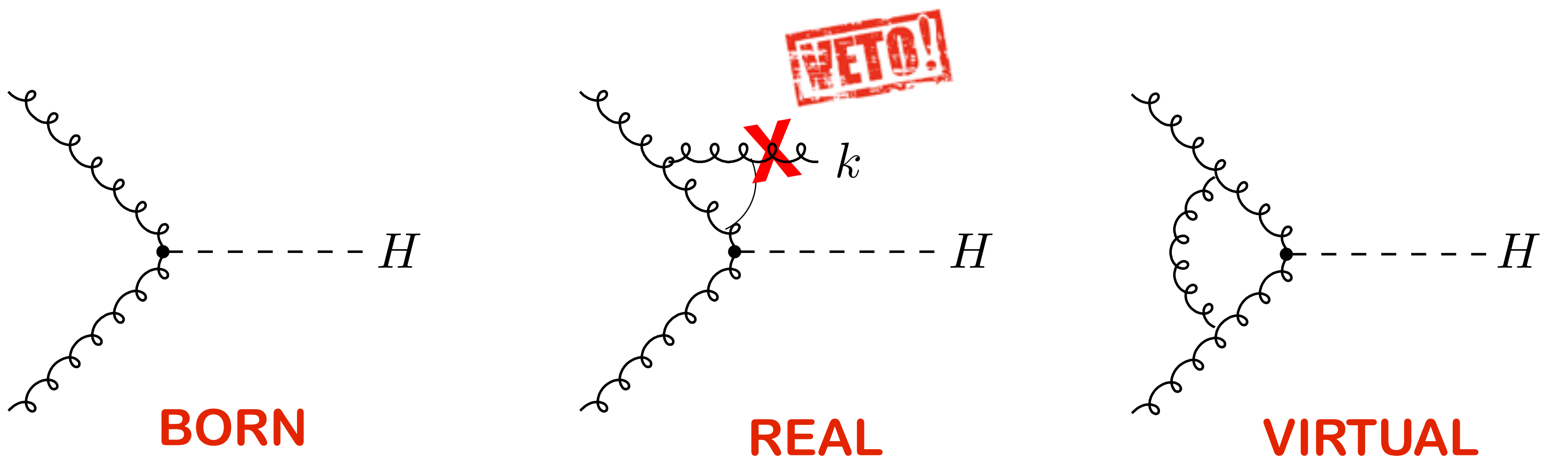
- The zero-jet cross section is characterised by two scales, the Higgs mass m_H and the jet resolution $p_{\mathrm{t,veto}}$



- In QCD, large logarithms such as $\ln(m_H/p_{\mathrm{t,veto}})$ appear whenever the phase space for the emission of soft and/or collinear gluons is restricted

One gluon emission

- Example: veto one soft ($E \ll m_H$) and collinear ($\theta \ll 1$) gluon k



soft

collinear

$$\sigma_0 \left[1 \quad + \quad C_A \frac{\alpha_s}{\pi} \int \frac{dE}{E} \frac{d\theta^2}{\theta^2} \Theta\left(p_{\mathrm{t,veto}} - E\theta\right) \quad - \quad C_A \frac{\alpha_s}{\pi} \int \frac{dE}{E} \frac{d\theta^2}{\theta^2} \right]$$

factorisation of soft radiation

$$\boxed{\sigma_{0-\mathrm{jet}} = \sigma_0 \left[1 - C_A \frac{\alpha_s}{\pi} \ln^2 \left(\frac{m_H}{p_{\mathrm{t,veto}}} \right) \right]}$$

All-order resummation

- The zero-jet cross section contains logarithmic contributions which can become large when $p_{\mathrm{t,veto}} \ll m_H$

$$\sigma_{0-\mathrm{jet}} \simeq \sigma_0 \left(\underset{\text{LO}}{1} - \underset{\text{NLO}}{2C_A \frac{\alpha_s(m_H)}{\pi} \ln^2 \frac{m_H}{p_{\mathrm{t,veto}}}} + \ldots \right)$$

breakdown of perturbation theory!

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with $L = \ln(m_H/p_{\mathrm{t,veto}})$

$$\sigma_{0-\mathrm{jet}} \sim \sigma_0 \exp\left[\underbrace{L g_1(\alpha_s L)}_{\mathrm{LL}} + \underbrace{g_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s g_3(\alpha_s L)}_{\mathrm{NNLL}} + \dots\right]$$

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$$\sigma_{0-\mathrm{jet}} \sim \sigma_0 \, e^{\overbrace{Lg_1(\alpha_s L)}^{\mathrm{LL}}} \times \left(\begin{array}{c} 1 \quad + \quad \alpha_s \quad + \dots \\ \underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} + \dots \end{array} \right)$$

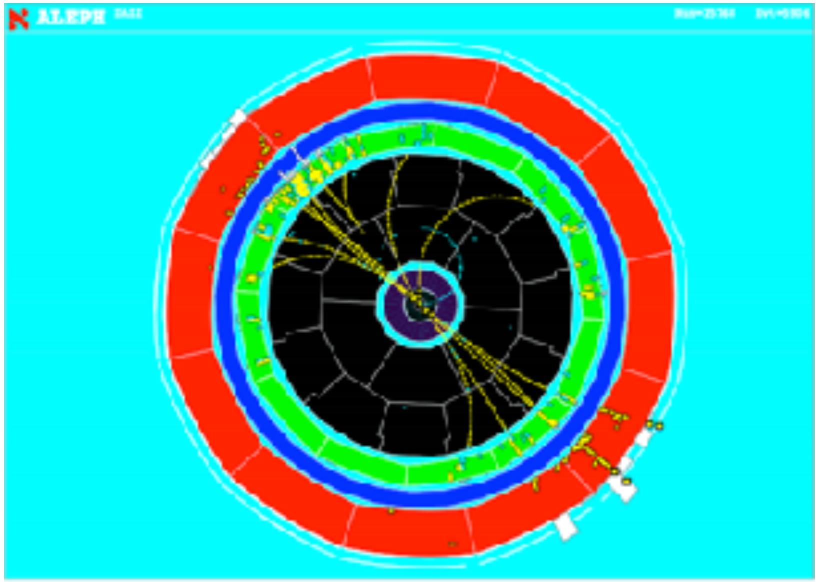
- To achieve NLL accuracy we have to consider
  - Double logarithms $\alpha_s L^2$: they come from soft *and* collinear contributions, and have to exponentiate
  - Single logarithms $\alpha_s L$: they come from soft and/or collinear contributions, and have to factorise from double logarithms

Final-state observables

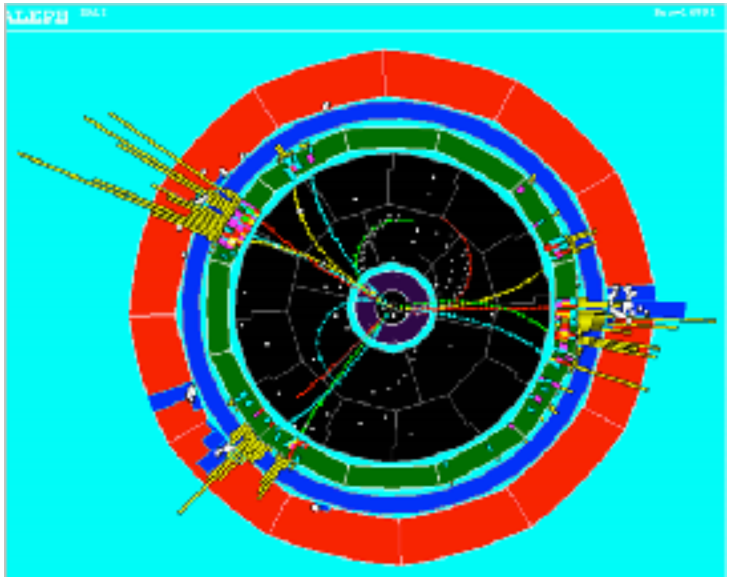
- We consider a generic final-state observable, a function $V(p_1, \ldots, p_n)$ of all final-state momenta $p_1, \ldots, p_n$
- Examples: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \text{hadrons}$

$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \mathrm{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

Pencil-like events $T \lesssim 1$

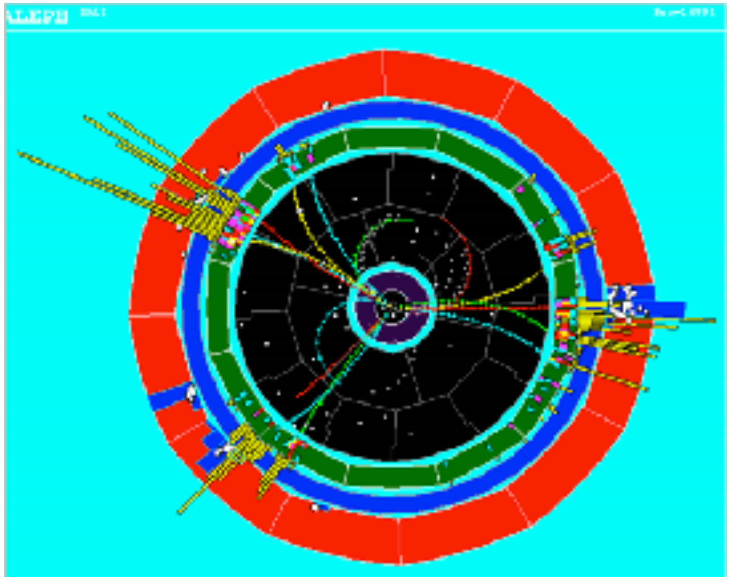


Planar events $T \gtrsim 2/3$

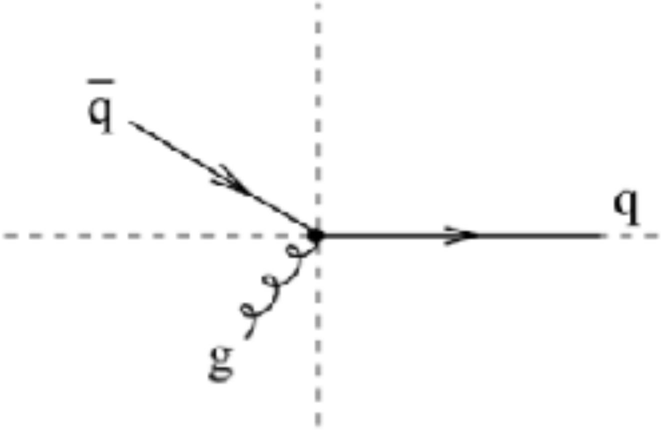


Collinear and infrared safety

- All final-state observables we consider are infrared and collinear (IRC) safe, so we can safely compute their distributions using the quark-gluon language of perturbative QCD



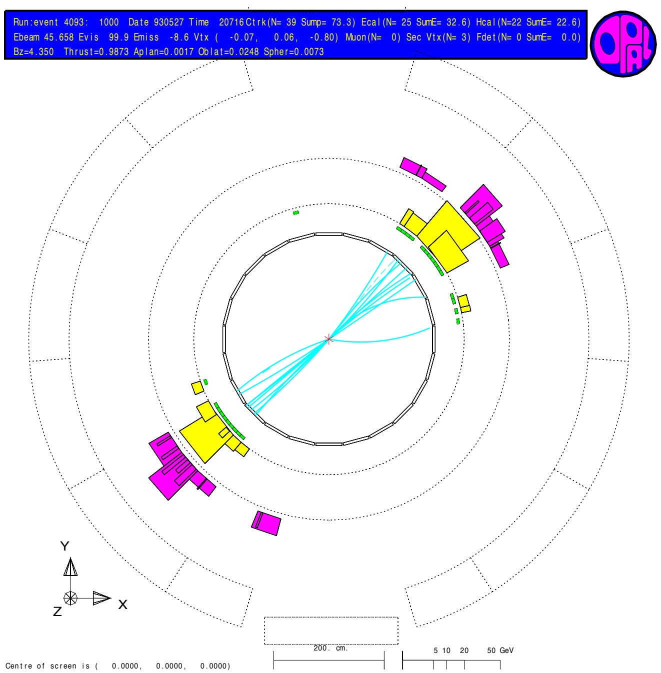
$\Rightarrow$



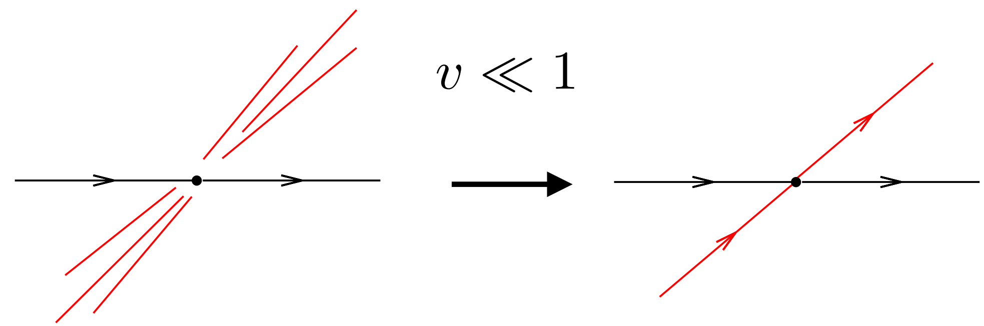
- Example: jets obtained from parton momenta are close to those obtained from hadron momenta if they do not change after
  - the addition of any number of soft patrons
  - any number of collinear splittings

Departure from the Born limit

- Final-state observables have the property that, for configurations close to the Born limit (e.g. a back-to-back $q\bar{q}$ pair), their value is close to zero
- Example: in two-jet events, one minus the thrust is the sum of the invariant masses of the two jets, which vanishes in the Born limit



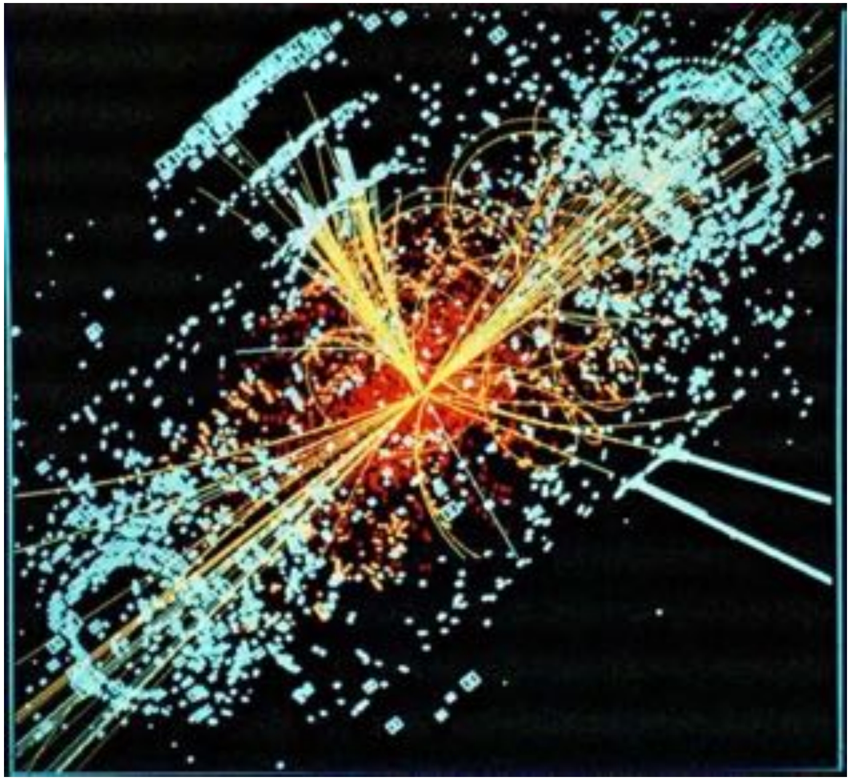
$$\Sigma(v) = \mathrm{Prob}[V(p_1, \ldots, p_n) < v]$$

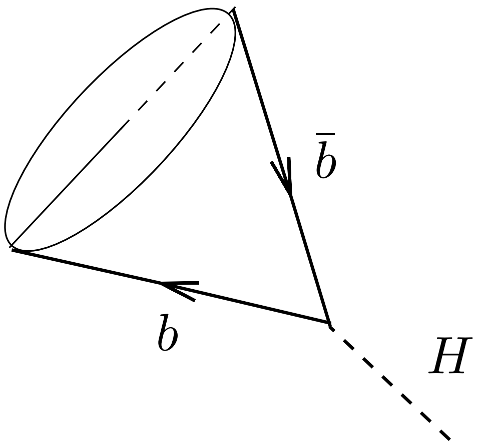


- To quantify the departure from the Born limit, we consider $\Sigma(v)$, the fraction of events such that $V(p_1, \ldots, p_n) < v$

Boosted object searches

- At the LHC it is possible to look for a boosted Higgs decaying into a $b\bar{b}$ pair

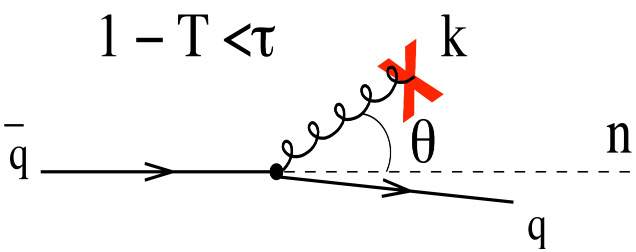




- The decay products of the Higgs tend to fall into the same jet $\Rightarrow$ consider the invariant mass of a fat jet and look for a peak for $m_{\rm jet} \sim m_H$
- If $p_{\rm t,jet} \sim 1\,\mathrm{TeV}$ we have a two-scale problem because $m_{\rm jet} \ll p_{\rm t,jet}$

The Lund plane

- For resummations, it is very useful to visualise emissions as points in the Lund plane



Sudakov decomposition

$$P_1 = \frac{Q}{2}(1, \vec{n}) \qquad P_2 = \frac{Q}{2}(1, -\vec{n})$$

$$k = z^{(1)} P_1 + z^{(2)} P_2 + k_t$$

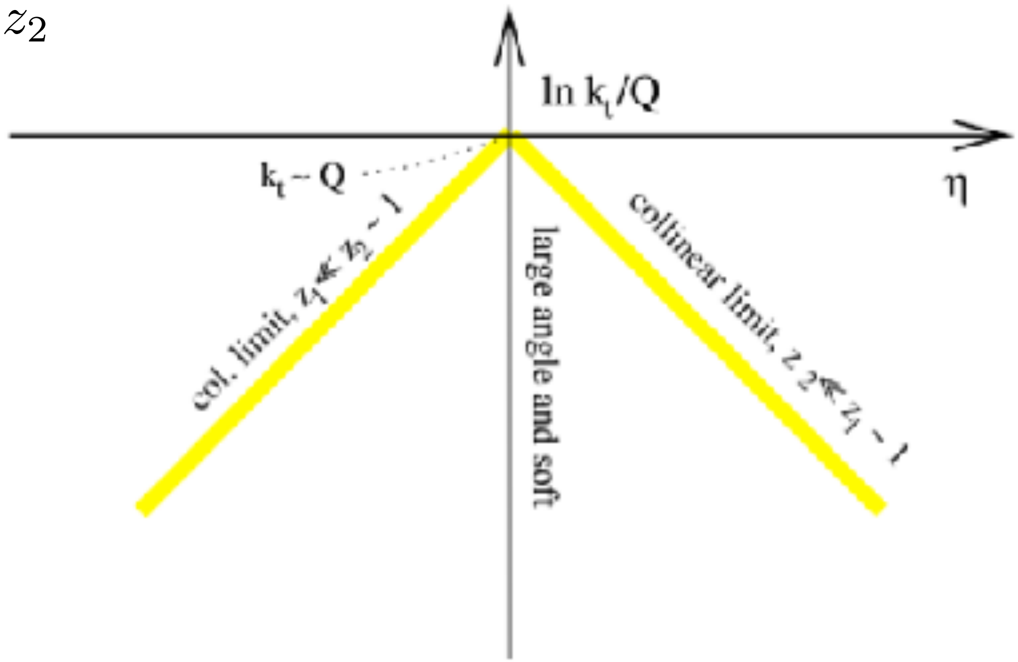
$$\eta = \frac{1}{2} \ln\left(\frac{z^{(1)}}{z^{(2)}}\right) \simeq \ln\frac{1}{\theta} \qquad z_1 \gg z_2$$

collinear limit

$$z_1, z_2 < 1 \Rightarrow |\eta| < \ln\left(\frac{Q}{k_t}\right)$$

soft-collinear matrix element

$$[dk] M^2(k) \simeq 2C_F \frac{\alpha_s}{\pi} \frac{dk_t}{k_t} d\eta \frac{d\phi}{2\pi}$$



The thrust in the Lund plane

- Behaviour of the thrust in the soft-collinear limit

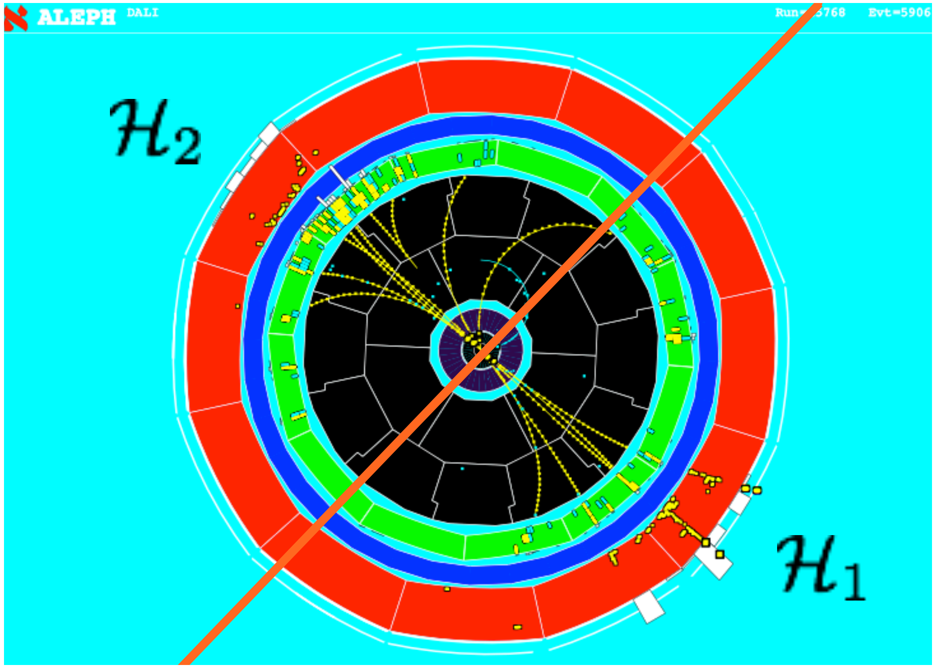
recoiling $q\bar{q}$ pair

$$1 - T(\{\tilde{p}\}, k_1, \ldots, k_n) \simeq \sum_i \frac{k_{ti}}{Q} e^{-|\eta_i|} + \sum_{\ell=1,2} \frac{1}{Q^2} \frac{\left|\sum_{i\in\mathcal{H}_\ell} \vec{k}_{ti}\right|^2}{1 - \sum_{i\in\mathcal{H}_\ell} z_i^{(\ell_i)}}$$

- Soft and collinear

- Soft and large angle

- Hard and collinear



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- Soft and collinear

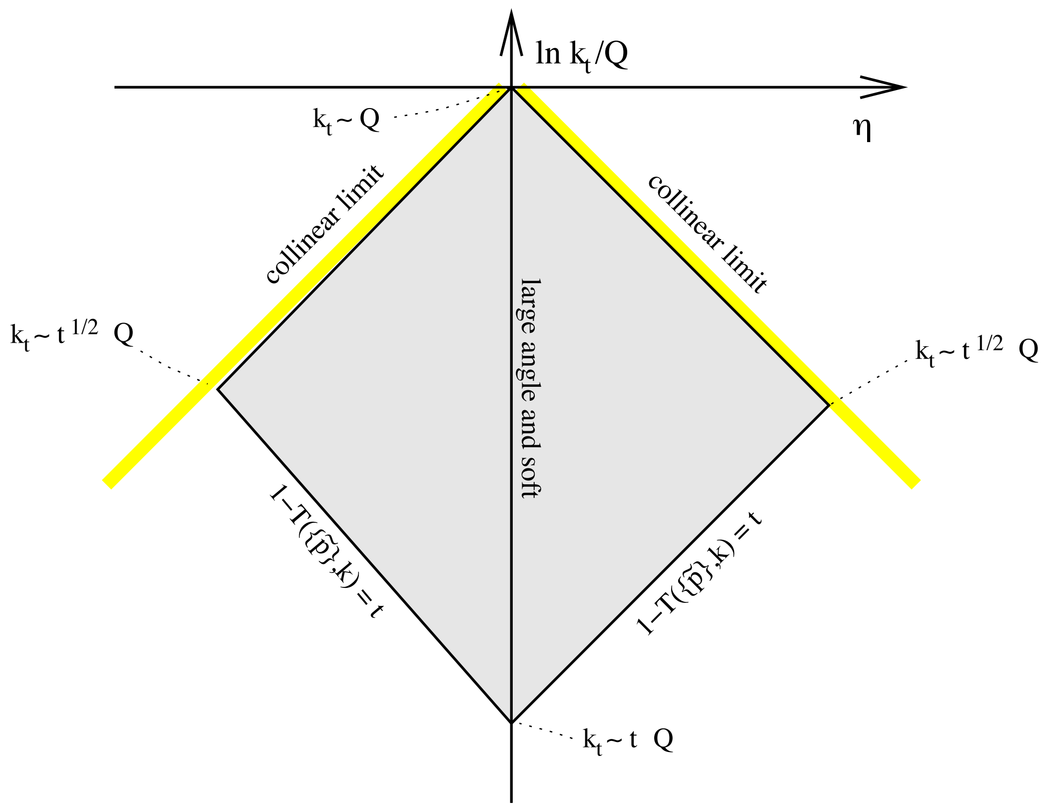
$$1 - T(\{\tilde{p}\}, k) \simeq \frac{k_t}{Q} e^{-|\eta|}$$

- Soft and large angle

$$1 - T(\{\tilde{p}\}, k) \sim k_t$$

- Hard and collinear

$$1 - T(\{\tilde{p}\}, k) \sim k_t^2$$



An IRC observable in the Lund plane

- For a single soft/collinear emission, the behaviour of an IRC safe observable is as follows

- Soft and collinear to leg $\ell = 1, 2$

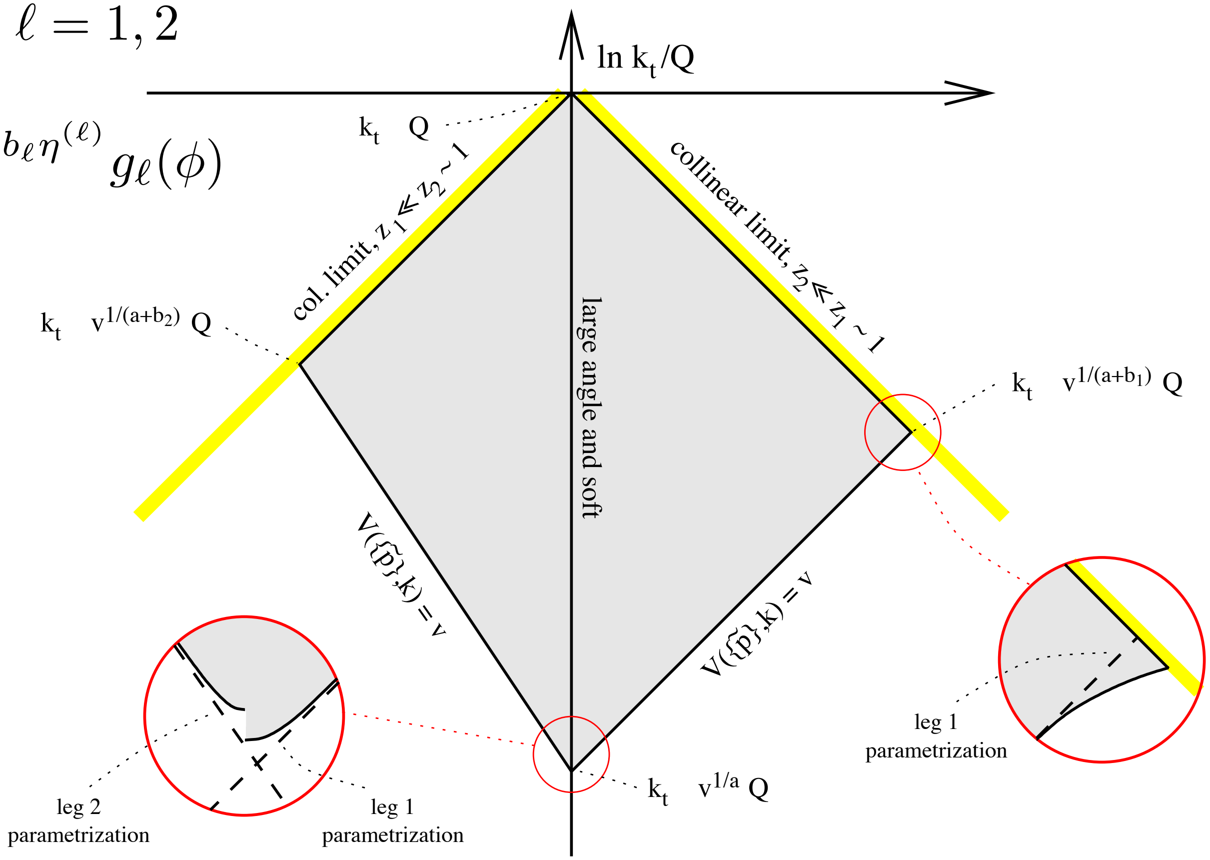
$$V(\{\tilde{p}\}, k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi)$$

- Soft and large angle

$$V(\{\tilde{p}\}, k) \sim k_t^a$$

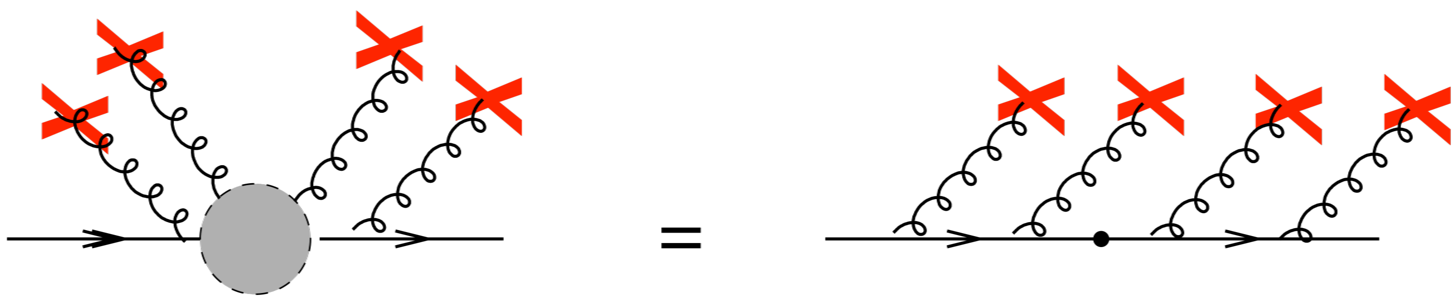
- Hard and collinear

$$V(\{\tilde{p}\}, k) \sim k_t^{a+b_\ell}$$



Multiple soft-collinear emissions

- We first consider an ensemble of soft-collinear emissions widely separated in angle (rapidity)
- Due to QCD coherence, the multi-gluon matrix element factorises into the product of single-emission matrix elements



- Contribution of multiple soft-collinear emissions to $\Sigma(v)$

$$\Sigma(v) = e^{-\int [dk] M^2(k)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_i [dk_i] M^2(k_i)\, \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_n))$$

virtual corrections, ensure that the inclusive sum over all emissions gives one

Leading logarithmic resummation

- Strategy: split the exponent in two parts

$$\int [dk] M^2(k) = \int_v [dk] M^2(k) + \int^v [dk] M^2(k) \qquad \int_v [dk] M^2(k) \equiv R(v)$$

$$\Sigma(v) = e^{-R(v)} \left\{ e^{-\int^v [dk] M^2(k)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_i [dk_i] M^2(k_i)\, \Theta(v - V(\{\tilde{p}\}, k_1, \dots, k_n)) \right\}$$

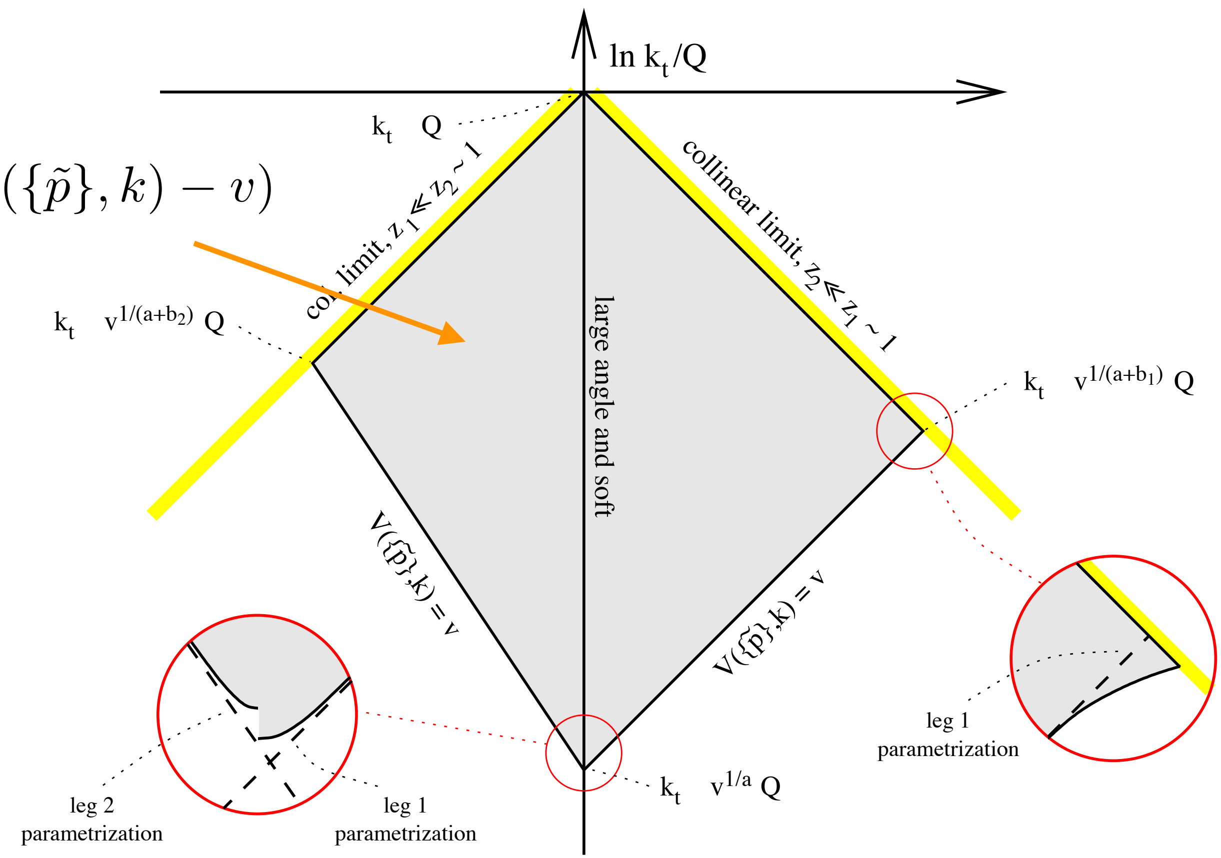
Sudakov form factor

multiple-emission correction

Leading logarithmic resummation

- The Sudakov exponent, a.k.a. as radiator, is just the area of the shaded region in the Lund plane

$$R(v) = \int [dk] M^2(k)\, \Theta(V(\{\tilde{p}\},k) - v)$$



- Since it is an area in the Lund plane, its contribution is double logarithmic

Exponentiation of double logarithms

- Double logarithms exponentiate if the contribution of multiple emissions is single-logarithmic

$$\mathcal{F}(v) = e^{-\int_{\epsilon v}^{v}[dk]M^2(k)} \sum_{n=0}^{\infty}\frac{1}{n!}\int_{\epsilon v}\prod_i [dk_i]M^2(k_i)\,\Theta(v - V(\{\tilde{p}\},k_1,\ldots,k_n)) \ + \ldots.$$

cutoff

- Phase-space massage

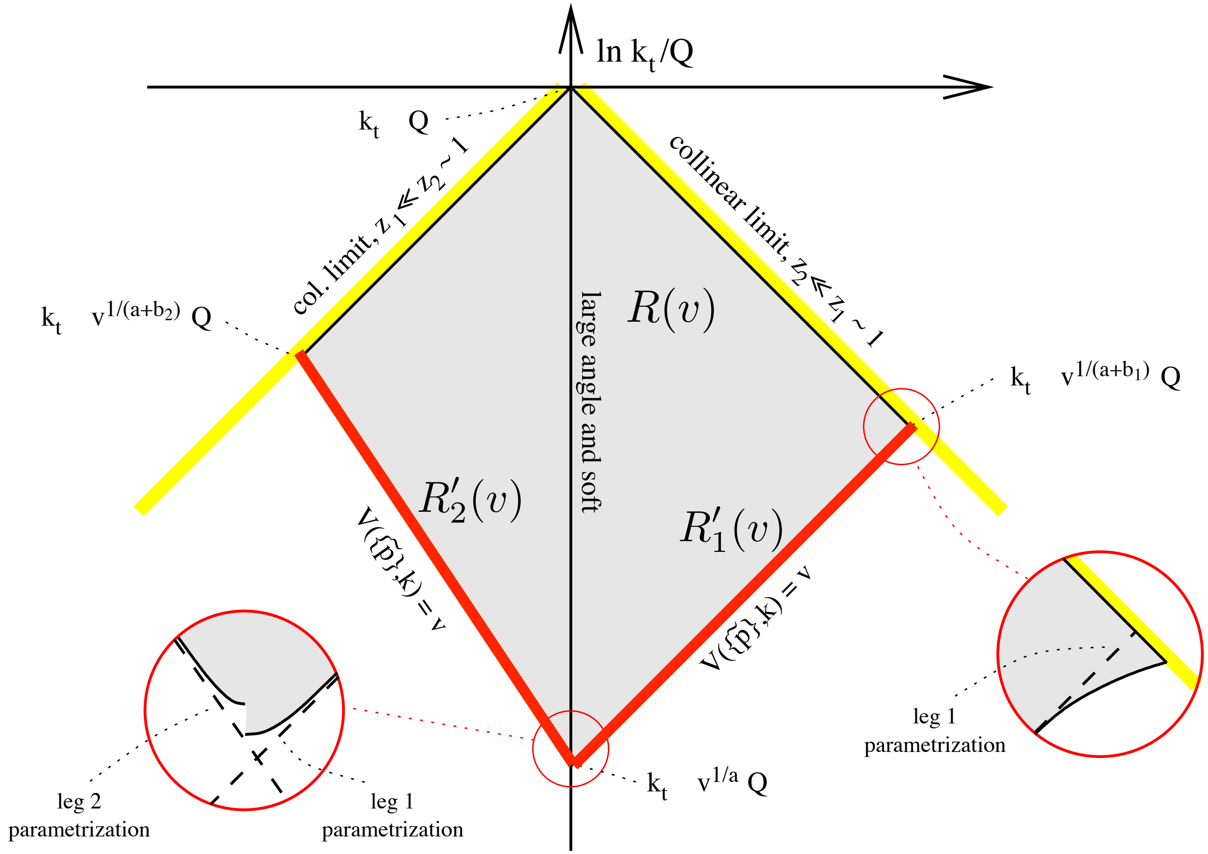
$$V(\{\tilde{p}\},k) = \zeta v \qquad \eta^{(\ell)} = \xi^{(\ell)}\,\eta^{(\ell)}_{\max} \qquad \eta^{(\ell)}_{\max} \sim \ln\frac{1}{v}$$

$$[dk]M^2(k) \simeq \sum_\ell \underbrace{R'_\ell(v)}_{\sim\alpha_s L}\frac{d\zeta}{\zeta}d\xi^{(\ell)}\frac{d\phi}{2\pi} \qquad R' = \sum_\ell R'_\ell = -v\frac{dR}{dv}$$

$$\mathcal{F}(v) \simeq \epsilon^{R'}\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\left(\sum_{\ell_i}R'_{\ell_i}\int_\epsilon^\infty\frac{d\zeta_i}{\zeta_i}\int_0^1 d\xi_i^{(\ell_i)}\int_0^{2\pi}\frac{d\phi_i}{2\pi}\right)\Theta\left(1 - \frac{V(\{\tilde{p}\},k_1,\ldots k_n)}{v}\right)$$

Exponentiation of double logarithms

- The logarithmic derivative of the radiator is the boundary of the shaded region in the Lund plane



- Since it is a line in the Lund plane, its contribution is single logarithmic

Perfectly exponentiating observable

- Consider an observable that takes contribution only from the emission for which $V(\{\tilde{p}\}, k)$ is the largest

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = \max_i V(\{\tilde{p}\}, k_i) \quad \text{, e.g.} \quad \frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \mathrm{jets}} \frac{p_{t,j}}{m_H}$$

$$\Theta\left(1 - \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) = \Theta\left(1 - \max_{i=1,\ldots,n} \underbrace{\frac{V(\{\tilde{p}\}, k_i)}{v}}_{=\zeta_i}\right) = \prod_{i=1}^{n} \Theta(1 - \zeta_i)$$

$$\mathcal{F}(v) \simeq \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left(R' \int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i} \Theta\left(1 - \zeta_i\right)\right) = 1$$

- For such observables the cumulative distribution is a Sudakov form factor!

$$\Sigma(v) = e^{-R(v)}$$

Additive observable

- Consider an observable that is the sum of the contributions of individual emissions

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = \sum_i V(\{\tilde{p}\}, k_i) \text{ , e.g. } 1 - T(\{\tilde{p}\}, k_1, \ldots, k_n) \simeq \sum_i \frac{k_{ti}}{Q} e^{-|\eta_i|}$$

$$\Theta\left(1 - \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) = \Theta\left(1 - \sum_{i=1}^{n} \underbrace{\frac{V(\{\tilde{p}\}, k_i)}{v}}_{=\zeta_i}\right) = \Theta\left(1 - \sum_{i=1}^{n} \zeta_i\right)$$

$$\mathcal{F}(v) \simeq \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \left(R' \int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i}\right) \Theta\left(1 - \sum_{i=1}^{n} \zeta_i\right) = \frac{e^{-\gamma_E R'}}{\Gamma(1 + R')}$$

- The crucial property that ensures that $\mathcal{F}(v)$ does not give rise to double logarithms is the fact that $V(\{\tilde{p}\}, k_1, \ldots, k_n) \sim v$ whenever $V(\{\tilde{p}\}, k_i) \sim v$

Non-exponentiating double logarithms

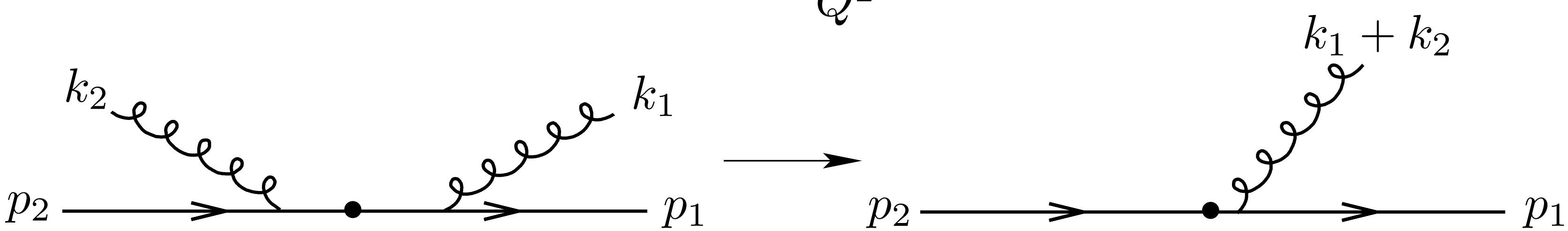
- In the case of the two-jet rate in the JADE algorithm, double logarithms do not exponentiate

Brown Stirling '90

$$\Sigma(y_{\mathrm{cut}}) = 1 - \frac{C_F\alpha_s}{\pi}\ln^2\left(\frac{1}{y_{\mathrm{cut}}}\right) + \frac{1}{2!}\times\frac{5}{6}\times\left(\frac{C_F\alpha_s}{\pi}\ln^2\left(\frac{1}{y_{\mathrm{cut}}}\right)\right)^2$$

- This is due to the peculiar way in which the JADE algorithm performs sequential recombinations

$$y_{ij} = \frac{(p_i+p_j)^2}{Q^2}$$



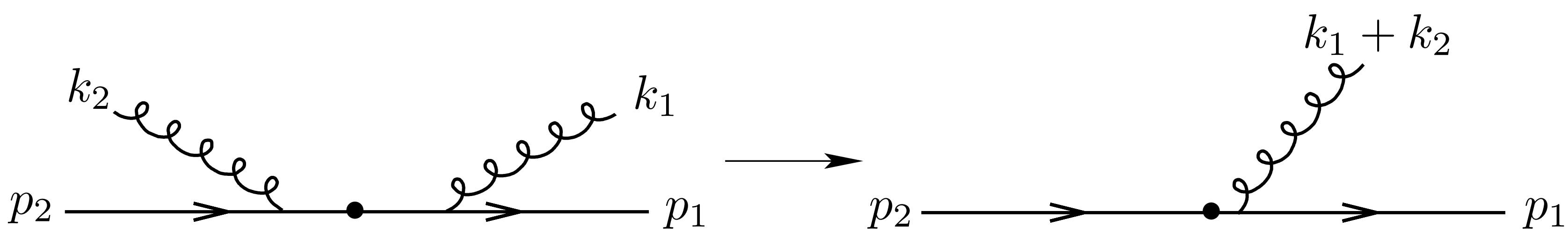
- The JADE algorithm is able to recombine together two soft emissions collinear to two different legs

Non-exponentiating double logarithms

- The way in which the JADE algorithm changes the scaling properties of the three-jet resolution

$$y_{k_1 p_1} = y_3(\{\tilde{p}\}, k_1) = \frac{(k_1 + p_1)^2}{Q^2} \simeq \frac{k_{t1}}{Q} e^{-\eta_1} = y_{\mathrm{cut}}$$

$$y_{k_2 p_2} = y_3(\{\tilde{p}\}, k_2) = \frac{(k_2 + p_2)^2}{Q^2} \simeq \frac{k_{t2}}{Q} e^{+\eta_2} = y_{\mathrm{cut}}$$



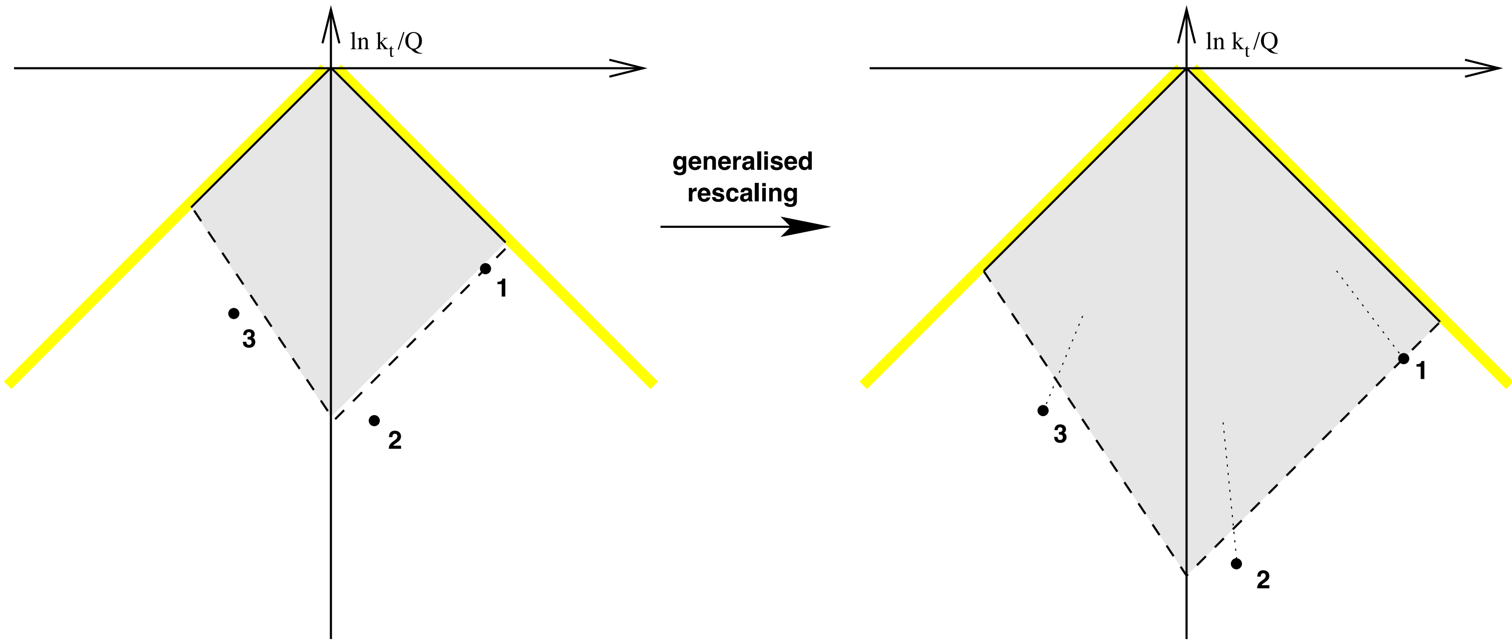
$$y_{k_1 k_2} = \frac{(k_1 + k_2)^2}{Q^2} \simeq y_{\mathrm{cut}}^{2-\xi_1-\xi_2} < y_{\mathrm{cut}} \quad \Leftrightarrow \quad \xi_1 + \xi_2 < 1$$

$$\frac{y_3(\{\tilde{p}\}, k_1, k_2)}{y_{\mathrm{cut}}} = y_{\mathrm{cut}}^{1-\xi_1-\xi_2} \Rightarrow \quad \text{depends on } y_{\mathrm{cut}} \Rightarrow \mathcal{F}(y_{\mathrm{cut}}) \text{ gives double logs}$$

Recursive IRC safety condition 1

- The requirement that the observable scales in the same way irrespectively of the number of emission is formalised as follows

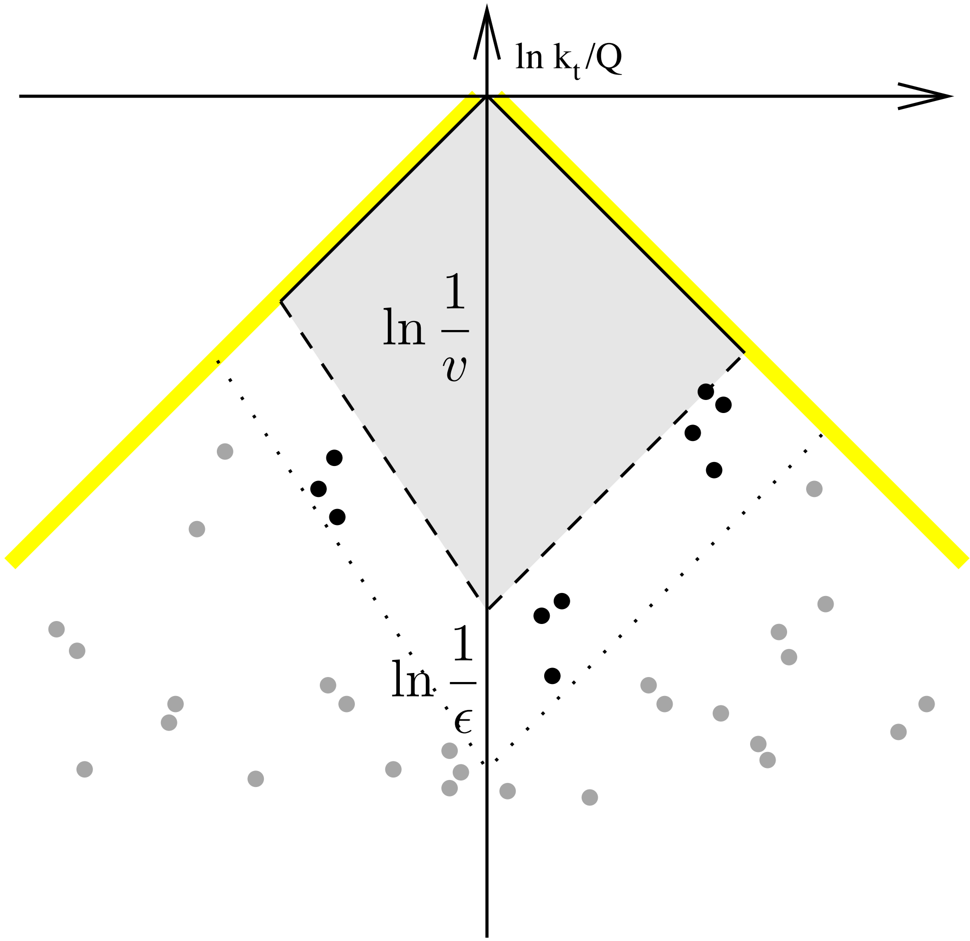
$$\lim_{v \to 0} \frac{V(\{\tilde{p}\}, k_1[v\zeta_1], \ldots, k_n[v\zeta_n])}{v} = \text{finite, and non-zero}$$



- This is the first of the requirements known as “recursive” IRC safety
- rIRC safe observables are the only ones that can be resummed so far

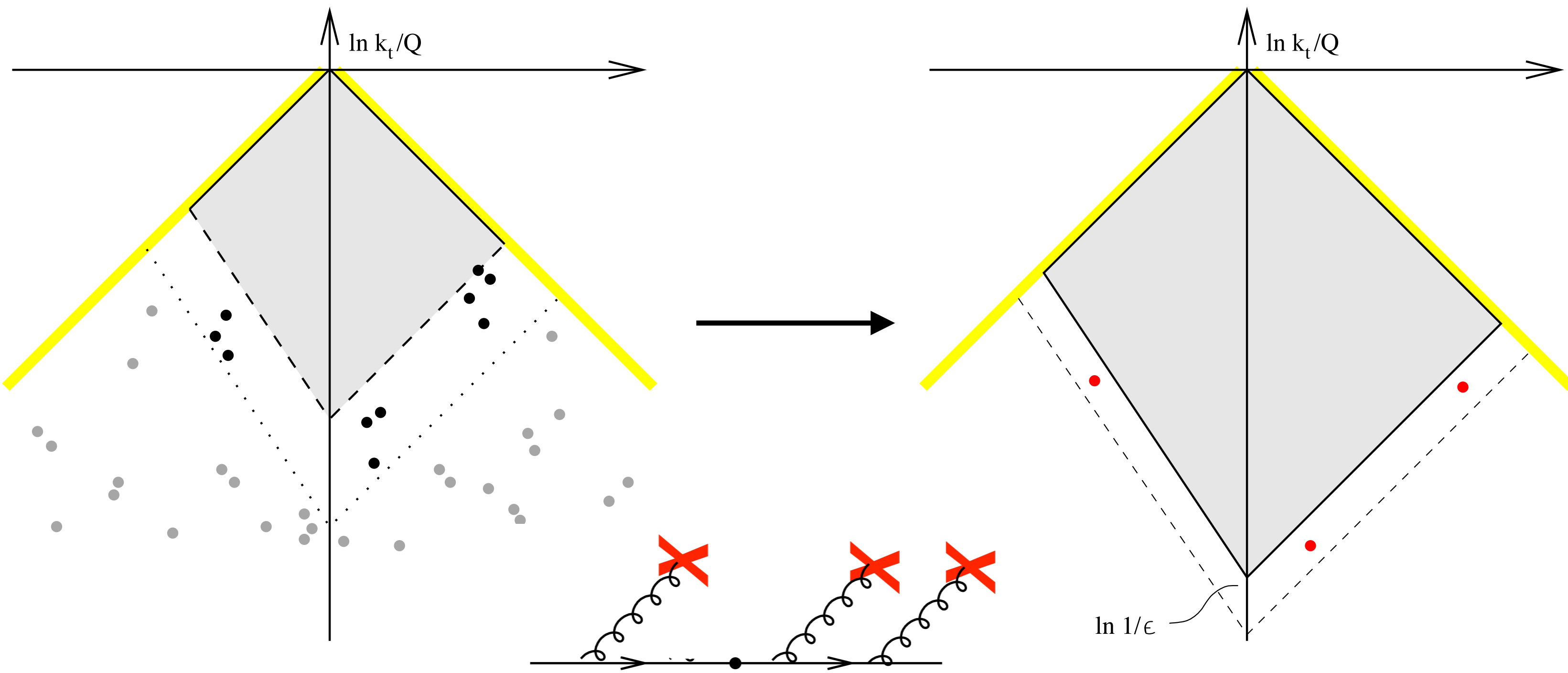
Recursive IRC safety condition 2a

- This condition ensures that all emissions with $V(\{\tilde{p}\}, k_i) < \epsilon v$ can be neglected, and furthermore $\epsilon \gg v$, independent of v



Recursive IRC safety condition 2b

- This condition ensures that the contribution of correlated gluon emissions, hard collinear and soft large-angle emissions is NNLL

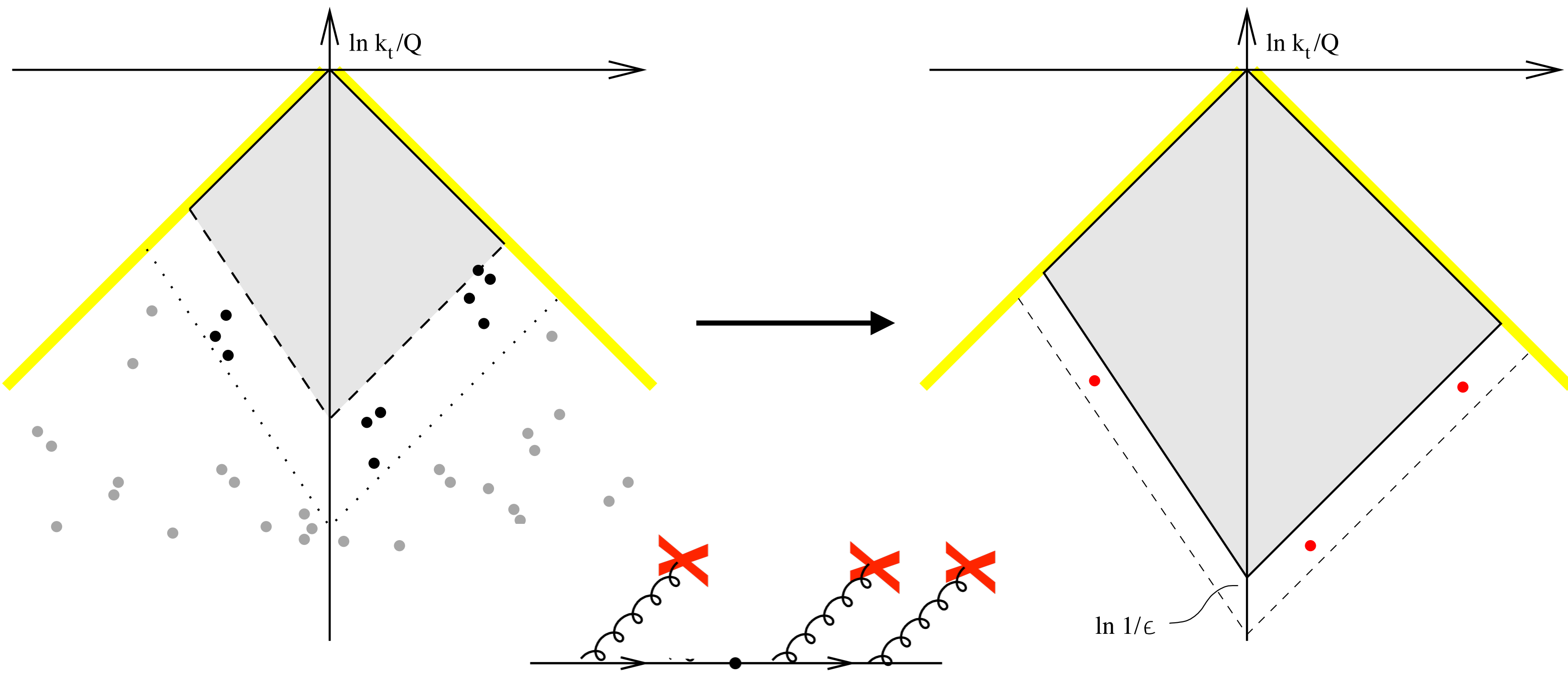


- At NLL accuracy, relevant emissions are soft and collinear, widely separated in angle, and in a strip of size $\ln v \times \ln \epsilon$
- This is a line in the Lund plane, hence a single logarithmic contribution

NLL resummation of rIRC safe observables

- NLL resummation of rIRC safe observables can be performed with a universal master formula

Banfi Salam Zanderighi '05

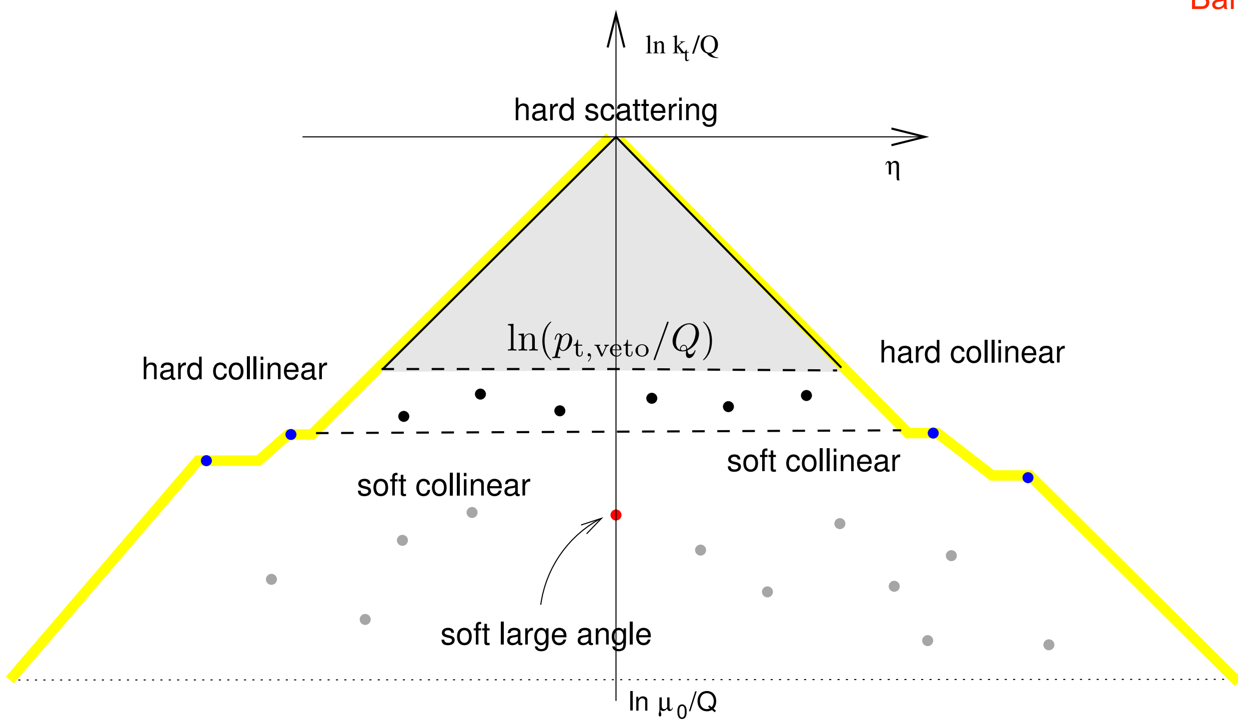


$$\Sigma(v) = e^{-R(v)} \underbrace{\left\{ \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \left(\sum_{\ell_i} R'_{\ell_i} \int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i} \int_0^1 d\xi_i^{(\ell_i)} \int_0^{2\pi} \frac{d\phi}{2\pi} \right) \Theta\left(1 - \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) \right\}}_{\text{single-logarithmic correction } \mathcal{F}_{\rm NLL}(R')}$$

Higgs plus zero jets at NLL

- In the presence of initial state radiation, the zero-jet cross section inclusive with respect to hard-collinear emission up to the scale $p_{\mathrm{t,veto}}$

Banfi Salam Zanderighi '12



- No k_t-type algorithm can recombine gluons that are widely separated in angle $\Rightarrow$ perfectly exponentiating observable

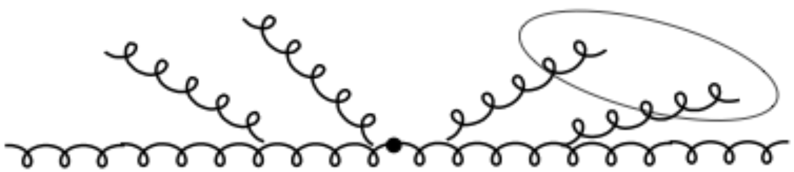
$$\sigma_{0-\mathrm{jet}} \simeq \mathcal{L}_{gg}(p_{\mathrm{t,veto}})\, e^{-R(p_{\mathrm{t,veto}})}$$

Higgs plus zero jets at NNLL

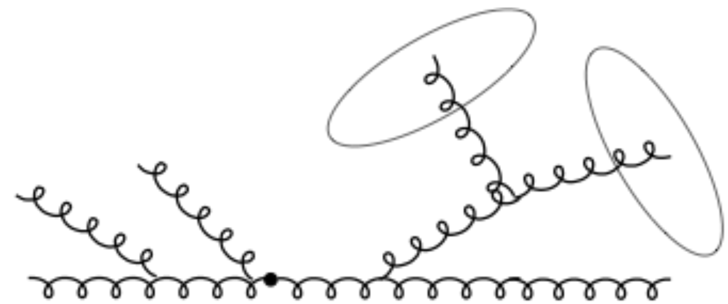
- Recombination effects start to matter at NNLL accuracy

Banfi Monni Salam Zanderighi '12

Two nearby gluons clustered in one jet



One gluon giving two jets



$$\sigma_{0-\mathrm{jet}} \simeq \mathcal{L}_{gg}(p_{\mathrm{t,veto}}) \left(1 + \underbrace{\alpha_s(p_{\mathrm{t,veto}})\, R'(p_{\mathrm{t,veto}})\, f(R)}_{\mathrm{NNLL}} \right) e^{-R(p_{\mathrm{t,veto}})}$$

- The function $f(R) \sim \ln R$ due to a cut off collinear singularity in gluon splitting. Leading logarithm of the jet radius can be also resummed at all orders

Dasgupta Dreyer Salam Soyez '15

Resummation uncertainties

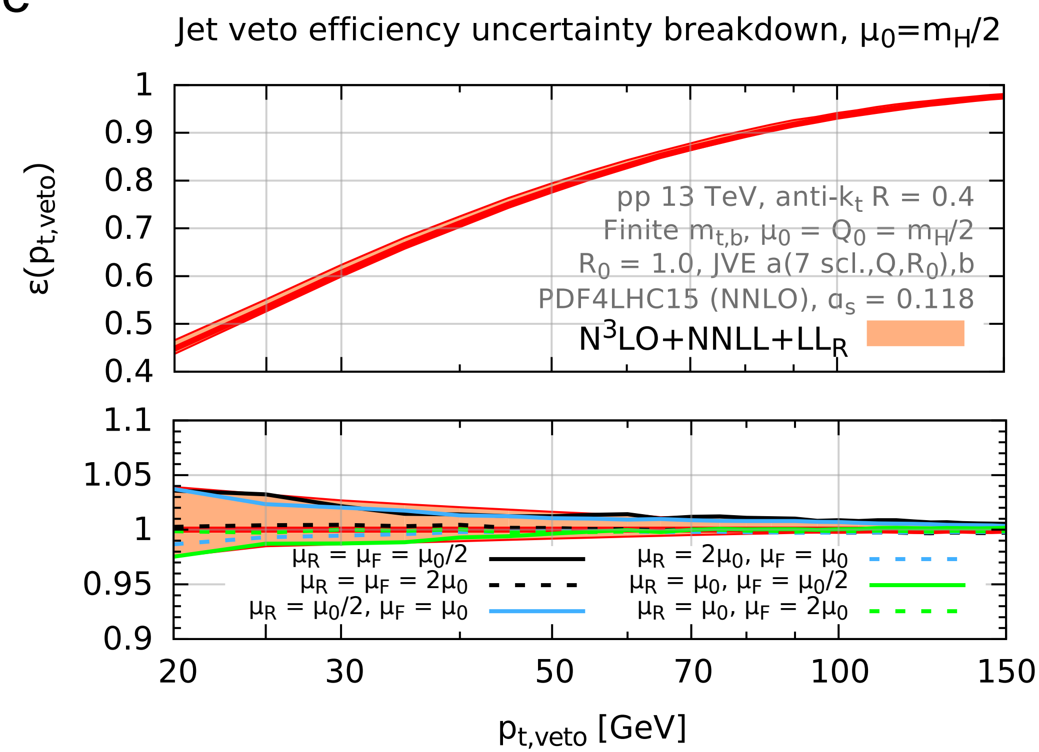
- Resummation has more handles to assess theoretical uncertainties

Banfi Caola Dreyer Monni Salam Zanderighi Dulat '16

- Variation of renormalisation and factorisation scales in the range

$$m_H/4 \leq \mu_R, \mu_F < m_H$$

$$1/2 \leq \mu_R/\mu_F < 2$$



Resummation uncertainties

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Banfi Caola Dreyer Monni Salam Zanderighi Dulat '16

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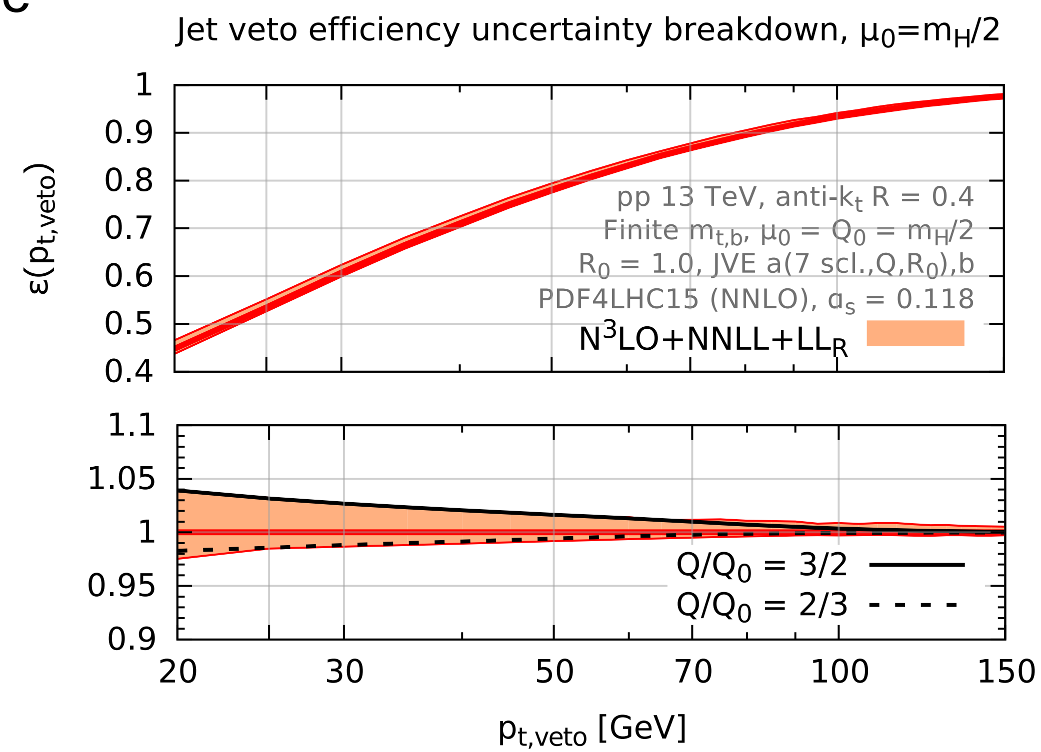
- Resummation scale: change the log to be resummed, to probe subleading logs

$$\ln(m_H/p_{\mathrm{t,veto}}) \to \ln(Q/p_{\mathrm{t,veto}})$$



Resummation uncertainties

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Banfi Caola Dreyer Monni Salam Zanderighi Dulat ‘16

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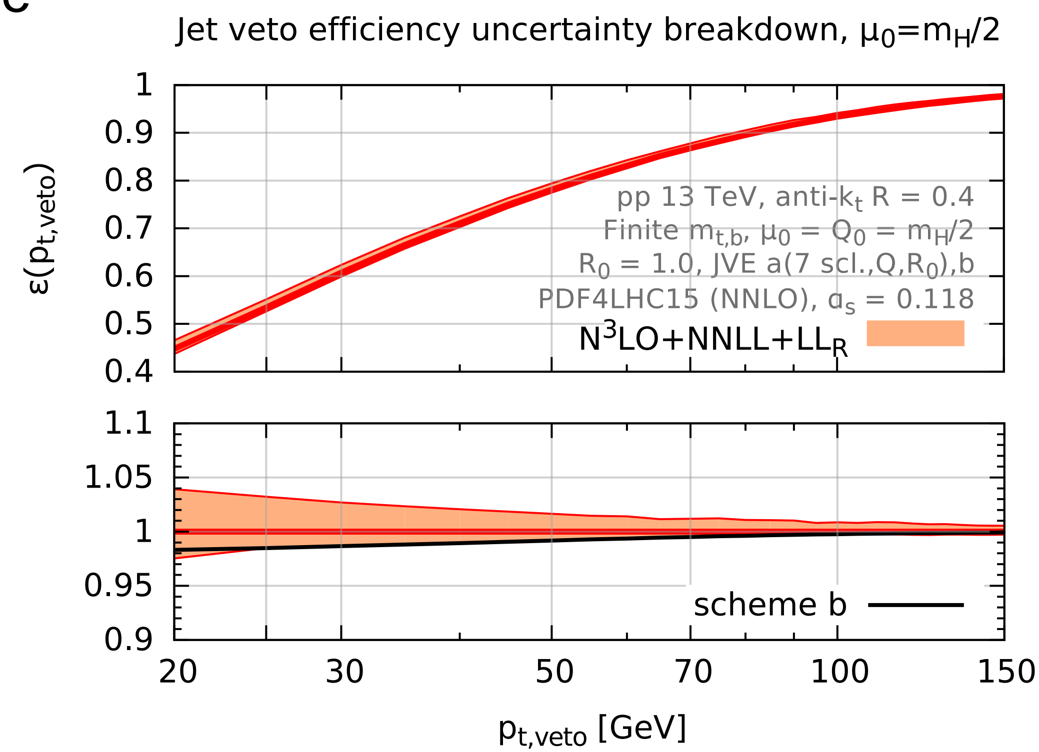
- Prescription to match to exact fixed-order



Resummation uncertainties

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Banfi Caola Dreyer Monni Salam Zanderighi Dulat '16

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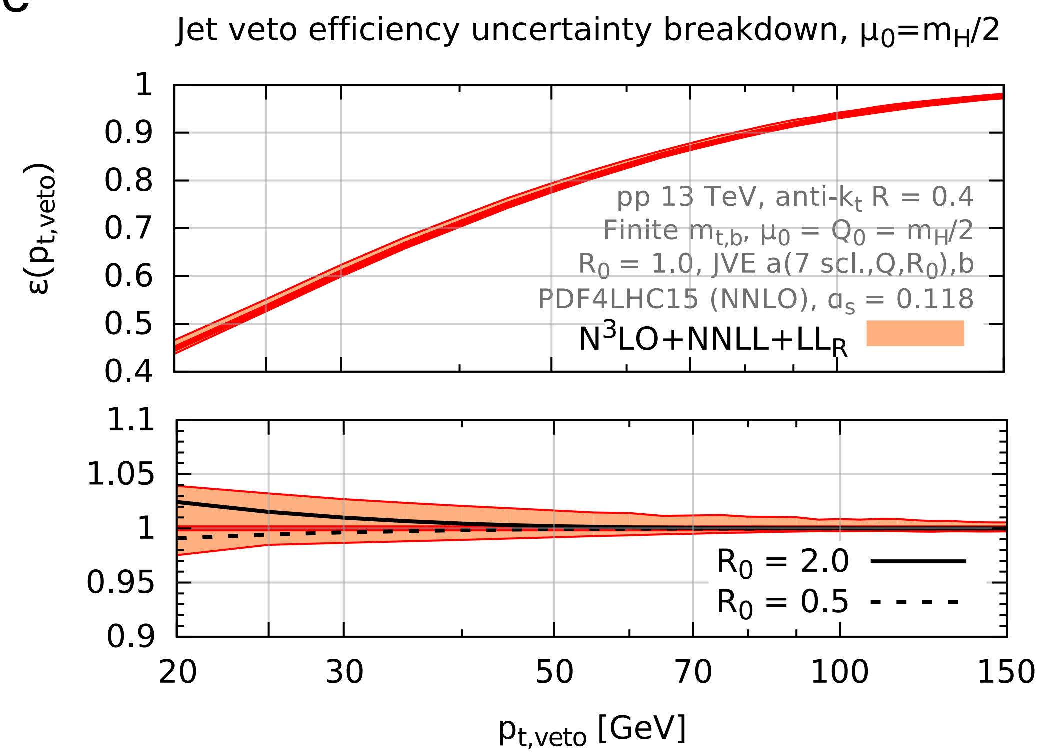
- Upper scale for small-R resummation

$$\ln(1/R) \to \ln(R_0/R)$$



Resummation uncertainties

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Banfi Caola Dreyer Monni Salam Zanderighi Dulat ‘16

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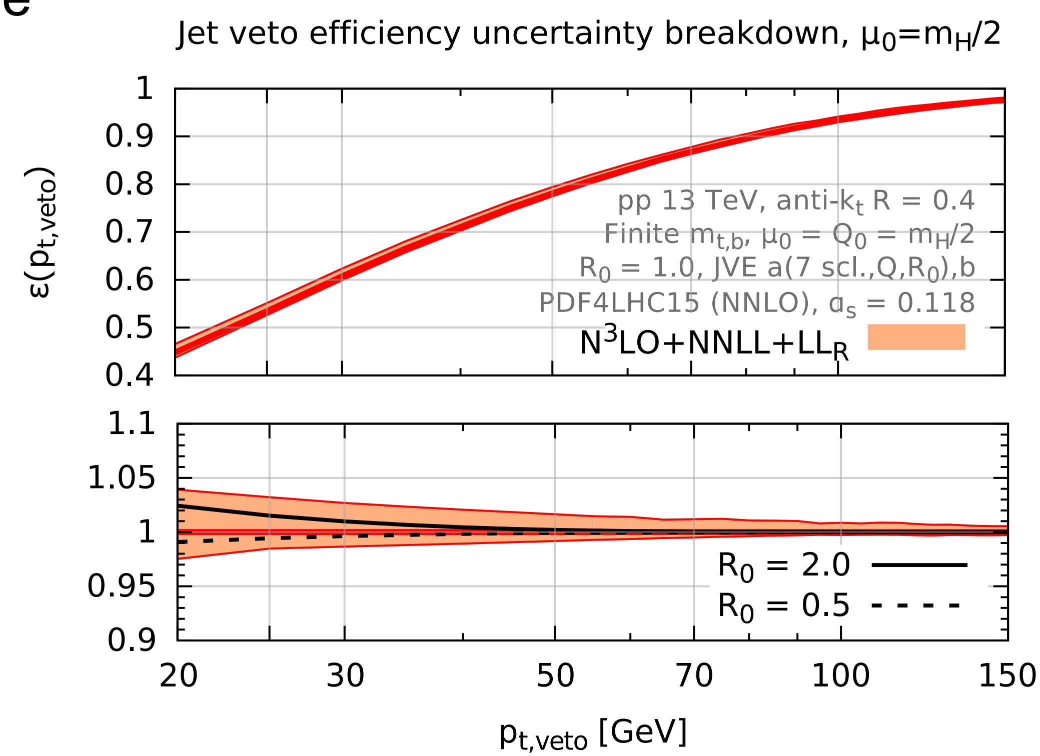


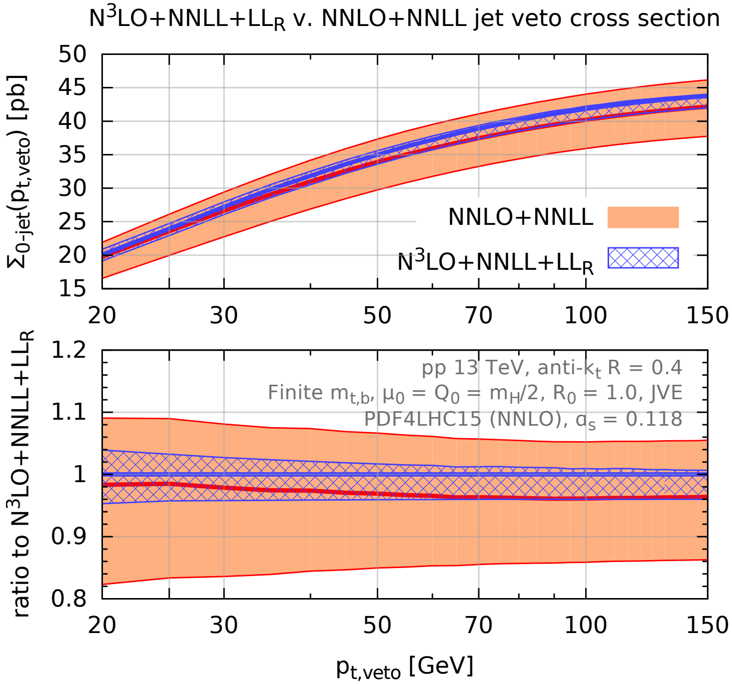
- Total uncertainty: envelope of all these curves

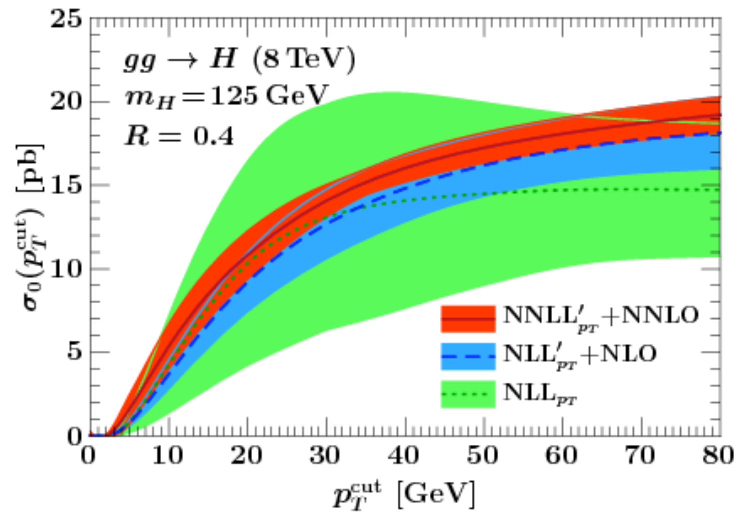
Resummation uncertainties

- In all resummed predictions for Higgs plus zero jets, theoretical uncertainties consistently reduce with increasing order

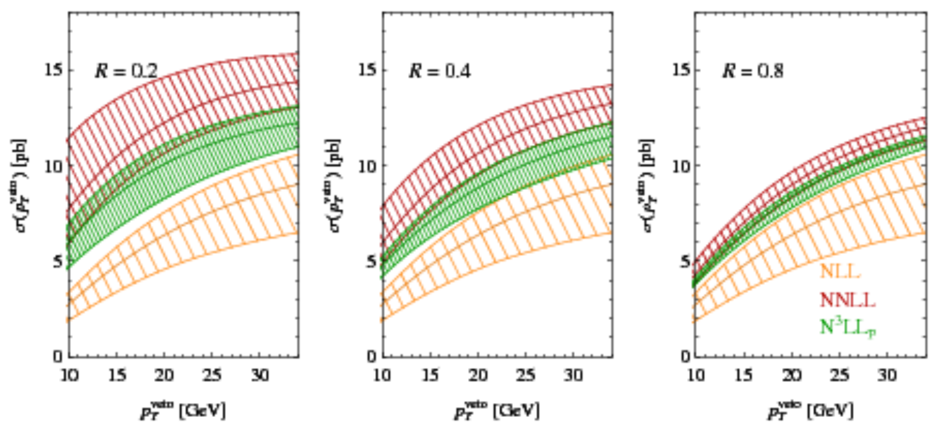
Stewart Tackmann Walsh Zuberi '14

Banfi Caola Dreyer Monni Salam Zanderighi Dulat '16





Becher Neubert Rothen '13



Learning outcomes

At the and of this lecture you should be able to

- give arguments for the choice of renormalisation and factorisation scales in fixed-order calculations
- understand the basic principles of final-state resummations
- provide strategies to estimate theory uncertainties in fixed-order and resummed calculations