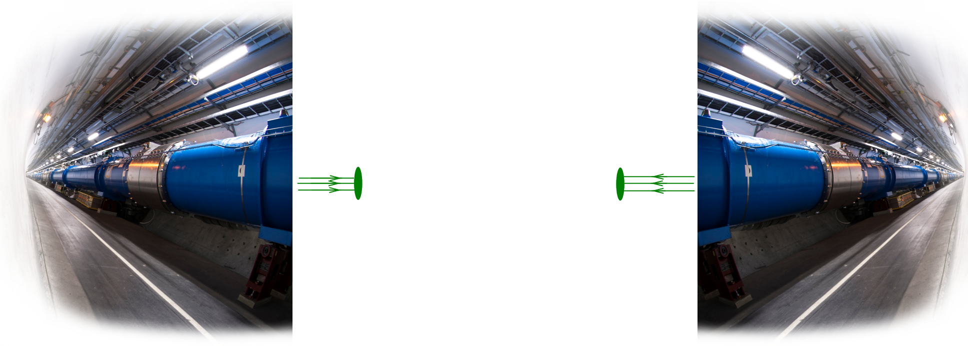
TECHNISCHE UNIVERSITÄT DRESDEN

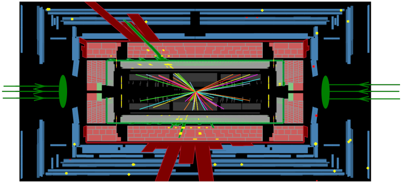
Fakultät Mathematik und Naturwissenschaften Institut für Kern- und Teilchenphysik

Introduction to Monte Carlo event generators

Frank Siegert, TU Dresden

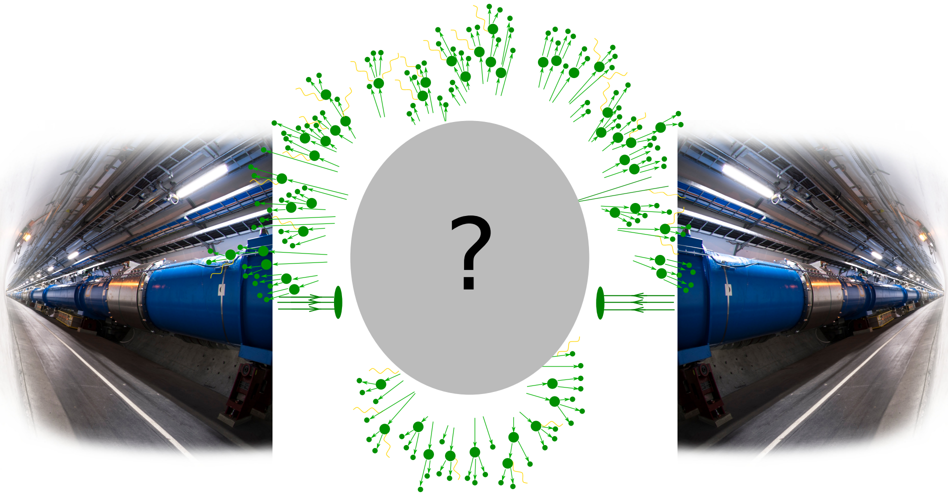
Terascale MC school, March 2017



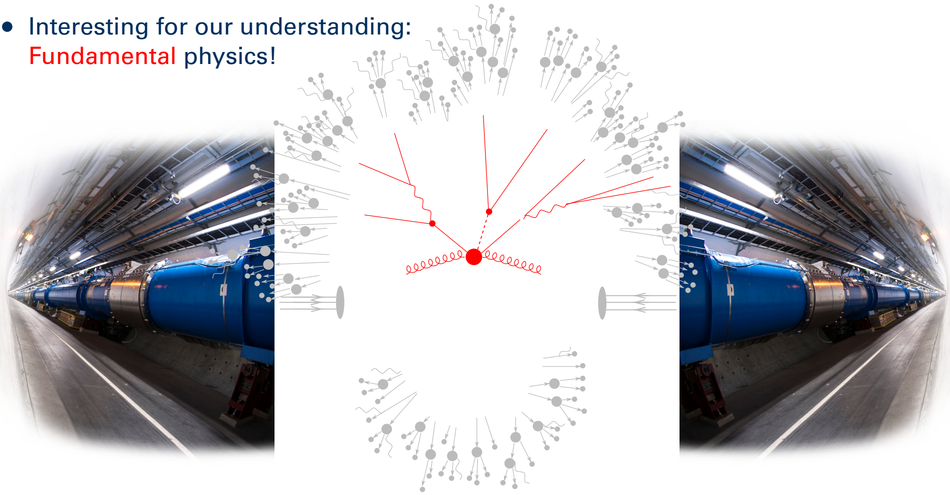


[ATLAS event display from 13 TeV collisions]

- In our detectors:
  stable hadrons

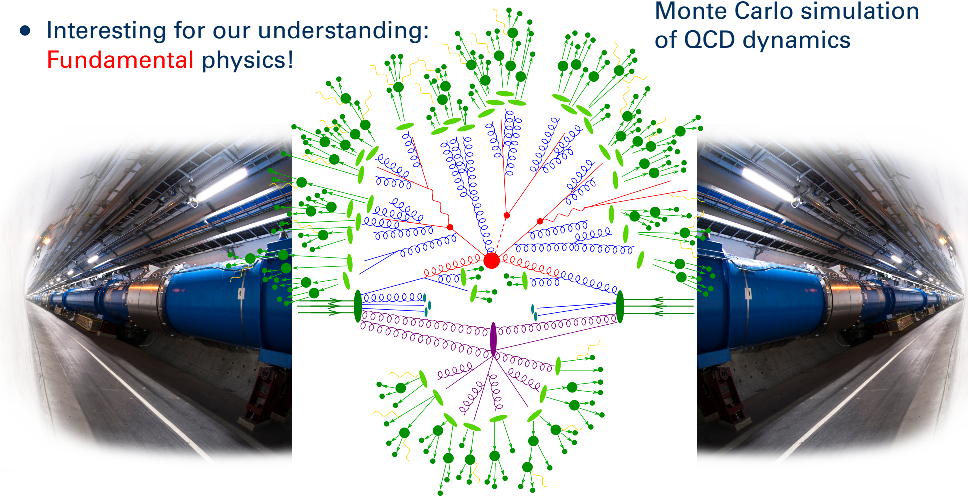


- In our detectors:
  stable hadrons
- Interesting for our understanding:
  Fundamental physics!

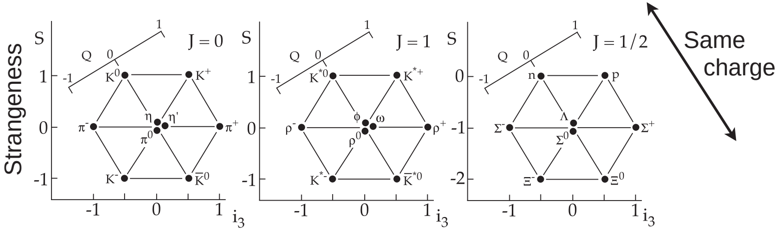


- In our detectors:
  stable hadrons
- Interesting for our understanding:
  Fundamental physics!

- Connection:
  Monte Carlo simulation
  of QCD dynamics



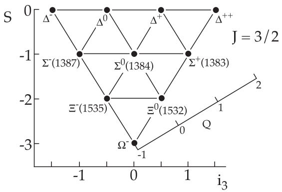
- 1960's → large zoo of hadrons dicovered, systematization needed
- Gell-Mann, Ne'eman '62 → "eight-fold way", prediction of Ω



- Use SU(3) of flavor (isospin & strangeness) to classify hadrons
- Describes structure of low-lying multiplets very well

Basic idea of QCD

- Decuplets contains state with three identical quarks



Isospin third component

- Corresponds to all symmetric state $\Delta^{++} = |u^\uparrow u^\uparrow u^\uparrow\rangle$
  $\rightarrow$ forbidden by Fermi statistics
- Rescue by postulating new degree of freedom $\rightarrow$ "color" charge

$$\Delta^{++} = N \sum \varepsilon_{ijk} |u_i^\uparrow u_j^\uparrow u_k^\uparrow\rangle$$

  $\rightarrow$ described by SU(3) of color, i.e. 3 charges ("red", "green", "blue")
- Color "unobserved" experimentally $\rightarrow$ SU(3) invariance of Lagrangian

Some definitions: Representations

- Lie algebra of SU(3) is $[T^a, T^b] = if^{abc}T_c$
- Fundamental representation usually in terms of $T^a = \lambda^a/2$ with Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- $\lambda_a^\dagger = \lambda_a$, $\mathrm{Tr}(\lambda^a) = 0$, $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$
- Fierz identity

$$\lambda_{ij}^a \lambda_{kl}^a = 2\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right)$$

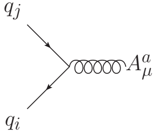
- Adjoint representation: $F_{bc}^a = -if_{abc}$

QCD Lagrangian

- Behaviour of quark field under gauge transformation

$$|u\rangle = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \rightarrow |u'\rangle = \exp\left\{ig_s\alpha_a T^a\right\} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

- Covariant derivative $D_\mu = \delta_\mu + ig_s T^a A^a_\mu$ with A_μ →gauge fields
$\rightarrow$ quark kinetic and quark-gluon interaction term: $\mathcal{L} = \bar{q}(i\not{D} - m)q$



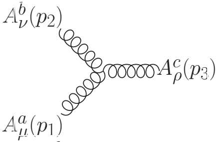
$$= \quad g_s\bar{q}_j\gamma^\mu T^a_{ji} q_i A^a_\mu$$

- Gluon kinetic and self-interaction term from QCD analog $\mathcal{L} = \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$

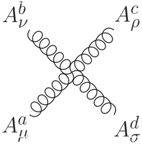
$$F^a_{\mu\nu} = \delta_\mu A^a_\nu - \delta_\nu A^a_\mu - g_s f_{abc} A^a_\mu A^b_\nu$$

$\rightarrow$ 3- and 4-gluon interaction terms

- Non-abelian structure of QCD manifests itself in 3- and 4-gluon vertices



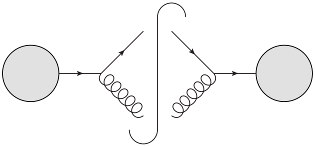
$$= g_s f_{abc}\big[\; g_{\mu\nu}(p_1-p_2)_\rho + g_{\nu\rho}(p_2-p_3)_\mu + g_{\rho\mu}(p_3-p_1)_\nu\big]$$



$$= -ig_s^2\big[\; f_{abe}f_{ecd}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho}) + f_{ade}f_{ecb}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\nu}g_{\rho\sigma}) + f_{ace}f_{ebd}(g_{\mu\sigma}g_{\nu\rho}-g_{\mu\nu}g_{\rho\sigma})\big]$$

External quark splitting

- Consider process $q \to qg$ attached to some other diagrams



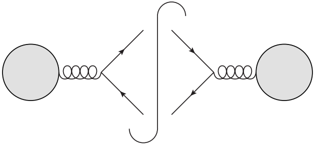
- Proportional to $g_s^2 T^a_{ij} T^a_{jk} = 4\pi\alpha_s C_F \delta_{ik}$, where

$$C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

- Identical to QED case except for color factor C_F

External gluon splitting

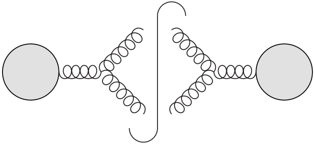
- Consider process $g \to q\bar{q}$ attached to some other diagrams



- Proportional to $g_s^2 T_{ij}^a T_{ji}^b = 4\pi\alpha_s T_R \delta^{ab}$, where

$$T_R = \frac{1}{2}$$

- Consider process $g \to gg$ attached to some other diagrams



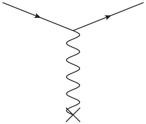
- Proportional to $g_s^2 f^{abc} f^{bcd} = 4\pi\alpha_s C_A \delta^{ad}$, where

$$C_A = N_c = 3$$

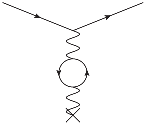
- $\to$ Gluons couple stronger to gluons than to quarks

Screening in QED

- At low energy, QED potential looks like $V(R) = -\frac{\alpha}{R}$



- When $R \lesssim 1/m_e$, vacuum polarization effects set in



  Potential changes to

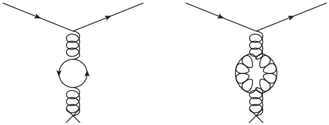
$$V(R) = -\frac{\alpha}{R}\left[1 + \frac{2\alpha}{3\pi}\log\frac{1}{m_e R} + \mathcal{O}(\alpha^2)\right] = \frac{\bar{\alpha}(R)}{R}$$

  where $\bar{\alpha}(R) \rightarrow$ effective (running) coupling

- Understand effective coupling in analogy to solid state physics:
  - In insulators, charge screened by polarization of atoms
  - In QED, charge screened by newly created e^+e pairs

Anti-Screening in QCD and the β-function

- In QCD, gauge bosons carry color $\rightarrow$ screening & anti-screening



- Sum of contributions defines running of strong coupling at 1-loop

$$\mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2} = \beta(\alpha_s) \quad \text{where} \quad \beta(\alpha_s) = -\alpha_s \sum_{n=0} \frac{\alpha_s}{4\pi} \beta_n$$

- Coefficients known up to four loops

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_R n_f$$

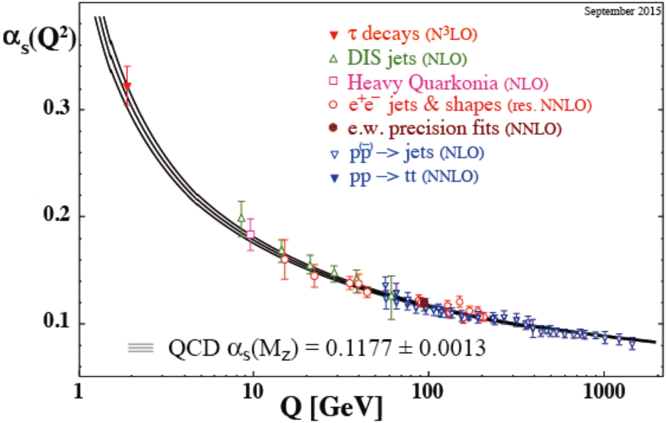
$$\beta_1 = \frac{17}{12} C_A^2 - \left(\frac{5}{6} C_A + \frac{1}{2} C_F \right) T_R n_f$$

$$\ldots$$

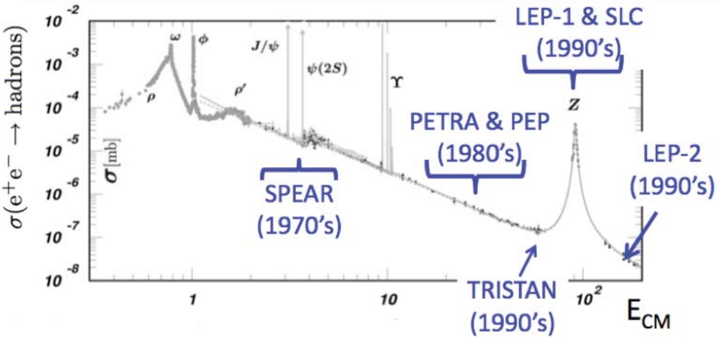
- Unless $n_f \geq 17$, 1-loop beta function is negative $\rightarrow$ confinement

Running of the strong coupling

[Bethke] Proc. HP α_s (2015)

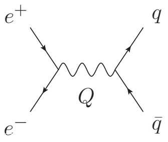


QCD in e^+e^- annihilation

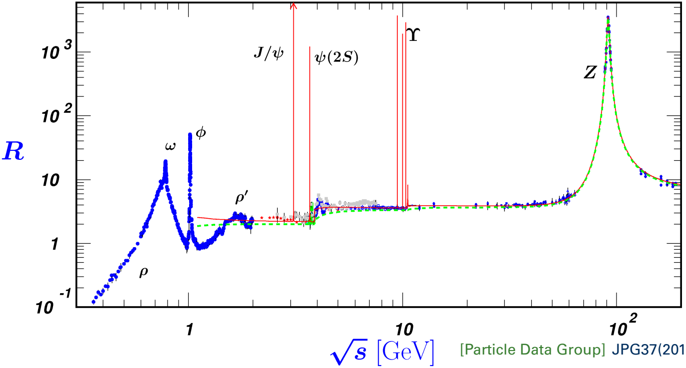


- SPEAR (SLAC): Discovery of quark jets
- PETRA (DESY) & PEP (SLAC): First high energy (>10 GeV) jets Discovery of gluon jets (PETRA) & pioneering QCD studies
- LEP (CERN) & SLC (SLAC): Large energies → more reliable QCD calculations, smaller hadronization uncertainties
  Large data samples → precision tests of QCD

Basic process for $e^+e^- \to$hadrons



- Prediction for $e^+e^- \to q\bar{q}$ at leading perturbative order differs from $e^+e^- \to \mu^+\mu^-$ only by quark charges
- Define ratio $R = \dfrac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \xrightarrow{\text{LO}} \sum_i e_{q,i}^2$

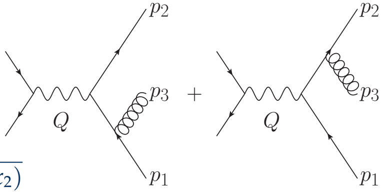


[Particle Data Group] JPG37(2010)075021

QCD corrections to $e^+e^- \to$ hadrons

- Kinematic variables $x_i = \frac{2p_i \cdot Q}{Q^2}$

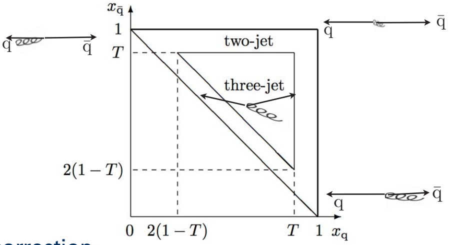
  $\to x_i < 1, \quad x_1 + x_2 + x_3 = 2$



- Differential cross section

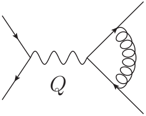
$$\frac{d^2\sigma}{dx_1 dx_2} = \sigma_0 \frac{\alpha_s}{\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- Divergent as
  - $x_1 \to 1$ ($p_3 \parallel p_1$)
  - $x_2 \to 1$ ($p_3 \parallel p_2$)
  - $(x_1, x_2) \to (1, 1)$ ($x_3 \to 0$)



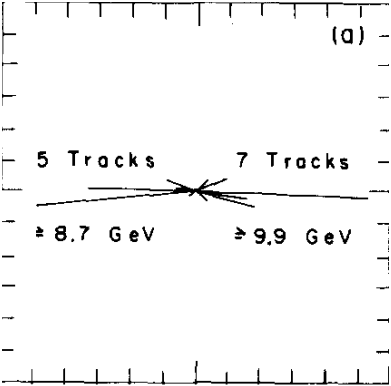
- Divergences canceled by virtual correction
  Total correction to $e^+e^- \to$ hadrons:

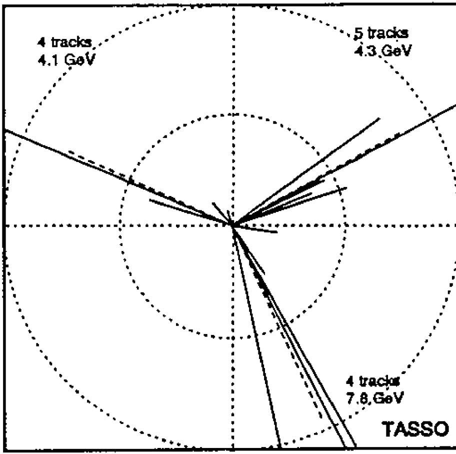
$$\sigma^{\mathrm{NLO}} = \sigma_0 \left(1 + \frac{3}{4} C_F \frac{\alpha_s}{\pi}\right)$$



Discovery of the gluon

[TASSO] PLB86(1979)243 & Proc. Neutrino ’79, Vol.1, p.113

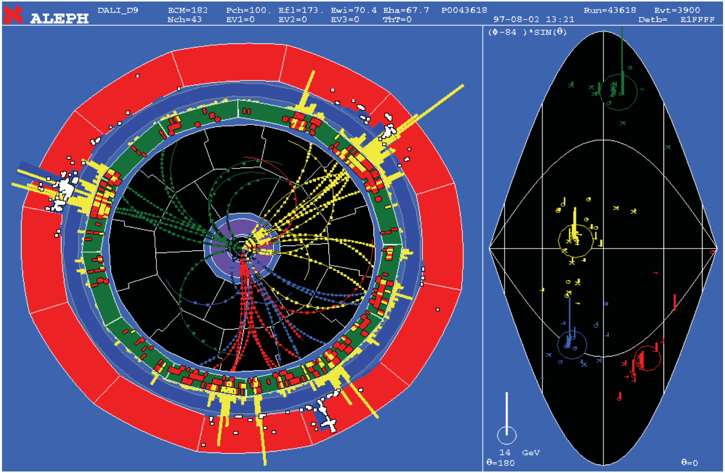


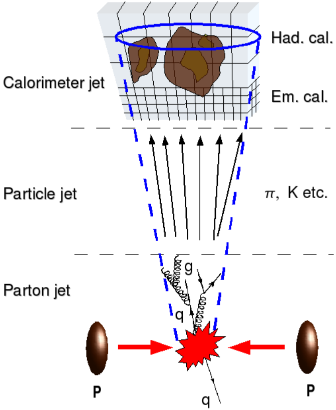


- Gluon discovery at the PETRA collider at DESY
- Typical three-jet event (right) vs. two-jet event (left)

High-energy colliders and jets

[ALEPH]





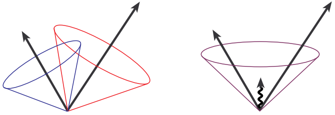
- Identify hadronic activity in experiment with partonic activity in pQCD theory

$\Rightarrow$ Requirements

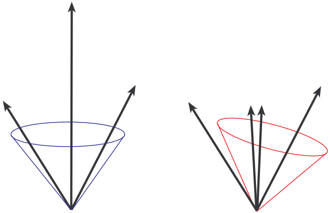
- Applicable both to data and theory
  - partons
  - stable particles
  - measured objects (calorimeter objects, tracks, etc.)
- Gives close relationship between jets constructed from any of the above
- Independent of the details of the detector, e.g. calorimeter granularity

Further requirements from QCD

- Infrared safety $\rightarrow$ no change when adding a soft particle

Counterexample: 

- Collinear safety $\rightarrow$ no change when substituting particle with two collinear particles

Counterexample: 

Cluster sequence of input momenta

Most widely used jet algorithms today of sequential recombination type

1. Calculate all d_{ij} and d_{iB}
2. Find their minimum, $d_{\min}$
   a) If $d_{\min}$ is a d_{ij}, combine i and j
   b) If $d_{\min}$ is a d_{iB}, remove i from list $\rightarrow$ jet.
3. Return to step 1 or stop when no particle left

Example: (anti-)k_t algorithm

- Distance between two momenta p_i, p_j:

$$d_{ij} = \min(p_{ti}^{\pm 2}, p_{tj}^{\pm 2}) \frac{\Delta R_{ij}^2}{R^2}$$

Distance of p_i to the beams:

$$d_{iB} = p_{ti}^{\pm 2}$$

- Further ambiguities: recombination, $p_{\perp}^{\min}$

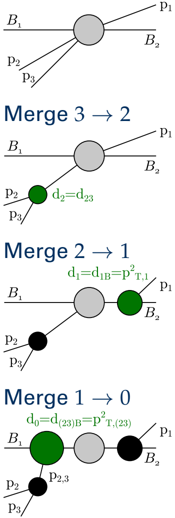
Example

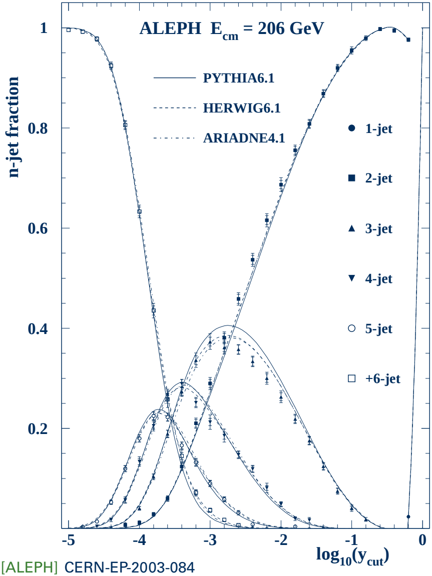
Step 0: Input momenta

Step 1: Merge $3 \rightarrow 2$

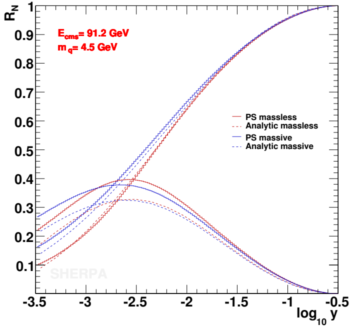
Step 2: Merge $2 \rightarrow 1$

Step 3: Merge $1 \rightarrow 0$





[ALEPH] CERN-EP-2003-084



- Comparison between theory and Monte-Carlo simulation

Event shape variables

- Shape variables characterize event as a whole
- Thrust (introduced 1978 at PETRA)

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_j |\vec{p}_j|}$$

  - $T \to 1$ – back-to-back event
  - $T \to 1/2$ – spherically symmetric event

  Vector for which maximum is obtained $\to$ thrust axis $\vec{n}_T$

- Thrust major/minor

$$T_{\mathrm{maj}} = \max_{\vec{n}_{\mathrm{maj}} \perp \vec{n}_T} \frac{\sum_i |\vec{p}_i \cdot \vec{n}_{\mathrm{maj}}|}{\sum_j |\vec{p}_j|}$$

$$T_{\mathrm{min}} = \frac{\sum_i |\vec{p}_i \cdot (\vec{n}_T \times \vec{n}_{\mathrm{maj}})|}{\sum_j |\vec{p}_j|}$$

- Jet broadening, computed for two hemispheres w.r.t. $\vec{n}_T$:

$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_j |\vec{p}_j|}$$

  - $B_W = \max(B_1, B_2)$ – Wide jet broadening
  - $B_N = \min(B_1, B_2)$ – Narrow jet broadening

- Jet mass

$$M_i^2 = \frac{1}{E_{cm}^2}\left(\sum_{k\in H_i} p_k\right)^2$$

Computed for two hemispheres w.r.t. $\vec{n}_T$, then
  - $\rho = \max(M_1^2, M_2^2)$ – Heavy jet mass
  - $M_L = \min(M_1^2, M_2^2)$ – Light jet mass

- Quadratic momentum tensor

$$M^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_j |\vec{p}_j|^2}\ , \qquad \alpha,\beta = 1,2,3$$

Eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$ used to define
  - $S = \frac{3}{2}(\lambda_2 + \lambda_3)$ – Sphericity
  - $A = \frac{3}{2}\lambda_3$ – Aplanarity
  - $P = \lambda_2 - \lambda_3$ – Planarity

- C-Parameter

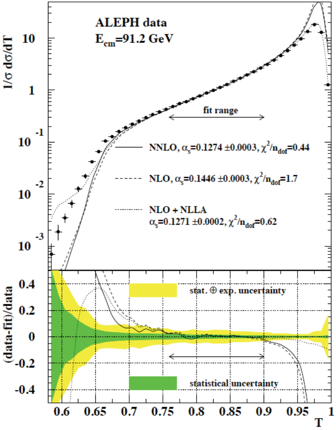
Linearized momentum tensor

$$\Theta^{\alpha\beta} = \frac{1}{\sum_j |\vec{p}_j|}\sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|}\ ,$$

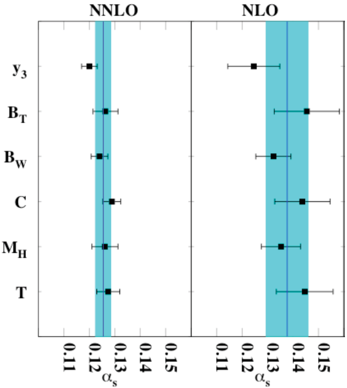
Eigenvalues λ_i define $C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$

Application of event shape variables

- Discovery of quark and gluon jets – Sphericity & Oblateness
- Measurement of strong coupling constant – T, C, B, ρ, Durham jet rates

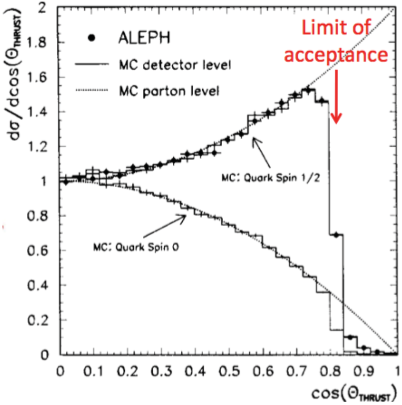


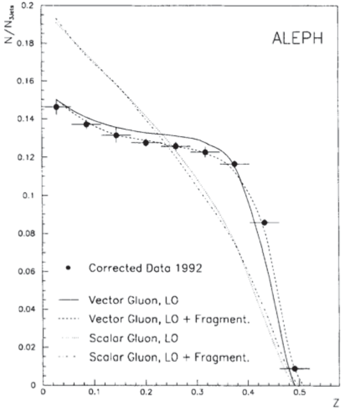
[Dissertori et al.] arXiv:0906.3436



Application of event shape variables

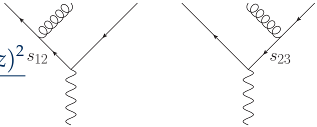
- Measurement of quark (and gluon) spin – Thrust axis
- Measurement of triple-gluon vertex – BZ angle





Parton evolution in the collinear limit

- Consider $e^+e^- \to 3$ partons



$$\frac{1}{\sigma_{2\to 2}} \frac{\mathrm{d}\sigma_{2\to 3}}{\mathrm{d}\cos\theta \mathrm{d}z} \sim C_F \frac{\alpha_s}{2\pi} \frac{2}{\sin^2\theta} \frac{1+(1-z)^2}{z}$$

θ - angle of gluon emission
z - fractional energy of gluon

- Divergent in
  - Collinear limit: $\theta \to 0, \pi$
  - Soft limit: $z \to 0$
- Separate into two independent jets

$$2\frac{\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}} \approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

- Independent evolution with θ

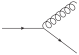
$$\mathrm{d}\sigma_3 \sim \mathrm{d}\sigma_2 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2} \mathrm{d}z \, \frac{1+(1-z)^2}{z}$$

Parton evolution in the collinear limit

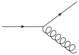
- Same equation for any variable with same limiting behavior
  - Transverse momentum $k_T^2 = z^2(1-z)^2\theta^2 E^2$
  - Virtuality $t = z(1-z)\theta^2 E^2$
- Call this the "evolution variable"

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}k_T^2}{k_T^2} = \frac{\mathrm{d}t}{t} \qquad \leftrightarrow \qquad \text{collinear divergence}$$

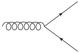
- Absorb z-dependence into flavor-dependent splitting kernel $P_{ab}(z)$



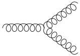
$$= C_F \frac{1+z^2}{1-z}$$



$$= C_F \frac{1+(1-z)^2}{z}$$



$$= T_R \left[z^2 + (1-z)^2\right]$$



$$= C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right]$$

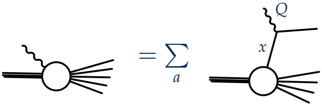
- DGLAP evolution equation emerges, but so far only pQCD, no PDF

$$\mathrm{d}\sigma_{n+1} \sim \mathrm{d}\sigma_n \sum_{\text{jets}} \frac{\mathrm{d}t}{t} \mathrm{d}z \, \frac{\alpha_s}{2\pi} P_{ab}(z)$$

The DGLAP equation (→ PDF talk on Friday)

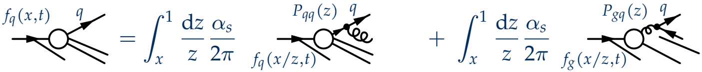
- Hadronic cross section factorizes into perturbative & non-perturbative piece

$$\sigma = \sum_{a=q,g} \int dx\, f_a(x, Q^2)\hat{\sigma}_a(Q^2)$$

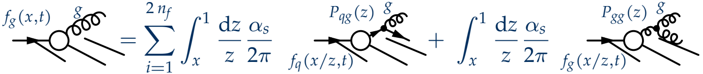


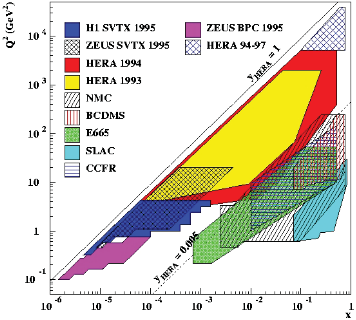
- Evolution from previous slide turns into evolution equation for $f_a(x, Q^2)$
- $f_a(x, Q^2)$ cannot be predicted as function of x
  but dependence on Q^2 can be computed order-by-order in pQCD
- DGLAP equation

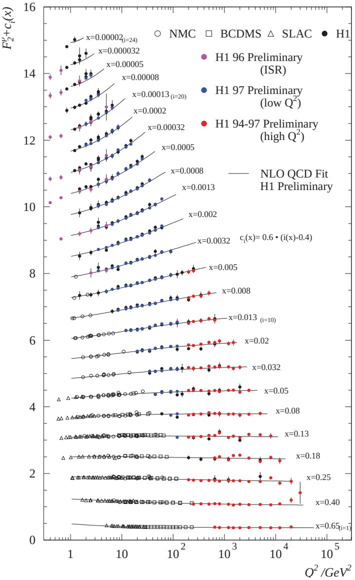
$$\frac{d}{d\log(t/\mu^2)} f_q(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{qq}(z)\, f_q(x/z,t) + \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{gq}(z)\, f_g(x/z,t)$$



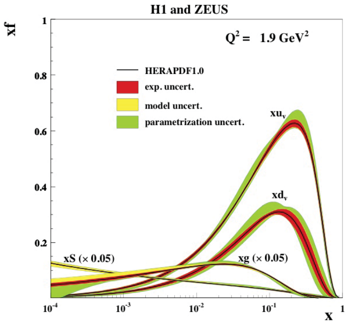
$$\frac{d}{d\log(t/\mu^2)} f_g(x,t) = \sum_{i=1}^{2n_f} \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{qg}(z)\, f_q(x/z,t) + \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{gg}(z)\, f_g(x/z,t)$$

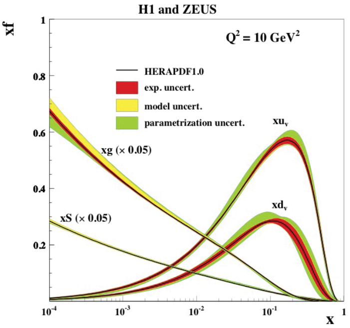



H1 SVTX 1995
ZEUS SVTX 1995
HERA 1994
HERA 1993
NMC
BCDMS
E665
SLAC
CCFR
ZEUS BPC 1995
HERA 94-97
y HERA = 1
y HERA = 0.005
Q² (GeV²)
x


NMC
BCDMS
SLAC
H1
H1 96 Preliminary (ISR)
H1 97 Preliminary (low Q²)
H1 94-97 Preliminary (high Q²)
NLO QCD Fit H1 Preliminary
c_i(x)= 0.6 • (i(x)-0.4)
Q² /GeV²

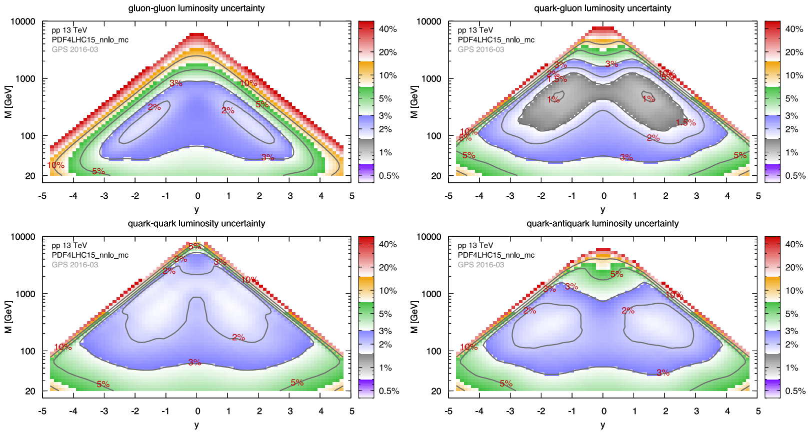
PDF measurements





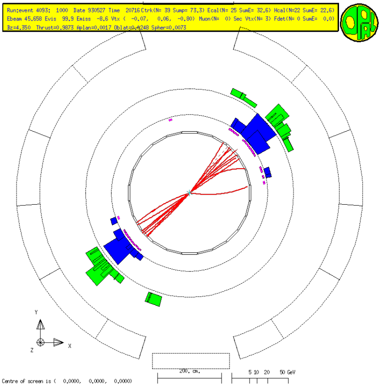
PDF uncertainties

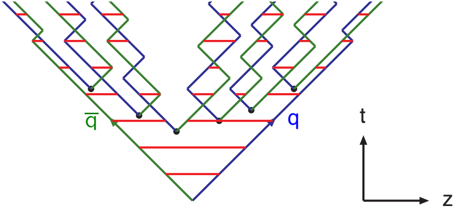
[plots from Gavin Salam]



Event generators in 1978

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31





- Lund string model: $\sim$ like rubber band that is pulled apart and breaks into pieces, or like a magnet broken into smaller pieces.
- Complete description of 2-jet events in $e^+e^- \rightarrow$hadrons

Event generators in 1978

[Andersson,Gustafson,Ingelman,Sjöstrand] Phys.Rept.97(1983)31

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END


      SUBROUTINE LIST(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      WRITE(6,110)
      DO 100 I=1,N
      IF(K(I,1).GT.0) C1=CHA1(K(I,1))
      IF(K(I,1).LE.0) IC1=-K(I,1)
      C2=CHA2(K(I,2))
      C3=CHA3((47-K(I,2))/20)
      IF(K(I,1).GT.0) WRITE(6,120) I, C1, C2, C3, (P(I,J), J=1,5)
  100 IF(K(I,1).LE.0) WRITE(6,130) I, IC1, C2, C3, (P(I,J), J=1,5)
      RETURN
  110 FORMAT(////T11,'I',T17,'ORI',T24,'PART',T32,'STAB',
     &T44,'PX',T56,'PY',T68,'PZ',T80,'E',T92,'M'/)
  120 FORMAT(10X,I2,4X,A2,1X,2(4X,A4),5(4X,F8.1))
  130 FORMAT(10X,I2,4X,1X,I2,2(4X,A4),5(4X,F8.1))
      END
```

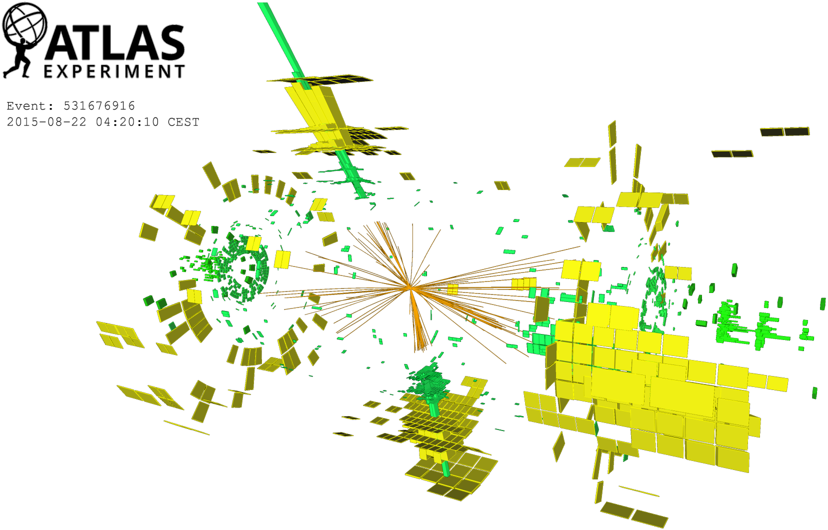
```
      SUBROUTINE DECAY(IPD,I)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      DIMENSION U(3), BE(3)
C 1 DECAY CHANNEL CHOICE, GIVES DECAY PRODUCTS
      TBR=RANF(0)
      IDC=IDCO(K(IPD,2)-7)
  100 IDC=IDC+1
      IF(TBR.GT.CBR(IDC)) GOTO 100
      ND=(59+KDP(IDC,3))/20
      DO 110 I1=I+1,I+ND
      K(I1,1)=-IPD
      K(I1,2)=KDP(IDC,I1-I)
  110 P(I1,5)=PMAS(K(I1,2))
C 2 IN THREE-PARTICLE DECAY CHOICE OF INVARIANT MASS OF PRODUCTS 2+3
      IF(ND.EQ.2) GOTO 130
      SA=(P(IPD,5)+P(I+1,5))**2
      SB=(P(IPD,5)-P(I+1,5))**2
      SC=(P(I+2,5)+P(I+3,5))**2
      SD=(P(I+2,5)-P(I+3,5))**2
      TDU=(SA-SD)*(SB-SC)/(4.*SQRT(SB*SC))
      IF(K(IPD,2).GE.11) TDU=SQRT(SB*SC)*TDU**3
  120 SX=SC+(SB-SC)*RANF(0)
      TDF=SQRT((SX-SA)*(SX-SB)*(SX-SC)*(SX-SD))/SX
      IF(K(IPD,2).GE.11) TDF=SX*TDF**3
      IF(RANF(0)*TDU.GT.TDF) GOTO 120
      P(100,5)=SQRT(SX)
C 3 TWO-PARTICLE DECAY IN CM, TWICE TO SIMULATE THREE-PARTICLE DECAY
  130 DO 160 IL=1,ND-1
      IO=(IL-1)*100-(IL-2)*IPD
      I1=I+IL
      I2=(ND-IL-1)*100-(ND-IL-2)*(I+IL+1)
      PA=SQRT((P(IO,5)**2-(P(I1,5)+P(I2,5))**2)*
     &(P(IO,5)**2-(P(I1,5)-P(I2,5))**2))/(2.*P(IO,5))
  140 U(3)=2.*RANF(0)-1.
      PHI=6.2832*RANF(0)
      U(1)=SQRT(1.-U(3)**2)*COS(PHI)
      U(2)=SQRT(1.-U(3)**2)*SIN(PHI)
      TDA=1.-(U(1)*P(IO,1)+U(2)*P(IO,2)+U(3)*P(IO,3))**2/
     &(P(IO,1)**2+P(IO,2)**2+P(IO,3)**2)
      IF(K(IPD,2).GE.11.AND.IL.EQ.2.AND.RANF(0).GT.TDA) GOTO 140
      DO 150 J=1,3
      P(I1,J)=PA*U(J)
  150 P(I2,J)=-PA*U(J)
      P(I1,4)=SQRT(PA**2+P(I1,5)**2)
  160 P(I2,4)=SQRT(PA**2+P(I2,5)**2)
C 4 DECAY PRODUCTS LORENTZ TRANSFORMED TO LAB SYSTEM
      DO 190 IL=ND-1,1,-1
      IO=(IL-1)*100-(IL-2)*IPD
      DO 170 J=1,3
  170 BE(J)=P(IO,J)/P(IO,4)
      GA=P(IO,4)/P(IO,5)
      DO 190 I1=I+IL,I+ND
      BEP=BE(1)*P(I1,1)+BE(2)*P(I1,2)+BE(3)*P(I1,3)
      DO 180 J=1,3
  180 P(I1,J)=P(I1,J)+GA*(GA/(1.+GA)*BEP+P(I1,4))*BE(J)
  190 P(I1,4)=GA*(P(I1,4)+BEP)
      I=I+ND
      RETURN
      END
```

≈ 200 punched cards

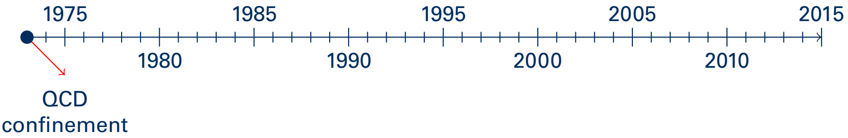
Fortran code

```
      SUBROUTINE EDIT(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      REAL ROT(3,3), PR(3)
C 1 THROW AWAY NEUTRALS OR UNSTABLE OR WITH TOO LOW PZ OR P
      I1=0
      DO 110 I=1,N
      IF(ITHROW.GE.1.AND.K(I,2).GE.8) GOTO 110
      IF(ITHROW.GE.2.AND.K(I,2).GE.6) GOTO 110
      IF(ITHROW.GE.3.AND.K(I,2).EQ.1) GOTO 110
      IF(P(I,3).LT.PZMIN.OR.P(I,4)**2-P(I,5)**2.LT.PMIN**2) GOTO
      I1=I1+1
      K(I1,1)=IDIM(K(I,1),0)
      K(I1,2)=K(I,2)
      DO 100 J=1,5
  100 P(I1,J)=P(I,J)
  110 CONTINUE
      N=I1
C 2 ROTATE TO GIVE JET PRODUCED IN DIRECTION THETA, PHI
      IF(THETA.LT.1E-4) GOTO 140
      ROT(1,1)=COS(THETA)*COS(PHI)
      ROT(1,2)=-SIN(PHI)
      ROT(1,3)=SIN(THETA)*COS(PHI)
      ROT(2,1)=COS(THETA)*SIN(PHI)
      ROT(2,2)=COS(PHI)
      ROT(2,3)=SIN(THETA)*SIN(PHI)
      ROT(3,1)=-SIN(THETA)
      ROT(3,2)=0.
      ROT(3,3)=COS(THETA)
      DO 130 I=1,N
      DO 120 J=1,3
  120 PR(J)=P(I,J)
      DO 130 J=1,3
  130 P(I,J)=ROT(J,1)*PR(1)+ROT(J,2)*PR(2)+ROT(J,3)*PR(3)
C 3 OVERALL LORENTZ BOOST GIVEN BY BETA VECTOR
  140 IF(BETA(1)**2+BETA(2)**2+BETA(3)**2.LT.1E-8) RETURN
      GA=1./SQRT(1.-BETA(1)**2-BETA(2)**2-BETA(3)**2)
      DO 160 I=1,N
      BEP=BETA(1)*P(I,1)+BETA(2)*P(I,2)+BETA(3)*P(I,3)
      DO 150 J=1,3
  150 P(I,J)=P(I,J)+GA*(GA/(1.+GA)*BEP+P(I,4))*BETA(J)
  160 P(I,4)=GA*(P(I,4)+BEP)
      RETURN
      END


      BLOCK DATA
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /EDPAR/ ITHROW, PZMIN, PMIN, THETA, PHI, BETA(3)
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      COMMON /DATA2/ IDCO(12), CBR(29), KDP(29,3)
      COMMON /DATA3/ CHA1(9), CHA2(19), CHA3(2)
      DATA PUD/0.4/, PS1/0.5/, SIGMA/350./, CX2/0.77/,
     &EBEG/10000./, WFIN/100./, IFLBEG/1/
      DATA ITHROW/1/, PZMIN/0./, PMIN/0./, THETA,PHI,BETA/5*0./
      DATA MESO/7,1,3,2,8,5,4,6,9,7,2,4,1,8,6,3,5,9/
      DATA CMIX/2*0.5,1.,2*0.5,1.,2*0.25,0.5,2*0.,1./
      DATA PMAS/0.,2*139.6,2*493.7,2*497.7,135.,548.8,957.6,
     &2*765.9,2*892.2,2*896.3,770.2,782.6,1019.6/
      DATA IDCO/0,1,6,11,12,13,15,17,19,21,22,25/
      DATA CBR/1.,0.381,0.681,0.918,0.969,1.,0.426,0.662,0.959,
     &0.980,1.,1.,1.,0.667,1.,0.667,1.,0.667,1.,0.667,1.,1.,
     &0.899,0.987,1.,0.486,0.837,0.984,1./
      DATA KDP/1,1,8,2,1,1,2,8,1,1,1,2,3,6,4,7,5,4,6,5,7,2,2,
     &1,2,4,6,2,1,1,1,8,3,2,1,3,8,17,18,1,8,8,2,8,3,8,3,8,2,8,
     &3,3,8,3,5,7,3,9,0,0,8,8,3,8,9,9,14*0,8,4*0,8,0/
      DATA CHA1/'UD','DU','US','SU','DS','SD','UU','DD','SS'/
      DATA CHA2/'GAMM','PI+','PI-','K+','K-','K0','KB0','PI0','ETA
     &'ETAP','RHO+','RHO-','K*+','K*-','K*0','KB*0','RHO0','OMEG',
      DATA CHA3/'    ','STAB'/
      END
```

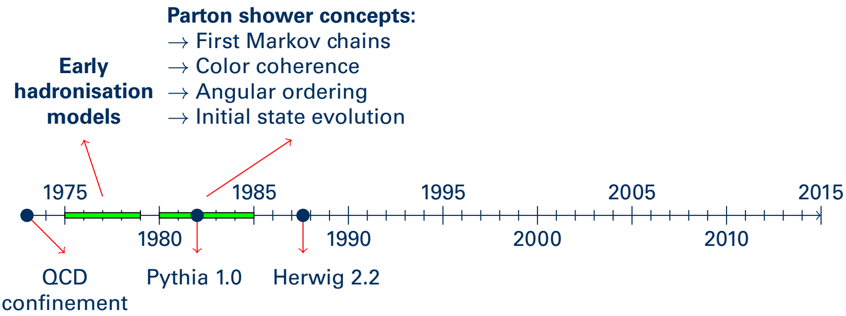

ATLAS
EXPERIMENT
Event: 531676916
2015-08-22 04:20:10 CEST

Event generators over the years

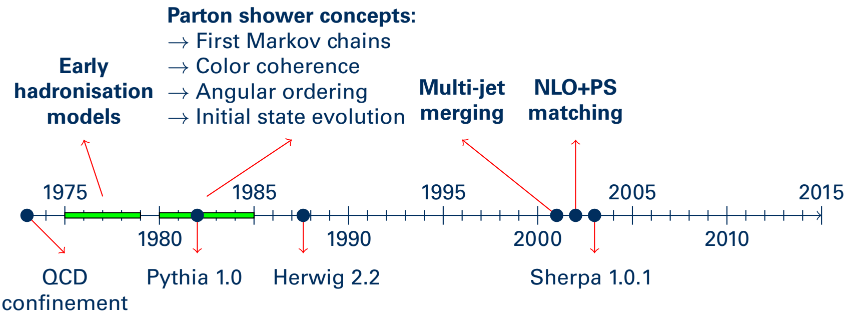


TECHNISCHE UNIVERSITÄT DRESDEN

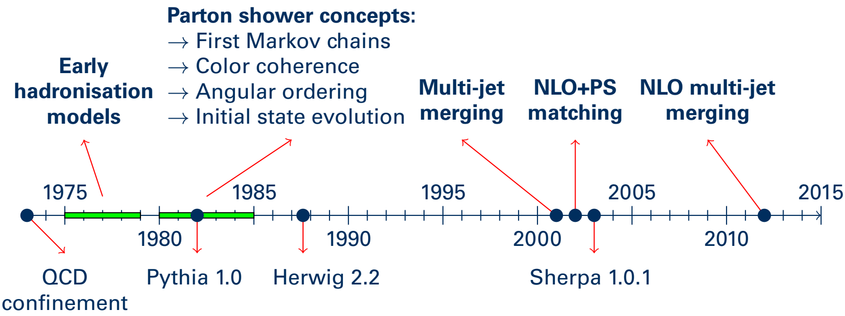
Event generators over the years



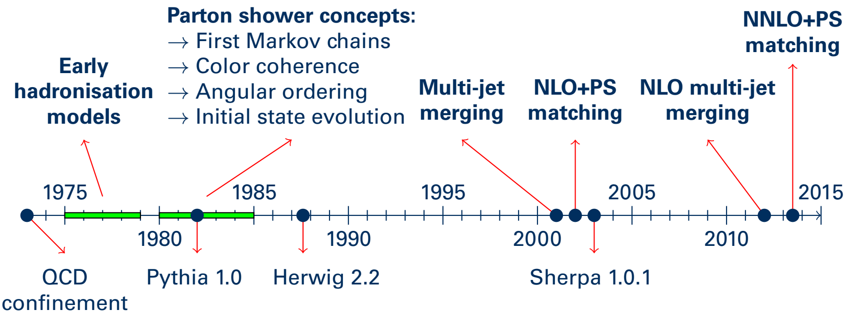
Event generators over the years



Event generators over the years



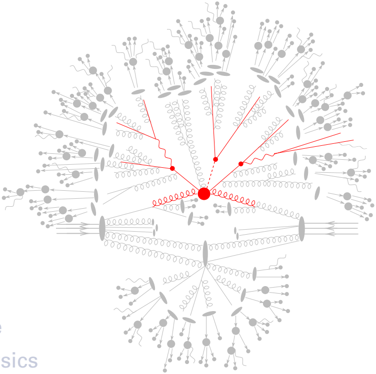
Event generators over the years



Event generators in 2017

Need to cover large dynamic range

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



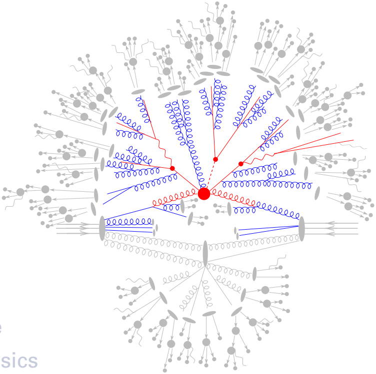
Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1 p_2 \to X} = \sum_{i,j \in \{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \to X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

Need to cover large dynamic range

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



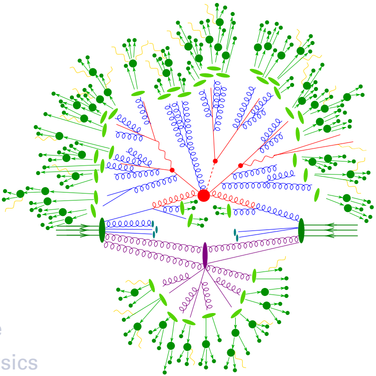
Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1p_2\to X} = \sum_{i,j\in\{q,g\}} \int dx_1 dx_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij\to X}(x_1x_2,\mu_F^2)}_{\text{short distance}}$$

Need to cover large dynamic range

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays



Divide and Conquer

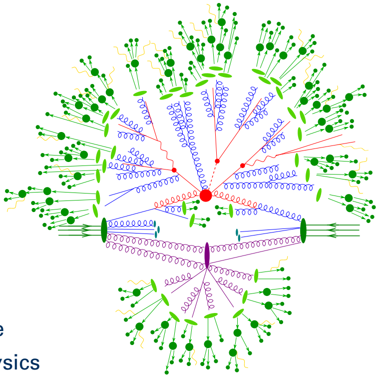
- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

$$\sigma_{p_1p_2\to X} = \sum_{i,j\in\{q,g\}} \int dx_1 dx_2 \underbrace{f_{p_1,i}(x_1,\mu_F^2) f_{p_2,j}(x_2,\mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij\to X}(x_1 x_2,\mu_F^2)}_{\text{short distance}}$$

Event generators in 2017

Need to cover large dynamic range

- Short distance interactions
  - Signal process
  - Radiative corrections
- Long-distance interactions
  - Hadronization
  - Particle decays

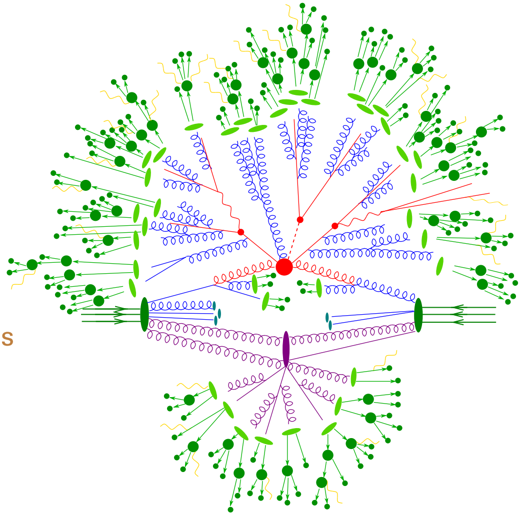


Divide and Conquer

- Quantity of interest: Total interaction rate
- Convolution of short & long distance physics

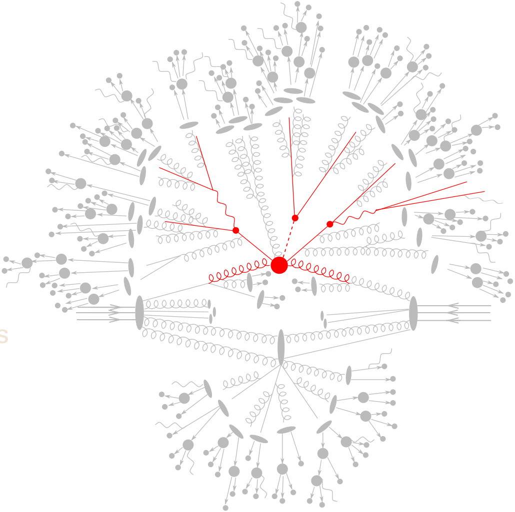
$$\sigma_{p_1 p_2 \to X} = \sum_{i,j \in \{q,g\}} \int \mathrm{d}x_1 \mathrm{d}x_2 \underbrace{f_{p_1,i}(x_1, \mu_F^2) f_{p_2,j}(x_2, \mu_F^2)}_{\text{long distance}} \underbrace{\hat{\sigma}_{ij \to X}(x_1 x_2, \mu_F^2)}_{\text{short distance}}$$

- Hard interaction
- Parton shower
- Multiple parton interactions
- Hadronisation
- Hadron decays
- Higher-order QED corrections



The structure of MC events

- Hard interaction
- Parton shower
- Multiple parton interactions
- Hadronisation
- Hadron decays
- Higher-order QED corrections



General $2 \to n$ hard scattering cross section

$$\hat{\sigma}_N = \int_{\text{cuts}} \mathrm{d}\hat{\sigma}_N = \int_{\text{cuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}\right] \delta^4\left(p_1 + p_2 - \sum_i^N q_i\right) \, |\mathcal{M}(p_1, p_2, q_1, \ldots, q_N)|^2$$

General $2 \to n$ hard scattering cross section

$$\hat{\sigma}_N = \int_{\text{cuts}} d\hat{\sigma}_N = \int_{\text{cuts}} \left[\prod_{i=1}^{N} \frac{d^3 q_i}{(2\pi)^3 2E_i} \right] \delta^4 \left(p_1 + p_2 - \sum_i^N q_i \right) \, |\mathcal{M}(p_1, p_2, q_1, \ldots, q_N)|^2$$

- Hard scattering matrix element

General $2 \to n$ hard scattering cross section

$$\hat{\sigma}_N = \int_{\text{cuts}} \mathrm{d}\hat{\sigma}_N = \int_{\text{cuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}\right] \delta^4\left(p_1 + p_2 - \sum_i^N q_i\right) \; |\mathcal{M}(p_1, p_2, q_1, \ldots, q_N)|^2$$

- Hard scattering matrix element
- Phase space integration including cuts

General $2 \to n$ hard scattering cross section

$$\hat{\sigma}_N = \int_{\text{cuts}} \mathrm{d}\hat{\sigma}_N = \int_{\text{cuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i} \right] \delta^4 \left(p_1 + p_2 - \sum_i^N q_i \right) \; |\mathcal{M}(p_1, p_2, q_1, \ldots, q_N)|^2$$

- Hard scattering matrix element
- Phase space integration including cuts

Monte Carlo task

1. Numerical integration for total cross section
   - Needs MC methods due to high dimensionality $D \gtrsim 4$
2. Event generation
   - $\to$ $(3 \cdot N - 4)$ random numbers
   - $\to$ N final state momenta
   - $\to$ natural "event" for $2 \to n$ scattering

$\Rightarrow$ Simply histogram any observable of interest

$\Rightarrow$ No need for dedicated calculations

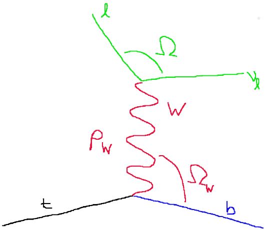
Simple example: $t(\to bW) \to b\bar{l}\nu_l$

- Matrix element for hard process

$$|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_W}\right)^2 \frac{p_t \cdot p_\nu \; p_b \cdot p_l}{(p_W^2 - m_W^2)^2 + \Gamma_W^2 m_W^2}$$

- Phase space integration in 5 dimensions

$$\Gamma = \frac{1}{2m_t}\frac{1}{128\pi^3}\int \mathrm{d}p_W^2 \frac{\mathrm{d}^2\Omega_W}{4\pi}\frac{\mathrm{d}^2\Omega}{4\pi}\left(1 - \frac{p_W^2}{m_t^2}\right)|\mathcal{M}|^2$$



- 5 random nos. for $p_W^2, \theta_W, \phi_W, \theta_{l,\nu}, \phi_{l,\nu}$
  $\to$ 3 momenta = event

Toy model: 1-dimensional integration

$$\int_a^b \mathrm{d}x\, f(x) \;\approx\; (b-a)\cdot\langle f\rangle$$

Toy model: 1-dimensional integration

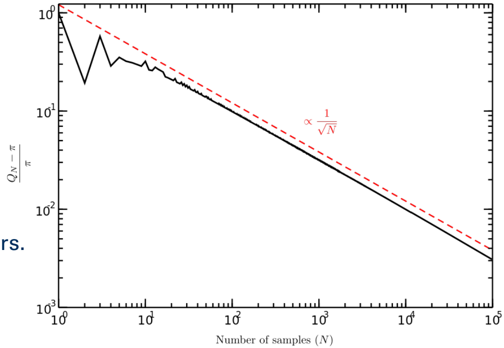
$$\int_a^b \mathrm{d}x\, f(x) \;\approx\; (b-a)\cdot\langle f\rangle \;\pm\; (b-a)\cdot\sqrt{\frac{\langle f^2\rangle - \langle f\rangle^2}{N}}$$

with

$$\langle f\rangle = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

$$\langle f^2\rangle = \frac{1}{N}\sum_{i=1}^{N} f^2(x_i)$$

$x_i \in [a,b]$ = uniform random numbers.



Multi-dimensional integration

- What are “uniform random numbers” in

$$\int \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}\right] \delta^4\left(p_1 + p_2 - \sum_i^N q_i\right) |\mathcal{M}(p_1, p_2, q_1, \ldots, q_N)|^2 \quad ?$$

$\rightarrow$ RAMBO algorithm as generator of flat phase space points

Multi-dimensional integration

- What are “uniform random numbers” in

$$\int \left[\prod_{i=1}^{N} \frac{d^3 q_i}{(2\pi)^3 2E_i}\right] \delta^4\left(p_1 + p_2 - \sum_i^N q_i\right) |\mathcal{M}(p_1, p_2, q_1, \dots, q_N)|^2 \quad ?$$

$\rightarrow$ RAMBO algorithm as generator of flat phase space points

Improving convergence

Several ways to improve the convergence behaviour by reducing the variance:

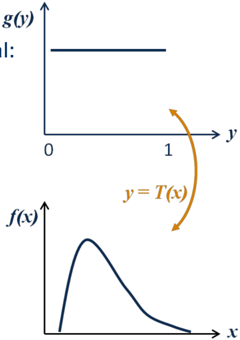
- Importance sampling

$$\int dx\, f(x) = \int dx\, g(x) \frac{f(x)}{g(x)} \equiv \int dG\, \frac{f(x)}{g(x)}$$

  using random numbers distributed according to $G(x) = \int_0^x dx' g(x')$
- Multi-channel integration
- VEGAS algorithm

Inverse transformation method

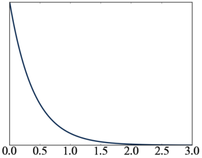
- Goal: Sample x according to $f(x)$ using flat random numbers y as input
- Prescription: $x = F^{-1}(y)$
- Corresponds to variable transformation of the differential:

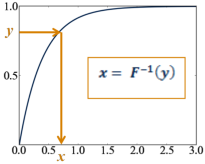


$$\begin{aligned} |\mathrm{d}F(x)| &= |\mathrm{d}G(x)| \\ |f(x)\mathrm{d}x| &= |g(y)\mathrm{d}y| \\ f(x) &= g(y)\left|\frac{\mathrm{d}y}{\mathrm{d}x}\right| = c\left|\frac{\mathrm{d}T(x)}{\mathrm{d}x}\right| \end{aligned}$$

which is solved by $T(x) = y = F(x)$

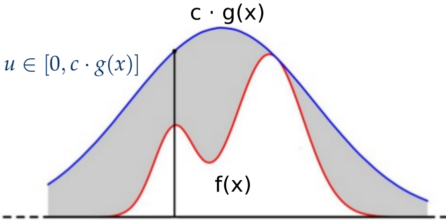
- Example: $f(x) = \frac{1}{\tau}e^{-x/\tau} \quad \rightarrow \quad F(x) = 1 - e^{-x/\tau}$





Hit-or-miss method (Rejection method)

- Most often it is not possible to determine F^{-1}
- Workaround: Find simple helper function $c \cdot g(x)$ which overestimates $f(x)$ and can be sampled
- Algorithm:
  1. Draw a sample x from $g(x)$
  2. Draw uniform random number $u \in [0, c \cdot g(x)]$
  3. If $\frac{f(x)}{c \cdot g(x)} \geq u$:
     Accept x
  4. Else:
     Reject, restart at 1

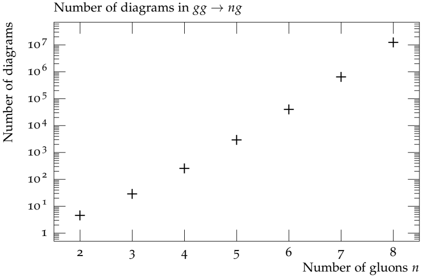


- The closer the functions match, the more efficient
  $\Rightarrow$ can use multiple estimators = multi-channel

Matrix element complexity

Example: Growth of the number of diagrams contributing to the tree-level $gg \to ng$ amplitude.

n	$\#_{\text{diagrams}}$
2	4
3	25
4	220
5	2485
6	34300
7	559405
8	10525900



Amplitude generation

- **Textbook**: Use completeness relations to square amplitudes
  sum/average over external states (helicity and color)
  Computational effort grows quadratically with number of diagrams
- **Real life**: Amplitudes are complex numbers
  first compute them, then add and square
  Effort grows linearly with number of diagrams

- Applies to dynamical degrees of freedom only
  - Consider helicity: Polarizations depend on momenta
    need to recompute for each phase-space point
  - Consider color: Mostly summed over at low multiplicity
    independent of other d.o.f. $\to$ no need to recompute

[Dixon] hep-ph/9601359
[Dittmaier] hep-ph/9805445

- Weyl-van-der-Waerden spinors for helicity states $+/-$

$$\chi_+(p) = \begin{pmatrix} \sqrt{p^+} \\ \sqrt{p^-} e^{i\phi_p} \end{pmatrix} \qquad \chi_-(p) = \begin{pmatrix} \sqrt{p^-} e^{i\phi_p} \\ -\sqrt{p^+} \end{pmatrix} \qquad \begin{aligned} p^\pm &= p^0 \pm p^3 \\ p_\perp &= p^1 + ip^2 \end{aligned}$$

  Basic building blocks for all amplitudes
  $+, -, \perp$ directions define "spinor gauge"

- Massive Dirac spinors in terms of WvdW spinors

$$u_+(p,m) = \frac{1}{\sqrt{2\bar{p}}} \begin{pmatrix} \sqrt{p_0 - \bar{p}}\ \chi_+(\hat{p}) \\ \sqrt{p_0 + \bar{p}}\ \chi_+(\hat{p}) \end{pmatrix} \qquad \bar{p} = \mathrm{sgn}(p_0)\,|\vec{p}|$$

$$u_-(p,m) = \frac{1}{\sqrt{2\bar{p}}} \begin{pmatrix} \sqrt{p_0 + \bar{p}}\ \chi_-(\hat{p}) \\ \sqrt{p_0 - \bar{p}}\ \chi_-(\hat{p}) \end{pmatrix} \qquad \hat{p} = (\bar{p}, \vec{p})$$

- γ^5 conveniently defined in Weyl representation

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\sigma^0 & 0 \\ 0 & \sigma^0 \end{pmatrix}$$

  Projection operator $P_{R,L} = P_\pm = (1 \pm \gamma^5)/2$ identifies
  lower/upper component of Dirac spinors as right-/left-handed

- Massless polarizations constructed from $u_\pm(p)$ and $u_\pm(k)$ with external light-like gauge vector k

$$\varepsilon^\mu_\pm(p,k) = \pm\frac{\bar{u}_\mp(k)\gamma^\mu u_\mp(p)}{\sqrt{2}\,\bar{u}_\mp(k)u_\pm(p)}\,.$$

  Defines light-like axial gauge

- For massive particles decompose momentum p using k

$$b = p - \kappa k \qquad \kappa = \frac{p^2}{2pk} \qquad \Rightarrow \qquad b^2 = 0$$

  Transverse polarizations as in massless case ($p \to b$) plus longitudinal

$$\varepsilon^\mu_0(p,k) = \frac{1}{m}\left(\bar{u}_-(b)\gamma^\mu u_-(b) - \kappa\,\bar{u}_-(k)\gamma^\mu u_-(k)\right)$$

- Vertices & propagators already known

- Building blocks for Standard model complete!

[Maltoni,Stelzer,Willenbrock] hep-ph/0209271
[Duhr,SH,Maltoni] hep-ph/0607057

- QCD amplitudes can be stripped of color factors
- Fundamental representation for n-gluons

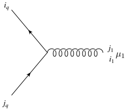
$$\mathcal{A}_n(p_1,\ldots,p_n) = \sum_{\vec{\sigma}\in P(2,\ldots,n)} \mathrm{Tr}(\lambda^{a_1}\lambda^{a_{\sigma_2}}\ldots\lambda^{a_{\sigma_n}})\, A(p_1,p_{\sigma_2},\ldots,p_{\sigma_n})$$

- Adjoint representation for n-gluons

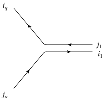
$$\mathcal{A}_n(p_1,\ldots,p_n) = \sum_{\vec{\sigma}\in P(2,\ldots,n-1)} \left[F^{a_{\sigma_2}}\ldots F^{a_{\sigma_{n-1}}}\right]^{a_1}_{a_n} A(p_1,p_{\sigma_2},\ldots,p_{\sigma_{n-1}},p_n)$$

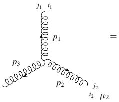
- Color-flow representation for n-gluons

$$\mathcal{A}_n(p_1,\ldots,p_n) = \sum_{\vec{\sigma}\in P(2,\ldots,n)} \delta^{i_1}_{j_{\sigma_2}}\delta^{i_{\sigma_2}}_{j_{\sigma_3}}\ldots\delta^{i_{\sigma_n}}_{j_1}\, A(p_1,p_{\sigma_2},\ldots,p_{\sigma_n})$$

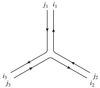


$$= i\frac{g}{\sqrt{2}}\gamma^{\mu_1}\ \underbrace{\delta^{i_q}_{j_1}\delta^{i_1}_{j_q}}$$





$$= i\frac{g}{\sqrt{2}}\sum\left[(p_1-p_2)_{\mu_3}g_{\mu_1\mu_2}+(p_2-p_3)_{\mu_1}g_{\mu_2\mu_3}+(p_3-p_1)_{\mu_2}g_{\mu_3\mu_1}\right]$$
$$\times\ \underbrace{\delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}}$$



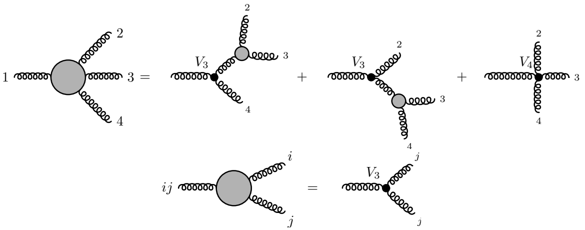
- We can sample colors just like we sample momenta
- Assign one in (r, g, b) / $(\bar{r}, \bar{g}, \bar{b})$ to each external (anti-)quark & gluon
- Average number of partial amplitudes is then smallest in color-flow basis

	Average # of partials		
n	Gell-Mann	Color-flow	Adjoint
4	4.83	1.28	1.15
5	15.2	1.83	1.52
6	56.5	3.21	2.55
7	251	6.80	5.53
8	1280	17.0	15.8
9	7440	48.7	56.4
10	47800	158	243

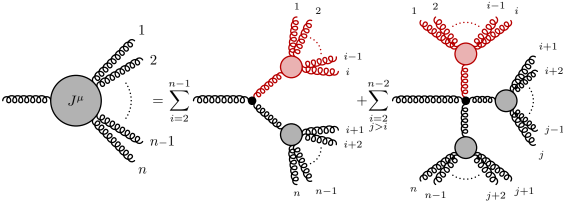
	Time [s/10^4pt]	
n	CO	CD
4	1.20	1.04
5	3.78	2.69
6	14.2	7.19
7	58.5	23.7
8	276	82.1
9	1450	270
10	7960	864

- Computational effort reduced further by not stripping amplitudes of color factors
- Evaluate dynamically at each vertex $\rightarrow$ straightforward computer algorithm
- Color dressing (CD) vs. color ordering (CO)

Example: Diagrams for
$g(1)g(2) \to g(3)g(4)$



[Berends,Giele] NPB306(1988)759



Example: Currents for
$g(1)g(2) \to g(3)g(4)$

Step 1 $J_1 = \varepsilon(1)$ $J_2 = \varepsilon(2)$ $J_3 = \varepsilon(3)$ $J_4 = \varepsilon(4)$

Step 2 J_{12} J_{13} J_{23}

Step 3 J_{123}

Step 4 $A(1,2,3,4) = J_4^* J_{123}$

Phase space

[James] CERN-68-15
[Byckling,Kajantie] NPB9(1969)568

- Need to evaluate in a process-independent way

$$d\Phi_n(p_a, p_b; p_1, .., p_n) = \left[\prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3\, 2E_i}\right] \delta^4\left(p_a + p_b - \sum_{i=1}^{n} p_n\right)$$

- Use factorization properties of phase-space integral

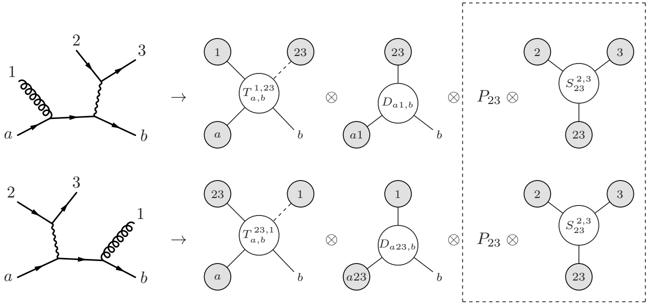
$$d\Phi_n(p_a, p_b; p_1, .., p_n) = d\Phi_{n-m+1}(p_a, p_b; p_{1m}, p_{m+1}, .., p_n) \times \frac{ds_{1m}}{2\pi}\, d\Phi_m(p_{1m}; p_1, .., p_m)$$

- Apply repeatedly until only 2-particle phase spaces remain

$$d\Phi_2 = \frac{\lambda(s_{ij}, m_i^2, m_j^2)}{16\pi^2\, 2s_{ij}} d\cos\theta_i d\phi_i$$

$\lambda^2(a, b, c) = (a - b - c)^2 - 4bc$ - Källen function

Phase space

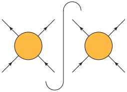
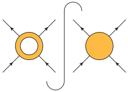
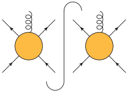


- Construct one integrator (= importance sampling) per diagram and combine into multi-channel
- Intuitive notion of pole structure, multi-channel determines balance
- Factorial growth with number of diagrams can be tamed by recursion

TECHNISCHE UNIVERSITÄT DRESDEN

NLO matrix elements ($\rightarrow$ talk + tutorial!)

NLO calculation

- Born term: $\mathrm{B} =$ 
- Virtual terms: $\mathrm{V} = \sum 2\,\mathrm{Re}\{$  $\}$
- Real terms: $\mathrm{R} = \sum$ 

- UV divergences in V removed by renormalization procedure
- V and R both still infrared divergent
- IR divergences cancel between V and R (KLN theorem)
- Exploit this fact to construct finite integrand for MC
  $\Rightarrow$ NLO subtraction

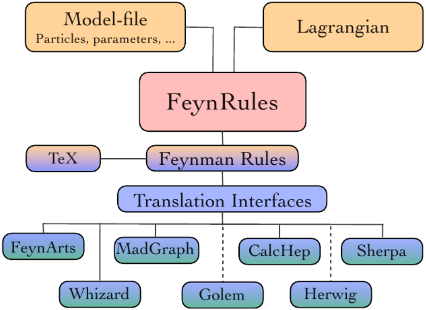
- Commonly used ME generators

	Built-in models	$2 \to$	$\lvert M_n \rvert^2$	$d\Phi_n$	NLO
ALPGEN	SM	8	recursive	Multi	-
AMEGIC	SM,MSSM,ADD	6	diagrams	Multi	sub
Comix	SM	8	recursive	Multi	sub
CompHEP	SM,MSSM	4	textbook	Single	-
HELAC	SM	8	recursive	Multi	sub+loop
MadEvent	SM,MSSM,UED	6	diagrams	Multi	sub+loop
Whizard	SM,MSSM,LH	8	recursive	Multi	sub

New physics models

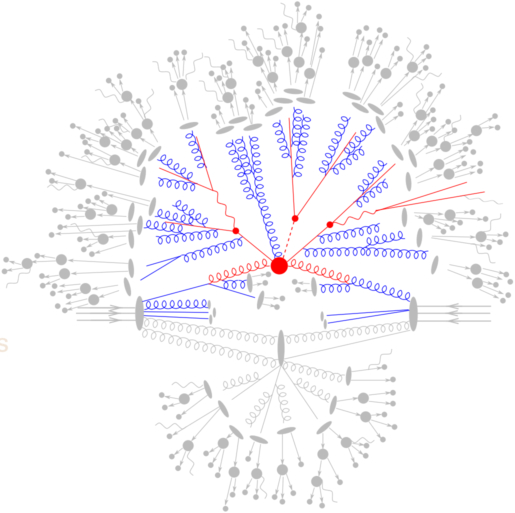
[Christensen,Duhr] arXiv:0806.4194

- Most ME generators suited for any physics model, but implementing Feynman rules tedious and error-prone
- Automated by FeynRules package
- Extracts vertices from Lagrangian based on minimal information about particle content
- Writes generator-specific output permitting easy cross-checks

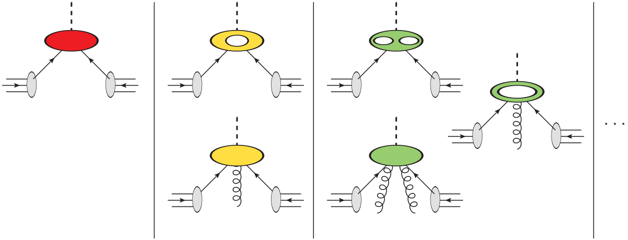


The structure of MC events

- Hard interaction
- Parton shower
- Multiple parton interactions
- Hadronisation
- Hadron decays
- Higher-order QED corrections



- Cannot solve QCD and calculate e.g. $pp \to t\bar{t}H$ exactly
- But can calculate parts of the perturbative series in α_s:



~ 1 ("LO") $\sim \alpha_s$ ("NLO") $\sim \alpha_s^2$ ("NNLO")

- Going beyond $\mathcal{O}(\alpha_s^2)$ non-trivial, only $gg \to H$ so far!
- $\alpha_s^2 \approx 1\% \Rightarrow$ high enough precision, right?
- Why is that not always true?

From fixed order to resummation

- Inclusive observables calculable at fixed-order ($\rightsquigarrow$ KLN theorem for cancellation of infrared divergences)
- But if not inclusive $\to$ finite remainders of infrared divergences:

  $$\text{logarithms of } \frac{\mu_{\text{hard}}^2}{\mu_{\text{cut}}^2} \text{ with each } \mathcal{O}(\alpha_s)$$

  can become large and spoil perturbative convergence

  Examples:

  - Observables that resolve soft emissions, like $p_\perp^Z = \mathcal{O}(1 \text{ GeV})$ in DY
  - Hadron-level predictions: confinement at $\mu_{\text{had}} \approx 1$ GeV

$\Rightarrow$ Need to resum the series to all orders

  - Problem: We are not smart enough for that.
  - Approximation in the collinear limit:
    Resum only the logarithmically enhanced terms in the series

- What do they look like and how do we “resum” them?

Resummation of multiple emissions

- Higher terms $\equiv$ additional QCD emissions
  $\rightarrow$ partons survive (no emissions) or split (real emission)

$\Rightarrow$ Resummation in analogy to radioactive decay, but:

  - evolution in energy scale "t"
  - generalisation to non-constant decay probability

Resummation of multiple emissions

Radioactive decay

- Constant differential decay probability

$$f(t) = \text{const} \equiv \lambda$$

- Survival probability $\mathcal{N}(t)$

$$-\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}t} = \lambda \mathcal{N}(t)$$

$$\Rightarrow \mathcal{N}(t) \sim \exp(-\lambda t)$$

- Resummed decay probability $\mathcal{P}(t)$

$$\mathcal{P}(t) = f(t)\,\mathcal{N}(t) \sim \lambda \exp(-\lambda t)$$

Resummation of multiple emissions

Radioactive decay

- Constant differential decay probability

$$f(t) = \text{const} \equiv \lambda$$

- Survival probability $\mathcal{N}(t)$

$$-\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}t} = \lambda \mathcal{N}(t)$$

$$\Rightarrow \mathcal{N}(t) \sim \exp(-\lambda t)$$

- Resummed decay probability $\mathcal{P}(t)$

$$\mathcal{P}(t) = f(t)\,\mathcal{N}(t) \sim \lambda \exp(-\lambda t)$$

QCD emissions

- Differential branching probability

$$f(t) \equiv \frac{\mathrm{d}\sigma_{n+1}^{(\text{approx})}(t)}{\mathrm{d}\sigma_n} (\rightsquigarrow \text{ later})$$

- Survival probability $\mathcal{N}(t)$

$$-\frac{\mathrm{d}\mathcal{N}}{\mathrm{d}t} = f(t)\,\mathcal{N}(t)$$

$$\Rightarrow \mathcal{N}(t) \sim \exp\left(-\int_0^t \mathrm{d}t'\, f(t')\right)$$

- Resummed branching probability $\mathcal{P}(t)$

$$\mathcal{P}(t) = f(t)\,\mathcal{N}(t) \sim f(t) \exp\left(-\int_0^t \mathrm{d}t'\, f(t')\right)$$

Universal structure at all orders (cf. $ee \to 3$ jets)

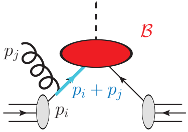
- Factorisation of QCD real emission ME for collinear partons (i, j):

$$|\mathcal{M}_{n+1}|^2 \stackrel{\text{approx}}{\longrightarrow} |\mathcal{M}_n|^2 \times \left[8\pi\alpha_s \, \frac{1}{2p_ip_j} \, \mathcal{K}_{ij}(p_i, p_j)\right]$$

Universal structure at all orders (cf. $ee \to 3$ jets)

- Factorisation of QCD real emission ME for collinear partons (i, j):

$$|\mathcal{M}_{n+1}|^2 \xrightarrow{\text{approx}} |\mathcal{M}_n|^2 \times \left[8\pi\alpha_s \frac{1}{2p_ip_j} \mathcal{K}_{ij}(p_i, p_j)\right]$$



  - $\mathcal{M}_{n+1}$ = real emission matrix element
  - $\mathcal{M}_n$ = Born matrix element
  - Massless propagator $\frac{1}{2p_ip_j}$

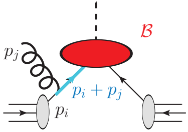
    $\to$ Evolution variable of shower $t \sim 2p_ip_j$, e.g. $k_\perp$, angle, ...
  - $\mathcal{K}_{ij}$ splitting kernel for branching $(ij) \to i + j$

    Specific form depends on factorisation scheme (DGLAP, Catani-Seymour, Antenna, ...)

Universal structure at all orders (cf. $ee \to 3$ jets)

- Factorisation of QCD real emission ME for collinear partons (i,j):

$$|\mathcal{M}_{n+1}|^2 \xrightarrow{\text{approx}} |\mathcal{M}_n|^2 \times \left[8\pi\alpha_s \, \frac{1}{2p_ip_j} \, \mathcal{K}_{ij}(p_i,p_j)\right]$$



  - $\mathcal{M}_{n+1}$ = real emission matrix element
  - $\mathcal{M}_n$ = Born matrix element
  - Massless propagator $\frac{1}{2p_ip_j}$

    $\to$ Evolution variable of shower $t \sim 2p_ip_j$, e.g. $k_\perp$, angle, ...
  - $\mathcal{K}_{ij}$ splitting kernel for branching $(ij) \to i + j$

    Specific form depends on factorisation scheme (DGLAP, Catani-Seymour, Antenna, ...)

- Factorisation of phase space element

$$\mathrm{d}\Phi_{n+1} \;\to\; \mathrm{d}\Phi_n \, \mathrm{d}\Phi_1 = \mathrm{d}\Phi_n \; \mathrm{d}t \, \frac{1}{16\pi^2} \, \mathrm{d}z \, \frac{\mathrm{d}\phi}{2\pi}$$

$\Rightarrow$ Combination gives differential branching probability

$$\frac{\mathrm{d}\sigma_{n+1}^{(\text{approx})}}{\mathrm{d}\sigma_n} \sim \sum_{ij} \mathrm{d}t \left[8\pi\alpha_s \, \frac{1}{2p_ip_j} \, \mathcal{K}_{ij}(p_i,p_j)\right] \sim \frac{\mathrm{d}t}{t} \, \frac{\alpha_s}{2\pi} \, \mathcal{K}_{ij}$$

TECHNISCHE UNIVERSITÄT DRESDEN

Summary of main parton shower ingredients

- "Sudakov form factor" $\equiv$ Survival probability of parton ensemble:

$$\mathcal{N}(t') \sim \exp\left(-\int_0^{t'} \mathrm{d}t\, f(t)\right) \quad \rightarrow \quad \Delta(t', t'') = \prod_{\{ij\}} \exp\left(-\int_{t'}^{t''} \frac{\mathrm{d}t}{t}\, \frac{\alpha_s}{2\pi}\, \mathcal{K}_{ij}\right)$$

- Evolution variable t: not time, but scale of collinearity from hard to soft
  $t \sim 2p_i p_j \sim$ e.g. angle θ, virtuality Q^2, relative transverse momentum $k_\perp^2$
- Starting scale μ_Q^2 (time $t = 0$ in radioactive decay) defined by hard ME
- Cutoff scale related to hadronisation scale $t_0 \sim \mu_{\text{had}}^2$
- Other variables (z, ϕ) generated directly according to $\mathrm{d}\sigma_{ij}^{(\text{PS})}(t, z, \phi)$

Summary of main parton shower ingredients

- "Sudakov form factor" $\equiv$ Survival probability of parton ensemble:

$$\mathcal{N}(t') \sim \exp\left(-\int_0^{t'} \mathrm{d}t\, f(t)\right) \quad \rightarrow \quad \Delta(t', t'') = \prod_{\{ij\}} \exp\left(-\int_{t'}^{t''} \frac{\mathrm{d}t}{t}\, \frac{\alpha_s}{2\pi}\, \mathcal{K}_{ij}\right)$$

- Evolution variable t: not time, but scale of collinearity from hard to soft
  $t \sim 2p_ip_j \sim$ e.g. angle θ, virtuality Q^2, relative transverse momentum $k_\perp^2$
- Starting scale μ_Q^2 (time $t = 0$ in radioactive decay) defined by hard ME
- Cutoff scale related to hadronisation scale $t_0 \sim \mu_{\text{had}}^2$
- Other variables (z, ϕ) generated directly according to $\mathrm{d}\sigma_{ij}^{(\mathrm{PS})}(t, z, \phi)$

⇒ Differential cross section (up to first emission)

$$\mathrm{d}\sigma^{(\mathrm{B})} = \mathrm{d}\Phi_B\, \mathcal{B}$$

Summary of main parton shower ingredients

- "Sudakov form factor" ≡ Survival probability of parton ensemble:

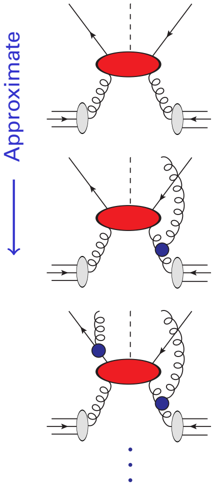
$$\mathcal{N}(t') \sim \exp\left(-\int_0^{t'} \mathrm{d}t\, f(t)\right) \quad \rightarrow \quad \Delta(t', t'') = \prod_{\{ij\}} \exp\left(-\int_{t'}^{t''} \frac{\mathrm{d}t}{t} \frac{\alpha_s}{2\pi} \mathcal{K}_{ij}\right)$$

- Evolution variable t: not time, but scale of collinearity from hard to soft
  $t \sim 2p_i p_j \sim$ e.g. angle θ, virtuality Q^2, relative transverse momentum $k_\perp^2$
- Starting scale μ_Q^2 (time $t = 0$ in radioactive decay) defined by hard ME
- Cutoff scale related to hadronisation scale $t_0 \sim \mu_{\mathrm{had}}^2$
- Other variables (z, ϕ) generated directly according to $\mathrm{d}\sigma_{ij}^{(\mathrm{PS})}(t, z, \phi)$

⇒ Differential cross section (up to first emission)

$$\mathrm{d}\sigma^{(\mathrm{PS})} = \mathrm{d}\Phi_B\, \mathcal{B} \left[\underbrace{\Delta^{(\mathrm{PS})}(t_0, \mu_Q^2)}_{\text{unresolved}} + \underbrace{\sum_{\{ij\}} \int_{t_0}^{\mu_Q^2} \mathrm{d}t\, \frac{\mathrm{d}\sigma_{ij}^{(\mathrm{PS})}}{\mathrm{d}t} \Delta^{(\mathrm{PS})}(t, \mu_Q^2)}_{\text{resolved}} \right]$$

Parton showers in a nutshell



$$\sigma_{\text{incl}} \Big[\Delta(t_0, \mu_Q^2)$$

$$+ \int_{t_0}^{\mu_Q^2} \frac{\mathrm{d}t}{t} \int \mathrm{d}z \frac{\alpha_s}{2\pi} P(z)\, \Delta(t, \mu_Q^2)$$

$$+ \frac{1}{2} \left(\int_{t_0}^{\mu_Q^2} \frac{\mathrm{d}t}{t} \int \mathrm{d}z \frac{\alpha_s}{2\pi} P(z) \right)^2 \Delta(t, \mu_Q^2)$$

$$+ \ldots$$

Simulation of parton shower cascade

- Start with parton ensemble from hard scattering
- Recursively generate branchings of each parton according to

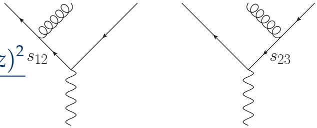
$$\mathcal{P}(t) = f(t)\,\mathcal{N}(t) \sim f(t)\exp\left(-\int_0^t \mathrm{d}t'\,f(t')\right)$$

Veto algorithm

- Generate next branching "time" t with probability
  $$\mathcal{P}(t, t_{\text{previous}}) = f(t)\exp\left(-\int_{t_{\text{previous}}}^{t} f(t')\mathrm{d}t'\right)$$
- Analytically:
  $t = F^{-1}\left[F(t_{\text{previous}}) + \log(\#_{\text{random}})\right]$ with $F(t) = \int_{t_0}^{t} \mathrm{d}t' f(t')$
- If integral/its inverse are not known: "Veto algorithm" = extension of hit-or-miss
  - Overestimate $g(t) \geq f(t)$ with known integral $G(t)$
    $\rightarrow t = G^{-1}\left[G(t_{\text{previous}}) + \log(\#_{\text{random}})\right]$
  - Accept t with probability $\frac{f(t)}{g(t)}$ using hit-or-miss

(Reminder: $e^+e^- \to q\bar{q}$)

- Consider $e^+e^- \to 3$ partons



$$\frac{1}{\sigma_{2\to 2}}\frac{\mathrm{d}\sigma_{2\to 3}}{\mathrm{d}\cos\theta \mathrm{d}z} \sim C_F \frac{\alpha_s}{2\pi}\frac{2}{\sin^2\theta}\frac{1+(1-z)^2}{z}$$

θ - angle of gluon emission
z - fractional energy of gluon

- Divergent in
  - Collinear limit: $\theta \to 0, \pi$
  - Soft limit: $z \to 0$
- Separate into two independent jets

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}} \approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

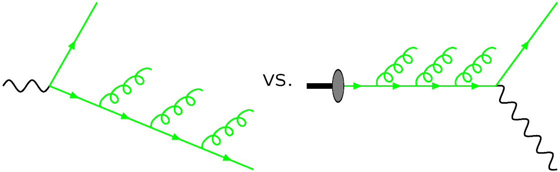
- Independent jet evolution

$$\mathrm{d}\sigma_3 \sim \mathrm{d}\sigma_2 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2}{\theta^2}\mathrm{d}z\,\frac{1+(1-z)^2}{z} \to \mathrm{d}\sigma_2 \sum_{\text{jets}} \frac{\mathrm{d}t}{t}\mathrm{d}z\,\frac{\alpha_s}{2\pi}P_{ab}(z)$$

Initial-state evolution

[Sjöstrand] PLB175(1985)321

- Iteration leads to tree-like approximation of higher-order configuration
- Slight difference between final-state and initial-state evolution



- Initial-state emission probability must account for probability to resolve (different) parton at larger x

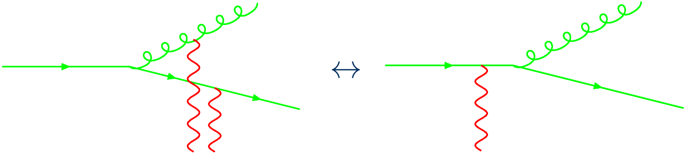
$$d\mathcal{P}_{\text{emit}}(x,t) = \frac{dt}{t}\int \frac{dz}{z}\,\frac{\alpha_s}{2\pi} P_{ab}(z)\,\frac{f_b(x/z,t)}{f_a(x,t)}$$

- Hard to implement in forward evolution (increasing t)
- Standard method is to evolve backward in initial state

Color coherence and angular ordering

[Marchesini,Webber] NPB310(1988)461

- Gluons with large wavelength not capable of resolving charges of emitting color dipole individually



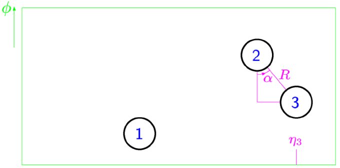
- Emission occurs with combined charge of mother parton instead
- Net effect is destructive interference outside cone with opening angle defined by emitting color dipole
- Can be implemented directly by angular ordering variable or additional ordering criterion in parton showers

Color coherence and angular ordering

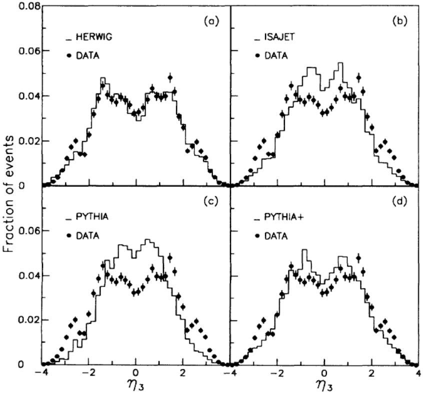
[CDF] PRD50(1994)5562

- Color coherence observed experimentally in 3-jet events

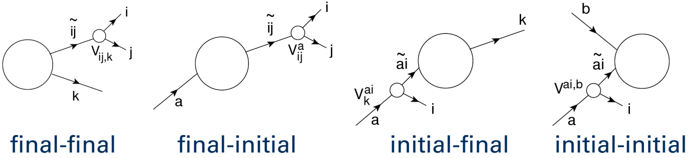
- Purely virtuality ordered PS's produce too much radiation in central region
- Angular ordered / angular vetoed PS's ok



$E_{T1} > 110$ GeV, $E_{T3} > 10$ GeV.



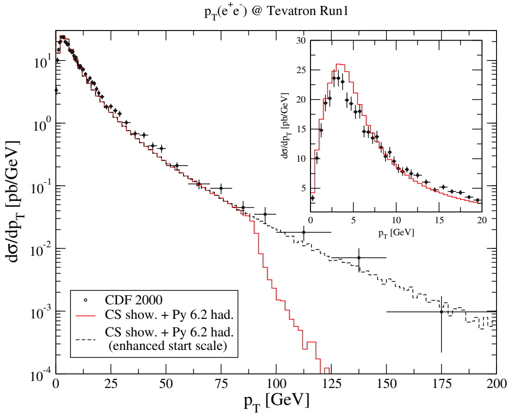
- In parton showers, the collinear/soft limit is never reached
  But who absorbs recoil when a splitting parton goes off mass-shell?
- No answer in DGLAP evolution equations $\leftrightarrow$ collinear limit
  Ambiguity introduces large uncertainties, especially at large t
- Natural solution provided by $2 \to 3$ splittings
  Spectator kinematics enters splitting probability
- Basic concept of dipole showers



- Publicly available generators

	Evolution variable	Splitting variable	Coherence
Ariadne	dipole-$k_\perp^2$	Rapidity	Antenna
Herwig	$E^2\theta^2$	Energy fraction	AO
Herwig++	$(t-m^2)/z(1-z)$	LC mom fraction	AO/Dipole
Pythia $<$6.4	t	Energy fraction	Enforced
Pythia $\geq$6.4	$k_\perp^2$	LC mom fraction	Enforced
Sherpa $<$1.2	t	Energy fraction	Enforced
Sherpa $\geq$1.2	$k_\perp^2$	LC mom fraction	Dipole
Vincia	variable	variable	Antenna

Effects of the parton shower



- Example: Drell-Yan lepton pair production at Tevatron
- If ME computed at leading order, then parton shower is only source of transverse momentum
- Any emissions softer than μ_F in terms of ordering parameter
- Significantly harder emissions experimentally, e.g. Drell-Yan

“Traditional” approach:

$$\delta_{\text{PS}} = |\text{Pythia} - \text{Herwig}|$$

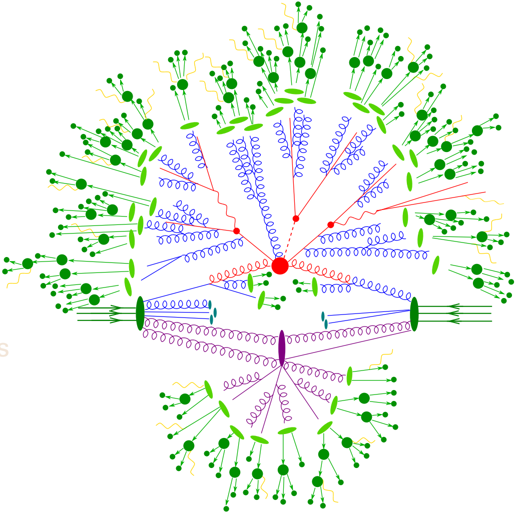
More rigorous assessment:

Systematic variations of ambiguities:

- Splitting kernels
  (finite pieces)
- Recoil scheme
  (onshell $1 \to 2$ splittings with 4-mom conservation!)
- Evolution variable
  (with correct collinear behaviour)
- Shower starting scale
  (often connected to μ_f)
- Shower cutoff scale
  transition to hadronisation (often part of tuning)
- α_S and PDFs
  (might be connected with tuning)

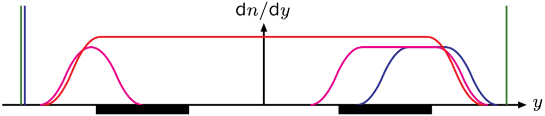
The structure of MC events

- Hard interaction
- Parton shower
- **Multiple parton interactions**
- Hadronisation
- Hadron decays
- Higher-order QED corrections

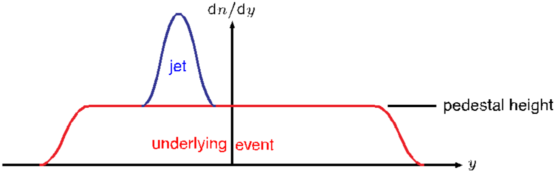


- Soft inclusive collision

$$\sigma_{\text{tot}} = \sigma_{\text{elastic}} + \sigma_{\text{single diffractive}} + \sigma_{\text{double diffractive}} + \sigma_{\text{non-diffractive}}$$



- Underlying event



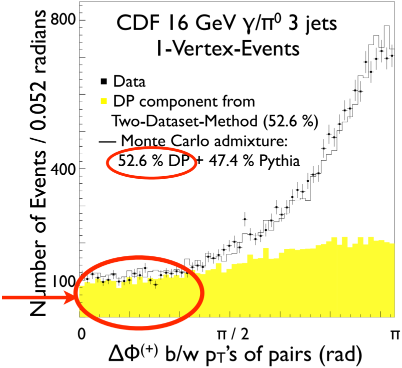
First evidence

- First experimental evidence for double-parton scattering: γ + 3 jets in CDF paper (1997)
- DPS component fitted to 53%
- Extraction of DPS cross section

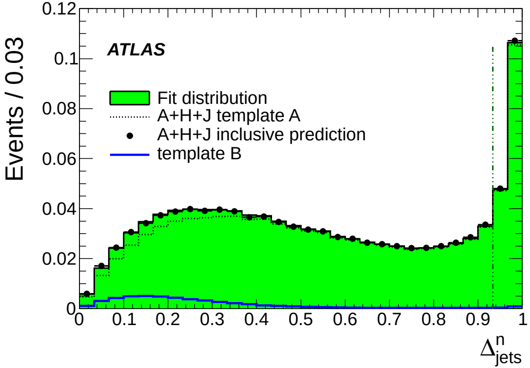
$$\sigma_{\text{DPS}} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\text{eff}}}$$

with

$$\sigma_{\text{eff}} = 14 \pm 4 \text{ mb}$$



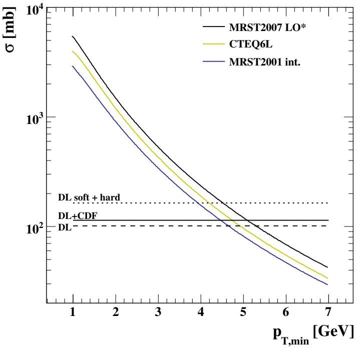
- Example: ATLAS measurement of W + 2 jets (2013)



- Extraction of DPS cross section
  $\sigma_{DPS} = \frac{\sigma_W \sigma_{jj}}{\sigma_{eff}}$ with $\sigma_{eff} = 15 \pm 3$ mb

Modeling the pedestal

[Sjöstrand,Zijl] PRD36(1987)2019

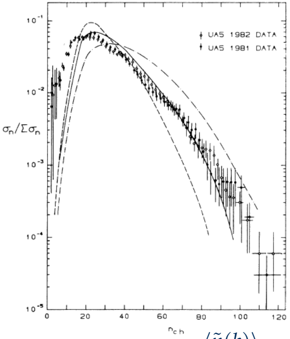


- Partonic cross sections diverge roughly like $\mathrm{d}p_T^2/p_T^4$
- Total cross section at LHC exceeded for $p_T \approx$ 2-5 GeV
- Interpretation as possibility for multiple hard scatters with

$$\langle n \rangle = \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$

- Main free parameter is $p_{T,\min}$ Determines size of σ_{hard}

Modeling the pedestal

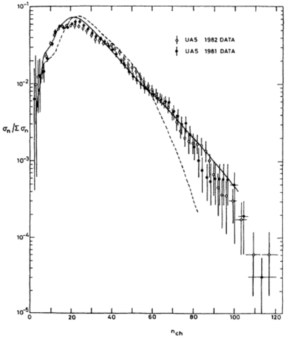


- Despite MPI wrong charged multi distribution
  Impact parameter dependent model needed
- Various hadron shape models in b-space
  (Exponential, Gaussian, double Gaussian)

$$\langle n \rangle = \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$

$$\downarrow$$

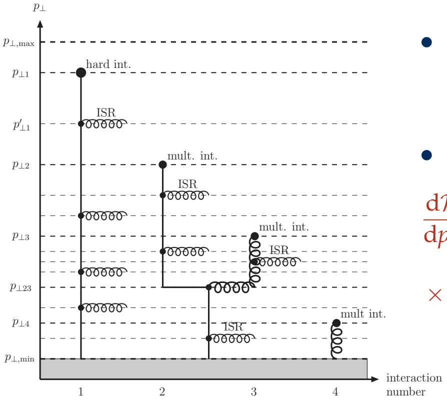
$$\langle \tilde{n}(b) \rangle = f_c f(b) \frac{\sigma_{\text{hard}}}{\sigma_{\text{non-diffractive}}}$$



- Hardness of the collision determines overlap
  Collisions with large overlap in turn
  have more secondary interactions

Combination with the parton shower

[Sjöstrand,Skands] hep-ph/0408302



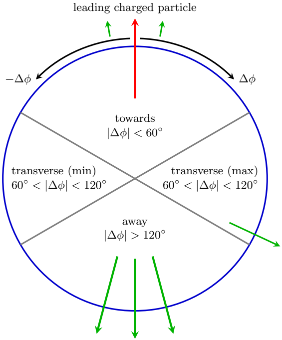
- When attaching IS shower to secondary scattering can ask at each point whether emission or new interaction is more likely
- New evolution equation

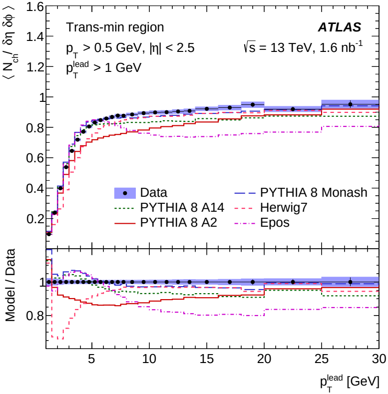
$$\frac{d\mathcal{P}}{dp_T} = \left(\frac{d\mathcal{P}_{\text{MI}}}{dp_T} + \frac{d\mathcal{P}_{\text{ISR}}}{dp_T}\right) \times \exp\left\{-\int_{p_T} dp'_T \left(\frac{d\mathcal{P}_{\text{MI}}}{dp'_T} + \frac{d\mathcal{P}_{\text{ISR}}}{dp'_T}\right)\right\}$$

Testing different models

Typical UE analysis:

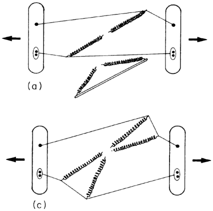
- Select hard direction of event, e.g. leading track
- Define towards/transverse/away regions
- Measure underlying activity in transverse region (uncorrelated)





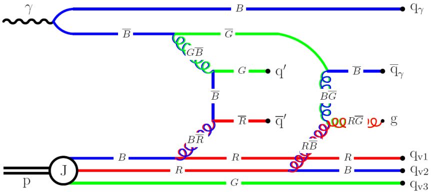
(Color connections and beam remnants)

[Sjöstrand,Skands] hep-ph/0402078



- Secondary scatterings need to be color-connected to something
- Simplest model would decouple them from proton remnants
- Next-to-simplest model would put all scatters on one color string

- New models embed scatters into existing color topology
- Three different options for string drawing
  - At random
  - Rapidity ordered
  - String length optimized



(A model for minimum bias collisions)

[Butterworth,Forshaw,Seymour] hep-ph/9601371
[Borozan,Seymour] hep-ph/0207283

- Assume parton distribution within beam hadron is

$$\frac{\mathrm{d}n_a(x,\mathbf{b})}{\mathrm{d}^2\mathbf{b}\mathrm{d}x} = f_a(x)\,G(\mathbf{b})$$

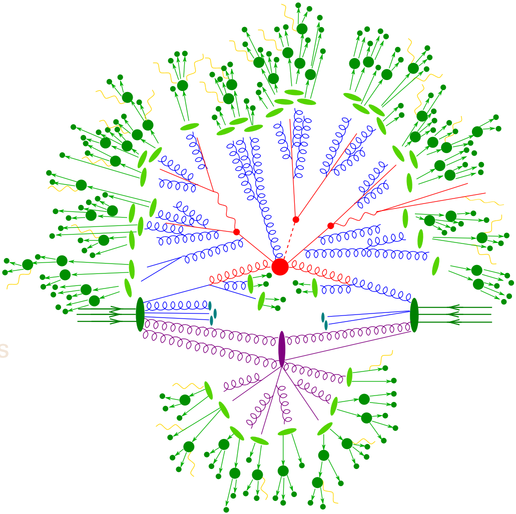
- Use electromagnetic form factor

$$G(\mathbf{b}) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}\,\frac{\exp(\mathbf{k}\cdot\mathbf{b})}{(1+\mathbf{k}^2/\mu^2)^2}$$

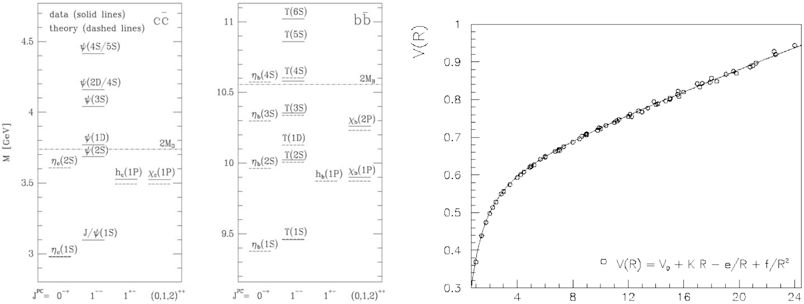
- EM measurements indicate $\mu_P = 0.71$ GeV
  μ is however left free in model $\rightarrow$ tuning
- Continue model below $p_{T,\mathrm{min}}$ with same b-space parametrization
  but cross section as Gaussian in p_T $\rightarrow$ inclusive non-diffractive events

The structure of MC events

- Hard interaction
- Parton shower
- Multiple parton interactions
- Hadronisation
- Hadron decays
- Higher-order QED corrections



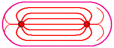
The inter-quark potential



- Measure QCD potential from quarkonia masses
- Or compute using lattice QCD
- Approximately linear potential ↔ QCD flux tube

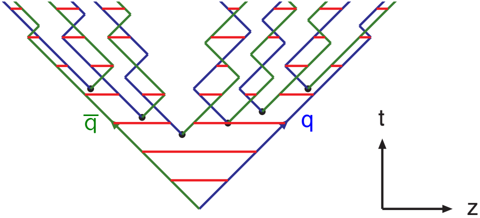
The Lund string model

[Andersson,Gustafson,Ingelman,Sjöstrand] PR97(1983)31

- Start with example $e^+e^- \to q\bar{q}$
- QCD flux tube with constant energy per unit rapidity $\leftrightarrow$ 
- New $q\bar{q}$-pairs created by tunneling (κ - string tension)

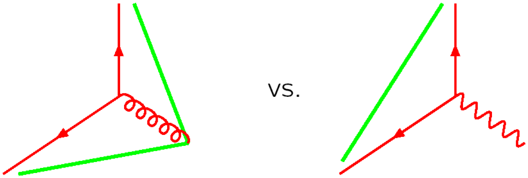
$$\frac{d\mathcal{P}}{dxdt} = \exp\left\{ -\frac{\pi^2 m_q^2}{\kappa} \right\}$$

- Expanding string breaks into hadrons, then yo-yo modes
- Baryons modeled as quark-diquark pairs



The Lund string model

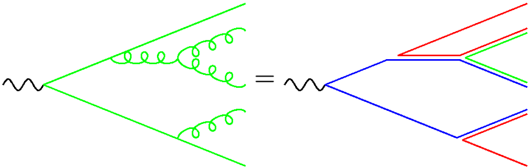
- String model very well motivated, but many parameters
- But also gives genuine prediction of "string effect"
- Gluons are kinks on string
  String accelerated in direction of gluon
- Infrared safe matching to parton showers
  Gluons with $k_T \lesssim 1/\kappa$ irrelevant



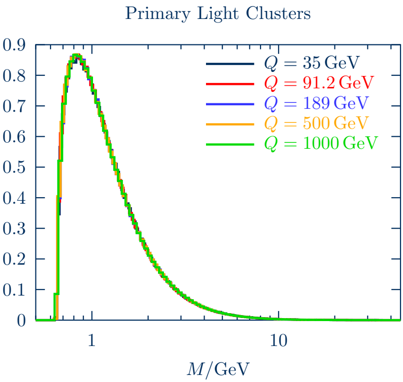
The cluster model

[Webber] NPB238(1984)492

- Underlying idea: Preconfinement
- Follow color structure of parton showers: color singlets end up close in phase space
- Mass of color singlets peaked at low scales ($\approx t_c$)



The cluster model



- Mass spectrum of primordial clusters independent of cm energy

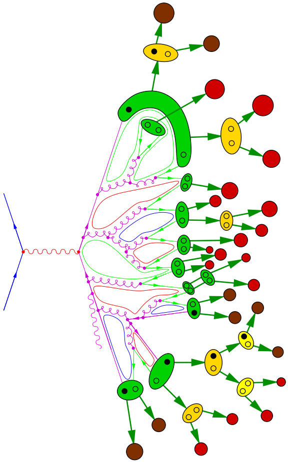
The cluster model

Naïve model

- Split gluons into $q\bar{q}$-pairs
  $\rightarrow$ flavour distribution important,
  momentum distribution not too much
- Color-adjacent pairs form primordial clusters
- Clusters decay into hadrons
  according to phase space
  $\rightarrow$ baryon & heavy quark production
  suppressed

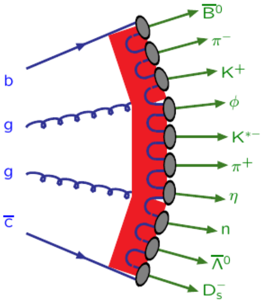
Improved model

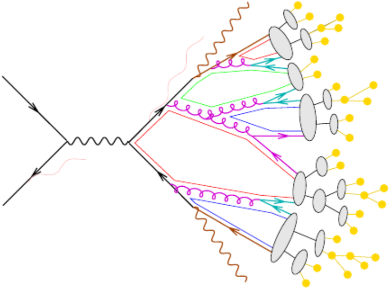
- Heavy clusters decay
  into lighter ones
- Three options: C$\rightarrow$CC,
  C$\rightarrow$CH & C$\rightarrow$HH
- Leading particle effects



String vs Cluster

[T.Sjöstrand, Durham'09]



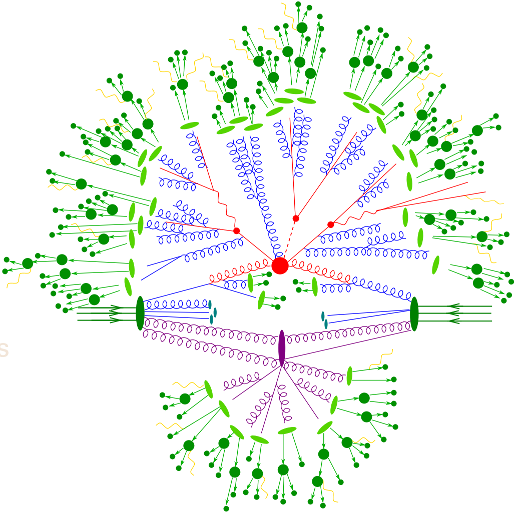


program	PYTHIA	HERWIG
model	string	cluster
energy–momentum picture	powerful	simple
	predictive	unpredictive
parameters	few	many
flavour composition	messy	simple
	unpredictive	in-between
parameters	many	few

"There ain't no such thing as a parameter-free *good* description"

The structure of MC events

- Hard interaction
- Parton shower
- Multiple parton interactions
- Hadronisation
- **Hadron decays**
- Higher-order QED corrections

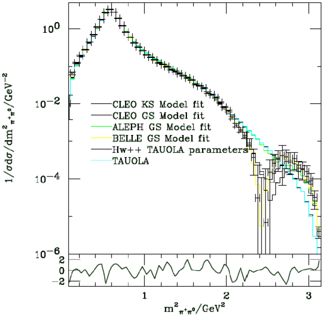


Secondary particle decays

- String and clusters decay to some stable hadrons
  but main outcome are unstable resonances
- These decay further according to the PDG decay tables
- Many hadron decays according to phase space
  but also a large variety of form factors known
- Not all branching ratios known precisely
  plus many BR's in PDG tables do not add up to one
- Significant effect on hadronization yields,
  hadronization corrections to event shapes, etc.

Secondary particle decays & photon radiation

- Previous generations of generators relied on external decay packages Tauola (τ-decays) & EvtGen (B-decays)
- New generation programs Herwig++ & Sherpa contain at least as complete a description
- Spin correlations and B-mixing built in
- No interfacing issues



- Previous generations of generators relied on external package Photos to simulate QED radiation
- New generation programs Herwig++ & Sherpa have simulation of QED radiation built in

General-purpose event generators

[Buckley et al.] arXiv:1101.2599

Herwig

- Originated in coherent shower studies → angular ordered PS
- Front-runner in development of Mc@Nlo and Powheg
- Simple in-house ME generator & spin-correlated decay chains
- Original framework for cluster fragmentation

Pythia

- Originated in hadronization studies → Lund string
- Leading in development of multiple interaction models
- Pragmatic attitude to ME generation → external tools
- Extensive PS development and earliest ME⊕PS matching

Sherpa

- Started with PS generator Apacic++ & ME generator Amegic++
- Current MPI model and hadronization pragmatic add-ons
- Leading in development of automated ME⊕PS merging
- Automated framework for NLO calculations and Mc@Nlo

Partial generators/calculators

MadGraph5_aMC@NLO [Allwall et al.] arXiv:1405.0301

- Tree-level and virtual matrix elements (NLO)
- NLO subtraction and matching

PowhegBox [Alioli et al.] arXiv:1002.2581

- Matrix elements at NLO (excluding virtuals)
- Hardest emission in Powheg formalism

OpenLoops [Cascioli et al.] arXiv:1111.5206

- Virtual matrix elements

Alpgen [Mangano et al.] arXiv:hep-ph/0206293

- Multi-jet matrix elements and merging at LO

EvtGen [Lange et al.] Nucl.Instrum.Meth. A462 (2001) 152-155

- Dedicated heavy-flavour hadron decays

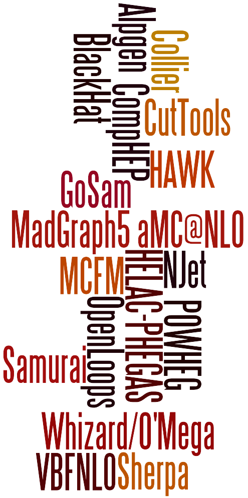
Tauola [Jadach et al.] Comput.Phys.Commun. 64 (1990) 275-299

- Dedicated tau decays

Photos [Barberio et al.] Comput.Phys.Commun. 66 (1991) 115-128

- QED radiation from final state leptons

. . . and many others for specialised purposes



Analysis and tuning tools

Rivet [Buckley et al.] arXiv:0103.0694

- LHC-successor to HZTool
  Collection of exp. data &
  matching analysis routines
- Spirit: “Right MC describes everything at the same time”

Professor [Buckley et al.] arXiv:0907.2973

- Tuning in multi-dimensional parameter space of MC
- Generate event samples at random parameter points
  Analyze them with Rivet
  Parametrize observables
  Minimize χ^2 and cross-check

Tune comparisons

Deviation metrics per gen/tune and observable group:

Gen	Tune	UE	Dijets	Multijets	Jet shapes	W and Z	Fragmentation	B frag
AlpGen	HERWIG6	–	1.83	5.36	2.48	0.91	–	–
	PYTHIA6-AMBT1	–	1.55	2.80	0.61	0.53	–	–
	PYTHIA6-D6T	–	1.38	2.67	2.31	1.67	–	–
	PYTHIA6-P2010	–	1.09	2.65	2.03	1.48	–	–
	PYTHIA6-P2011	–	1.12	2.60	0.48	0.24	–	–
	PYTHIA6-Z2	–	1.48	2.63	0.55	0.48	–	–
	PYTHIA6-profQ2	–	1.16	2.65	1.43	1.29	–	–
HERWIG	AUET2-CTEQ6L1	0.43	0.55	0.77	0.35	0.58	22.80	2.38
	AUET2-LOxx	0.25	0.71	0.60	0.39	0.88	22.13	2.29
Herwig++	2.5.1-UE-EE-3-CTEQ6L1	0.27	0.87	0.78	0.51	0.98	10.58	1.32
	2.5.1-UE-EE-3-MRSTLOxx	0.23	1.05	0.78	0.50	0.65	10.58	1.32
PYTHIA6	AMBT1	0.39	1.20	0.54	0.77	0.27	0.93	1.65
	AUET2B-CTEQ6L1	0.16	0.92	0.44	0.59	0.74	0.67	1.29
	AUET2B-LOxx	0.13	1.33	0.55	0.58	1.15	0.67	1.30
	D6T	0.58	0.79	0.50	0.56	1.25	0.36	2.63
	DW	0.81	0.78	0.61	0.56	1.33	0.36	2.63
	P2010	0.30	0.93	0.82	1.07	0.30	0.44	1.75
	P2011	0.12	0.89	0.67	1.02	0.53	0.43	2.13
	ProfQ2	0.51	0.67	0.81	0.51	0.64	0.30	1.65
	Z2	0.18	0.94	0.73	0.80	0.30	0.95	2.78
Pythia8	4C	0.30	0.97	0.93	0.50	0.90	0.38	1.12
Sherpa	1.3.1	0.68	0.47	0.34	0.71	0.36	0.75	2.48

[LH’11 SM WG] arXiv:1203.6803 [hep-ph]

Thank you for your attention and active participation!

Suggested reading

- L. Dixon, F. Petriello (Editors)
  Journeys Through the Precision Frontier
  Proceedings of TASI 2014, World Scientific, 2015
- R. K. Ellis, W. J. Stirling, B. R. Webber
  QCD and Collider Physics
  Cambridge University Press, 2003
- R. D. Field
  Applications of Perturbative QCD
  Addison-Wesley, 1995
- A. Buckley et al.
  General-Purpose Event Generators for LHC Physics
  Phys. Rept. 504 (2011) 145
- T. Sjöstrand, S. Mrenna, P. Z. Skands
  PYTHIA 6.4 Physics and Manual
  JHEP 05 (2006) 026