# *Parton distribution functions*

**Sven-Olaf Moch**

*Universität Hamburg*

Terascale alliance *Monte Carlo school*, DESY Hamburg, Mar 17, 2017

# *Based on work done in collaboration with:*

- *Parton Distribution Functions, $\alpha_s$ and Heavy-Quark Masses for LHC Run II*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė arXiv:1701.05838

- *A Critical Appraisal and Evaluation of Modern PDFs*
  A. Accardi, S. Alekhin, J. Blümlein, M.V. Garzelli, K. Lipka, W. Melnitchouk, S. M., J.F. Owens, R. Plačakytė, E. Reya, N. Sato, A. Vogt and O. Zenaiev arXiv:1603.08906

- *Iso-spin asymmetry of quark distributions and implications for single top-quark production at the LHC*
  S. Alekhin, J. Blümlein, S. M. and R. Plačakytė arXiv:1508.07923

- *Determination of Strange Sea Quark Distributions from Fixed-target and Collider Data*
  S. Alekhin, J. Blümlein, L. Caminada, K. Lipka, K. Lohwasser, S. M., R. Petti, and R. Plačakytė arXiv:1404.6469

- Many more papers of ABM and friends . . .
  2008 - ...

# PDF landscape

- Significant number of active groups ABMP16, CJ15, CT14, HERAPDF2.0, JR14, MMHT14, NNPDF3.0
  - PDFs accurate to NNLO in QCD, except for CJ15 (NLO)
  - different choices of data sets
  - different fitting procedures ($\Delta\chi^2$ criterium)

| PDF sets | $\Delta\chi^2$ criterion | data sets used in analysis |
|---|---|---|
| ABMP16 arXiv:1701.05838 | 1 | incl. DIS, DIS charm, DY, $t\bar{t}$, single $t$ |
| CJ15 arXiv:1602.03154 | 1 | incl. DIS, DY (incl. $p\bar{p} \to W^{\pm}X$), $p\bar{p}$ jets, $\gamma$+jet |
| CT14 arXiv:1506.07443 | 100 | incl. DIS, DIS charm, DY, $p\bar{p}$ jets, $pp$ jets |
| HERAPDF2.0 arXiv:1506.06042 | 1 | incl. DIS, DIS charm, DIS jets |
| JR14 arXiv:1403.1852 | 1 | incl. DIS, DIS charm, DY, $p\bar{p}$ jets, DIS jets |
| MMHT14 arXiv:1510.02332 | $2.3 \ldots 42.3$ (dynamical) | incl. DIS, DIS charm, DY, $p\bar{p}$ jets, $pp$ jets, $t\bar{t}$ |
| NNPDF3.0 arXiv:1410.8849 | n.a. | incl. DIS, DIS charm, DY, $p\bar{p}$ jets, $pp$ jets, $t\bar{t}$, $W+$charm |

# *Recommendations (I)*

## *PDF4LHC recommendations for LHC Run II*

- Recommendations by CT14, MMHT14, NNPDF3.0
- PDFs averaged in set PDF4LHC15
  - to be used for Higgs cross sections, in seaches, for PDF uncertainties and for Monte Carlo simulations

**PDF4LHC recommendations for LHC Run II**

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**Abstract:**

# Recommendations (II)

## *Recommendations for PDF usage in LHC predictions*

- Shortcomings in PDF4LHC recommendations addressed
- Recommendations by ABMP16, CJ15, HERAPDF2.0, JR14

**Recommendations for PDF usage in LHC predictions**



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W. Melnitchouk[b], S. Moch[c], R. Plačakytė[f], J.F. Owens[g], E. Reya[h], N. Sato[b], A. Vogt[i]
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# *Recommendations (II)*

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- Shortcomings in PDF4LHC recommendations addressed
- Recommendations by ABMP16, CJ15, HERAPDF2.0, JR14

(i) Precision theory predictions

Recommendation: Use the individual PDF sets ABMP16, CJ15, CT14, HERAPDF2.0, JR14, MMHT14, NNPDF3.0 (or as many as possible), together with the respective uncertainties for the chosen PDF set, the strong coupling $\alpha_s(M_Z)$ and the heavy quark masses $m_c$, $m_b$ and $m_t$.

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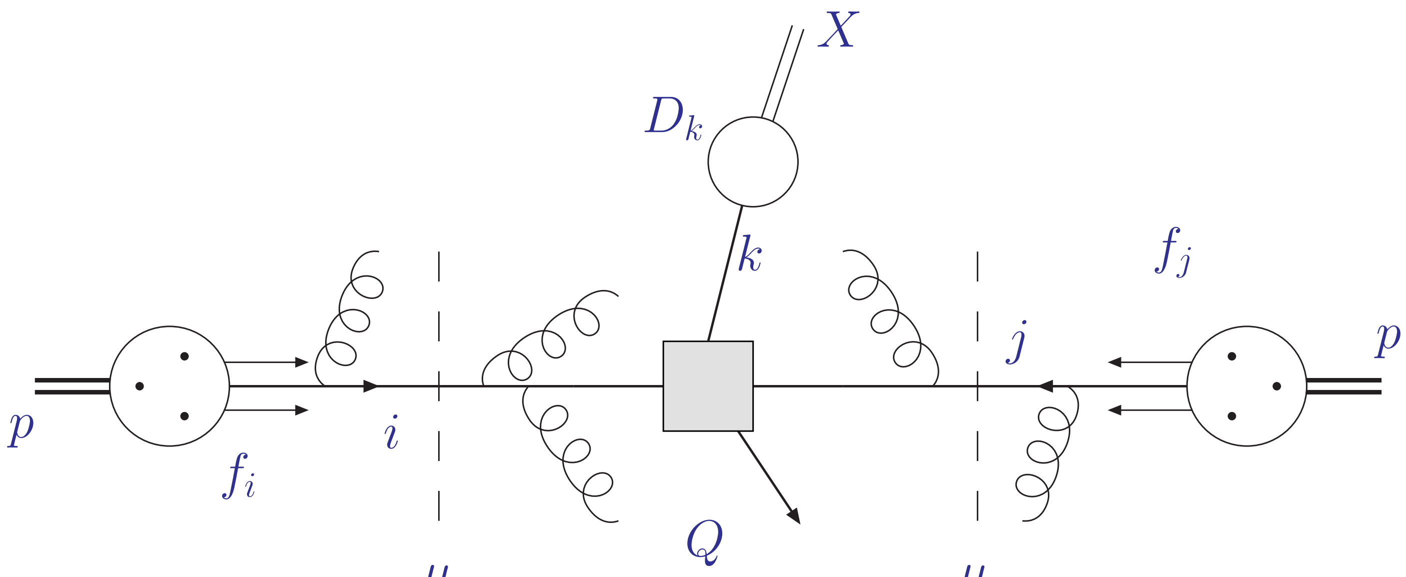
(ii) Other theory predictions

Recommendation: Use any one of the PDF sets listed in LHAPDF(v6).

# *QCD factorization*

# QCD factorization




$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Factorization at scale $\mu$
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types $i$, $j$ and hadronic final state $X$
- Parton distribution functions $f_i$, strong coupling $\alpha_s$, particle masses $m_X$
  - known from global fits to exp. data, lattice computations, . . .

# *Parton luminosity*

- Long distance dynamics due to proton structure

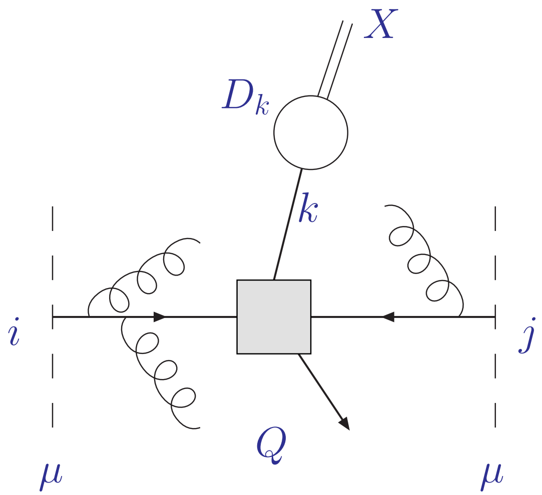

- Cross section depends on parton distributions $f_i$

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \Big[\ldots\Big]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage

# *Hard scattering cross section*

- Parton cross section $\hat{\sigma}_{ij\to k}$ calculable pertubatively in powers of $\alpha_s$
  - known to NLO, NNLO, ... ($\mathcal{O}(\mathrm{few}\%)$ theory uncertainty)

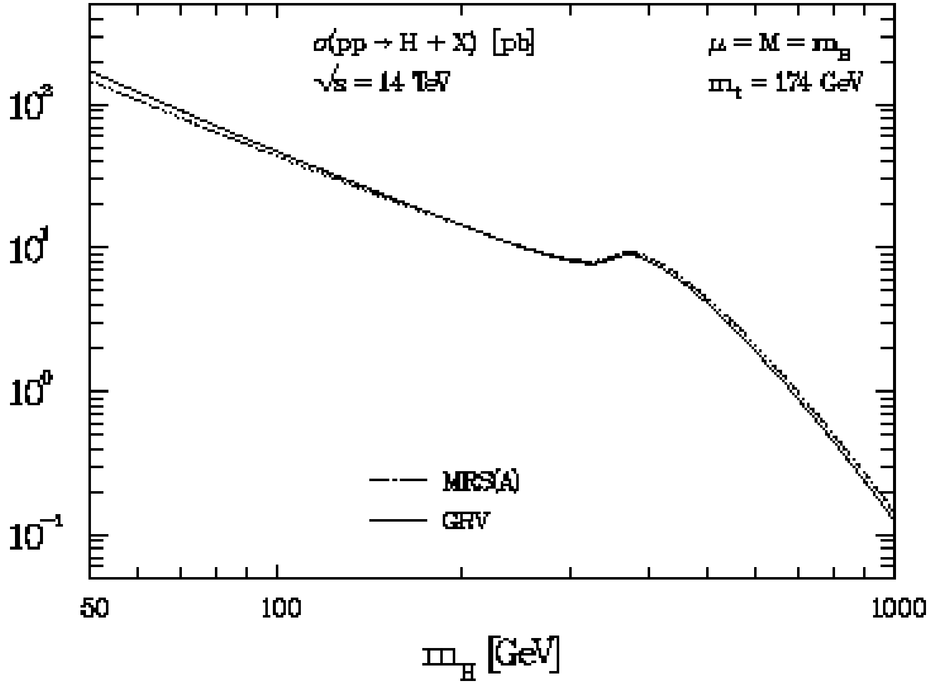



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50-100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10-30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ( $\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - $\mathrm{N}^3$LO (next-to-next-to-next-to-leading order)
  - ...

## *Higgs boson production*

# Higgs cross section (1995)

## NLO QCD corrections




MRS(A): Martin, Roberts and Stirling, Phys. Rev. D50 (1994) 6734

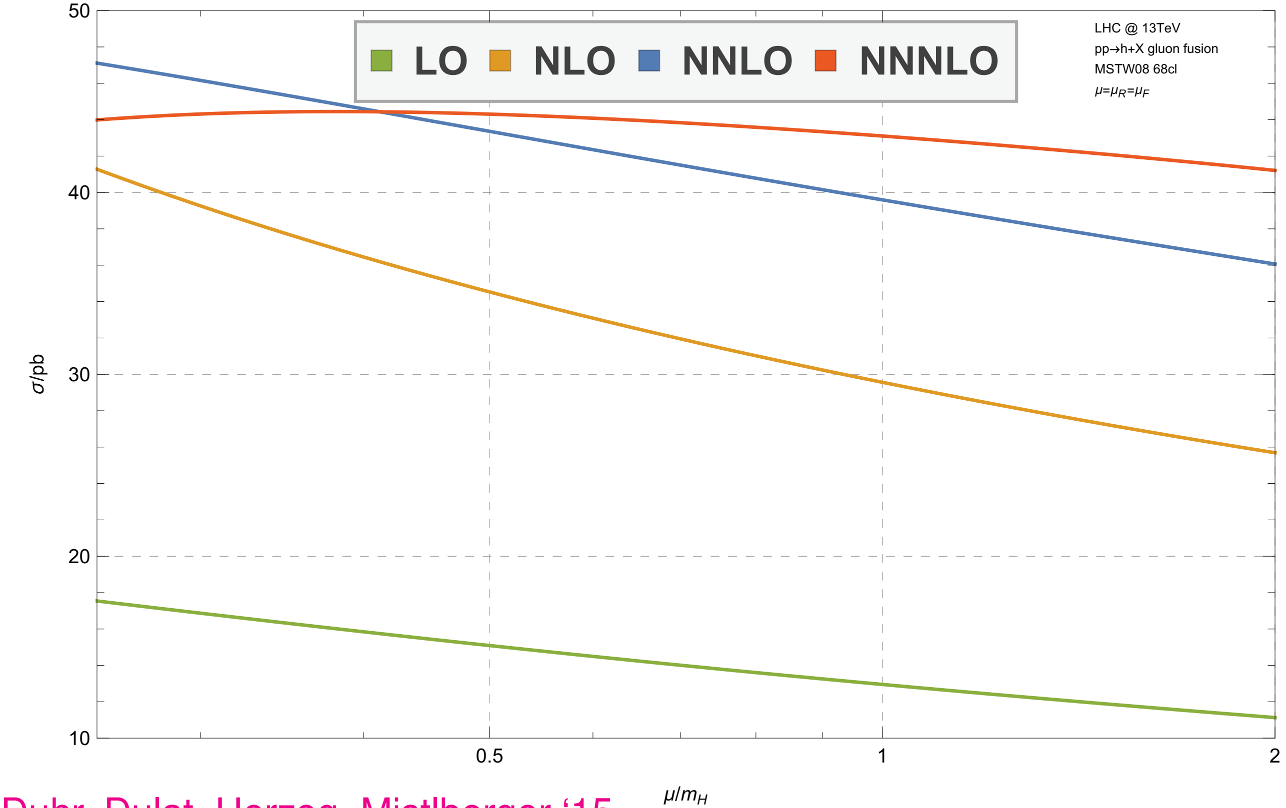
GRV: Glück, Reya and Vogt, Z. Phys. C53 (1992) 127

**One of the main uncertainties** *in the prediction of the Higgs production cross section is due to the* **gluon density***. [. . . ] Adopting a set of representative parton distributions [. . . ], we find a* **variation of about 7%** *between the maximum and minimum values of the cross section for Higgs masses above* $\sim$ *100 GeV.*

**Spira, Djouadi, Graudenz, Zerwas (1995)**
**hep-ph/9504378**

# *Higgs cross section (2017)*

## *Exact N$^3$LO QCD corrections*

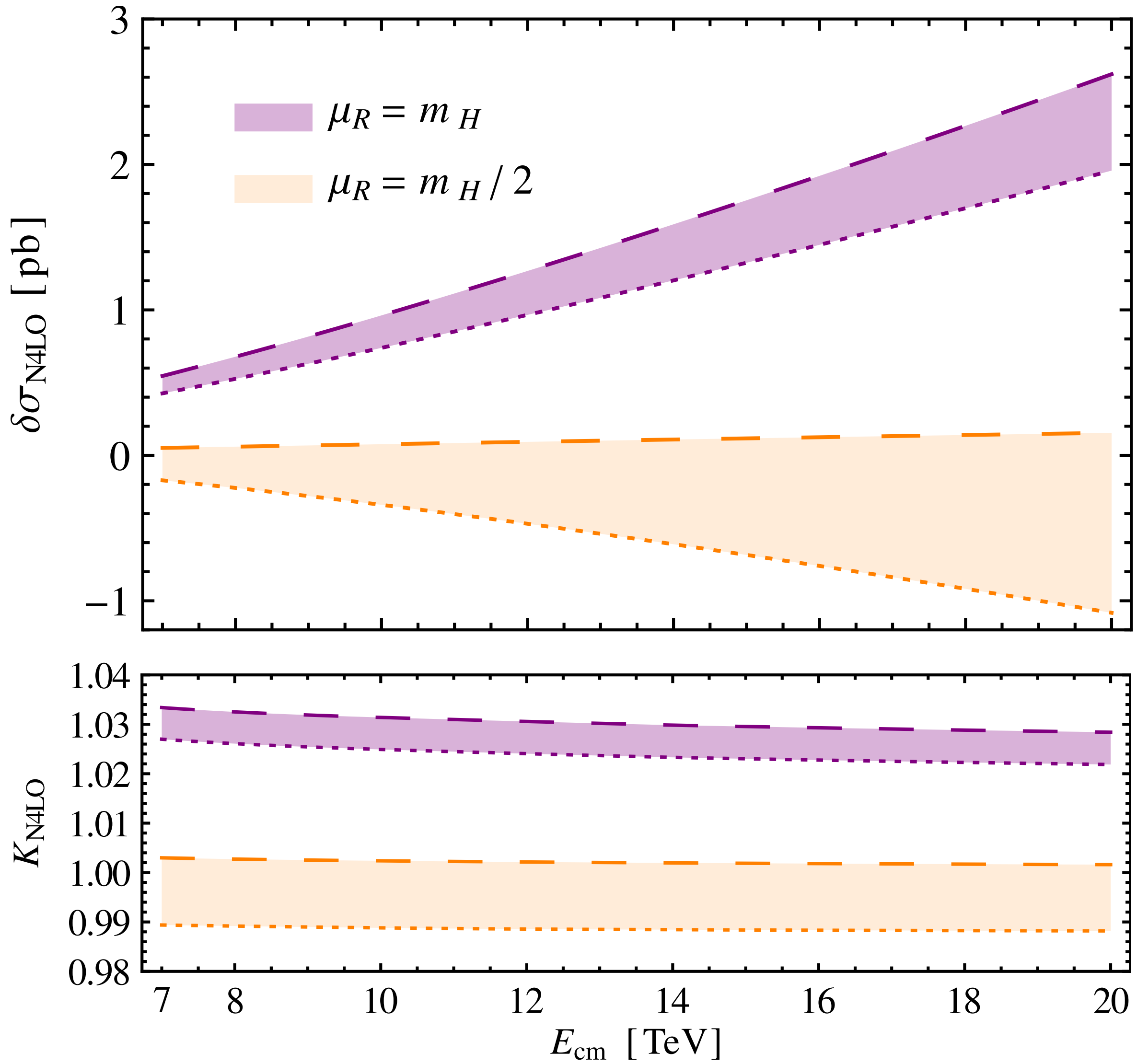



Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15

- Apparent convergence of perturbative expansion
- Scale dependence of exact N$^3$LO prediction with residual uncertainty $3\%$
- Minimal sensitivity at scale $\mu = m_H/2$

# *Approximate N$^4$LO QCD corrections*

- Consistency check with approximate N$^4$LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- $K$-factor $\simeq 1\%$ for $\mu = m_H/2$ with at $\sqrt{s} = 13$ TeV
  de Florian, Mazzitelli, S.M., Vogt ‘14



# Higgs cross section and PDFs

- Cross section $\sigma(H)$ at NNLO with uncertainties: $\sigma(H) + \Delta\sigma(\mathrm{PDF} + \alpha_\mathrm{s})$ for $m_H = 125.0$ GeV at $\sqrt{s} = 13$ TeV with $\mu_R, \mu_F = m_H$ and nominal $\alpha_s$

| PDF sets | $\sigma(H)^{\mathrm{NNLO}}$ [pb]<br>nominal $\alpha_s(M_Z)$ |
|---|---|
| ABMP16 Alekhin, Blümlein, S.M., Placakyte '17 | $40.20 \pm 0.63$ |
| CJ15 Accardi, Brady, Melnitchouk et al. '16 | $42.45\,^{+\,1.73}_{-\,1.12}$ |
| CT14 Dulat et al. '15 | $42.33\,^{+\,1.43}_{-\,1.68}$ |
| HERAPDF2.0 H1+Zeus Coll. | $42.62\,^{+\,0.35}_{-\,0.43}$ |
| JR14 (dyn) Jimenez-Delgado, Reya '14 | $38.01 \pm 0.34$ |
| MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14 | $42.36\,^{+\,0.56}_{-\,0.78}$ |
| NNPDF3.0 Ball et al. '14 | $42.59 \pm 0.80$ |
| PDF4LHC15 Butterworth et al. '15 | $42.42 \pm 0.78$ |

- Large spread for predictions from different PDFs $\sigma(H) = 38.0 \ldots 42.6$ pb
- PDF and $\alpha_s$ differences between sets amount to up to $11\%$
  - significantly larger than residual theory uncertainty at $\mathrm{N}^3$LO QCD

# $\alpha_s(M_Z)$ *in PDFs*

| PDF sets | $\alpha_s(M_Z)$ | method of determination |
|---|---|---|
| ABMP16 Alekhin, Blümlein, S.M., Placakyte '17 | $0.1147 \pm 0.0008$ | fit at NNLO |
| CJ15 Accardi, Brady, Melnitchouk et al. '16 | $0.118 \pm 0.002$ | fit at NLO |
| CT14 Dulat et al. '15 | $0.118$ | assumed at NNLO |
| HERAPDF2.0 H1+Zeus Coll. | $0.1183\ ^{+0.0040}_{-0.0034}$ | fit at NLO |
| JR14 Jimenez-Delgado, Reya '14 | $0.1136 \pm 0.0004$ | dynamical fit at NNLO |
| | $0.1162 \pm 0.0006$ | standard fit at NNLO |
| MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14 | $0.118$ | assumed at NNLO |
| | $0.1172 \pm 0.0013$ | best fit at NNLO |
| NNPDF3.0 Ball et al. '14 | $0.118$ | assumed at NNLO |
| PDF4LHC15 Butterworth et al. '15 | $0.118$ | assumed at NLO |
| | $0.118$ | assumed at NNLO |

- Values of $\alpha_s(M_Z)$ often assumed and not fitted (no correlations)
- Large spread of fitted values at NNLO: $\alpha_s(M_Z) = 0.1136 \ldots 0.1172$
- PDF4LHC: order independent recommendation
  - use $\alpha_s(M_Z) = 0.118$ at NLO and NNLO

## *Parton content of the proton*

# *Data in global PDF fits*

## *Data sets considered in ABMP16 analysis*

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC ($NDP = 2155$)
  - DIS heavy-quark production data
    HERA, CCFR, Chorus, NOMAD, NuTeV ($NDP = 313$)
  - Drell-Yan data (fixed target) E-605, E-866 ($NDP = 158$)
  - Tevatron & LHC data for $W^{\pm}$- and $Z$-boson production
    D0, ATLAS, CMS, LHCb ($NDP = 172$)
  - Top-quark production D0, ATLAS, CMS, LHCb ($NDP = 36$)

## *Iterative cycle of PDF fits*

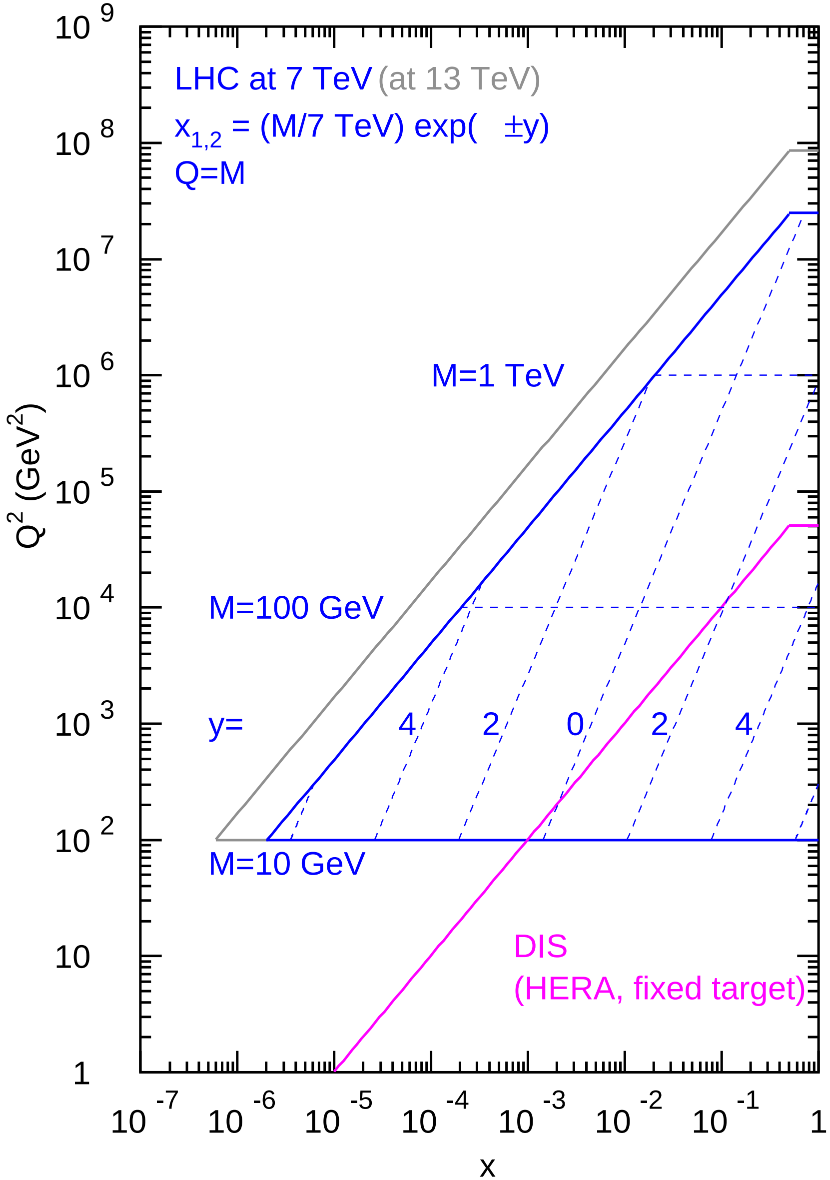
i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of parton distributions, strong coupling $\alpha_s(M_Z)$ and heavy quark masses

# *Parton kinematics at LHC*

- Information on proton structure depends on kinematic coverage




- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with $x_{1,2} = M/\sqrt{S}e^{\pm y}$
  - forward rapidities sensitive to small-$x$
- Cross section depends on convolution of parton distributions
  - small-$x$ part of $f_i$ and large-$x$ PDFs $f_j$

$$\sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \left[ \dots \right]$$

# *Theory considerations in PDF fits*

## *Theory considerations in ABMP16*

- Strictly NNLO QCD for determination of PDFs and $\alpha_s$
- Consistent scheme for treatment of heavy quarks
  - $\overline{\mathrm{MS}}$-scheme for quark masses and $\alpha_s$
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Full account of error correlations

## *Interplay with perturbation theory*

- Accuracy of determination driven by precision of theory predictions
- PDF parameters, $\alpha_s$, $m_c$, $m_b$ and $m_t$ sensitive to
  - radiative corrections at higher orders
  - chosen scheme (e.g. ($\overline{MS}$ scheme)
  - renormalization and factorization scales $\mu_R$, $\mu_F$
  - ...

# *Benchmark measurements*

## *DIS*

- Structure functions for neutral and charged current known to $\mathcal{O}(\alpha_s^3)$
  - $F_2$, $F_3$, known N$^3$LO, $F_L$ known NNLO S.M, Vermaseren, Vogt ‘04–‘08
- Heavy-quark structure functions
  - asymptotic NNLO terms at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein ‘09; Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock ‘14
  - approximate NNLO expressions for neutral and charged current Lo Presti, Kawamura, S.M., Vogt ‘12, Blümlein, A. Hasselhuhn, and T. Pfoh ‘14
- Dijet production in DIS at NNLO Currie, Gehrmann, Niehues ‘16

## *LHC*

- $W^{\pm}$- and $Z$-boson production at NNLO
  Hamberg, van Neerven, Matsuura ‘91; Harlander, Kilgore ’02
- Hadro-production of top-quark pairs Czakon, Fiedler, Mitov ‘13
- Single top-quark production ($t$-channel) Brucherseifer, Caola, Melnikov ‘14
- $Z + 1$jet Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello ‘15; Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan ‘15
- Hadroproduction of jets
  - all partonic channels; leading color only Currie, Glover, Pires ‘16

# *ABMP16 PDF ansatz*

- PDFs parameterization at scale $\mu_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M., Placakyte '17
  - ansatz for valence-/sea-quarks, gluon

$$
\begin{aligned}
xq_v(x,\mu_0^2) &= \frac{2\delta_{qu} + \delta_{qd}}{N_q^v} x^{a_q}(1-x)^{b_q} x^{P_{qv}(x)} \\
xq_s(x,\mu_0^2) &= x\bar{q}_s(x,\mu_0^2) \;=\; A_{qs}(1-x)^{b_{qs}} x^{a_{qs} P_{qs}(x)} \\
xg(x,\mu_0^2) &= A_g x^{a_g}(1-x)^{b_g} x^{a_g\, P_g(x)}
\end{aligned}
$$

  - strange quark is taken in charge-symmetric form
  - function $P_p(x)$

$$
P_p(x) \;=\; (1 + \gamma_{-1,p} \ln x)\left(1 + \gamma_{1,p}x + \gamma_{2,p}x^2 + \gamma_{3,p}x^3\right) \;,
$$

  - 29 parameters in fit including $\alpha_s^{(n_f=3)}(\mu_0 = 3$ GeV), $m_c$, $m_b$ and $m_t$
  - simultaneous fit of higher twist parameters (twist-4)
- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

# *Quality of fit*

## *Statistical tests*

- Goodness-of-fit estimator
  - $\chi^2$ values compared to number of data points (typically a few thousand in global fit)

## *Covariance matrix*

- Positive-definite covariance matrix
  - correlations for fit parameters of ABMP16 PDFs

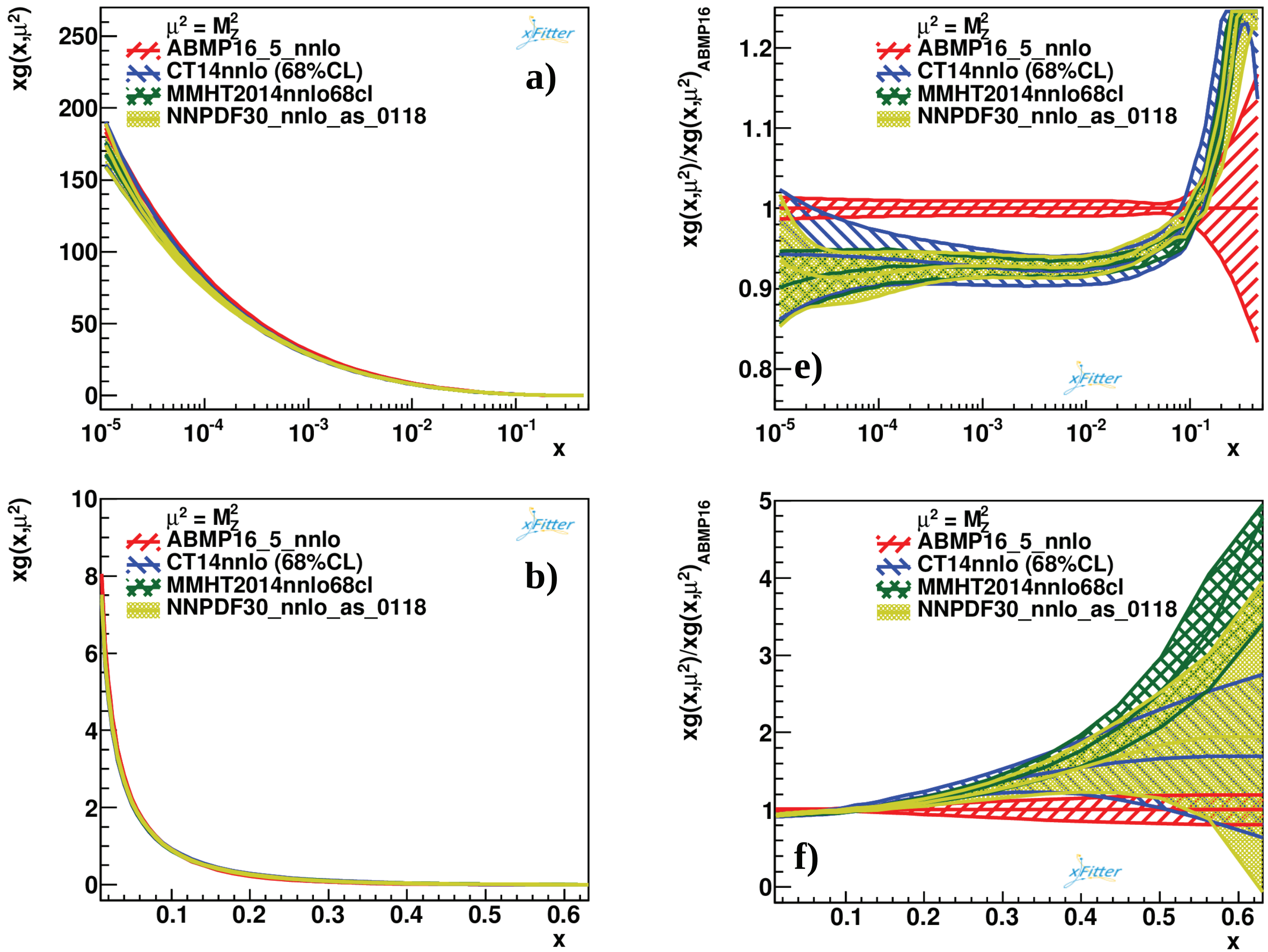
| | $a_u$ | $b_u$ | $\gamma_{1,u}$ | $\gamma_{2,u}$ | $\gamma_{3,u}$ | $a_d$ | $b_d$ | $\gamma_{1,d}$ | $\gamma_{2,d}$ | $\gamma_{3,d}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_u$ | 1.0 | 0.7617 | 0.9372 | -0.5078 | 0.4839 | 0.4069 | 0.3591 | 0.4344 | -0.3475 | 0.0001 |
| $b_u$ | 0.7617 | 1.0 | 0.6124 | -0.1533 | -0.0346 | 0.3596 | 0.2958 | 0.3748 | -0.2748 | 0.0001 |
| $\gamma_{1,u}$ | 0.9372 | 0.6124 | 1.0 | -0.7526 | 0.7154 | 0.2231 | 0.2441 | 0.2812 | -0.2606 | 0.0001 |
| $\gamma_{2,u}$ | -0.5078 | -0.1533 | -0.7526 | 1.0 | -0.9409 | 0.2779 | 0.2276 | 0.2266 | -0.1860 | 0.0 |
| $\gamma_{3,u}$ | 0.4839 | -0.0346 | 0.7154 | -0.9409 | 1.0 | -0.1738 | -0.1829 | -0.1327 | 0.1488 | 0.0 |
| $a_d$ | 0.4069 | 0.3596 | 0.2231 | 0.2779 | -0.1738 | 1.0 | 0.7209 | 0.9697 | -0.6529 | 0.0001 |
| $b_d$ | 0.3591 | 0.2958 | 0.2441 | 0.2276 | -0.1829 | 0.7209 | 1.0 | 0.7681 | -0.9786 | -0.0001 |
| $\gamma_{1,d}$ | 0.4344 | 0.3748 | 0.2812 | 0.2266 | -0.1327 | 0.9697 | 0.7681 | 1.0 | -0.7454 | 0.0002 |
| $\gamma_{2,d}$ | -0.3475 | -0.2748 | -0.2606 | -0.1860 | 0.1488 | -0.6529 | -0.9786 | -0.7454 | 1.0 | -0.0002 |
| $\gamma_{3,d}$ | 0.0001 | 0.0001 | 0.0001 | 0.0 | 0.0 | 0.0001 | -0.0001 | 0.0002 | -0.0002 | 1.0 |
| $a_{us}$ | -0.0683 | -0.0081 | -0.2094 | 0.3881 | -0.3206 | 0.2266 | 0.1502 | 0.2000 | -0.1293 | 0.0 |
| $b_{us}$ | -0.3508 | -0.3089 | -0.3462 | 0.0906 | -0.0537 | -0.1045 | -0.2000 | -0.2241 | 0.2798 | 0.0 |
| $\gamma_{-1,us}$ | 0.2296 | 0.1387 | 0.3367 | -0.4043 | 0.3474 | -0.1171 | -0.1127 | -0.0810 | 0.0767 | 0.0 |
| $\gamma_{1,us}$ | -0.4853 | -0.4119 | -0.3844 | -0.0365 | 0.0064 | -0.4380 | -0.3592 | -0.4957 | 0.3771 | -0.0001 |
| $A_{us}$ | 0.0506 | 0.0807 | -0.0949 | 0.3198 | -0.2560 | 0.2527 | 0.1648 | 0.2350 | -0.1509 | 0.0 |
| $a_{ds}$ | -0.0759 | -0.0443 | -0.0951 | 0.0263 | -0.0382 | -0.2565 | -0.2541 | -0.2666 | 0.2380 | 0.0 |
| $b_{bs}$ | 0.0452 | -0.0197 | 0.0345 | -0.0589 | 0.0683 | -0.2084 | 0.0190 | -0.1841 | -0.0522 | 0.0 |
| $\gamma_{1,ds}$ | -0.0492 | -0.0809 | 0.0101 | -0.1791 | 0.1309 | -0.5576 | -0.2029 | -0.4584 | 0.0946 | 0.0 |
| $A_{ds}$ | -0.1980 | -0.1262 | -0.2349 | 0.1526 | -0.1428 | -0.1113 | -0.2167 | -0.1739 | 0.2407 | 0.0 |
| $a_{ss}$ | -0.2034 | -0.1285 | -0.2362 | 0.2328 | -0.2080 | 0.0960 | 0.1596 | 0.0661 | -0.1054 | 0.0 |
| $b_{ss}$ | -0.1186 | -0.0480 | -0.1532 | 0.1549 | -0.1536 | 0.0486 | 0.1508 | 0.0267 | -0.1161 | 0.0 |
| $A_{ss}$ | -0.1013 | -0.0411 | -0.1458 | 0.1802 | -0.1625 | 0.1216 | 0.1678 | 0.0924 | -0.1196 | 0.0 |
| $a_g$ | 0.0046 | -0.0374 | 0.1109 | -0.1934 | 0.1653 | -0.0288 | -0.0122 | 0.0053 | 0.0059 | 0.0 |
| $b_g$ | 0.2662 | 0.3141 | 0.1579 | -0.0050 | -0.0207 | 0.0973 | 0.0870 | 0.0646 | -0.0666 | 0.0 |
| $\gamma_{1,g}$ | 0.2008 | 0.2274 | 0.0706 | 0.0876 | -0.0835 | 0.0919 | 0.0574 | 0.0493 | -0.0364 | 0.0 |
| $\alpha_s^{(n_f=3)}(\mu_0)$ | 0.1083 | -0.0607 | 0.0848 | -0.0250 | 0.0765 | 0.0763 | -0.0306 | 0.0725 | 0.0243 | 0.0 |
| $m_c(m_c)$ | -0.0006 | 0.0170 | -0.0104 | 0.0206 | -0.0201 | -0.0123 | -0.0161 | -0.0114 | 0.0108 | 0.0 |
| $m_b(m_b)$ | 0.0661 | 0.0554 | 0.0605 | -0.0367 | 0.0287 | -0.0116 | 0.0029 | -0.0074 | -0.0051 | 0.0 |
| $m_t(m_t)$ | -0.1339 | -0.2170 | -0.0816 | 0.0081 | 0.0250 | -0.0616 | -0.0813 | -0.0491 | 0.0736 | 0.0 |

| | $a_{us}$ | $b_{us}$ | $\gamma_{-1,us}$ | $\gamma_{1,us}$ | $A_{us}$ | $a_{ds}$ | $b_{bs}$ | $\gamma_{1,ds}$ | $A_{ds}$ | $a_{ss}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_u$ | -0.0683 | -0.3508 | 0.2296 | -0.4853 | 0.0506 | -0.0759 | 0.0452 | -0.0492 | -0.1980 | -0.2034 |
| $b_u$ | -0.0081 | -0.3089 | 0.1387 | -0.4119 | 0.0807 | -0.0443 | -0.0197 | -0.0809 | -0.1262 | -0.1285 |
| $\gamma_{1,u}$ | -0.2094 | -0.3462 | 0.3367 | -0.3844 | -0.0949 | -0.0951 | 0.0345 | 0.0101 | -0.2349 | -0.2362 |
| $\gamma_{2,u}$ | 0.3881 | 0.0906 | -0.4043 | -0.0365 | 0.3198 | 0.0263 | -0.0589 | -0.1791 | 0.1526 | 0.2328 |
| $\gamma_{3,u}$ | -0.3206 | -0.0537 | 0.3474 | 0.0064 | -0.2560 | -0.0382 | 0.0683 | 0.1309 | -0.1428 | -0.2080 |
| $a_d$ | 0.2266 | -0.1045 | -0.1171 | -0.4380 | 0.2527 | -0.2565 | -0.2084 | -0.5576 | -0.1113 | 0.0960 |
| $b_d$ | 0.1502 | -0.2000 | -0.1127 | -0.3592 | 0.1648 | -0.2541 | 0.0190 | -0.2029 | -0.2167 | 0.1596 |
| $\gamma_{1,d}$ | 0.2000 | -0.2241 | -0.0810 | -0.4957 | 0.2350 | -0.2666 | -0.1841 | -0.4584 | -0.1739 | 0.0661 |
| $\gamma_{2,d}$ | -0.1293 | 0.2798 | 0.0767 | 0.3771 | -0.1509 | 0.2380 | -0.0522 | 0.0946 | 0.2407 | -0.1054 |
| $\gamma_{3,d}$ | 0.0 | 0.0 | 0.0 | -0.0001 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $a_{us}$ | 1.0 | -0.3156 | -0.8947 | -0.5310 | 0.9719 | 0.2849 | 0.0241 | -0.0470 | 0.2983 | 0.4131 |
| $b_{us}$ | -0.3156 | 1.0 | 0.1372 | 0.8258 | -0.3995 | 0.0467 | -0.0221 | -0.1190 | 0.1856 | 0.0291 |
| $\gamma_{-1,us}$ | -0.8947 | 0.1372 | 1.0 | 0.2611 | -0.7829 | -0.1695 | 0.0156 | 0.0501 | -0.2117 | -0.7191 |
| $\gamma_{1,us}$ | -0.5310 | 0.8258 | 0.2611 | 1.0 | -0.6479 | 0.0086 | 0.0076 | 0.1460 | 0.0781 | -0.0010 |
| $A_{us}$ | 0.9719 | -0.3995 | -0.7829 | -0.6479 | 1.0 | 0.2983 | 0.0515 | -0.0404 | 0.3055 | 0.2811 |
| $a_{ds}$ | 0.2849 | 0.0467 | -0.1695 | 0.0086 | 0.2983 | 1.0 | -0.1608 | 0.0719 | 0.9152 | -0.2941 |
| $b_{bs}$ | 0.0241 | -0.0221 | 0.0156 | 0.0076 | 0.0515 | -0.1608 | 1.0 | 0.7834 | -0.3022 | -0.0390 |
| $\gamma_{1,ds}$ | -0.0470 | -0.1190 | 0.0501 | 0.1460 | -0.0404 | 0.0719 | 0.7834 | 1.0 | -0.1838 | -0.1373 |
| $A_{ds}$ | 0.2983 | 0.1856 | -0.2117 | 0.0781 | 0.3055 | 0.9152 | -0.3022 | -0.1838 | 1.0 | -0.1833 |
| $a_{ss}$ | 0.4131 | 0.0291 | -0.7191 | -0.0010 | 0.2811 | -0.2941 | -0.0390 | -0.1373 | -0.1833 | 1.0 |
| $b_{ss}$ | 0.2197 | 0.0643 | -0.4479 | 0.1286 | 0.1193 | -0.1579 | -0.0260 | 0.0169 | -0.0896 | 0.6522 |
| $A_{ss}$ | 0.3627 | 0.0261 | -0.6319 | 0.0102 | 0.2412 | -0.2688 | -0.0180 | -0.0960 | -0.1797 | 0.9280 |
| $a_g$ | -0.2570 | 0.0001 | 0.2196 | 0.0039 | -0.2493 | -0.2190 | -0.0454 | -0.1031 | -0.2571 | 0.0626 |
| $b_g$ | -0.1419 | 0.1266 | 0.0694 | 0.2648 | -0.1715 | -0.0515 | 0.0917 | 0.2130 | -0.0469 | -0.0092 |
| $\gamma_{1,g}$ | -0.0241 | 0.0332 | -0.0226 | 0.1296 | -0.0489 | -0.0137 | 0.0503 | 0.1409 | 0.0022 | -0.0279 |
| $\alpha_s^{(n_f=3)}(\mu_0)$ | 0.0954 | -0.2866 | -0.0341 | -0.3493 | 0.1110 | -0.0604 | -0.1265 | -0.1811 | -0.1330 | -0.0841 |
| $m_c(m_c)$ | 0.0704 | -0.0093 | -0.0033 | -0.0462 | 0.1182 | 0.0849 | 0.0547 | 0.0413 | 0.1193 | -0.0728 |
| $m_b(m_b)$ | -0.0183 | -0.0132 | 0.0044 | 0.0209 | -0.0298 | -0.0006 | 0.0332 | 0.0695 | -0.0432 | -0.0159 |
| $m_t(m_t)$ | 0.0641 | -0.1841 | -0.0408 | -0.2635 | 0.0755 | -0.0573 | -0.1067 | -0.2003 | -0.0869 | 0.0169 |

| | $b_{ss}$ | $A_{ss}$ | $a_g$ | $b_g$ | $\gamma_{1,g}$ | $\alpha_s^{(n_f=3)}(\mu_0)$ | $m_c(m_c)$ | $m_b(m_b)$ | $m_t(m_t)$ |
|---|---|---|---|---|---|---|---|---|---|
| $a_u$ | -0.1186 | -0.1013 | 0.0046 | 0.2662 | 0.2008 | 0.1083 | -0.0006 | 0.0661 | -0.1339 |
| $b_u$ | -0.0480 | -0.0411 | -0.0374 | 0.3141 | 0.2274 | -0.0607 | 0.0170 | 0.0554 | -0.2170 |
| $\gamma_{1,u}$ | -0.1532 | -0.1458 | 0.1109 | 0.1579 | 0.0706 | 0.0848 | -0.0104 | 0.0605 | -0.0816 |
| $\gamma_{2,u}$ | 0.1549 | 0.1802 | -0.1934 | -0.0050 | 0.0876 | -0.0250 | 0.0206 | -0.0367 | 0.0081 |
| $\gamma_{3,u}$ | -0.1536 | -0.1625 | 0.1653 | -0.0207 | -0.0835 | 0.0765 | -0.0201 | 0.0287 | 0.0250 |
| $a_d$ | 0.0486 | 0.1216 | -0.0288 | 0.0973 | 0.0919 | 0.0763 | -0.0123 | -0.0116 | -0.0616 |
| $b_d$ | 0.1508 | 0.1678 | -0.0122 | 0.0870 | 0.0574 | -0.0306 | -0.0161 | 0.0029 | -0.0813 |
| $\gamma_{1,d}$ | 0.0267 | 0.0924 | 0.0053 | 0.0646 | 0.0493 | 0.0725 | -0.0114 | -0.0074 | -0.0491 |
| $\gamma_{2,d}$ | -0.1161 | -0.1196 | 0.0059 | -0.0666 | -0.0364 | 0.0243 | 0.0108 | -0.0051 | 0.0736 |
| $\gamma_{3,d}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $a_{us}$ | 0.2197 | 0.3627 | -0.2570 | -0.1419 | -0.0241 | 0.0954 | 0.0704 | -0.0183 | 0.0641 |
| $b_{us}$ | 0.0643 | 0.0261 | 0.0001 | 0.1266 | 0.0332 | -0.2866 | -0.0093 | -0.0132 | -0.1841 |
| $\gamma_{-1,us}$ | -0.4479 | -0.6319 | 0.2197 | 0.0694 | -0.0226 | -0.0341 | -0.0034 | 0.0044 | -0.0408 |
| $\gamma_{1,us}$ | 0.1286 | 0.0102 | 0.0039 | 0.2648 | 0.1296 | -0.3493 | -0.0462 | 0.0209 | -0.2635 |
| $A_{us}$ | 0.1193 | 0.2412 | -0.2493 | -0.1715 | -0.0489 | 0.1110 | 0.1182 | -0.0298 | 0.0755 |
| $a_{ds}$ | -0.1579 | -0.2688 | -0.2190 | -0.0515 | -0.0137 | -0.0604 | 0.0849 | -0.0006 | -0.0573 |
| $b_{bs}$ | -0.0260 | -0.0180 | -0.0454 | 0.0917 | 0.0503 | -0.1265 | 0.0547 | 0.0332 | -0.1067 |
| $\gamma_{1,ds}$ | 0.0169 | -0.0960 | -0.1031 | 0.2130 | 0.1409 | -0.1811 | 0.0413 | 0.0695 | -0.2003 |
| $A_{ds}$ | -0.0896 | -0.1797 | -0.2571 | -0.0469 | 0.0022 | -0.1330 | 0.1193 | -0.0432 | -0.0869 |
| $a_{ss}$ | 0.6522 | 0.9280 | 0.0626 | -0.0092 | -0.0279 | -0.0841 | -0.0728 | -0.0159 | 0.0169 |
| $b_{ss}$ | 1.0 | 0.6427 | -0.0179 | 0.1967 | 0.1164 | -0.2390 | -0.0965 | 0.0169 | -0.1675 |
| $A_{ss}$ | 0.6427 | 1.0 | -0.0211 | 0.1403 | 0.0997 | -0.1385 | 0.0216 | 0.0072 | -0.1109 |
| $a_g$ | -0.0179 | -0.0211 | 1.0 | -0.5279 | -0.8046 | 0.1838 | -0.2829 | 0.0076 | 0.3310 |
| $b_g$ | 0.1967 | 0.1403 | -0.5279 | 1.0 | 0.8837 | -0.5124 | 0.1438 | 0.1255 | -0.7275 |
| $\gamma_{1,g}$ | 0.1164 | 0.0997 | -0.8046 | 0.8837 | 1.0 | -0.2511 | 0.1829 | 0.0814 | -0.5180 |
| $\alpha_s^{(n_f=3)}(\mu_0)$ | -0.2390 | -0.1385 | 0.1838 | -0.5124 | -0.2511 | 1.0 | -0.1048 | 0.0423 | 0.6924 |
| $m_c(m_c)$ | -0.0965 | 0.0216 | -0.2829 | 0.1438 | 0.1829 | -0.1048 | 1.0 | 0.0328 | -0.1577 |
| $m_b(m_b)$ | 0.0169 | 0.0072 | 0.0076 | 0.1255 | 0.0814 | 0.0423 | 0.0328 | 1.0 | -0.0900 |
| $m_t(m_t)$ | -0.1675 | -0.1109 | 0.3310 | -0.7275 | -0.5180 | 0.6924 | -0.1577 | -0.0900 | 1.0 |

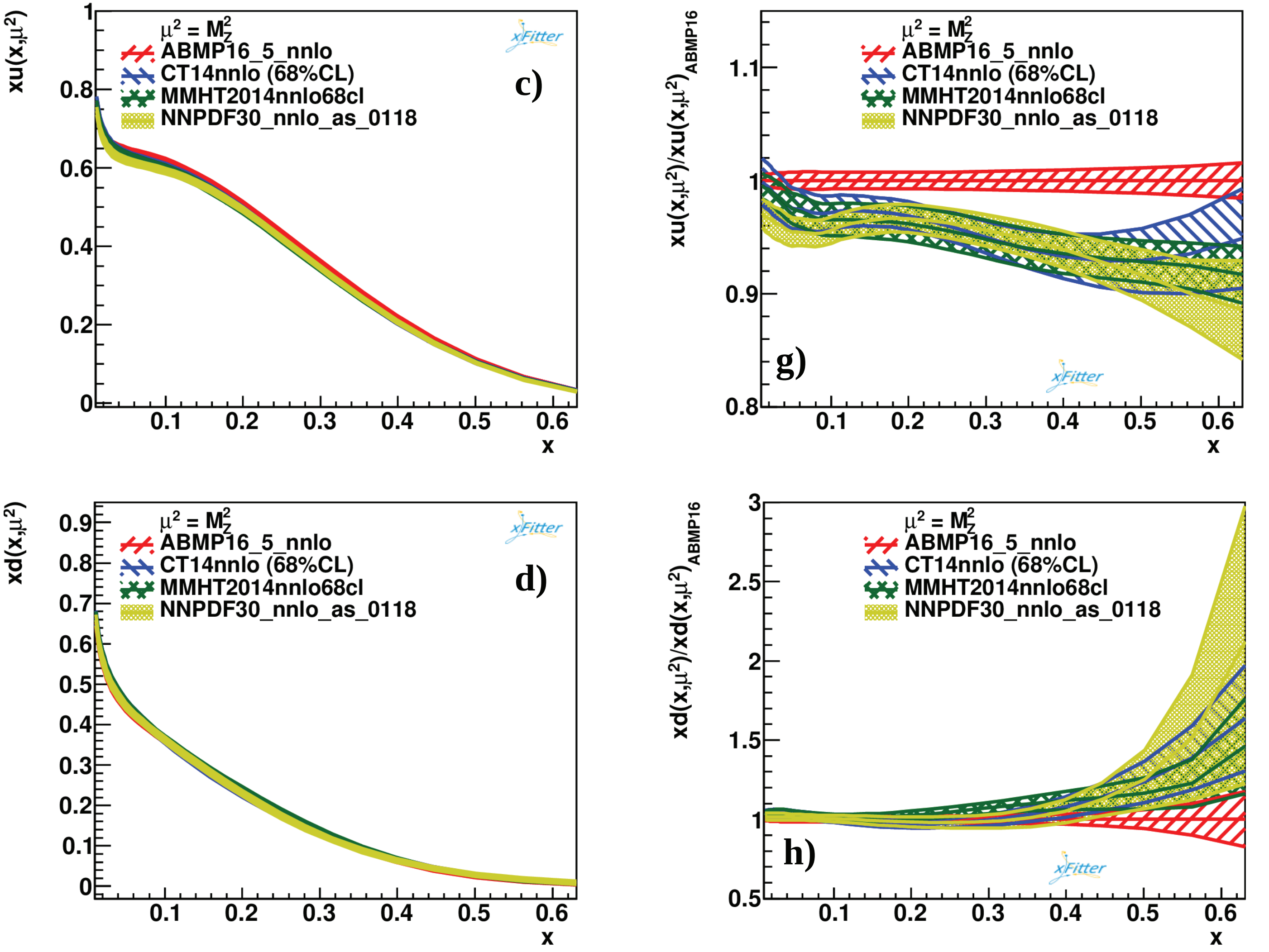
# *Results for parton distributions (I)*

- PDFs with $1\sigma$ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Gluon $g(x)$



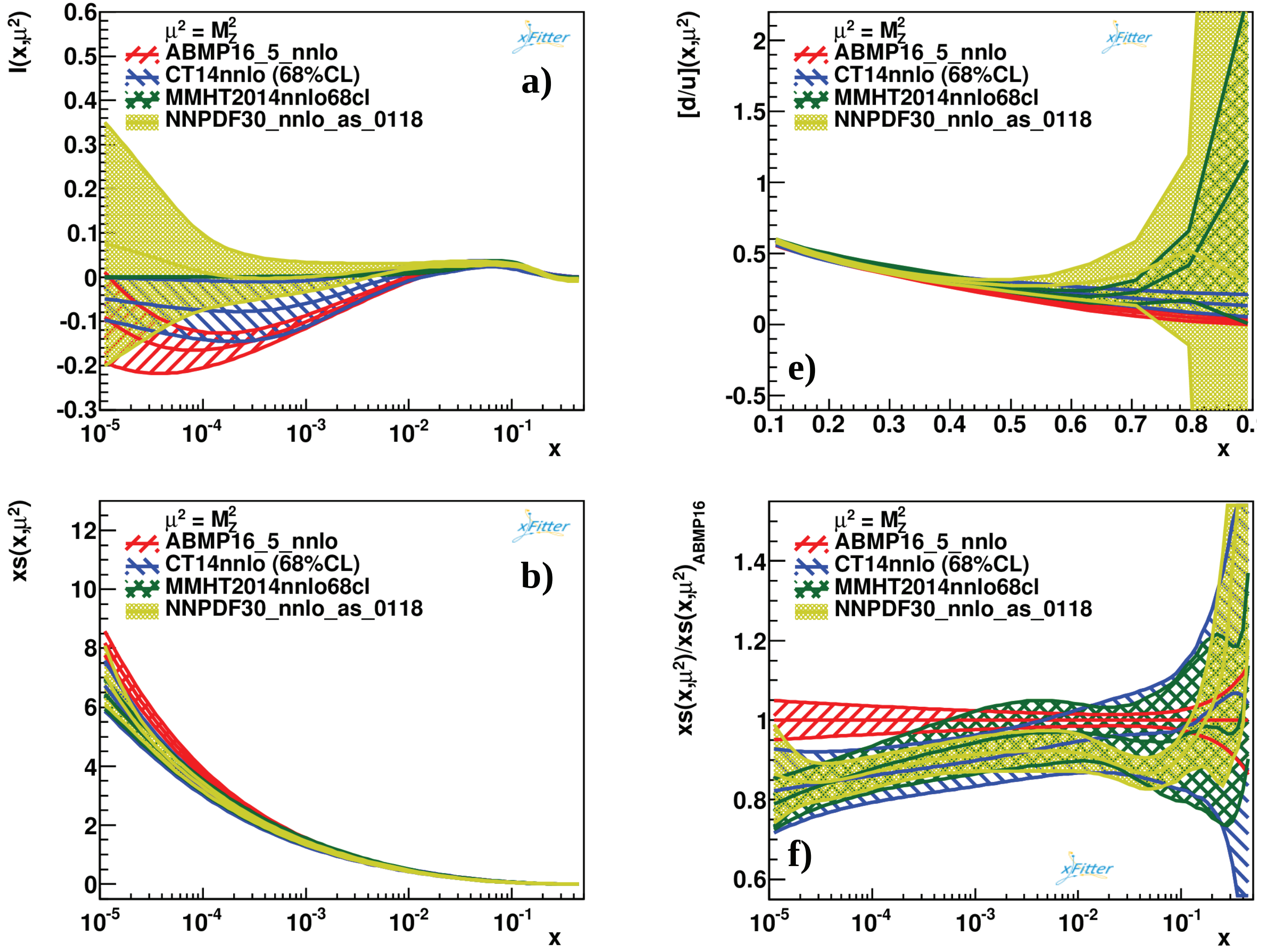
# *Results for parton distributions (II)*

- PDFs with $1\sigma$ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Light valence quarks $u(x)$, $d(x)$



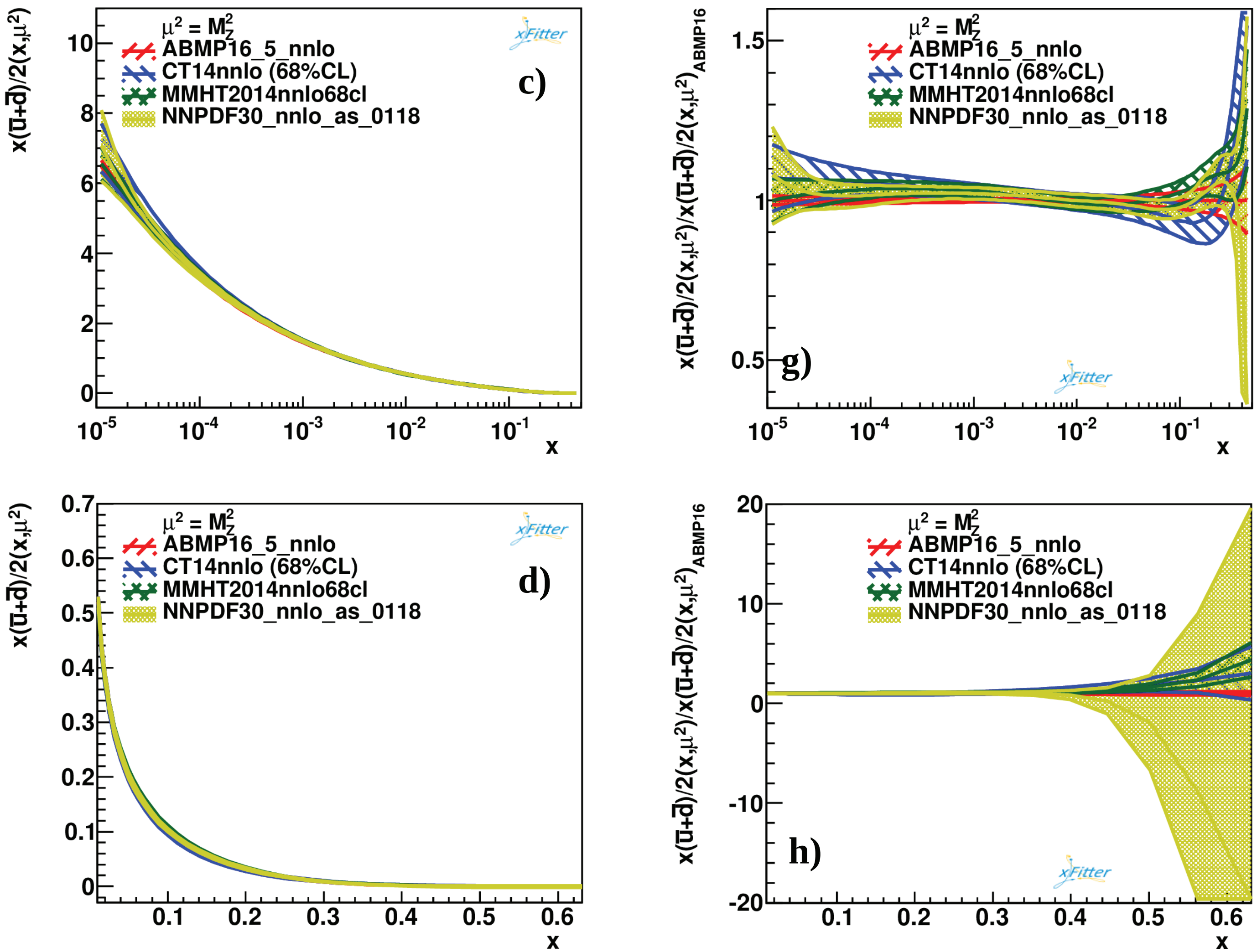
# *Results for parton distributions (III)*

- PDFs with $1\sigma$ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Iso-spin asymmetry $x(\bar{d}(x) - \bar{u}(x))$; ratio $d(x)/u(x)$; strange $s(x)$



# *Results for parton distributions (IV)*

- PDFs with $1\sigma$ uncertainty bands; compare ABMP16, CT14, MMHT14 NNPDF3.0
- Sea quarks $\bar{u}(x) + \bar{d}(x)$



# Tools

## Accessibility of PDFs

- PDFs stored as grid in $x$ and $Q^2$; size $\mathcal{O}(5)$ to $\mathcal{O}(100)$ MB
  - LHAPDF library for table look-up
    Buckley et al. '14; http://projects.hepforge.org/lhapdf
- PDF evolution 'on-the-fly'
  - use PDF parametrization (function of $x$) at initial scale $\mu_0^2$;
    solution of differential quation; little computational overhead
  - Public codes for NNLO evolution of PDFs
    PEGASUS Vogt hep-ph/0408244, QCDNUM Botje arXiv:1005.1481,
    HOPPET Salam, Rojo arXiv:0804.3755,
    CANDIA Cafarella, Coriano, Guzzi arXiv:0803.0462,
    APFEL Bertone, Carrazza, Rojo arXiv:1310.1394

## Reproducibility of PDF fits

- Public code OPENQCDRAD for cross section computations in ABMP16;
  http://www-zeuthen.desy.de/~alekhin/OPENQCDRAD
- XFitter library Alekhin et al. '14 for statistical tests ($\chi^2$ values) and fits of individual data sets; https://www.xfitter.org/xFitter

## *Heavy quarks in deep-inelastic scattering*

# *Treatment of heavy-quarks*

## *Light quarks*

- Neglect “light quark” masses $m_u, m_d \ll \Lambda_{QCD}$ and $m_s < \Lambda_{QCD}$ in hard scattering process
  - scale-dependent $u, d, s, g$ PDFs from mass singularities

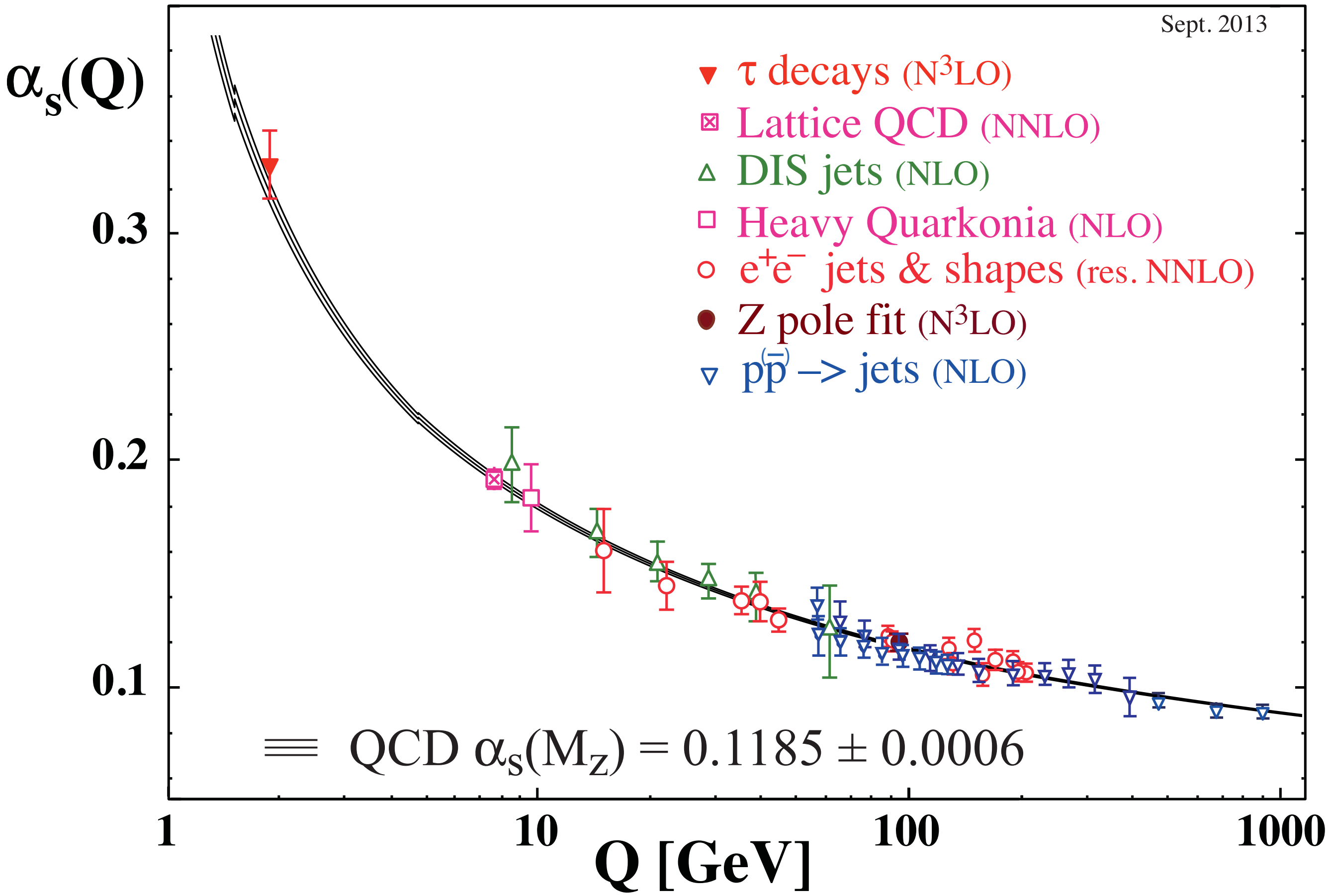
## *Heavy quarks*

- No mass singularities for $m_c, m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
  - $c$ and $b$ PDFs for $Q \ggg m_c, m_b$ generated perturbatively
  - matching of two distinct theories
    $\longrightarrow n_f$ light flavors + heavy quark of mass $m$ at low scales
    $\longrightarrow n_f + 1$ light flavors at high scales

# *Strong coupling with flavor thresholds*

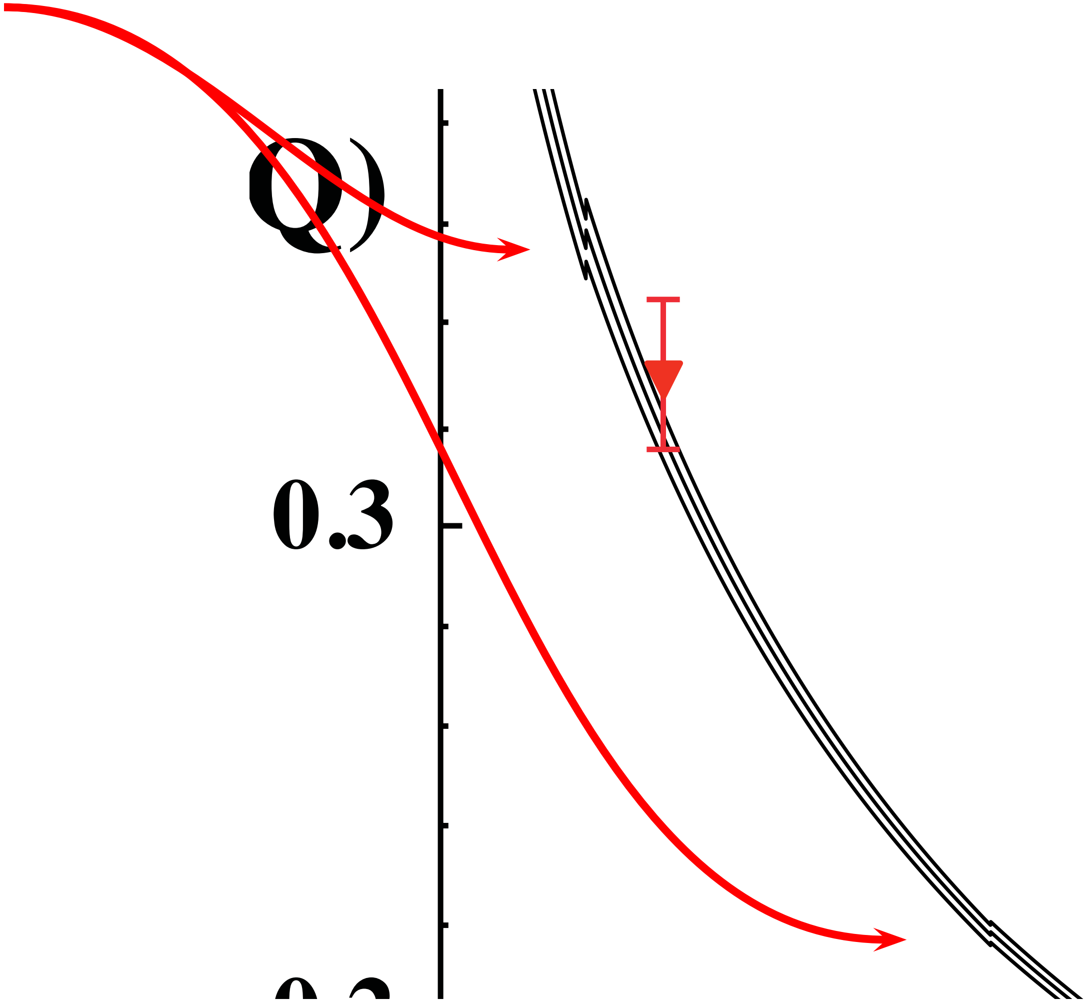
- Solution of QCD $\beta$-function for $\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$
  - discontinuities for $n_f = 3 \to n_f = 4 \to n_f = 5$
- Big picture

Bethke for PDG 2014



# *Strong coupling with flavor thresholds*

- Solution of QCD $\beta$-function for $\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$
  - discontinuities for $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$
- Zoom



# PDFs with flavor thresholds (I)

- Generate heavy-quark PDFs $h^{(n_f+1)}$ from light-flavor PDFs
  - heavy-quark operator matrix elements (OMEs) $A_{ji}$ at three loops
    Bierenbaum, Blümlein, Klein '09; Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider '14

$$h^{(n_f+1)}(x,\mu) + \bar{h}^{(n_f+1)}(x,\mu) = A_{hq}(x) \otimes \Sigma^{(n_f)}(x,\mu) + A_{hg}(x) \otimes g^{(n_f)}(x,\mu)$$

  - likewise light-quark PDFs $l_i^{(n_f)} \to l_i^{(n_f+1)}$ and gluon and the quark singlet PDFs $(\Sigma^{(n_f)}, g^{(n_f)}) \to (\Sigma^{(n_f+1)}, g^{(n_f+1)})$
- Perturbative expansion of OME $A_{hg}$

$$A_{hg}^{(1)}(x) = \underbrace{a_{hg}^{(10)}}_{=0} + \ln\left(\frac{\mu^2}{m^2}\right) P_{qg}^{(0)}$$

  - charm density at leading order with matching $c(x, \mu^2 = m_c^2) = 0$

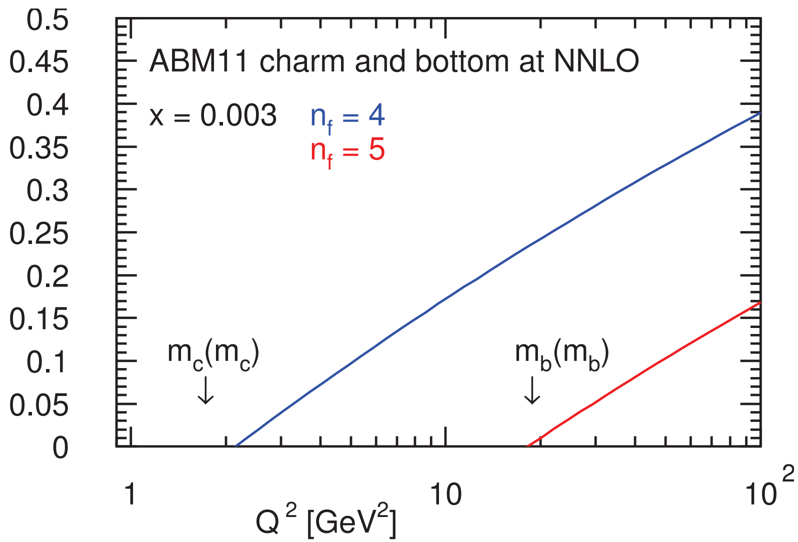
$$c(x,\mu^2)\Big|_{\rm LO} = a_s(\mu^2) \int_x^1 \frac{dz}{z} \ln\left(\frac{\mu^2}{m_c^2}\right) P_{qg}^{(0)}(z)\, g\left(\frac{x}{z}, \mu^2\right)$$

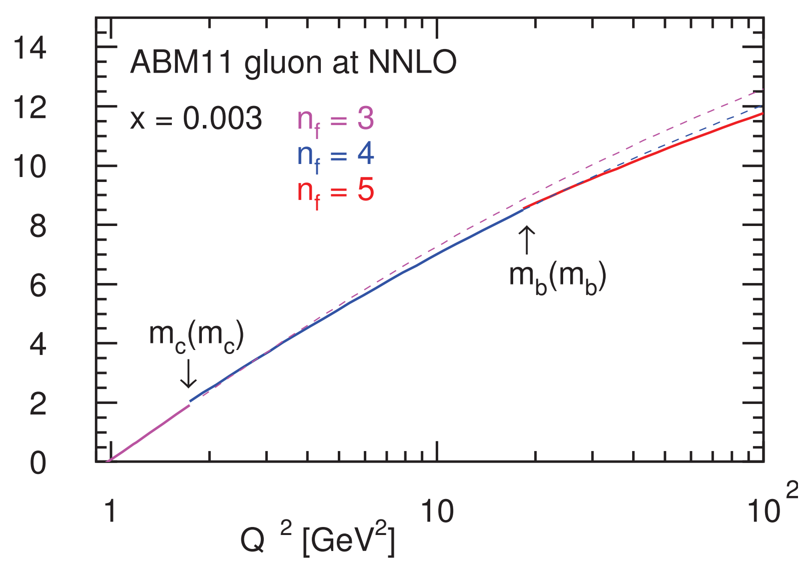
  - higher order matching $c(x, \mu^2 = m_c^2) \neq 0$

$$A_{hg}^{(2)}(x) = \underbrace{a_{hg}^{(20)}}_{\neq 0} + \ln\left(\frac{\mu^2}{m^2}\right) a_{hg}^{(21)} + \ln^2\left(\frac{\mu^2}{m^2}\right) a_{hg}^{(22)}$$

# *PDFs with flavor thresholds (II)*

- Solution of evolution equations between thresholds for $n_f \longrightarrow (n_f + 1)$ with fixed $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$
  - discontinuities in PDFs across flavor thresholds
  - matching conditions known to NLO; $A_{hg}^{(3)}$ currently unknown



# *Cross sections with flavor thresholds*

## *Fixed flavor number scheme (FFNS) ("fixed order $\ln(Q^2/m^2)$")*

- Cross section with massive quarks at scales $Q \not\gg m_c$
  - top-quark hadro-production ($t\bar{t}$ pairs, single top in 4FS or 5FS, ...]
- $F_2^c$ at HERA with $u, d, s, g$ partons and massive charm coeff. fcts.
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven '92
  - approximations at NNLO Bierenbaum, Blümlein, Klein '09; Lo Presti, Kawamura, S.M., Vogt '12; Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock '14

## *Variable flavor number scheme (VFNS) ("resum $\ln(Q^2/m^2)$")*

- (Smooth) matching of two distinct theories:
  $n_f$ light + heavy quark at low scales $\longrightarrow n_f + 1$ light flavors at high scales
  - Higgs boson production in $b\bar{b}$-annihilation ("Santander matching" Harlander, Krämer, Schumacher '11)
- $F_2^c$ at HERA with ACOT Aivazis, Collins, Olness, Tung '94, BMSN Buza, Matiounine, Smith, van Neerven '98, RT Thorne, Roberts '98, FONLL Forte, Laenen, Nason, Rojo '10
  - model assumptions in matching conditions
  - details of implementation matter in global fits

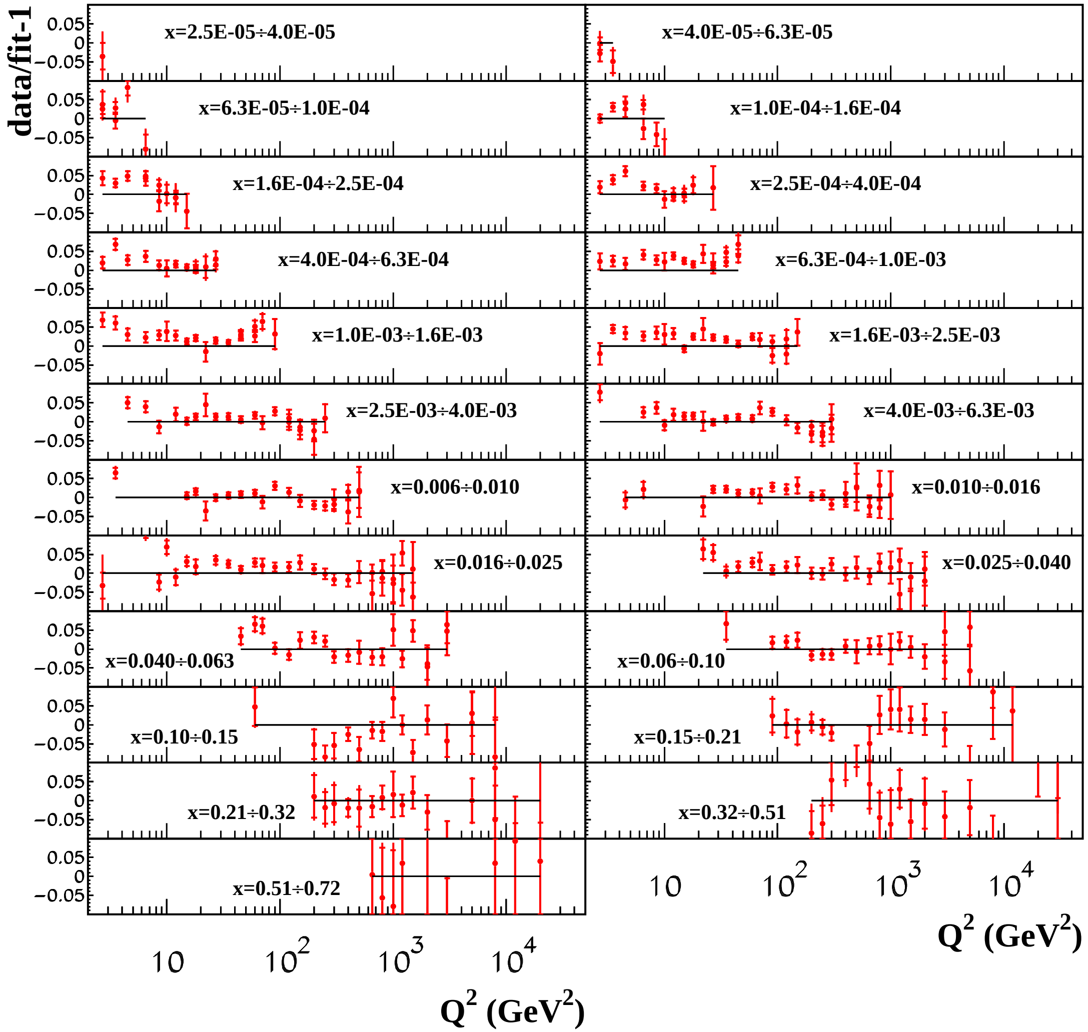
# GM-VFNS implementation

- GM-VFNS implementation using BSMN
  Buza, Matiounine, Smith, van Neerven '98
  - other variant: FONLL Cacciari, Greco, Nason '98; Forte, Laenen, Nason, Rojo '10
- DIS structure function $F_2^h$ for heavy-quark $h$

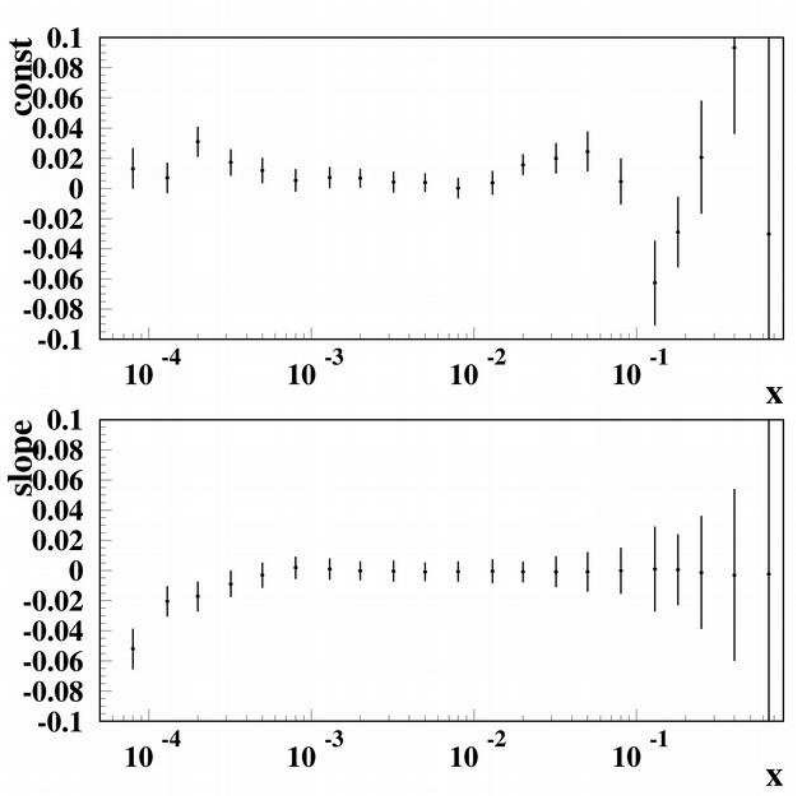
$$F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2) =$$
$$= \quad F_2^{h,\mathrm{exact}}(N_f,x,Q^2) + \left\{ F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2) - F_2^{h,\mathrm{asymp}}(N_f,x,Q^2) \right\}$$

  - $F_2^{h,\mathrm{exact}}$: massive heavy-quark structure function ($m \neq 0$)
  - $F_2^{h,\mathrm{ZMVFN}}$: DIS structure function with zero mass ($m = 0$)
  - $F_2^{h,\mathrm{asymp}}$: asymptotic expansion of heavy-quark structure function (logarithms $\ln(Q^2/m^2)$)
- Difference $\{\dots\}$ has to vanish at threshold $Q \simeq m$
  - details differ for other GM-VFNS implementations:
    ACOT: S-ACOT-$\chi$ for slow rescaling $x \to \chi(x) = x\left(1+\frac{4m^2}{Q^2}\right)$
    FONLL: suppression of $\{\dots\}$ with damping factor $\left(1+\frac{m^2}{Q^2}\right)^2$
    RT: continuity of physical observables in threshold region

# *Statistical check of big logarithms*




| $Q^2_{\min}$ (GeV$^2$) | $\chi^2$/NDP |
|---|---|
| 10 | 366/324 |
| 100 | 193/201 |
| 1000 | 95/83 |

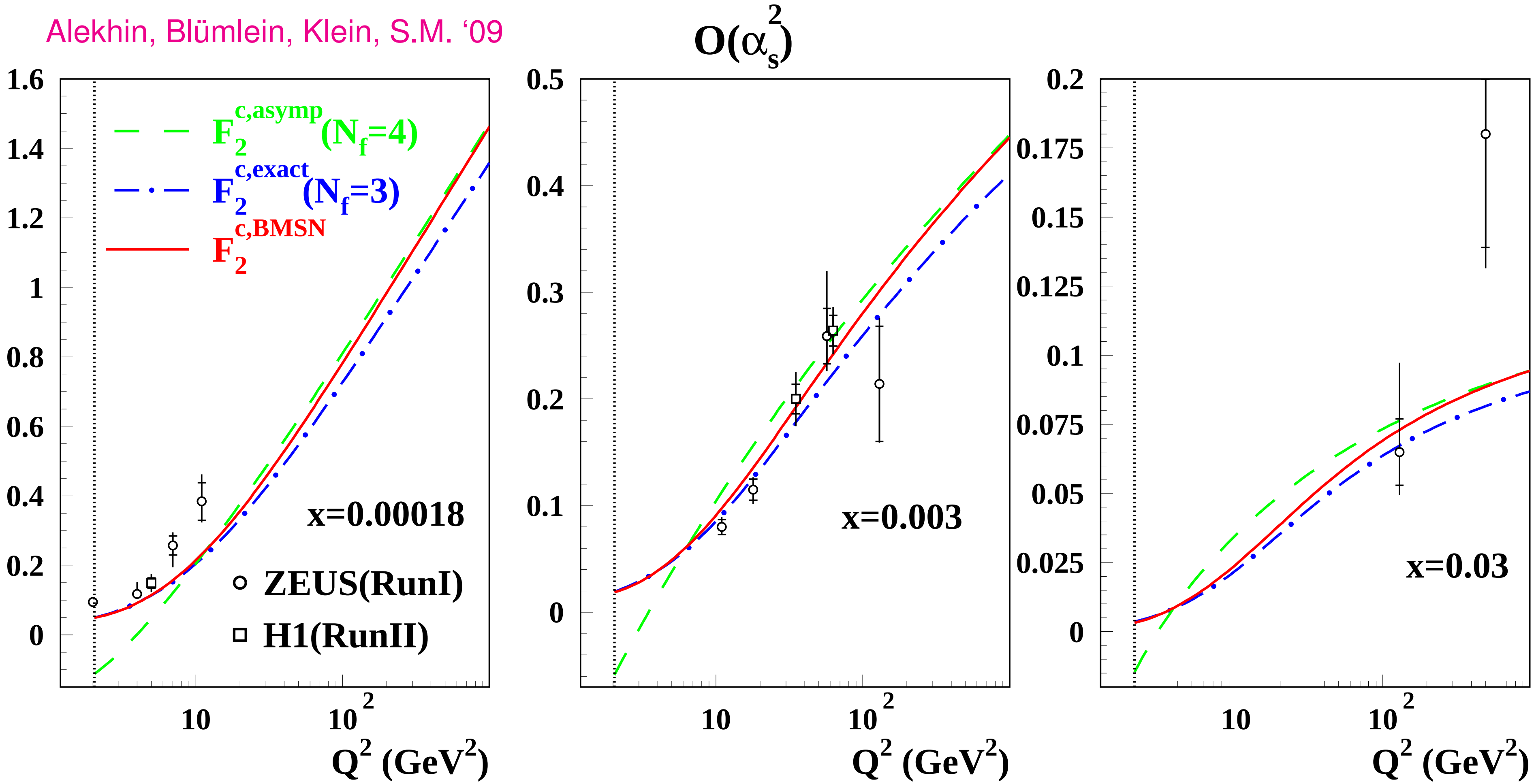



- Parametrization of pulls in ABM12

  $$\text{pulls} = \text{const} + \text{slope} \ln\left(\frac{Q^2}{Q_0^2}\right)$$

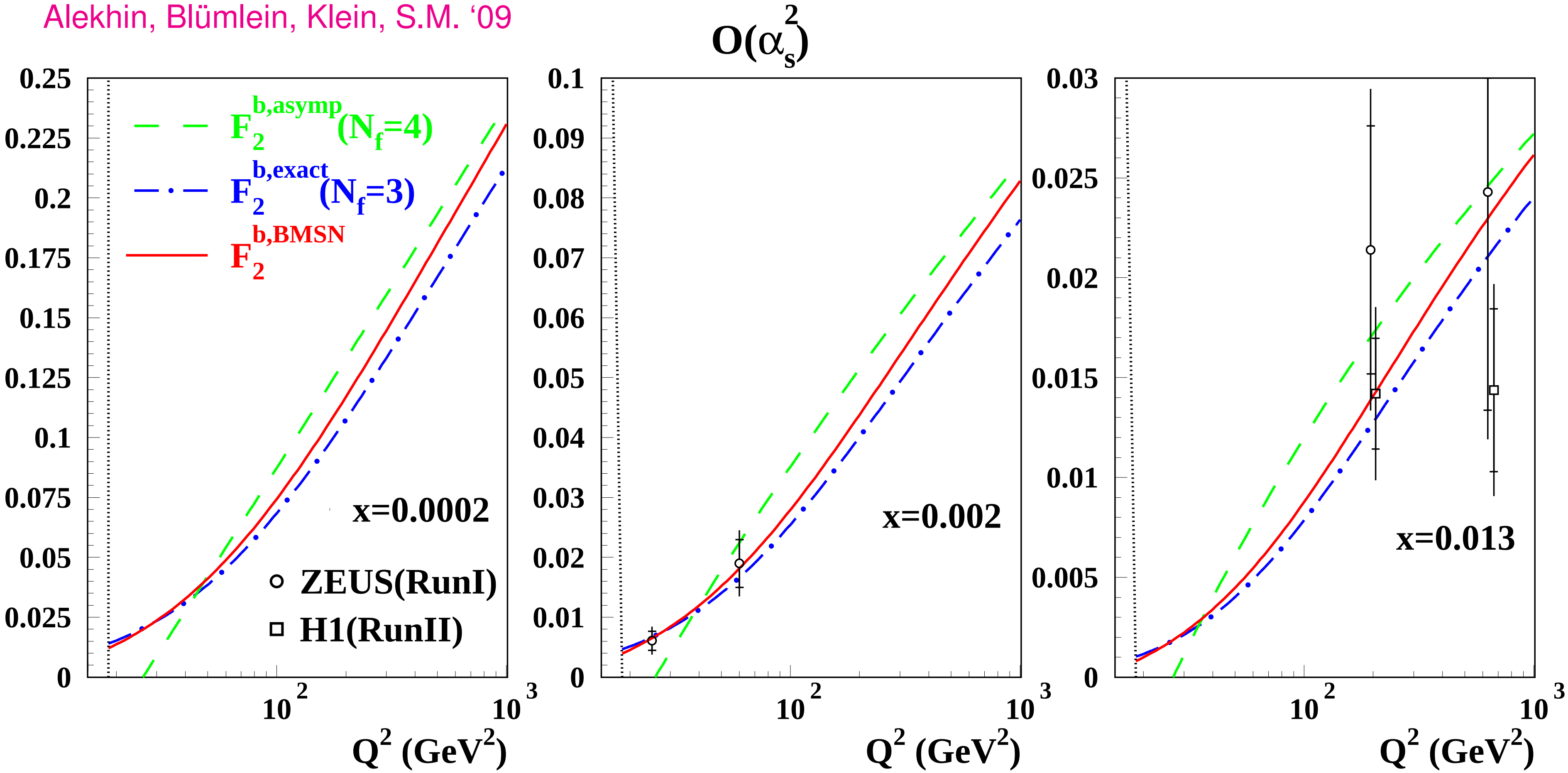
- No indications for big logarithms

Alekhin, Blümlein, Klein, S.M. '09



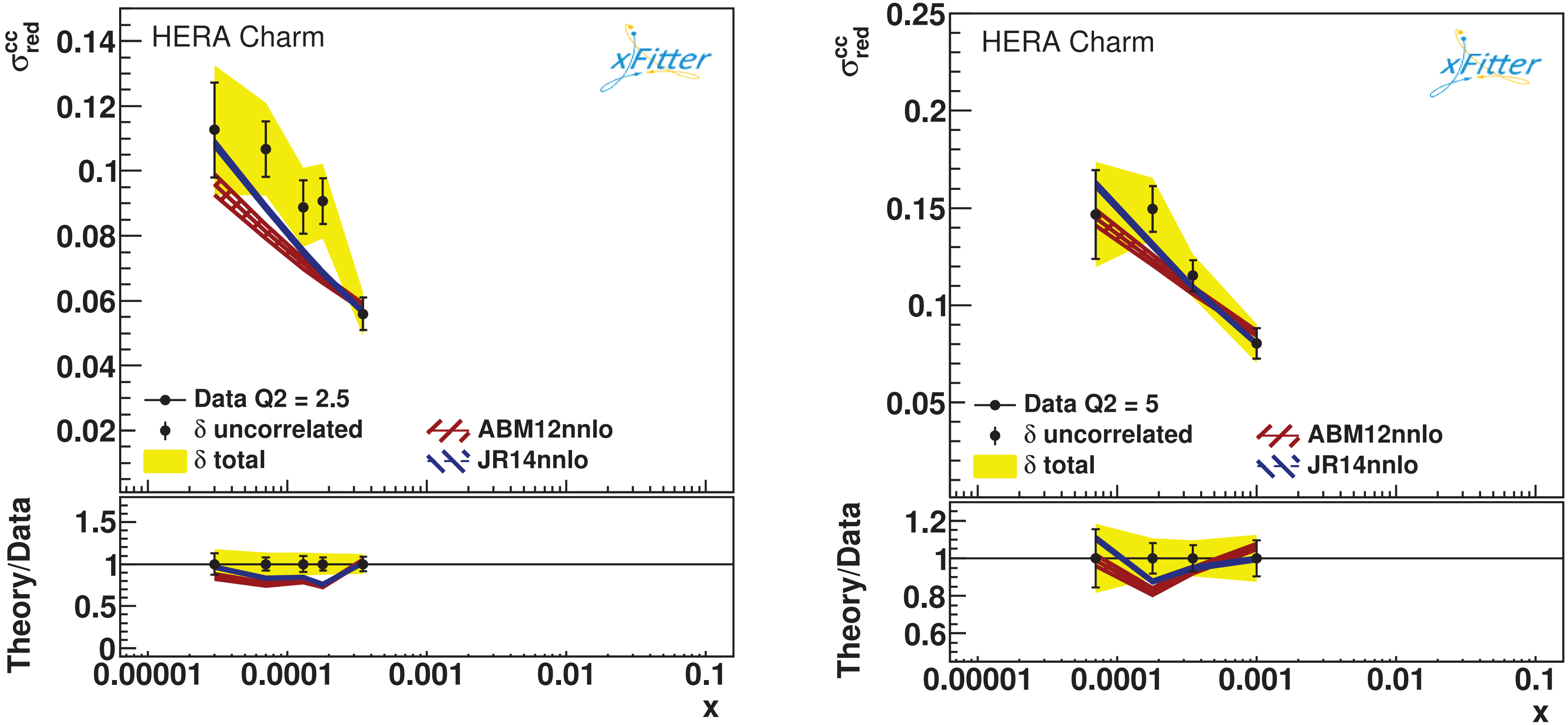

- $F_2^c$ in different schemes compared to H1- and ZEUS-data
  - GMVFN scheme in BMSN prescription (solid lines)
  - 3-flavor scheme (dash-dotted lines)
  - 4-flavor scheme (dashed lines)
  - charm-quark mass $m_c = 1.43$ GeV (vertical dotted line)

Alekhin, Blümlein, Klein, S.M. '09



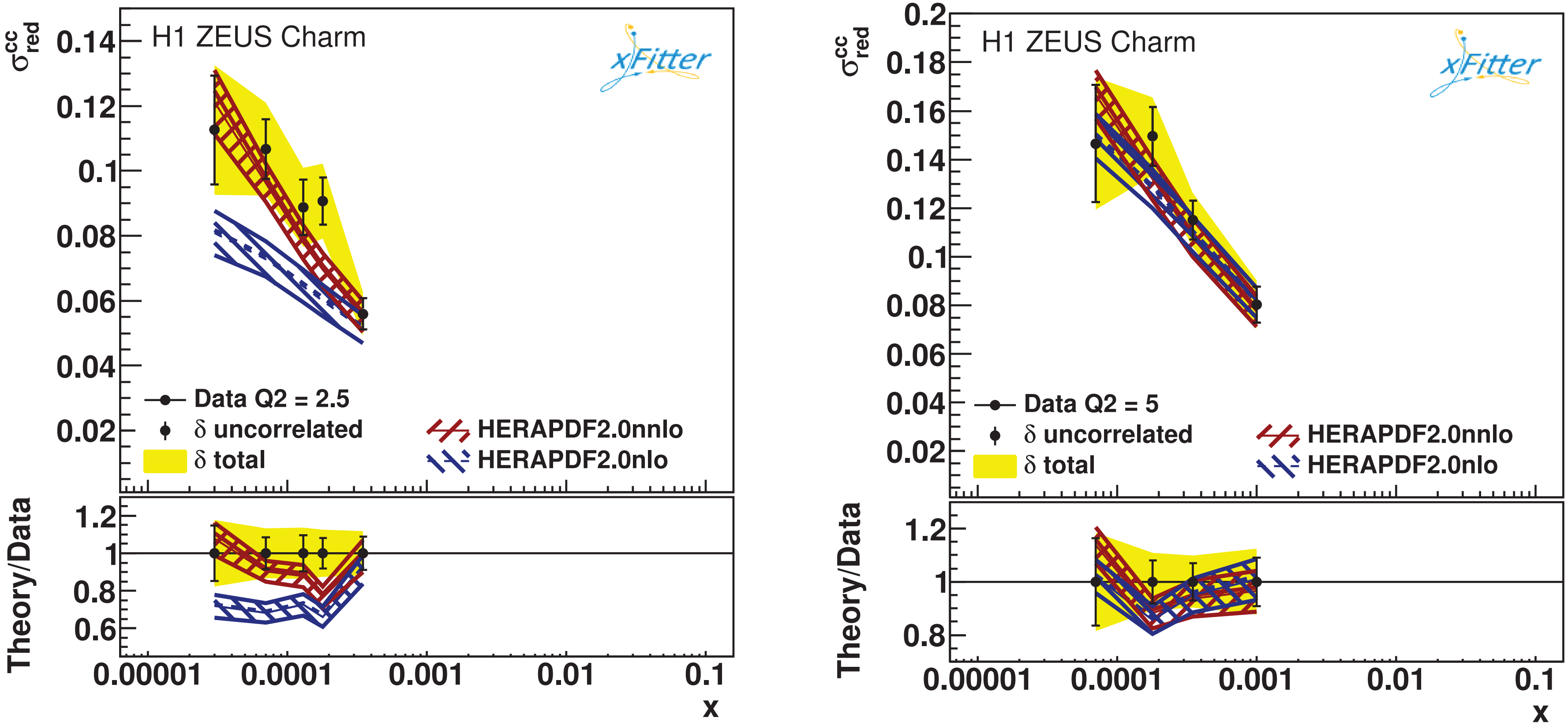

- $F_2^b$ in different schemes compared to H1- and ZEUS-data
  - GMVFN scheme in BMSN prescription (solid lines)
  - 3-flavor scheme (dash-dotted lines)
  - 4-flavor scheme (dashed lines)
  - bottom-quark mass $m_b = 4.30\ \mathrm{GeV}$ (vertical dotted line)

# *Comparision to data*



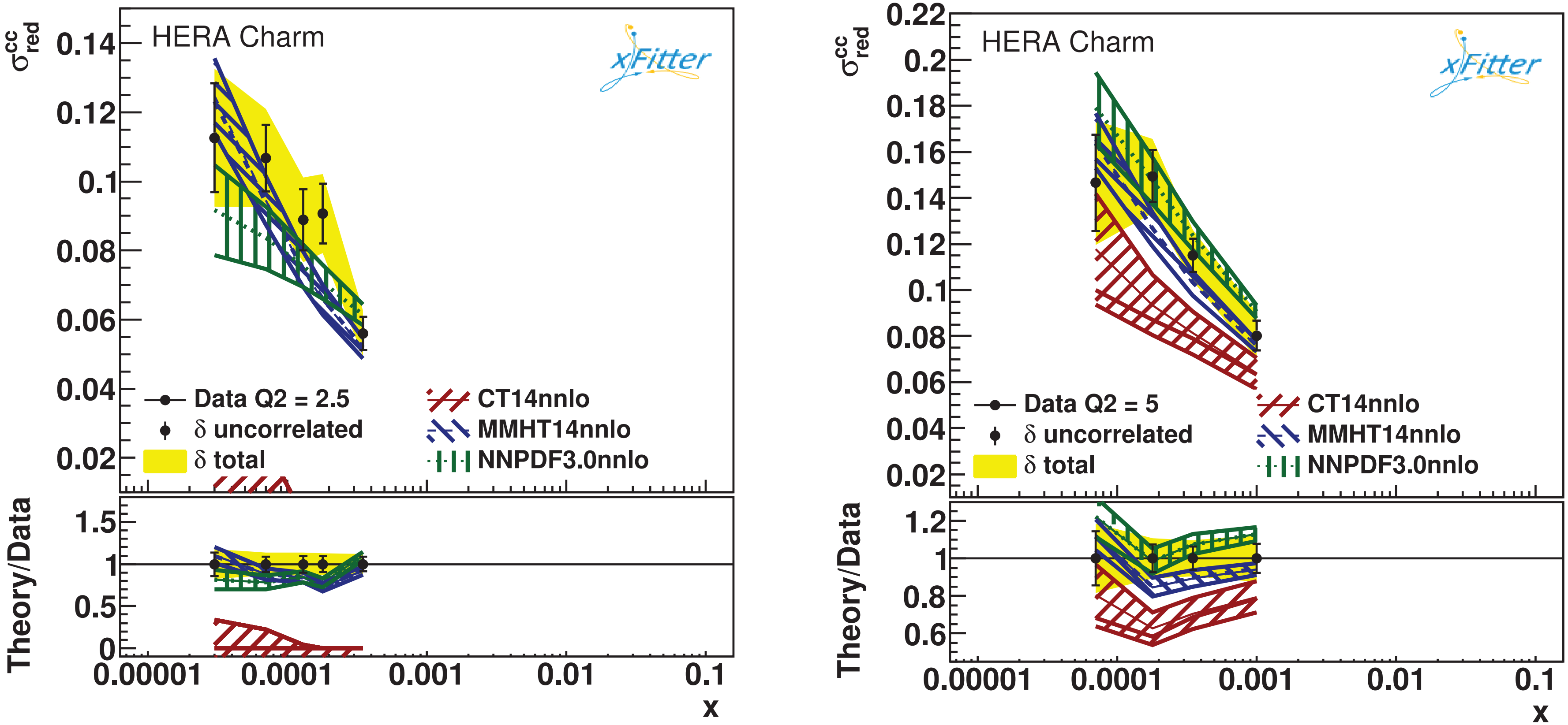

- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - ABM12 and JR14 using FFNS scheme

# *Comparision to data*



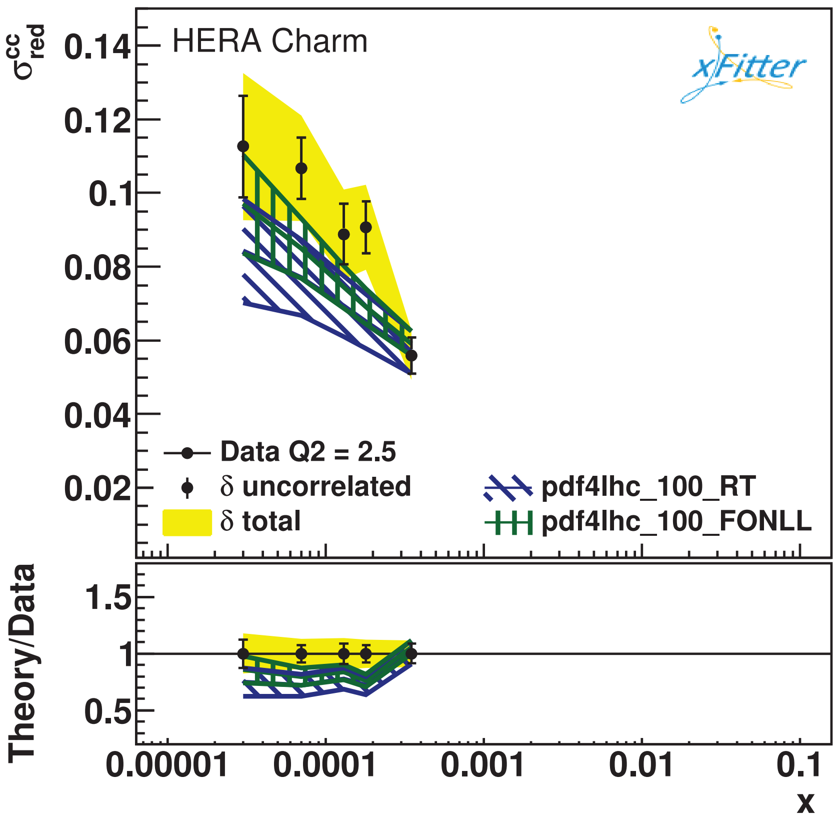

- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - HERAPDF2.0 using the RT optimal GM-VFNS scheme

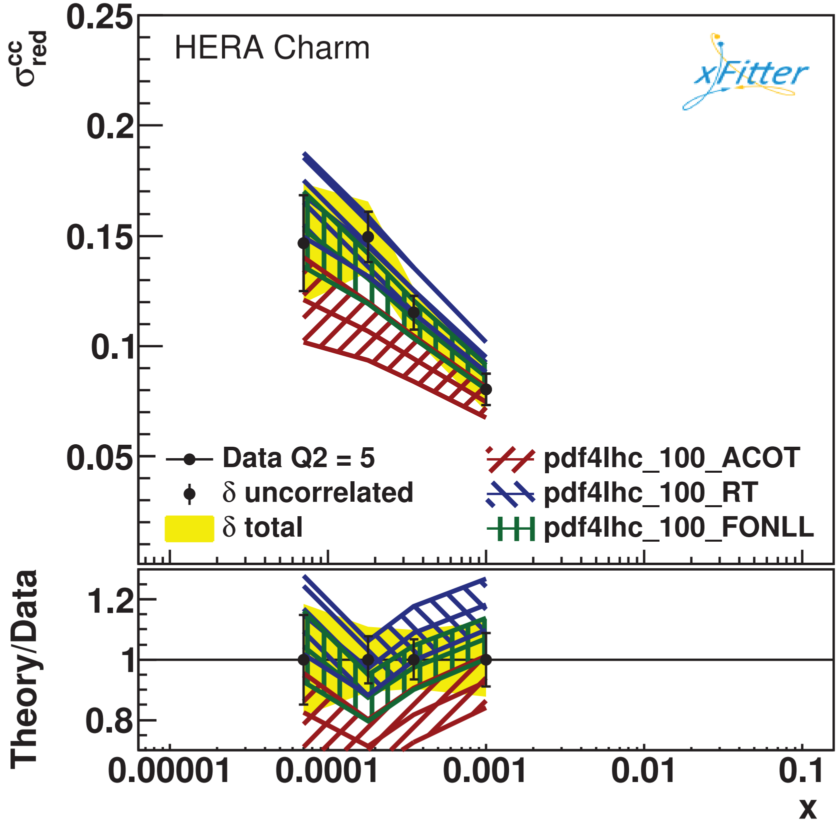
# *Comparision to data*




- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - CT14, MMHT14 and NNPDF3.0 using FFNS scheme

# *Comparision to data*




- Comparision of theory predictions for the DIS pair-production of charm quarks to the combined HERA data H1 & ZEUS coll. arXiv:1211.1182
  - ABM12 and JR14 using FFNS scheme

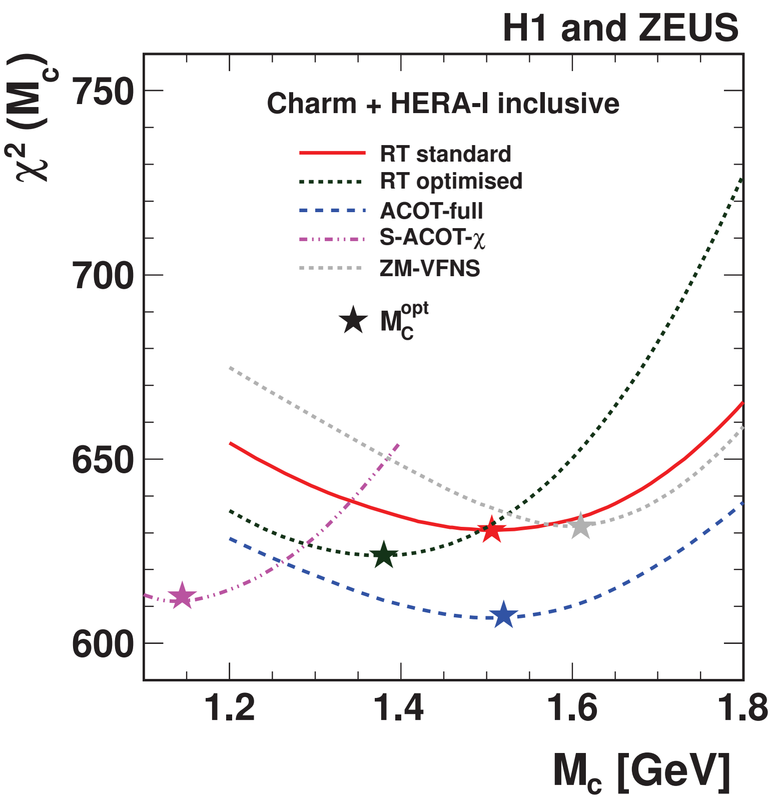
# Charm quark mass vs. data

- Data on $F_2^c$ at HERA has correlation of $m_c$, $\alpha_S(M_Z)$, gluon PDF

  $\sigma_{c\bar{c}} \sim \alpha_s \, m_c^2 \, g(x)$

- Comparison of measured data with predictions in various VFNS schemes
  - data shows very good sensitivity to value of $m_c$
  - fit of value of $m_c$ strongly dependent on particular choice of VFNS

    H1 coll. arxiv:1211.1182



# *Charm quark mass in PDF fits*

| | $m_c$ (GeV) | $m_c$ scheme | $\chi^2$/NDP (HERA data) | $F_2^c$ scheme | NNLO Wilson coeff. |
|---|---|---|---|---|---|
| ABMP16<br>arXiv:1701.05838 | $1.252 \pm 0.018$ | $m^{\overline{MS}}$ | 62/52 | FFNS($n_f = 3$) | **yes** |
| CT14<br>arXiv:1506.07443 | 1.3 (assumed) | $m^{\rm pole}$ | 582/52 (64/47) | S-ACOT-$\chi$ | **no** |
| MMHT<br>arXiv:1510.02332 | 1.25 | $m^{\rm pole}$ | 75/52 | RT optimal | **no** |
| NNPDF3.0<br>arXiv:1410.8849 | 1.275 (assumed) | $m^{\rm pole}$ | 67/52 | FONLL-C | **no** |
| PDF4LHC15<br>arXiv:1510.03865 | - | - | 58/52 | FONLL-B | - |
| | - | - | 71/52 | RT optimal | - |
| | - | - | 51/47 | S-ACOT-$\chi$ | - |

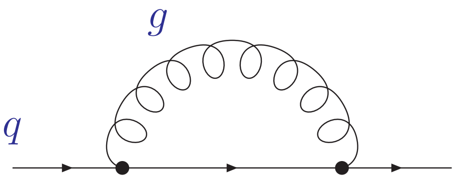
- PDG quotes running masses:
  charm: $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV, bottom: $m_b(m_b) = 4.20^{+0.17}_{-0.07}$ GeV
- Values of charm-quark pole mass for CT14, MMHT14 and NNPDF3.0 not compatible with world average of PDG

# *Quark mass renormalization*

## *Pole mass*

- Based on (unphysical) concept of heavy-quark being a free parton

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{p^2=m_q^2}$$




  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
- Renormalon ambiguity in definition of pole mass of $\mathcal{O}(\Lambda_{QCD})$
  Bigi, Shifman, Uraltsev, Vainshtein '94; Beneke, Braun '94; Smith, Willenbrock '97

## *$\overline{MS}$ mass*

- Free of infrared renormalon ambiguity
- Conversion between $m_{\rm pole}$ and $\overline{MS}$ mass $m(\mu_R)$ in perturbation theory known to four loops in QCD Marquard, Smirnov, Smirnov, Steinhauser '15
  - does not converge in case of charm quark

$$\begin{aligned} m_c(m_c) = 1.27 \text{ GeV} &\longrightarrow m_c^{\rm pole} = 1.47 \text{ GeV (one loop)} \\ &\longrightarrow m_c^{\rm pole} = 1.67 \text{ GeV (two loops)} \\ &\longrightarrow m_c^{\rm pole} = 1.93 \text{ GeV (three loops)} \\ &\longrightarrow m_c^{\rm pole} = 2.39 \text{ GeV (four loops)} \end{aligned}$$

# *Charm quark mass and the Higgs cross section*

## *MMHT14*

- "Tuning" of charm mass $m_c$ parameter effects the Higgs cross section
  - linear rise in $\sigma(H) = 40.5 \ldots 42.6$ pb for $m_c = 1.15 \ldots 1.55$ GeV with MMHT14 PDFs Martin, Motylinski, Harland-Lang, Thorne '15

| $m_c^{\text{pole}}$ [GeV] | $\alpha_s(M_Z)$ (best fit) | $\chi^2$/NDP (HERA data on $\sigma^{c\bar{c}}$) | $\sigma(H)^{\text{NNLO}}$ [pb] best fit $\alpha_s(M_Z)$ | $\sigma(H)^{\text{NNLO}}$ [pb] $\alpha_s(M_Z) = 0.118$ |
|---|---|---|---|---|
| 1.15 | 0.1164 | 78/52 | 40.48 | (42.05) |
| 1.2 | 0.1166 | 76/52 | 40.74 | (42.11) |
| 1.25 | 0.1167 | **75/52** | **40.89** | (42.17) |
| 1.3 | 0.1169 | 76/52 | 41.16 | (42.25) |
| 1.35 | 0.1171 | 78/52 | 41.41 | (42.30) |
| 1.4 | 0.1172 | **82/52** | 41.56 | **(42.36)** |
| 1.45 | 0.1173 | 88/52 | 41.75 | (42.45) |
| 1.5 | 0.1173 | 96/52 | 41.81 | (42.51) |
| 1.55 | 0.1175 | 105/52 | 42.08 | (42.58) |

# *Charm quark mass and the Higgs cross section*

## *NNPDF*

- Same trend: lighter charm mass implies smaller Higgs cross section
  - fit range for $m_c$ too small and no correlation with value of $\alpha_s(M_Z)$
  - best fits with NNPDF2.1 and NNPDF30 give range $\sigma(H) = 42.6 \ldots 44.2$ pb

| PDF sets | $m_c^{\text{pole}}$ [GeV] | $\alpha_s(M_Z)$ (fixed) | $\chi^2$/NDP (HERA data on $\sigma^{c\bar{c}}$) | $\sigma(H)^{\text{NNLO}}$ [pb] fixed $\alpha_s(M_Z)$ |
|---|---|---|---|---|
| NNPDF2.1 [arXiv:1107.2652] | $\sqrt{2}$ | 0.119 | **65/52** | $44.18 \pm 0.49$ |
| | 1.5 | 0.119 | 78/52 | $44.54 \pm 0.51$ |
| | 1.6 | 0.119 | 92/52 | $44.74 \pm 0.50$ |
| | 1.7 | 0.119 | 110/52 | $44.95 \pm 0.51$ |
| NNPDF2.3 [arXiv:1207.1303] | $\sqrt{2}$ | 0.118 | 71/52 | $43.77 \pm 0.41$ |
| NNPDF3.0 [arXiv:1410.8849] | 1.275 | 0.118 | **67/52** | $42.59 \pm 0.80$ |

# *Strong coupling constant*

# *Strong coupling constant (1992)*

| | $\alpha_s(\mathrm{M}_\mathrm{Z}^2)$ |
|---|---|
| $R_\tau$ | $0.117\ {}^{+\ 0.010}_{-\ 0.016}$ |
| DIS | $0.112 \pm 0.007$ |
| $\Upsilon$ Decays | $0.110 \pm 0.010$ |
| $R_{e^+e^-}(s < 62\text{GeV})$ | $0.140 \pm 0.020$ |
| $p\bar{p} \to W + jets$ | $0.121 \pm 0.024$ |
| $\Gamma(Z \to \text{hadrons})/\Gamma(Z \to l\bar{l})$ | $0.132 \pm 0.012$ |
| Jets at LEP | $0.122 \pm 0.009$ |
| **Average** | $0.118 \pm 0.007$ |

**G. Altarelli (1992)**
**in *QCD - 20 Years Later*,**
**CERN-TH-6623-92**

## *Essential facts*

- World average 1992 $\alpha_s(M_Z) = 0.118 \pm 0.007$
- Central value at NLO QCD
  - still right, but for very different reasons
- Error at NLO QCD
  - now down to $\sim 0.0050 - 0.0040$ (theory scale uncertainty)

# Strong coupling constant (2016)

## Measurements at NNLO

- Values of $\alpha_s(M_Z)$ at NNLO from PDF fits

| | | | |
|---|---|---|---|
| SY | $0.1166 \pm 0.013$ | $F_2^{ep}$ | Santiago, Yndurain '01 |
| | $0.1153 \pm 0.063$ | $xF_3^{\nu N}$ (heavy nucl.) | |
| A02 | $0.1143 \pm 0.013$ | DIS | Alekhin '01 |
| MRST03 | $0.1153 \pm 0.0020$ | | Martin, Roberts, Stirling, Thorne '03 |
| BBG | $0.1134\ {}^{+\,0.0019}_{-\,0.0021}$ | valence analysis, NNLO | Blümlein, Böttcher, Guffanti '06 |
| GRS | $0.112$ | valence analysis, NNLO | Glück, Reya, Schuck '06 |
| A06 | $0.1128 \pm 0.015$ | | Alekhin '06 |
| JR08 | $0.1128 \pm 0.0010$ | dynamical approach | Jimenez-Delgado, Reya '08 |
| | $0.1162 \pm 0.0006$ | including NLO jets | |
| ABKM09 | $0.1135 \pm 0.0014$ | HQ: FFNS $n_f = 3$ | Alekhin, Blümlein, Klein, S.M. '09 |
| | $0.1129 \pm 0.0014$ | HQ: BSMN | |
| MSTW | $0.1171 \pm 0.0014$ | | Martin, Stirling, Thorne, Watt '09 |
| Thorne | $0.1136$ | [DIS+DY, HT*] (2013) | Thorne '13 |
| $\text{ABM11}_J$ | $0.1134 \ldots 0.1149\ \pm 0.0012$ | Tevatron jets (NLO) incl. | Alekhin, Blümlein, S.M. '11 |
| NN21 | $0.1173 \pm 0.0007$ | (+ heavy nucl.) | NNPDF '11 |
| ABM12 | $0.1133 \pm 0.0011$ | | Alekhin, Blümlein, S.M. '13 |
| | $0.1132 \pm 0.0011$ | (without jets) | |
| CT10 | $0.1140$ | (without jets) | Gao et al. '13 |
| CT14 | $0.1150\ {}^{+\,0.0060}_{-\,0.0040}$ | $\Delta\chi^2 > 1$ (+ heavy nucl.) | Dulat et al. '15 |
| MMHT | $0.1172 \pm 0.0013$ | (+ heavy nucl.) | Martin, Motylinski, Harland-Lang, Thorne '15 |

# *Strong coupling constant (2017)*

## *Other measurements of $\alpha_s$ at NNLO*

- Values of $\alpha_s(M_Z)$ at NNLO from measurements at colliders

| | | | |
|---|---|---|---|
| 3-jet rate | $0.1175 \pm 0.0025$ | Dissertori et al. 2009 | arXiv:0910.4283 |
| $e^+e^-$ thrust | $0.1131\,^{+\,0.0028}_{-\,0.0022}$ | Gehrmann et al. | arXiv:1210.6945 |
| $e^+e^-$ thrust | $0.1140 \pm 0.0015$ | Abbate et al. | arXiv:1204.5746 |
| $C$-parameter | $0.1123 \pm 0.0013$ | Hoang et al. | arXiv:1501.04111 |
| CMS | $0.1151 \pm 0.0033$ | $t\bar{t}$ | arXiv:1307.1907 |
| NLO Jets ATLAS | $0.111\,^{+\,0.0017}_{-\,0.0007}$ | | arXiv:1312.5694 |
| NLO Jets CMS | $0.1148 \pm 0.0055$ | | arXiv:1312.5694 |

## *PDG average*

- Some tension with the PDG average at NNLO

  PDG (Bethke, Dissertori, Salam) ‘16

$$\alpha_s(M_Z) = 0.1181 \pm 0.0013$$

  - PDG value driven by lattice determinations (and low scale $\tau$-data)

# Differences in $\alpha_s$ determinations

## Why $\alpha_s$ values from MSTW and NNPDF are large

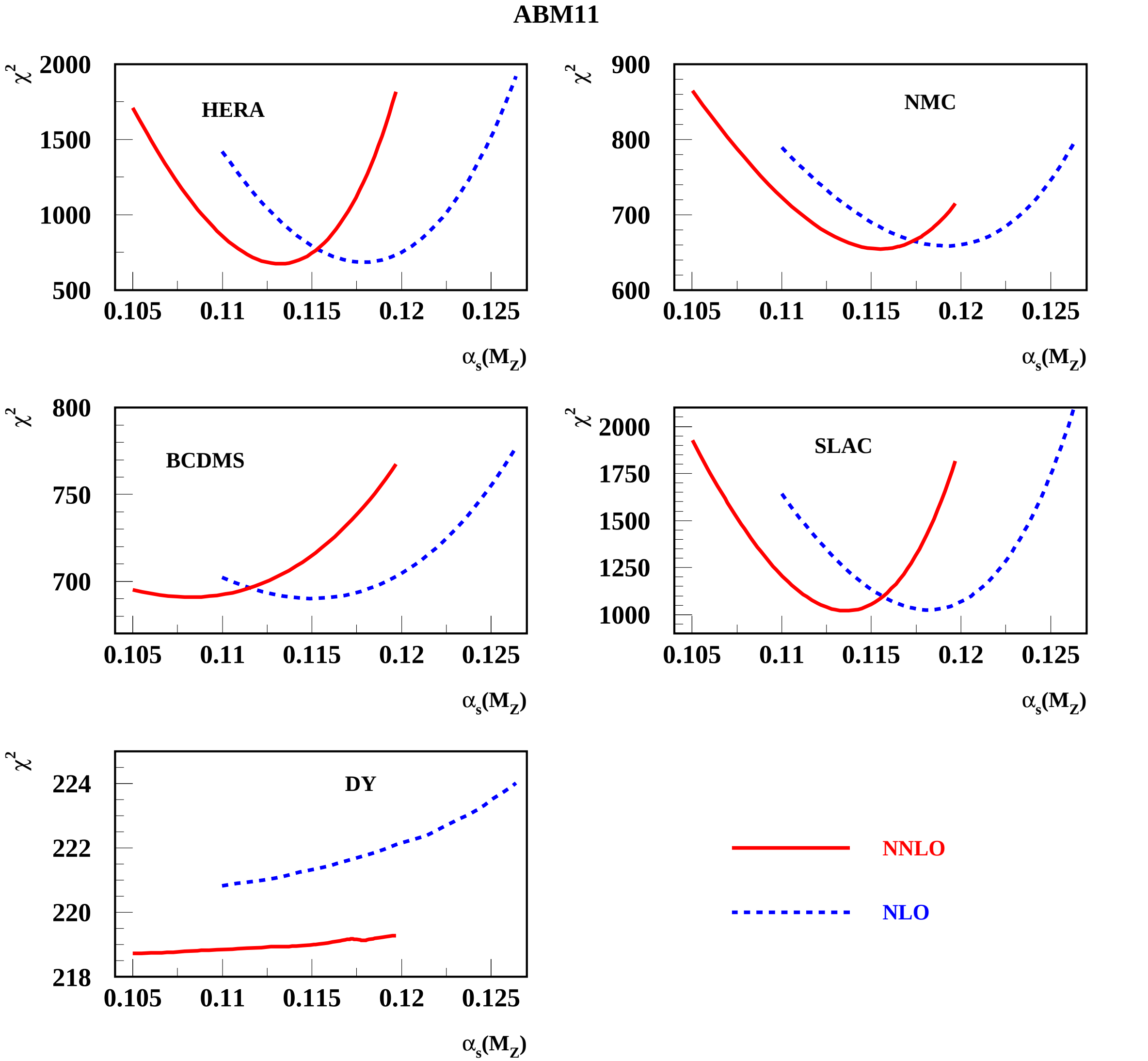
- Differences result from different physics models and analysis procedures
- Fits of DIS data
  - target mass corrections (powers of nucleon mass $M_N^2/Q^2$)
  - higher twist $F_2^{\rm ht} = F_2 + ht^{(4)}(x)/Q^2 + ht^{(6)}(x)/Q^4 + \dots$
  - correlation of errors among different data sets

| | $\alpha_s$ | NNLO | target mass corr. | higher twist | error correl. |
|---|---|---|---|---|---|
| ABM12 | $0.1132 \pm 0.0011$ | yes | yes | yes | yes |
| NNPDF21 | $0.1173 \pm 0.0007$ | (yes) | yes | no | yes |
| MSTW | $0.1171 \pm 0.0014$ | (yes) | no | no | no |
| MMHT | $0.1172 \pm 0.0013$ | (yes) | no | no | – |

- Effects for differences are understood
  - variants of ABM with no higher twist etc. reproduce larger $\alpha_s$ values
    Alekhin, Blümlein, S.M. '11

# *Zooming in on ABM*

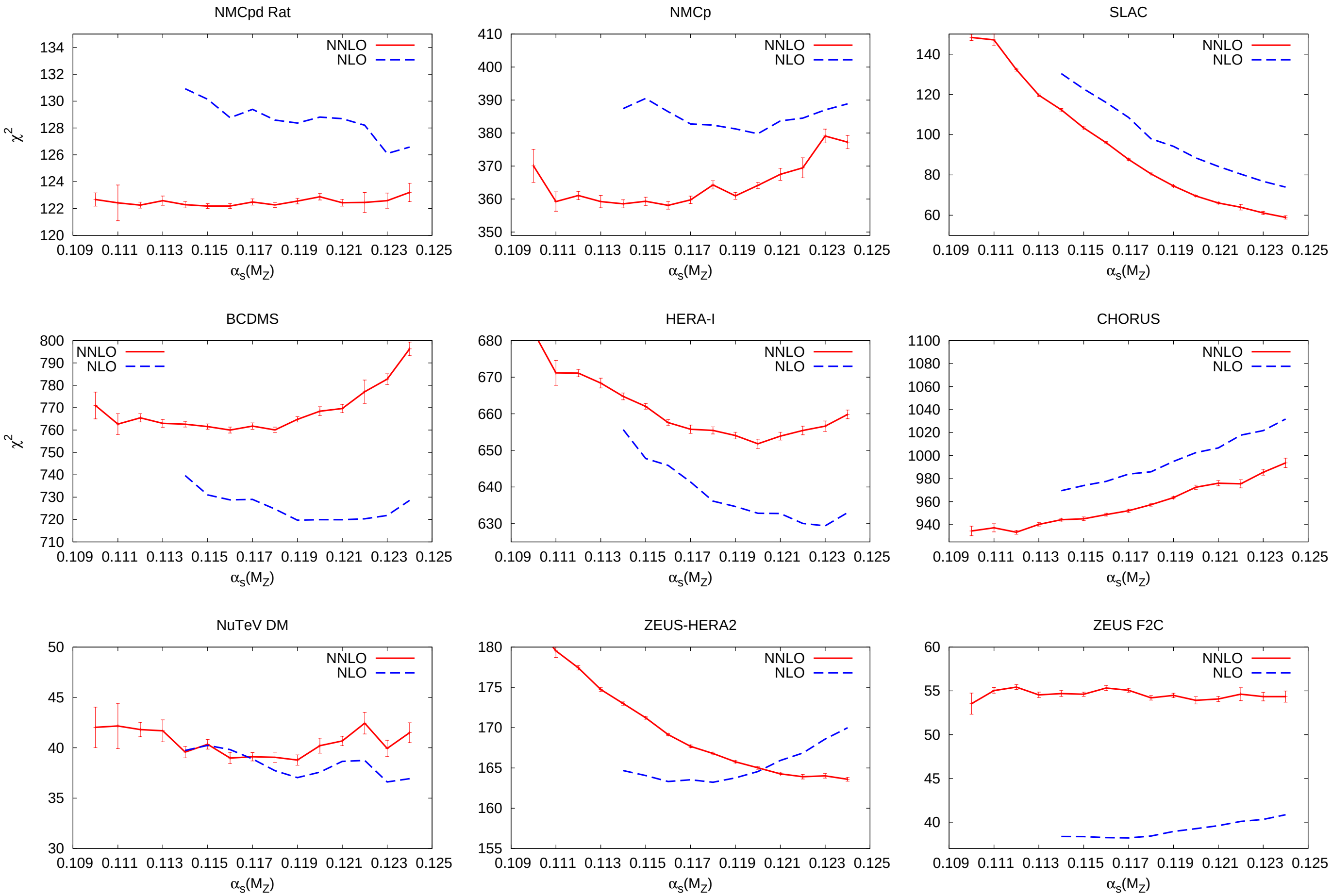
## $\alpha_s$ *from DIS and PDFs*




- Profile of $\chi^2$ for different data sets in ABM11 PDF fit Alekhin, Blümlein, S.M. '12

# *Zooming in on NNPDF*

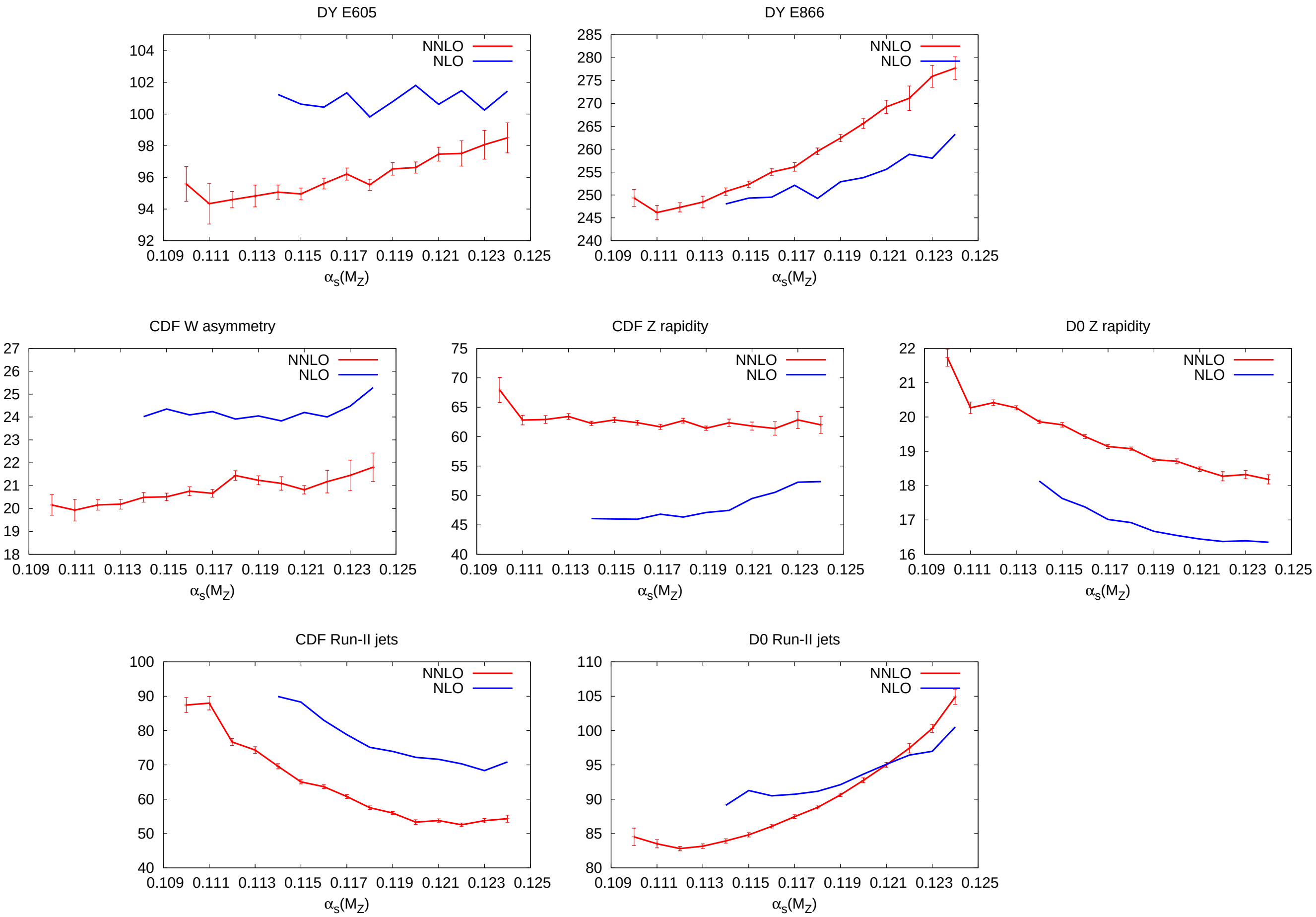
## $\alpha_s$ *from DIS and PDFs*




- Profile of $\chi^2$ for different data sets in NNPDF21 fit Ball et al. '11

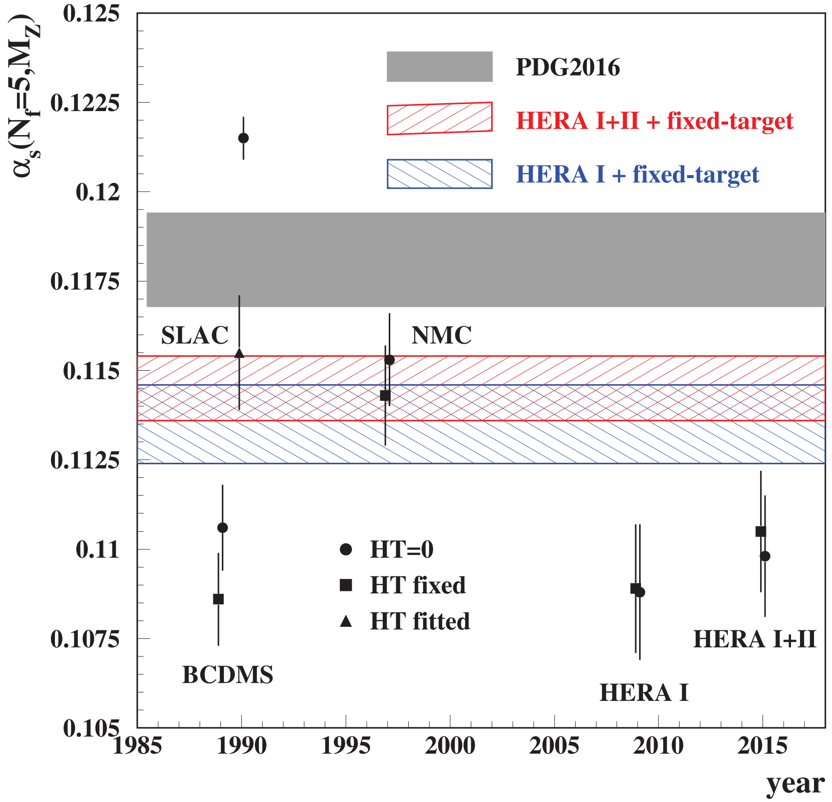
# *Zooming in on NNPDF*

## $\alpha_s$ *from DIS and PDFs*




- Profile of $\chi^2$ for different data sets in NNPDF21 fit Ball et al. '11

# World DIS data and the value of $\alpha_s$

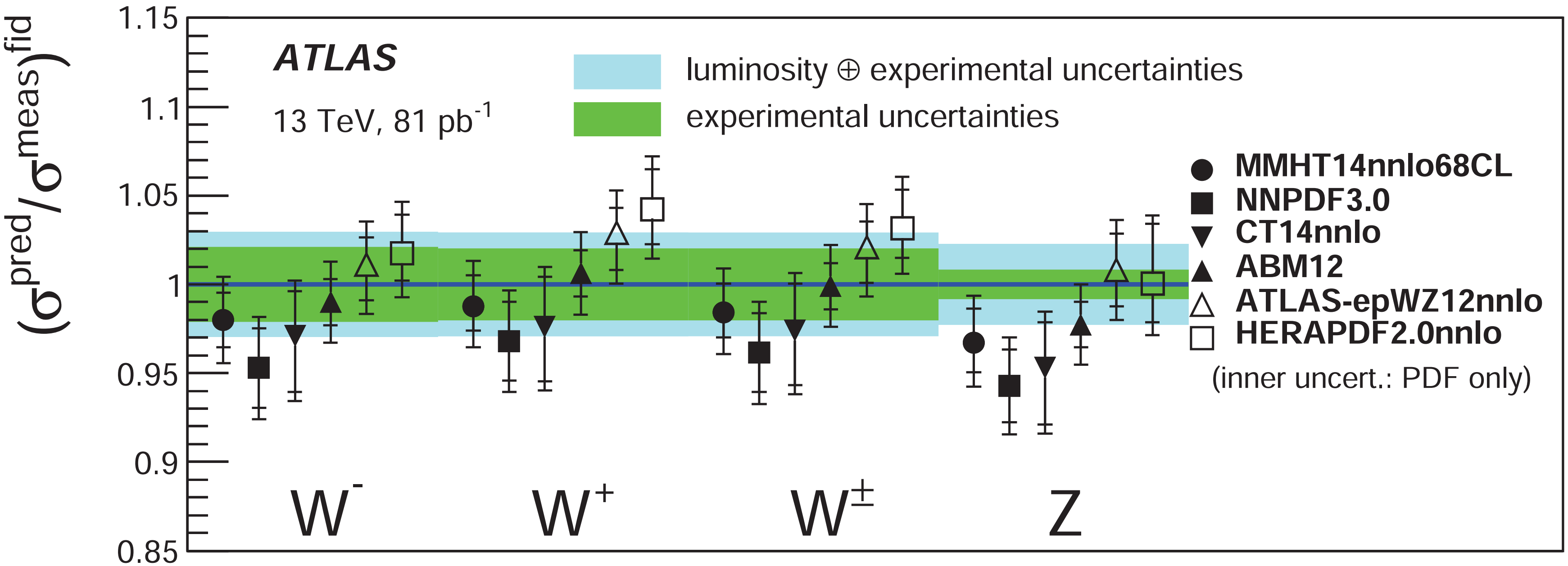



- Value of $\alpha_(M_Z)$ is lower than PDG average
  - value of $\alpha_(M_Z)$ is pulled up by SLAC and NMC and pulled down by BCDMS and HERA data
- Only $\alpha_(M_Z)$ preferred by SLAC data is compatible with PDG average (provided higher twist terms are accounted for)
- Update of the $\alpha_s$ determination with combined data HERA I+II
  - value of $\alpha_(M_Z)$ increases by $1\sigma$

## $W^{\pm}$- and $Z$-boson production

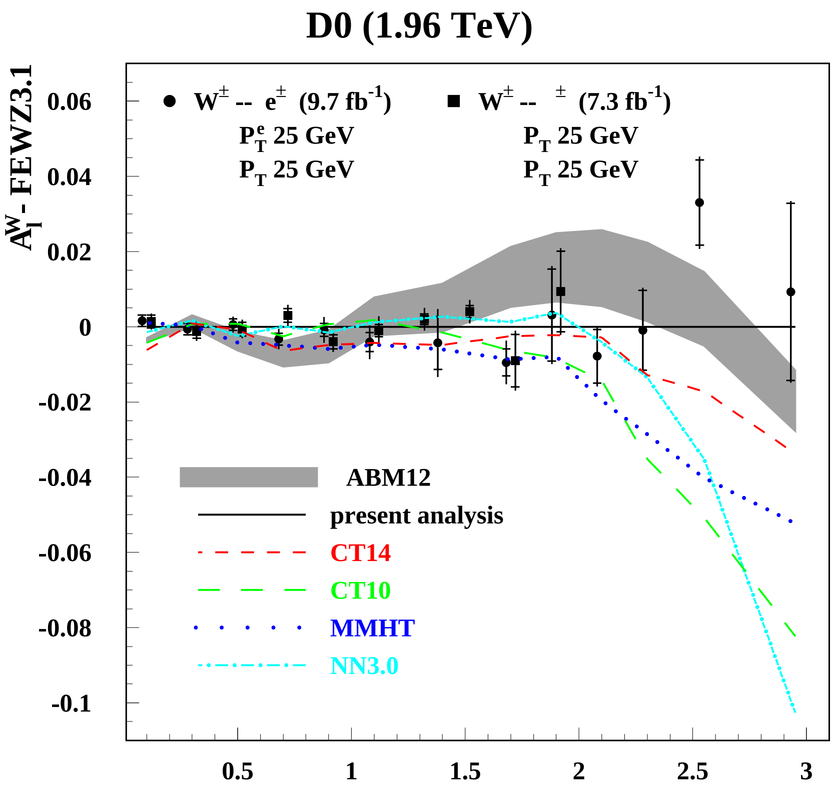
# $W$- and $Z$-boson cross sections

- High precision data from LHC ATLAS, CMS, LHCb and Tevatron D0
  - differential distributions extend to forward region
  - sensitivity to light quark flavors at $x \simeq 10^{-4}$
  - statistically significant: $NDP = 172$ in ABMP16
- ATLAS measurement at $\sqrt{s} = 13$ TeV from arXiv:1603.09222



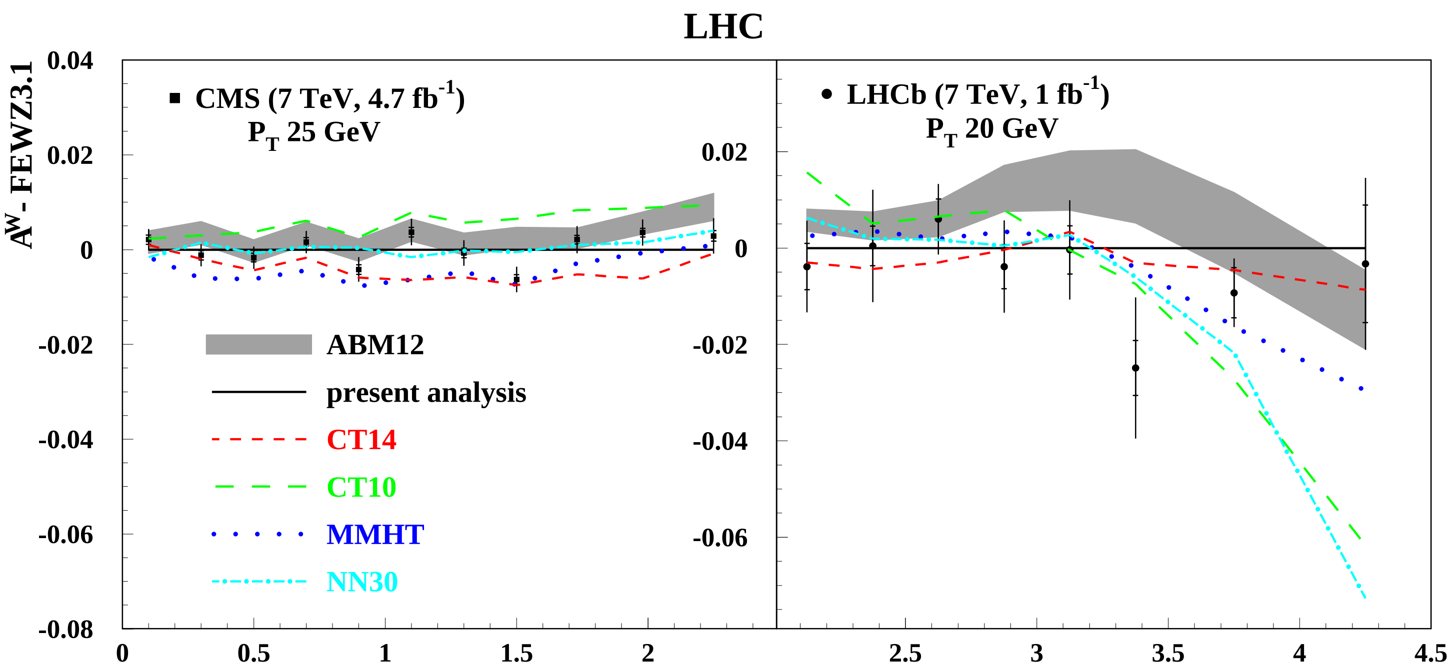

- Spread in predictions from different PDFs significantly larger than experimental precision

# *Tevatron charged lepton asymmetry*



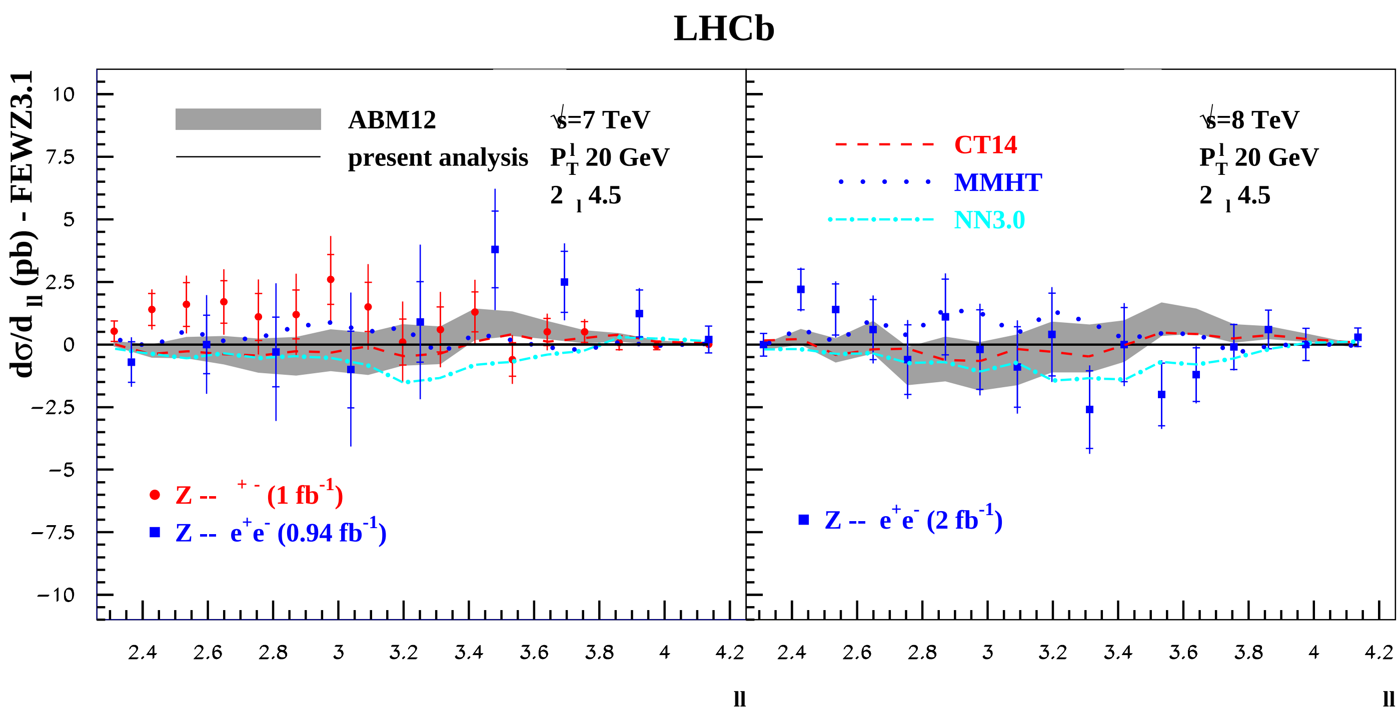

- D0 data for $p\bar{p} \to W^\pm + X \to l^\pm \nu$ (electrons and muons) at $\sqrt{s} = 1.96$ TeV
- Charged lepton asymmetry as function of pseudo-lepton rapidity $\eta_l$
- NNLO QCD predictions with FEWZ (version 3.1)
- Comparison with ABM12 (including combined PDF+$\alpha_s$ uncertainty), CT10, CT14, MMHT, and NN3.0

# *Muon charge asymmetry from LHC*




- CMS and LHCb data for $pp \to W^{\pm} + X \to \mu^{\pm}\nu$ at $\sqrt{s} = 7$ TeV
- Problematic data points at $\eta_\mu = 3.375$ in LHCb data are omitted in fit

# *Z*-boson production from LHC

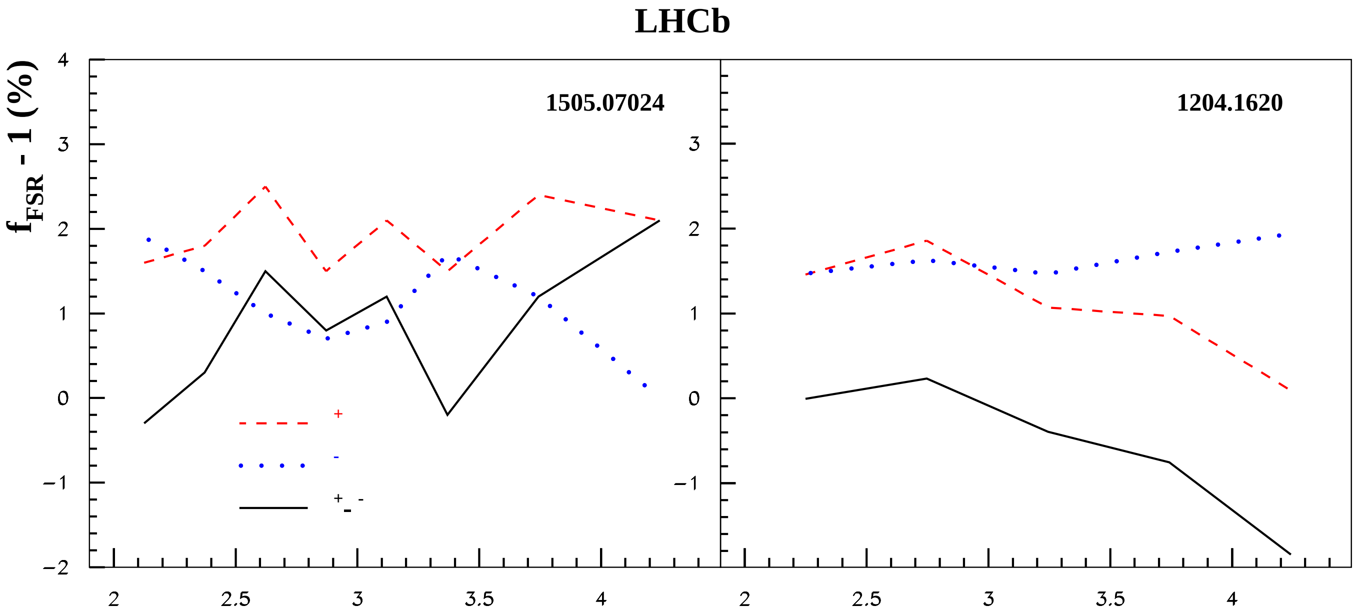



- LHCb data for $pp \to Z + X \to l\bar{l}$ (muon and electron) at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV
- Comparison with ABM12 (including combined PDF+$\alpha_s$ uncertainty), CT14, MMHT, and NNPDF3.0

# *Theory issues (I)*

## *Final-state-radiation effects*

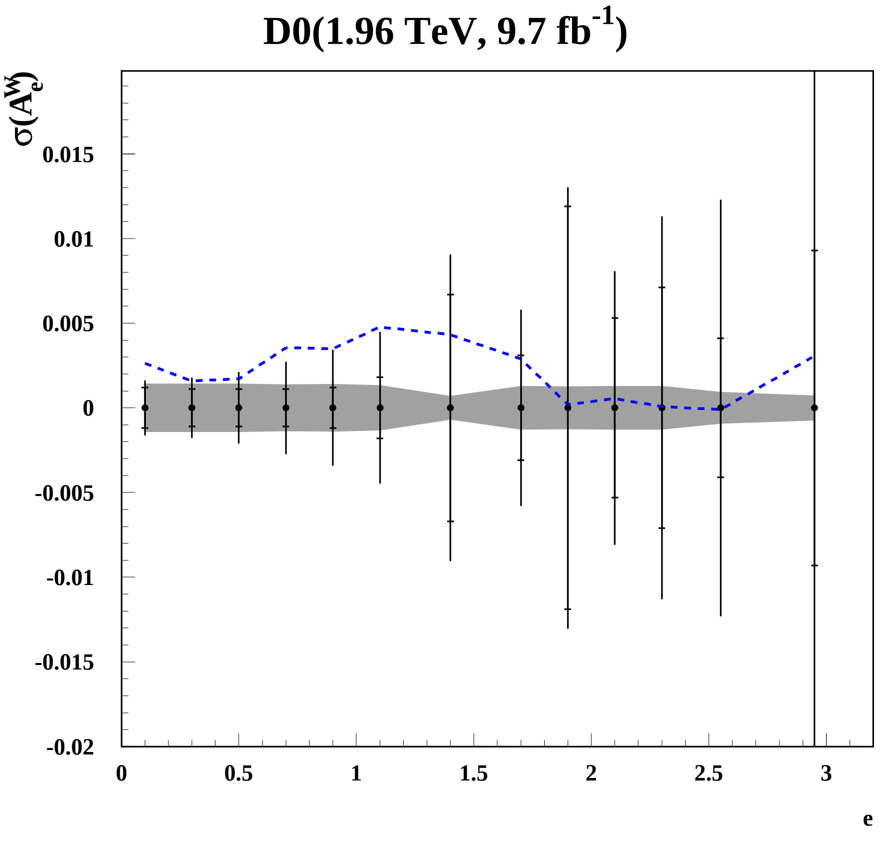
- QED corrections in $W^{\pm}$- and $Z$-boson decays applied to data of LHCb
  - left: FSR effects from mean of simulations with Herwig++ and Pythia8 with anomalous irregularity at $\eta_\mu = 3.375$
  - right: earlier analysis of LHCb with smooth FSR corrections from PHOTOS Monte Carlo Golonka, Was '05



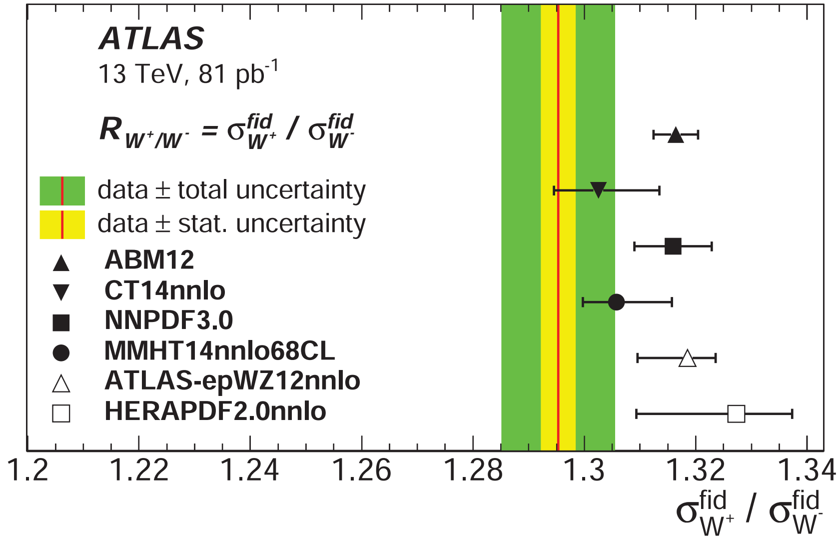

- Dropping problematic data points at $\eta_\mu = 3.375$ reduces $\chi^2$ value by some 10 units

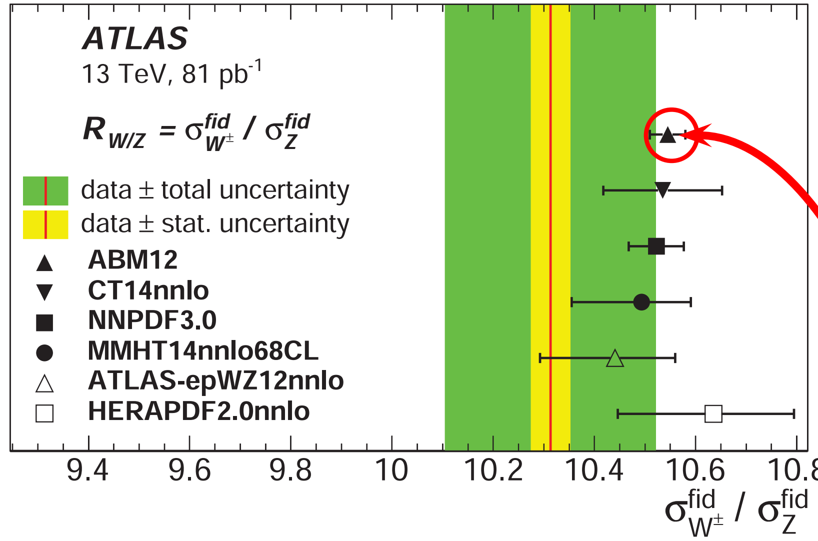
# *Theory issues (II)*

- Data on electron asymmetry with high precision at central rapidities D0
- NNLO corrections in coefficient functions not uniform in $\eta_e$ (dashed curve)
- Numerical accuracy at NNLO (shaded area) obtained with FEWZ (v3.1)
- Accuracy of $\mathcal{O}(1\ \mathrm{ppm})$ to meet uncertainties in experimental data requires $\mathcal{O}(10^4 \mathrm{h})$ of running FEWZ (v3.1) at NNLO

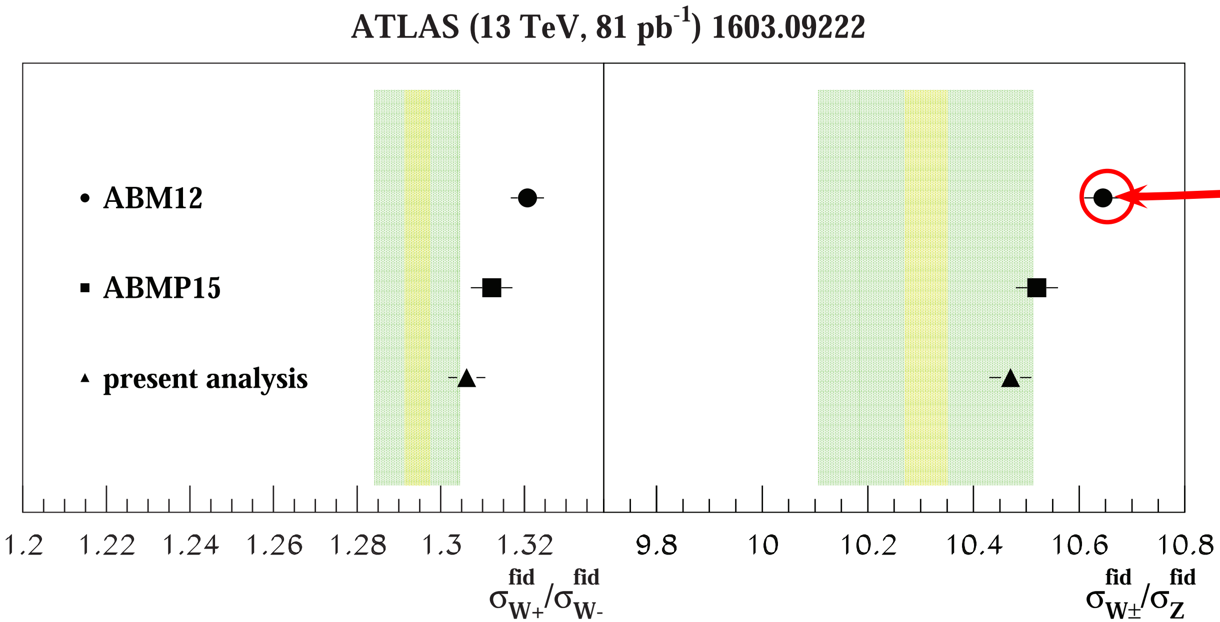


# *Theory issues (III)*




**DYNNLO (v1.5)**
Catani, Grazzini '07



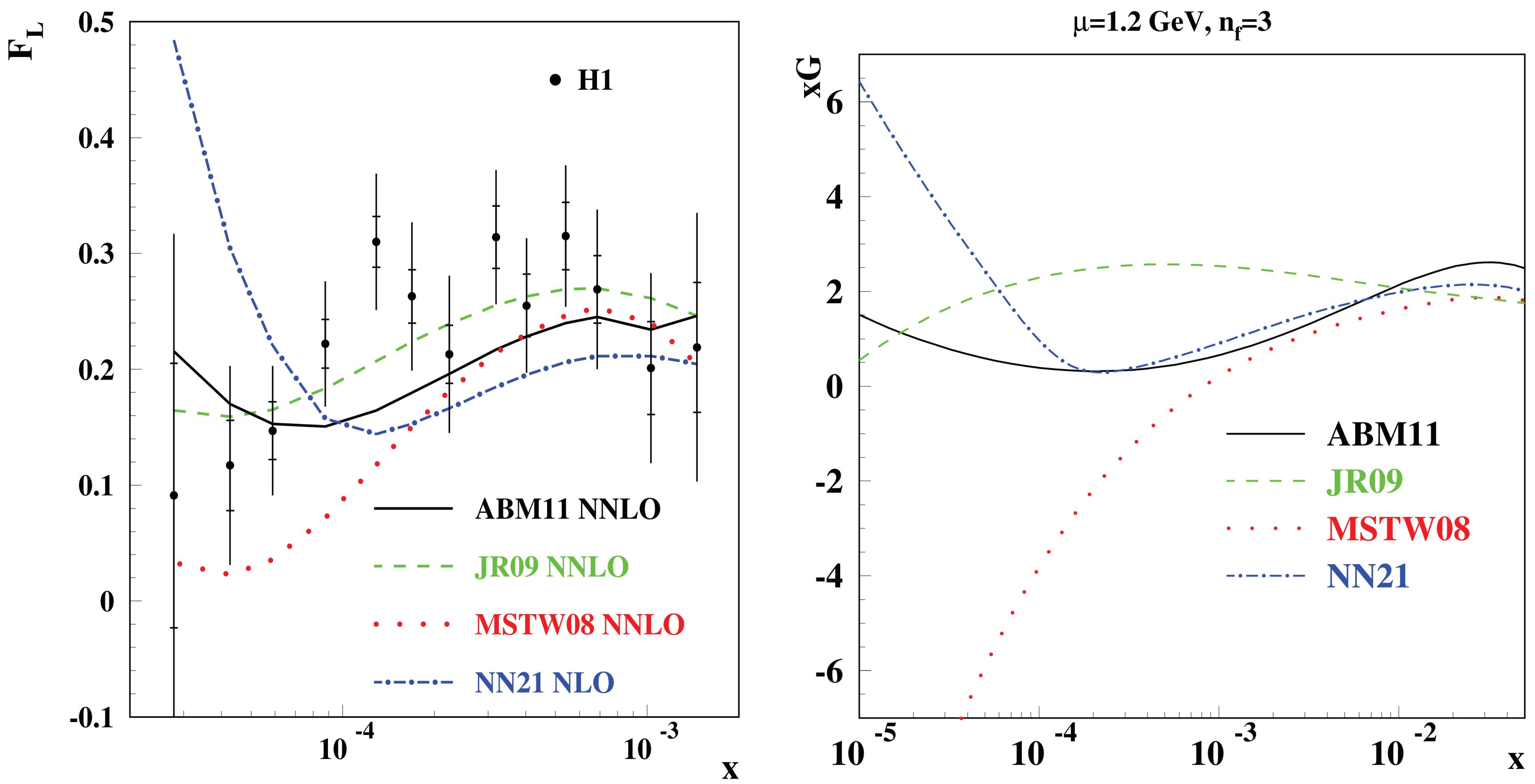

**FEWZ (v3.1)**
Gavin, Li, Petriello, Quackenbush '12

- Differences at NNLO between DYNNLO and FEWZ up to $\mathcal{O}(1\%)$ or more

## *Gluon PDF at small $x$*

# *Longitudinal structure function*

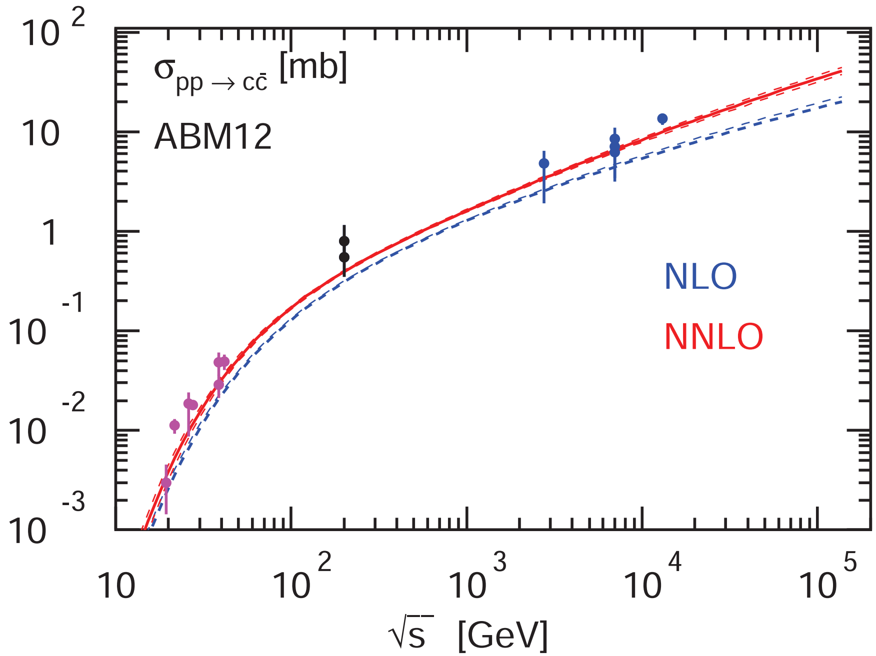
## Discrimination of the small-x gluons

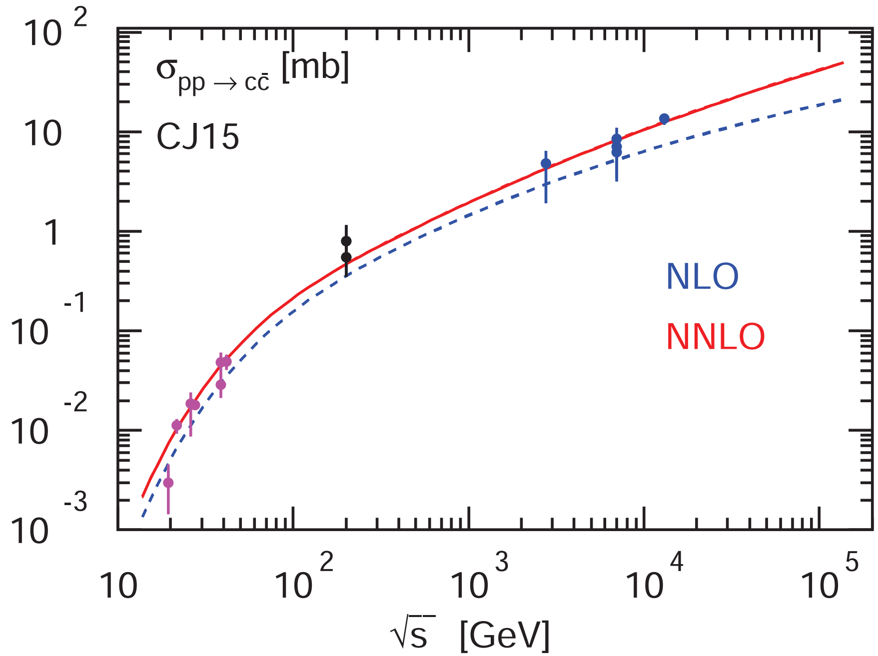



Slide from S. Alekhin

# *Charm quark hadro-production (I)*

- Charm-quark hadro-production at high energies
  - quark-gluon parton luminosity dominates
- Gluon PDF at small-$x$
  - fits yield $xg(x) \simeq x^a$; e.g. $a \simeq -0.2$ in ABM12
  - kinematic coverage of data down to $x \simeq 10^{-5}$ (DIS structure function $F_L$)
- Predictions compatible with LHC measurements (Alice, ATLAS, LHCb)

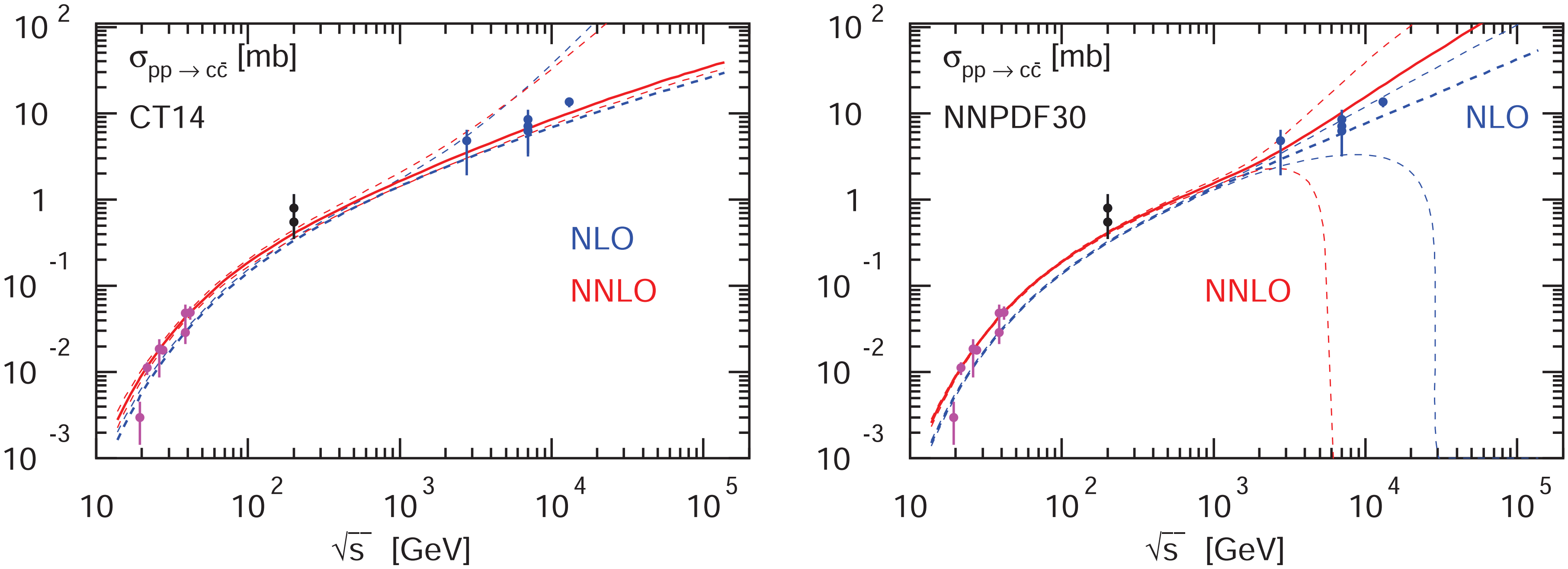


# *Charm quark hadro-production (II)*

## *Issues*

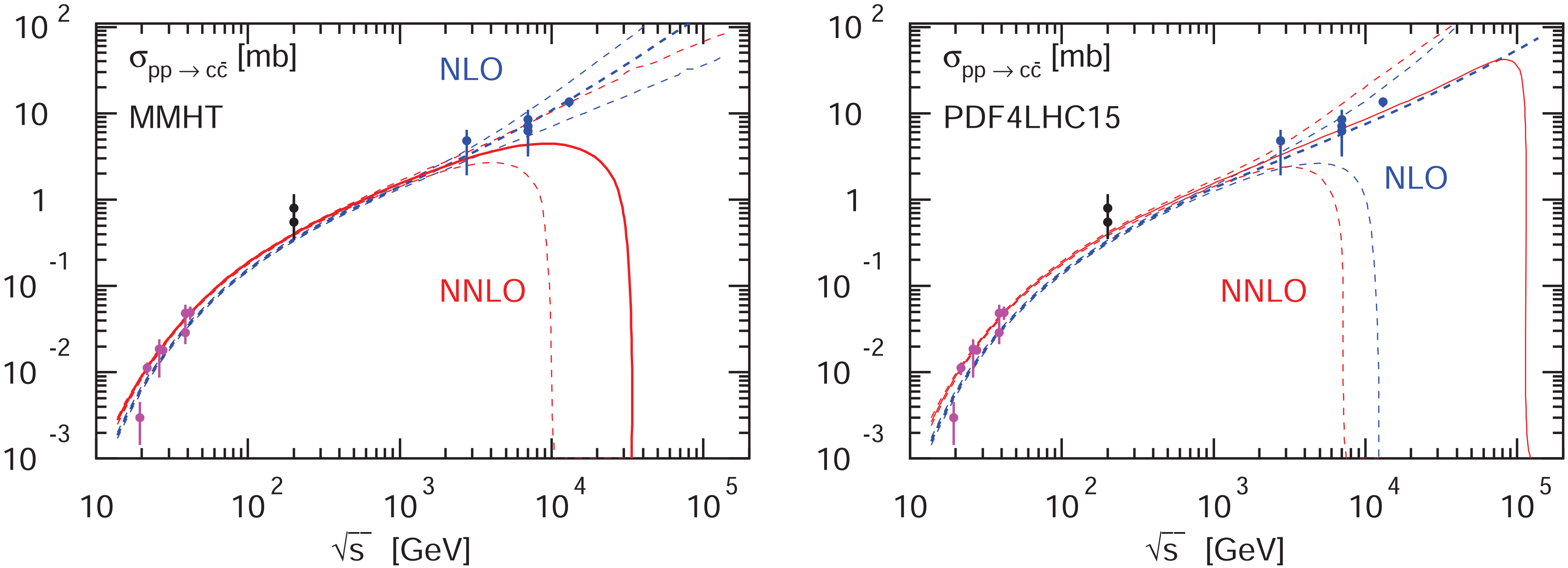
- Extrapolation of gluon PDF towards smaller $x$
  - some PDFs feature large uncertainities for extrapolation to unmeasured regions $\longrightarrow$ this invalidates predictive potential



# *Charm quark hadro-production (III)*

## *More issues*

- Some PDFs predict negative gluon PDF at small-$x$ and low scales $\mu_F \simeq 2m_c$
  - negative cross section is unphysical; consequence of modelling in variable flavor number schemes applied and description of structure function $F_L$ at NNLO
  - large differences between gluon PDFs fitted at NLO and NNLO



# Summary

- Precision determination of non-perturbative parameters is essential
  - parton content of proton (PDFs), strong coupling constant $\alpha_s(M_Z)$, quark masses $m_c$, $m_b$, $m_t$
- Experimental precision of $\lesssim 1\%$ makes theoretical predictions at NNLO in QCD mandatory
- Uncertainties due model assumption in PDF fits often neglected
  - implementations of variable flavor number schemes use charm-quark mass $m_c$ for tuning
  - low value of pole mass $m_c^{\rm pole} \simeq 1.25$GeV in contradiction to world average
- Values of $\alpha_s(M_Z)$ at NNLO from measurements at colliders lower than world average
  - $\alpha_s(M_Z) = 0.118$ at NNLO not preferred by data
  - data analysis with fixed value of $\alpha_s(M_Z)$ lacks correlation with parameters of PDF fits
- PDF4LHC recommendations introduce bias and inflated uncertainties
  - very difficult to quantify potential discrepancies between individual PDF sets