UNDERSTANDING the LANDSCAPE of $\mathcal{N}=2$ Super Conformal Field Theories

DESY 08/11/16

based on: [hep-th/1505.04814]
[hep-th/1601.00011]
[hep-th/1602.02764]
[hep-th/1609.04404]
[hep-th/1611.XXXXX]

in collaboration with **P. Argyres, M. Lotito** and **Y. Lü**

Why?

1) CFTs are the "fundamental bricks" of Effective Field Theory.

2) Conformal Bootstrap techniques revived the interest in Conformal Field Theories (CFTs)

3) SCFTs are a great "arena" to learn about Quantum Field Theory

We will Classify rank 1 $\mathcal{N}=2$ SCFTs

INTRO

"The novel phylosophy of our approach"

U(1)-gauge theories

w/ n_I fields w/ charge $\pm Q_I$

$$\boxed{\text{IR-free for all } n_I \ \& \ Q_I}$$

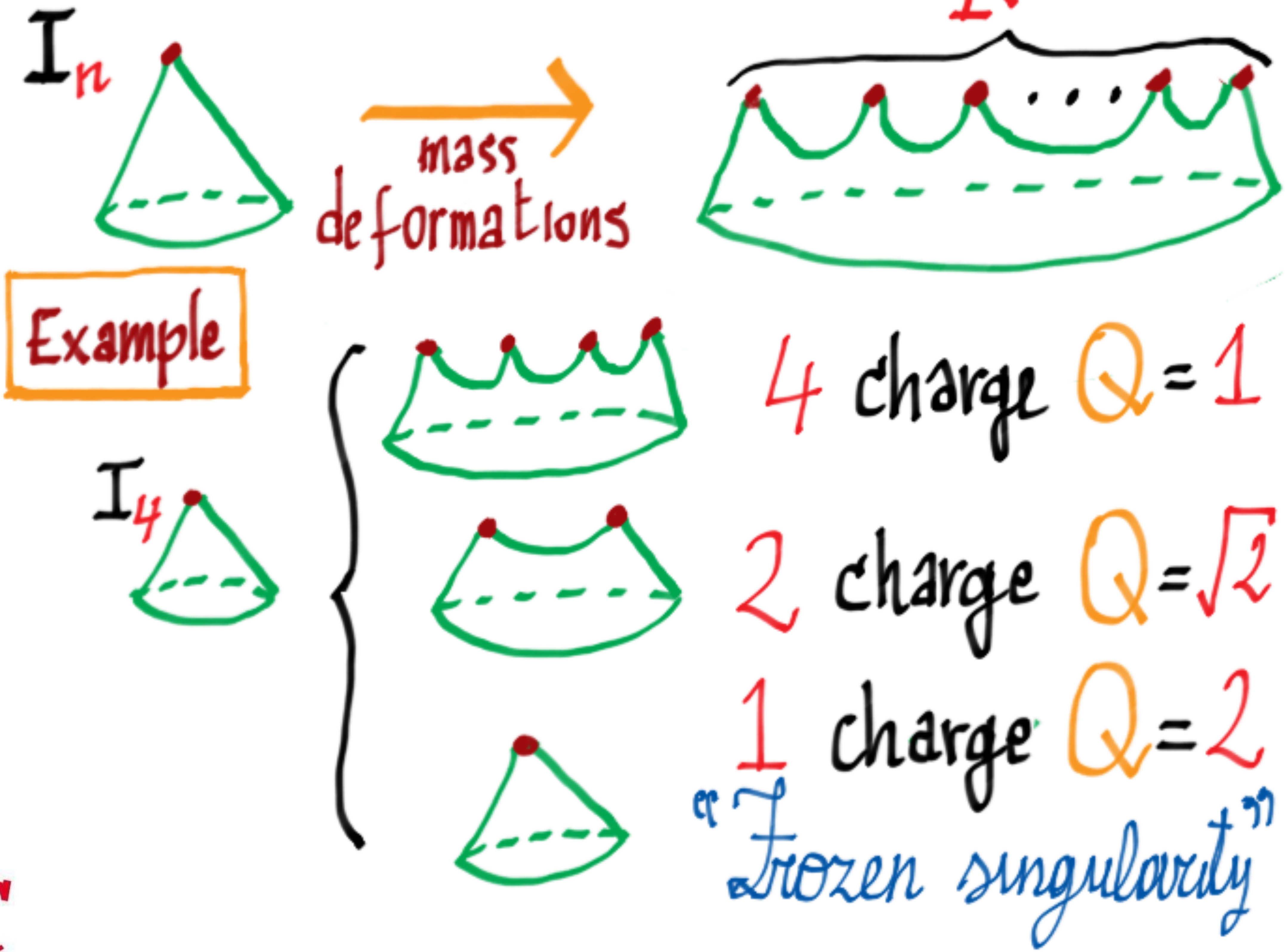
U(1)-gauge theories

a) β-function: $\boxed{n = \sum_I n_I Q_I^2}$

b) # of mass deformations $\left(m \Phi_{Q_I} \Phi_{-Q_I}\right)$

$$\boxed{N = \sum_I n_I}$$

Geometric Description



SU(2)-gauge theories

w/ fields in the R_i repr.$^{\text{ns}}$

$$\boxed{b_0 = -4 + \sum_i T(R_i)}$$

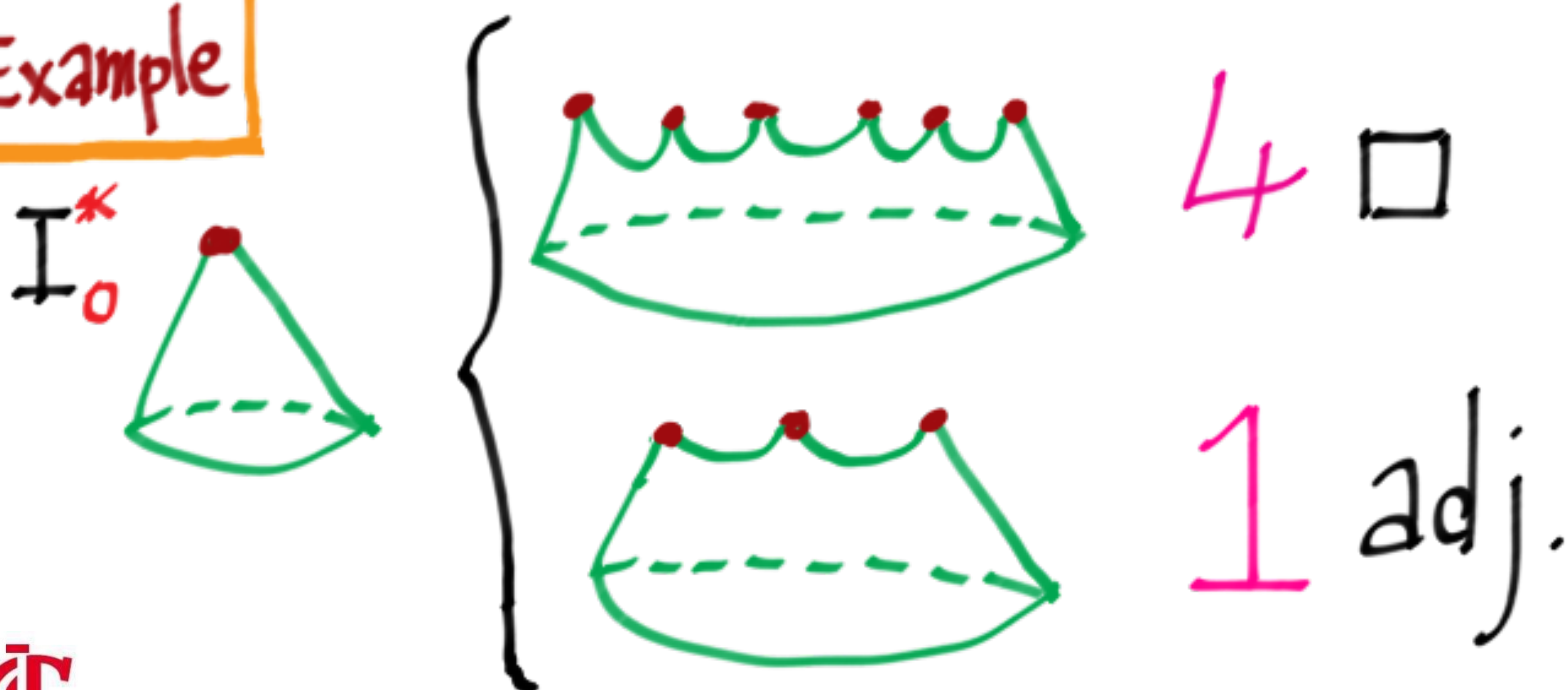
$T(\square) = 1, \; T(\text{adj.}) = 4$

SU(2)-gauge theories

$$b_0 \begin{cases} < 0 & \text{Asymptotically free} \\ = 0 & \boxed{4\,\square \text{ or } 1\,\text{adj}} \quad \text{Conformal} \\ > 0 & \text{IR-free} \end{cases}$$

Geometric Description

Example



Rank 1 $\mathcal{N}=2$ SCFTs

= $\mathcal{N}=3$ theories

= $\mathcal{N}=4$ theories

= Lagrangian theories

= IR-free theories

New Theories

series	SI sing.	$\Delta(u)$	deform.	$\mathfrak{f}$	$k_{\mathfrak{f}}$	$24a$	$12c$
I_1 series	II^*	6	$\{{I_1}^{10}\}$	E_8	12	95	62
	III^*	4	$\{{I_1}^{9}\}$	E_7	8	59	38
	IV^*	3	$\{{I_1}^{8}\}$	E_6	6	41	26
	I_0^*	2	$\{{I_1}^{6}\}$	D_4	4	23	14
	IV	3/2	$\{{I_1}^{4}\}$	A_2	3	14	8
	III	4/3	$\{{I_1}^{3}\}$	A_1	8/3	11	6
	II	6/5	$\{{I_1}^{3}\}$	$\varnothing$	–	43/5	22/5
	I_1	1	–	U_1	?	6	3
I_4 series	II^*	6	$\{{I_1}^{6}, I_4\}$	C_5	7	82	49
	III^*	4	$\{{I_1}^{5}, I_4\}$	C_3C_1	$(5,8)$	50	29
	IV^*	3	$\{{I_1}^{4}, I_4\}$	C_2U_1	$(4,?)$	34	19
	I_0^*	2	$\{{I_1}^{2}, I_4\}$	C_1	3	18	9
	I_4	1	–	U_1	?	6	3
I_1^* series	II^*	6	$\{{I_1}^{3}, I_1^*\}$	$A_3 \rtimes \mathbb{Z}_2$	14	75	42
	III^*	4	$\{{I_1}^{2}, I_1^*\}$	$A_1U_1 \rtimes \mathbb{Z}_2$	$(k,?)$	45	24
	IV^*	3	$\{I_1, I_1^*\}$	U_1	?	30	15
	I_1^*	2	–	$\varnothing$	–	17	8
$IV^*_{Q=1}$ ser.	II^*	6	$\{{I_1}^{2}, IV^*_{Q=1}\}$	$A_2 \rtimes \mathbb{Z}_2$	14	71	38
	III^*	4	$\{I_1, IV^*_{Q=1}\}$	$U_1 \rtimes \mathbb{Z}_2$	?	42	21
	$IV^*_{Q=1}$	3	–	$\varnothing$	–	55/2	25/2
I_2 ser.	I_0^*	2	$\{{I_2}^{3}\}$	C_1	3	18	9
	I_2	1	–	U_1	?	6	3

PART I "Geometric structure of the Coulomb Branch"

PART II "Scale invariant geometries and Mass deformations"

PART III "Classification of physical geometries"

PART IV "$\mathcal{N}=3$ theories"

PART I

"Geometric structure of the Coulomb Branch"

VECTOR MULTIPLET

$V^{\mathcal{N}=2}$:

$\mathcal{N}=1$ Vector Multiplet

$$\underbrace{\lambda_\alpha \longleftrightarrow A_\mu}$$

$$\underbrace{\Phi \longleftrightarrow \tilde{\lambda}_\alpha}$$

$\mathcal{N}=1$ Chiral Multiplet

Φ is a scalar which can acquire a VEV:

$$\boxed{\Phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N \end{pmatrix}, \quad \sum_i^N a_i = 0}$$

$\Phi \in \mathbb{C}^{N-1}$ & on the cB $SU(N) \to U(1)^{N-1}$

for $SU(2)$

$$\Phi = \begin{pmatrix} a & \\ & -a \end{pmatrix}$$

- a is not gauge invariant ✗
- $u = \frac{1}{2}\langle \mathrm{tr}\, \Phi^2 \rangle = a^2$ is ✓

Coulomb Branches

inherit "a lot" of

GEOMETRIC STRUCTURE from

$\mathcal{N}=2$ Supersymmetry!

Holomorphic gauge Coupling

$$\boxed{\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}}$$

1) For non-zero values of u, τ runs.

2) τ is thus a function of u, $\tau(u)$.

Eelectro-Magnetic Duality

1) $E \to -B \Rightarrow \tau \to -\frac{1}{\tau}$

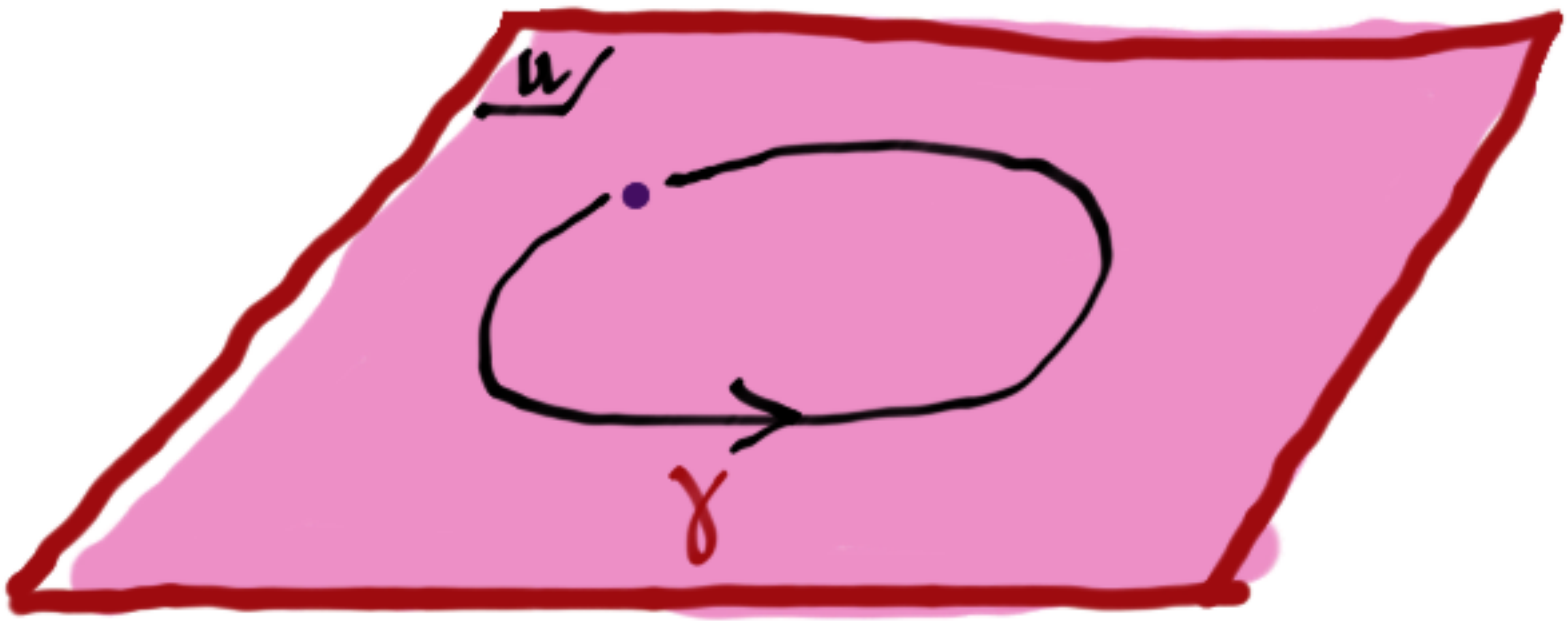
2) $\theta \to \theta + 2\pi \Rightarrow \tau \to \tau + 1$

Combination of 1) & 2) generates $SL(2,\mathbb{Z})$

$$\boxed{\tau \to \frac{a\tau + b}{c\tau + d}, \{a,b,c,d\} \in \mathbb{Z}, ad - bc = 1}$$

Seiberg-Witten in a nutshell

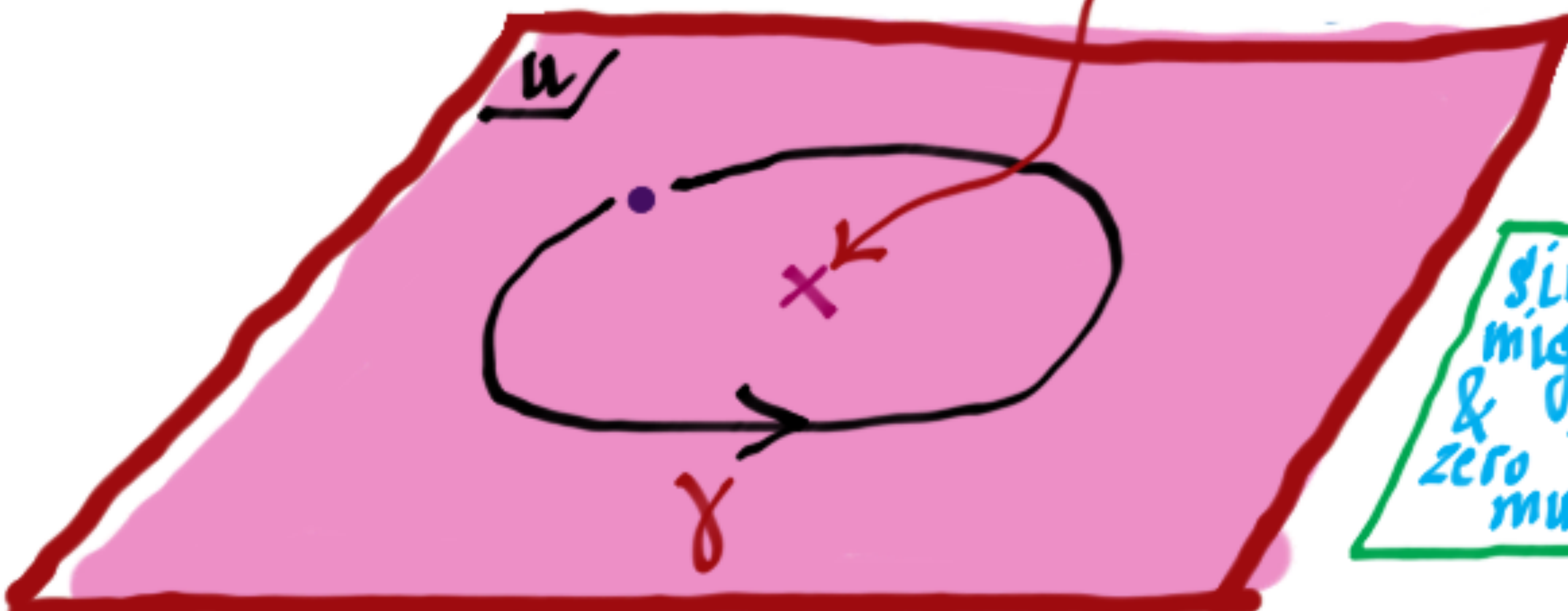
$$\tau \xrightarrow[\gamma]{SL(2,\mathbb{Z})} \tau' = \frac{a\tau+b}{c\tau+d} \quad \Bigg| \quad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$



Seiberg-Witten in a nutshell

singularities appear for values of the CB parameter which allow extra massless states in the theory.

there needs to be a singularity within the curve

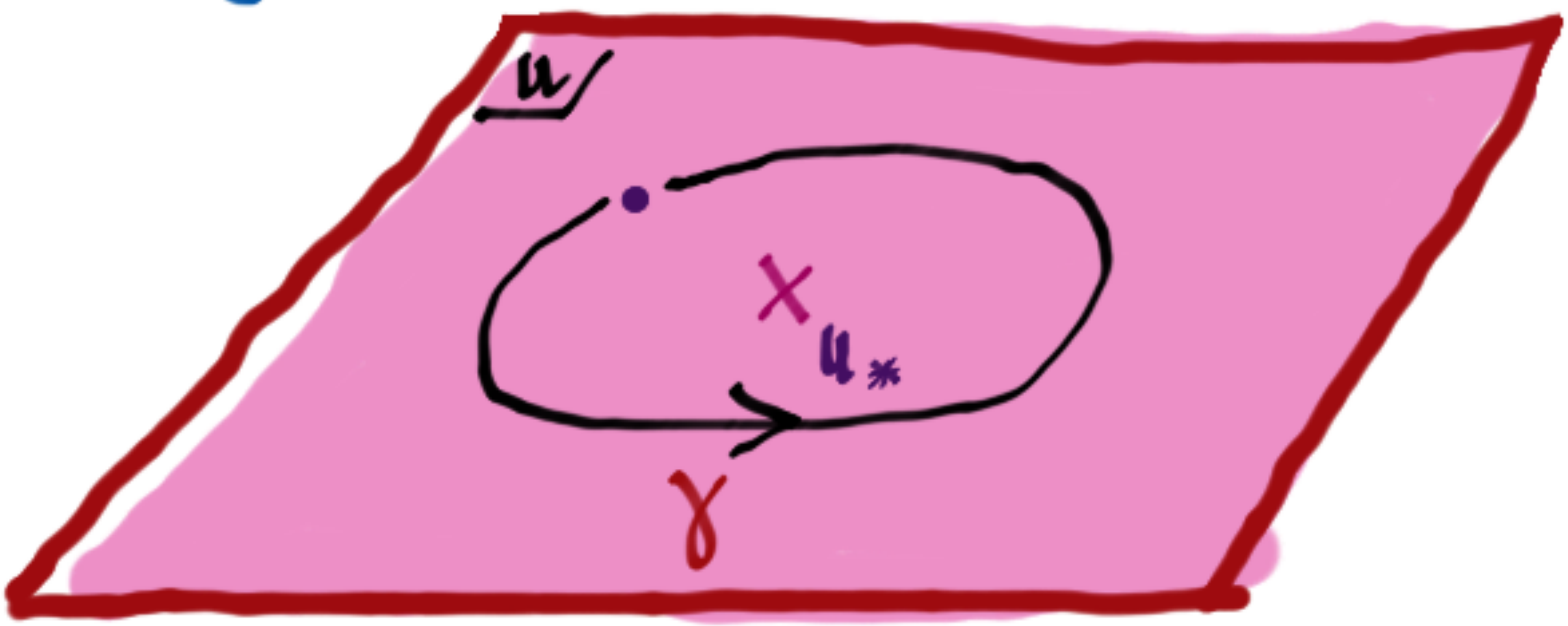


singularities might overlap & have non-zero multiplicity

Monodromies

1) In the rank 1 case $M_{u_*} \in SL(2,\mathbb{Z})$

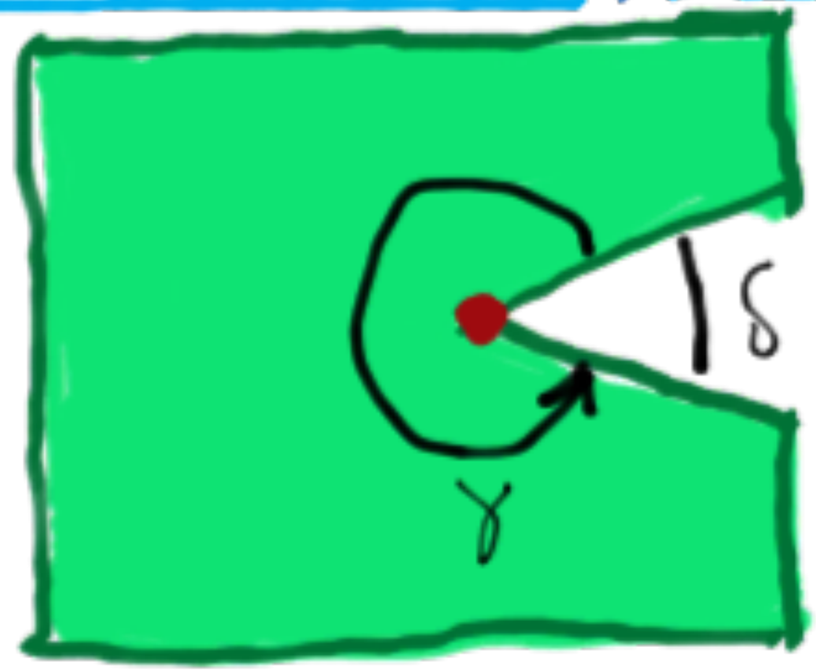
2) Monodromies uniquely characterize the singularity u_*.



Part II

"Scale invariant geometries and Mass deformations"

Conformal invariance restricts the CB to be Scale-Invariant for Rank 1 → Incredibly constraining



$\mathbb{C}$ w/ deficit angle δ

The deficit angle δ is fixed by the u scaling dimension

$$\left\{\begin{array}{c|ccccccc|c} \delta & \pi/3 & \pi/2 & 2\pi/3 & \pi & 4\pi/3 & 3\pi/2 & 5\pi/3 & 2\pi\,(\text{cusp}) \\ \Delta(u) & 6 & 4 & 3 & 2 & 3/2 & 4/3 & 6/5 & \mathbb{N} \end{array}\right\}$$

KODAIRA classification

these CB geometries arise in non Lagrangian theories which can be constructed through F-theory

This geometry arises in $SU(2)$ theories.

Name	curve ($y^2=\dots$)	$\Delta(u)$	M_0	#
II^*	x^3+u^5	6	ST	10
III^*	x^3+u^3x	4	S	9
IV^*	x^3+u^4	3	$-(ST)^{-1}$	8
I_0^*	$x^3+u^2x+gu^2$	2	$-\mathbb{1}$	6
IV	x^3+u^2	$3/2$	$-ST$	4
III	x^3+ux	$4/3$	S^{-1}	3
II	x^3+u	$6/5$	$(ST)^{-1}$	2
$I^*_{n>0}$	$x^3+ux^2+\Lambda^{-2n}u^{n+3}$	2	$-T^{-n}$	$n+6$
$I_{n>0}$	$(x-1)(x^2+\Lambda^{-n}u^n)$	1	T^{-n}	n

Is this list COMPLETE?

MASS DEFORMATIONS I/II

Consider a hyper-multiplet:

$$Q_i^{\mathcal{N}=2}=(Q_i^{\mathcal{N}=1},\tilde{Q}_i^{\mathcal{N}=1}) \qquad i\equiv \text{flavour}$$

the mass term is:

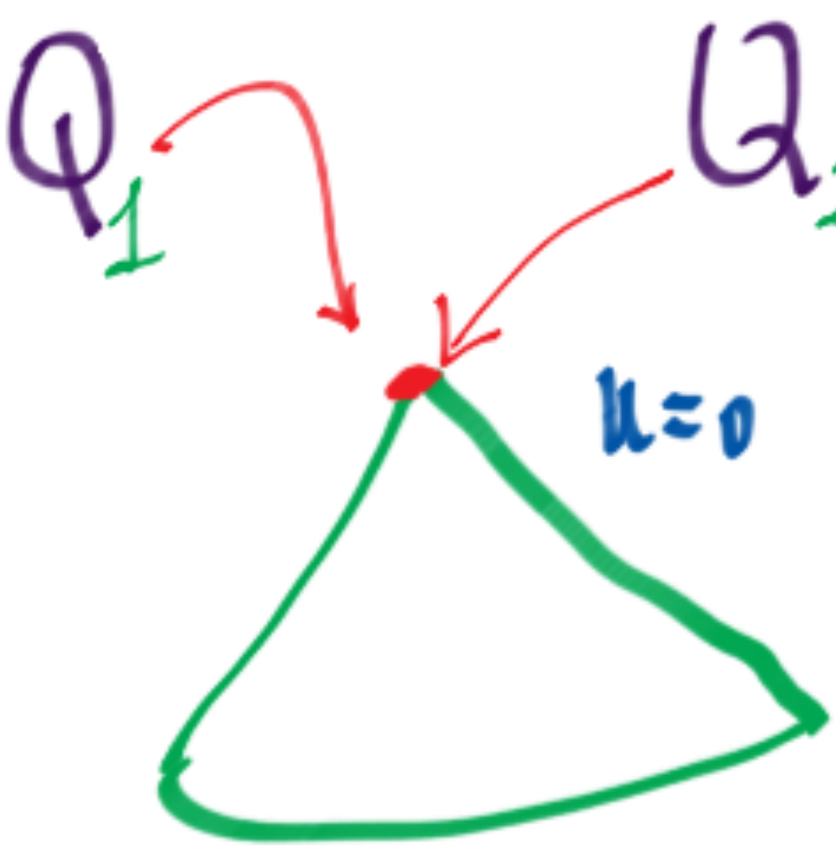
$$\tilde{Q}_i\Phi Q^i+\mu_i\tilde{Q}_iQ^i$$

with masses:

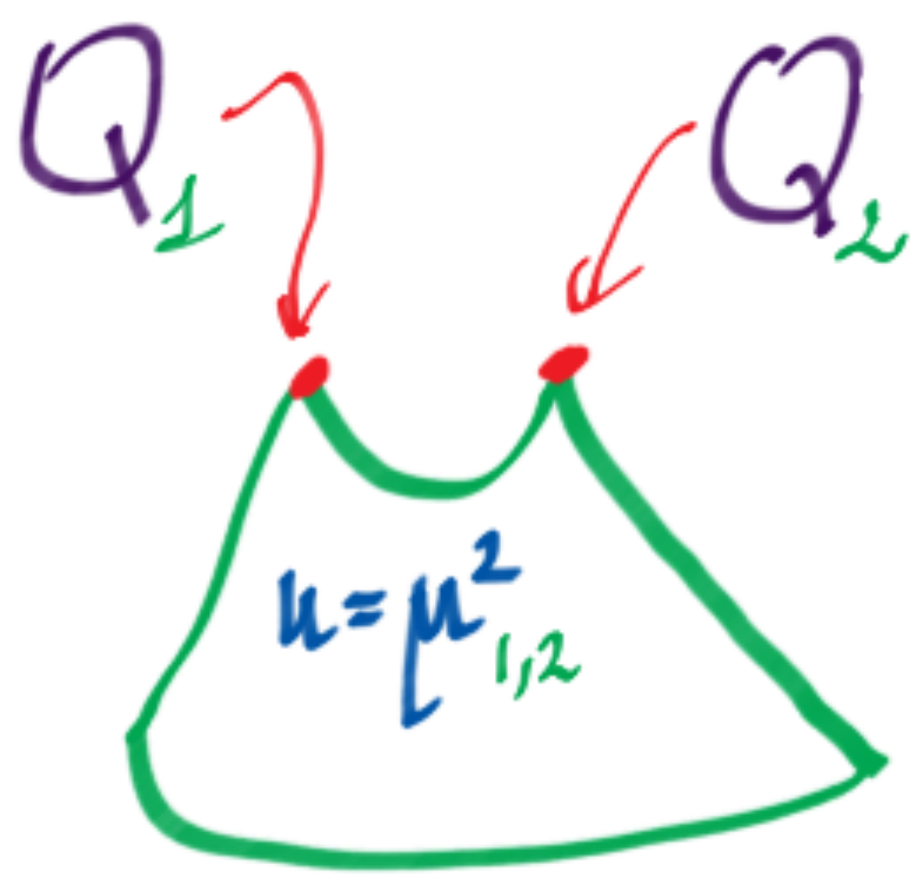
$$M_Q^i\sim|\mu_i+u|$$

MASS DEFORMATIONS $^{II}/_{II}$

$\mu = 0$



$\mu \neq 0$



Part III

"Classification of physical geometries"

CONJECTURE

$\mathcal{N}=2$ SCFT's are classified by scale-invariant geometries (i.e. KODAIRA) plus all "physical" splitting of the singularity @ $u=0$ (i.e. spectrum of mass deformations)

Recall KODAIRA...

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$I_{n>0}$	$(x-1)(x^2+\Lambda^{-n}u^n)$	1	T^{-n}	n

Only Deformations which Preserve the order of the singularities are Allowed

Geometric Notation

$$II^* \to \{I_1^6, I_4\}$$

"Initial Singularity"

"Deformation Pattern"

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Dirac Quantization CONDITION

3) The "ending" singularities have to satisfy Dirac Quantization condition:

$$\boxed{\text{The } \sqrt{n} \text{ need to be COMMENSURATE}}$$

OF DEFORMATION PATTERNS 389

$\cup$ $\downarrow$

$$\boxed{\text{\# DIRAC QUANTIZATION} \quad 34}$$

OF DEFORMATION PATTERNS 389

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DIRAC QUANTIZATION 34

$\cup$ $\downarrow$

$SL(2,\mathbb{Z})$ MONODROMIES 25

of DEFORMATION PATTERNS 389

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DIRAC QUANTIZATION 34

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$\boxed{\text{\# RG FLOWS} \quad 17}$

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PART IV

"$\mathcal{N}=3$ theories"

Recap on $\mathcal{N}=3$

i) Intrinsically non-lagrangian (isolated)

ii) $U(3)_R$ R-symmetry

iii) No flavor group (like $\mathcal{N}=4$)

iv) Moduli Space parametrized by $\mathcal{N}=3$ vector multiplet

v) $c=a$

See Them From $\mathcal{N}=2$

i) Can only appear as IV^*, III^* or II^*

ii) $U(2)_R \subset U(3)_R$

iii) Have a $U(1)$ flavor simmetry

iv) $V^{N=3} \to V^{N=2} \oplus Q^{N=2} \Rightarrow \dim_{IH}=1$ mixed branch

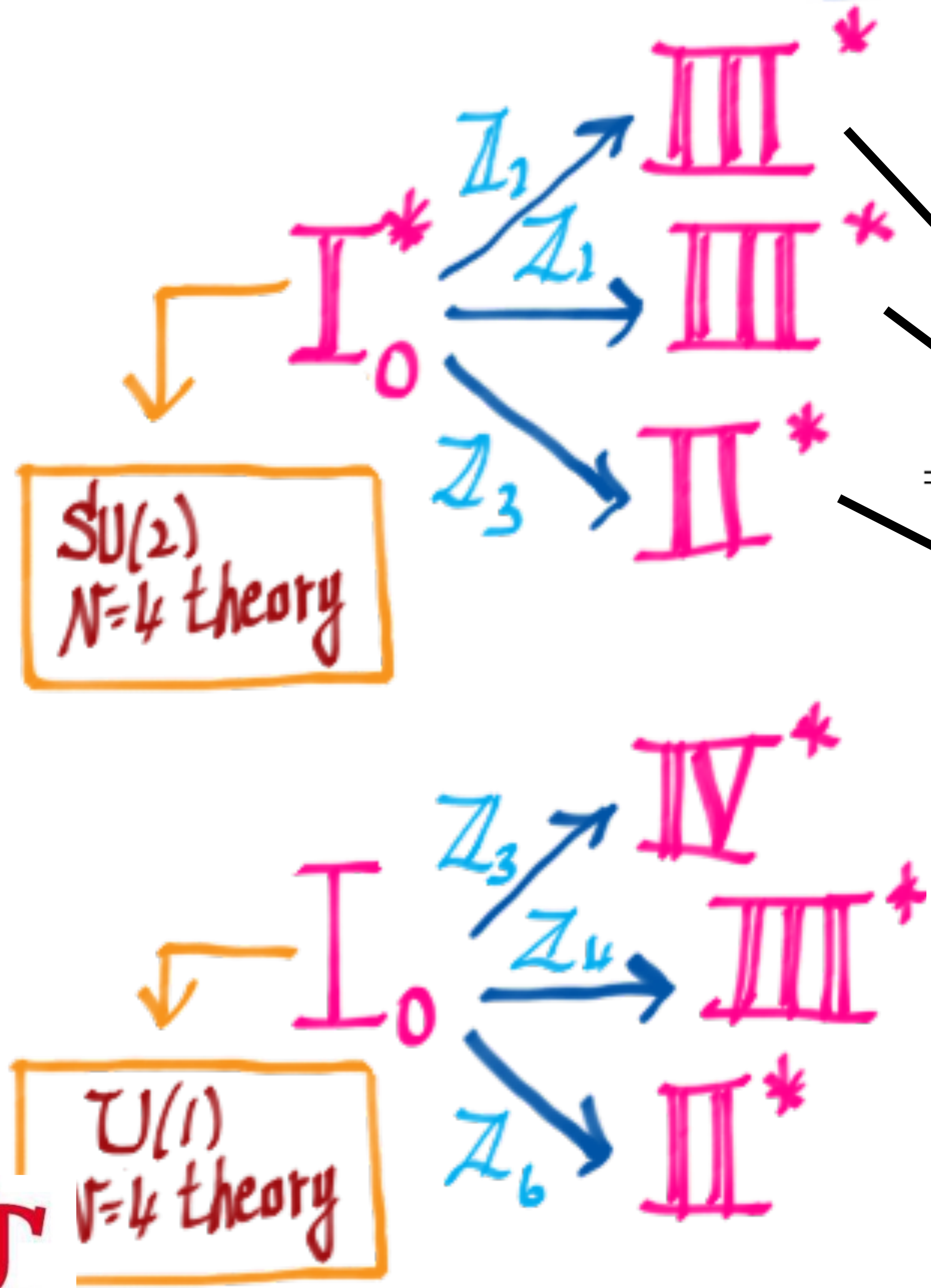
v) Appropriate central charges $\boxed{c=a}$

"Genuinely" $\mathcal{N}=3$

$$\left[IV^*, U(1), 12\ c=15, 24\ a=30, k_f=5\right]$$

$$\left[III^*, U(1)\times\mathbb{Z}_2, 12\ c=15, 24\ a=30, k_f=5\right]$$

Discretely Gauged



	$\ell = 1$	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 6$
$k = 2$	$-1/4$	$+1/4$			
$k = 3$	$-1/3$		$+1/3$		
$k = 4$	$-3/8$	$-1/8$		$+3/8$	
$k = 6$	$-5/12$	$-1/4$	$-1/12$		$+5/12$

From 1602.08638
O. Aharony & Y. Tachikawa

CONCLUSIONS & OPEN QUESTIONS

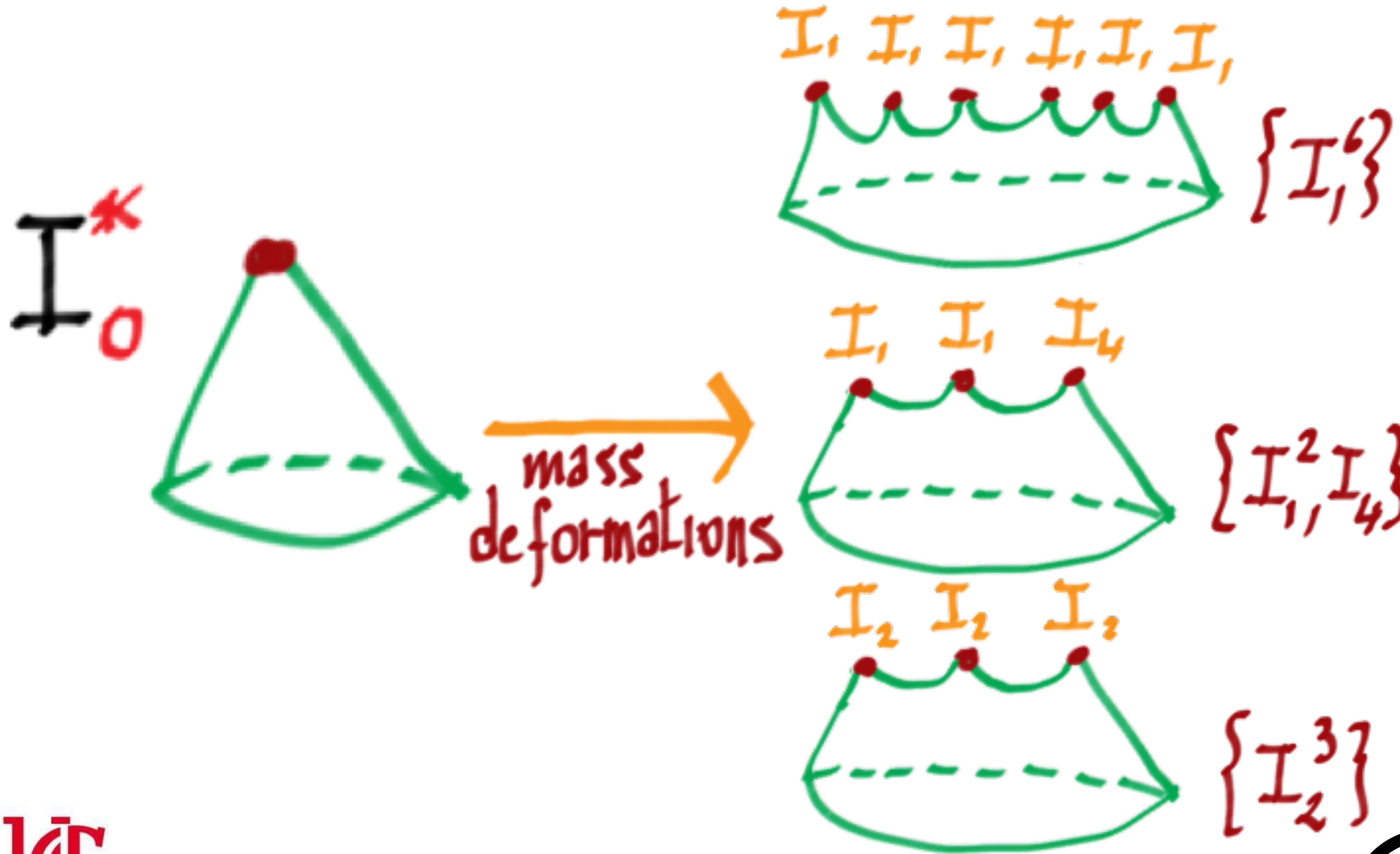
1) We proposed a set of "physical" conditions to construct consistent SW geometries. This can be turned in powerful constraints on $\mathcal{N}=2$ SCFTs

2) We have shown the possibility of the exsitence of many new $\mathcal{N}=2$ & $\mathcal{N}=3$ SCFTs

3) Can we generalize it to rank=2?

Thank you!

I_0^* Deformations



$\{I_1^6\}$-Geometry

"Pure-SU(2)" dyonic singularities

Singularities ass. to the 4 flavors

$\boxed{I_1,\ I_1}\ \boxed{I_1,\ I_1,\ I_1,\ I_1}$

Features

i) 4 mass deformations.

ii) $SO(8)/D_4$ flavour group.

iii) Appropriate central charges

$\{I_1^2 I_4\}$-Geometry

"Pure-$SU(2)$"
dyonic singularities

Singularities ass. to the adj. hyper

$I_1 \quad I_1$ $\quad$ I_4

Features

i) 1 mass deformation.

ii) $Sp(2)/SU(2)$ flavour group.

iii) Appropriate central charges $\boxed{c=a}$

iv) Existence of a $\dim_{IH}=1$ mixed branch

$\{I_2^3\}$-Geometry

$\boxed{I_2 \quad I_2 \quad I_2}$

???

Features

i) 1 mass deformation.

ii) $Sp(2)/SU(2)$ flavour group.

iii) Appropriate central charges $\boxed{c=a}$

iv) Existence of a $\dim_{\mathbb{H}}=1$ mixed branch

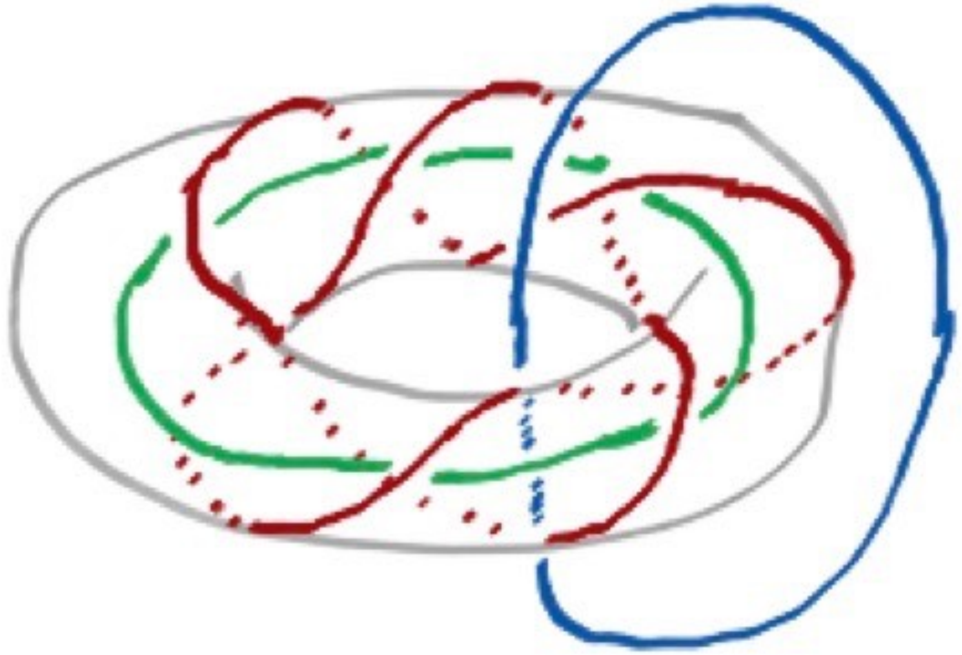
v) Existence of a $\mathbb{Z}_3 \in SL(2,\mathbb{Z})_{EM}$

how about RANK 2?

w/ P. Argyres
D. Kulkarni
C. Long

1) The CB is $\mathbb{C}^2 \sim \mathbb{R}^4$.

2) The scale inv. singularity is still co-dim 1.
not a point but a 1-complex dim. curve.



Generically
- Torus Links -

3) The E-M duality group: $SL(2,\mathbb{Z}) \rightarrow Sp(4,\mathbb{Z})$

Current Situation

w/ P. Argyres
D. Kulkarni
C. Long

1) The analog of the KODAIRA classification is not known
2) Many more scale-inv. geometries are known (>25 vs. 7 in rank 1)
3) They are classified by representations of the $\pi_1(\text{knot})$ in $Sp(4,\mathbb{Z})$
4) We have classified all the "single knot" geometries!

Extra