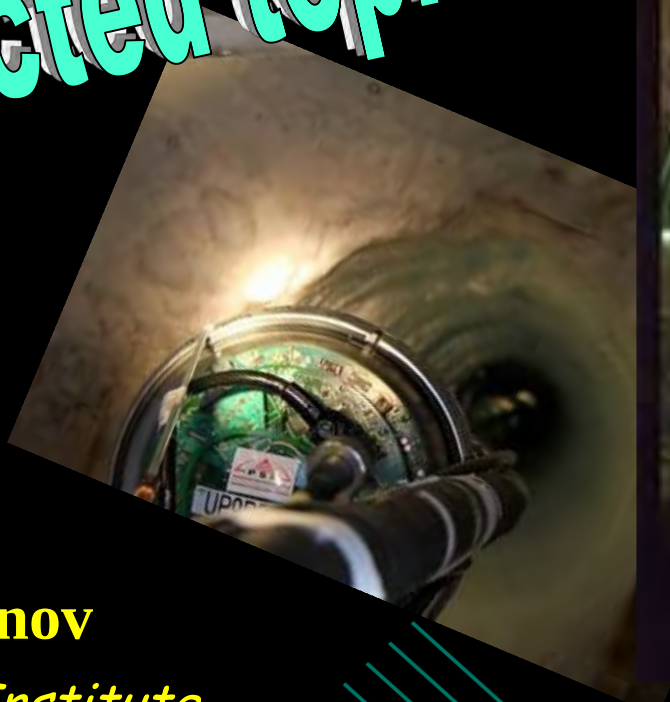
Neutrinos
selected topics



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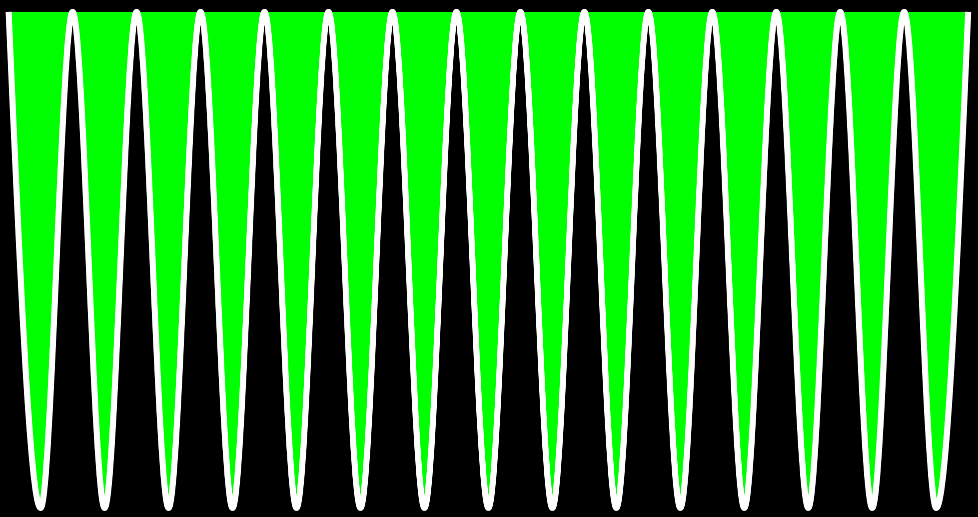
March 6 - 10, 2017

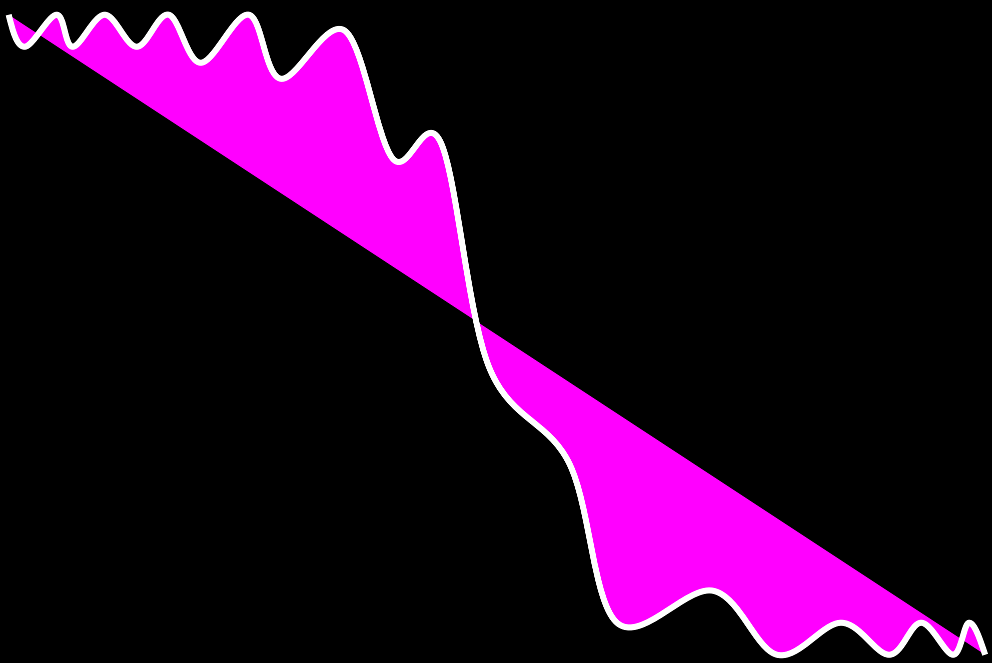
Matter effects

oscillations



adiabatic

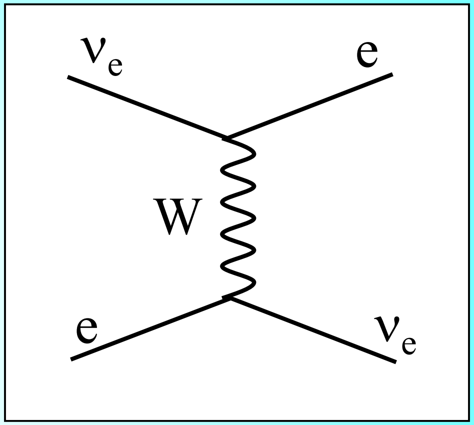
conversion



Matter potential

L. Wolfenstein, 1978

for ν_e ν_μ



at low energies Re A >> Im A
inelastic interactions can be neglected

Elastic forward scattering $\Rightarrow$ V_e, V_μ

potentials

difference of potentials

$$V = V_e - V_\mu = \sqrt{2}\, G_F\, n_e$$

$V \sim 10^{-13}$ eV inside the Earth

Refraction index:

$$n - 1 = V / p$$

for E = 10 MeV

$$n - 1 = \begin{cases} \sim 10^{-20} & \text{inside the Earth} \\ < 10^{-18} & \text{inside the Sun} \end{cases}$$

Refraction length:

$$l_0 = \frac{2\pi}{V}$$

Matter potential

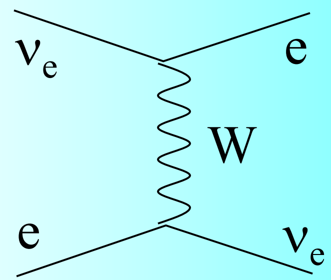
At low energies: neglect the inelastic scattering and absorption effect is reduced to the elastic forward scattering (refraction) described by the potential V (mean field approximation):

$$H_{int}(\nu) = \langle \psi \,|\, H_{int} \,|\, \psi \rangle = V \, \bar{\nu}\, \nu$$

ψ is the wave function of the medium

CC interactions with electrons

$$H_{int} = \frac{G_F}{\sqrt{2}} \, \bar{\nu}\, \gamma^{\mu}(1 - \gamma_5)\, \nu \; \bar{e}\, \gamma_{\mu}(1 - \gamma_5)\, e$$



$\langle \bar{e}\, \gamma_0 (1 - \gamma_5)\, e \rangle = n_e$ - electron number density

$\langle \bar{e}\, \vec{\gamma}\, e \rangle = n_e \vec{v}$ - velocity of electrons

$\langle \bar{e}\, \vec{\gamma}\, \gamma_5\, e \rangle = n_e \vec{\lambda}_e$ - averaged polarization vector of electrons

For unpolarized medium at rest:

$$\boxed{V = \sqrt{2}\, G_F n_e}$$

Mixing in matter

Mixing is determined with respect to the eigenstates of propagation

in vacuum:

H_0

ν_{mass} ν_1 ν_2

θ

ν_f ν_e ν_μ

θ_m

in matter:

$H(n_e, E) = H + V$

ν_H ν_{1m} ν_{2m}

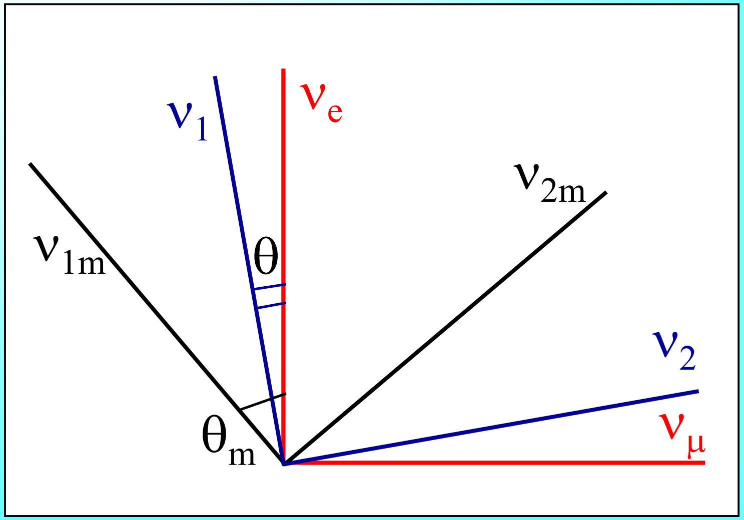
Eigenstates in vacuum H_0

Eigenstates in medium

Mixing angle determines flavors (flavor content) of eigenstates of propagation

θ_m depends on n_e, E

Flavor basis is the same,
Eigenstates basis changes



Hamiltonian, eigenstates and eigenvalues

$$H(n_e, E) = H + V$$

$$V = \mathrm{diag}(V_e\ ,\ 0)$$

In the flavor basis $(\nu_e\,,\,\nu_\mu)^T$

$$H_{tot} = \frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta + \xi & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\xi = \frac{4V_e E}{\Delta m^2} \qquad V_e = \sqrt{2}\,G_F n_e$$

Mixing in matter

Eigenstates and eigenvalues

Diagonalization of the Hamiltonian:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

$$V = \sqrt{2}\, G_F\, n_e$$

Mixing is maximal if

$$V = \frac{\Delta m^2}{2E} \cos 2\theta$$

Resonance condition

$$H_e = H_\mu$$

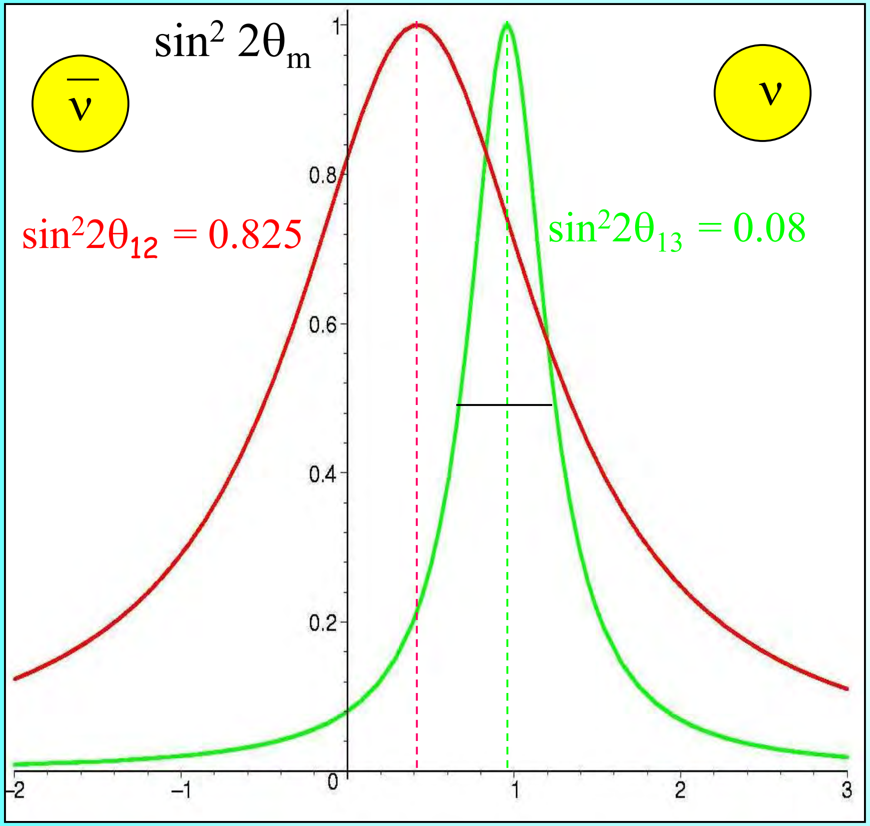
$$\sin^2 2\theta_m = 1$$

Difference of the eigenvalues

$$H_{2m} - H_{1m} = \frac{\Delta m^2}{2E} \sqrt{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

Resonance

Dependence of mixing on density, energy has a resonance character



In resonance: $\sin^2 2\theta_m = 1$

Flavor mixing is maximal

$l_\nu = l_0 \cos 2\theta$

Vacuum oscillation length $\approx$ Refraction length

Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$

density $l_\nu / l_0 \sim n E$

$\theta_m \rightarrow 0$ | $\theta_m = \theta$ | $\theta_m = \pi/4$ | $\theta_m \rightarrow \pi/2$

Mixing is suppressed at high densities

$|V| \gg \frac{\Delta m^2}{2E}$ → Flavor states coincide with eigenstates and vice versa

Mixing in matter: dynamical variable

θ_m depends on n_e, E

Changes when propagates in matter with varying density

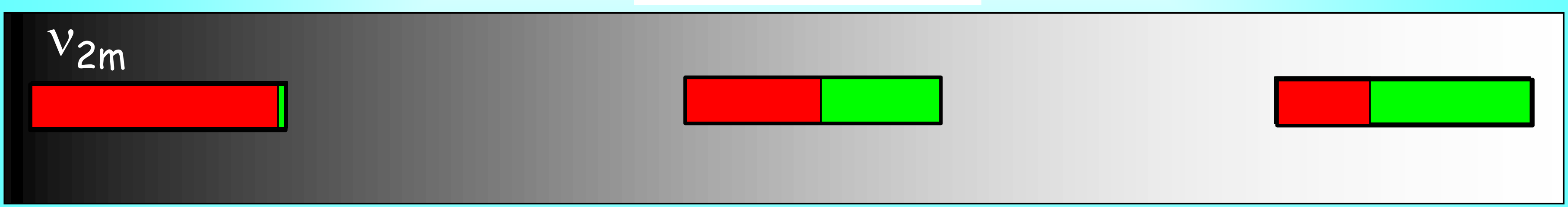
Mixing angle determines flavor content of eigenstates of propagation

High density
Mixing suppressed

Resonance:
Maximal mixing

$l_\nu = l_0 \cos 2\theta$

Low density
Vacuum mixing



Level crossing

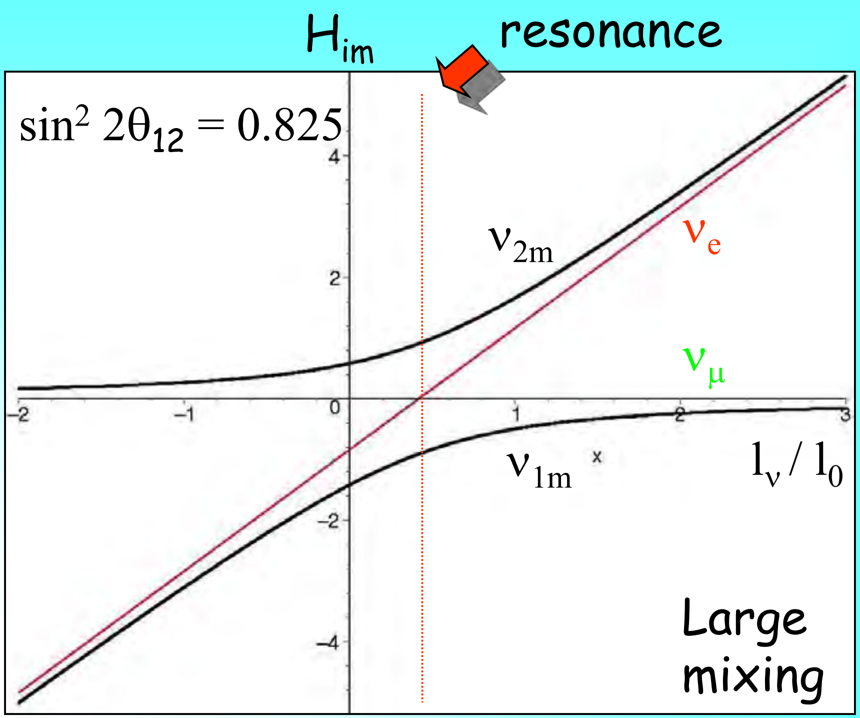
V. Rubakov, private comm.
N. Cabibbo, Savonlinna 1985
H. Bethe, PRL 57 (1986) 1271

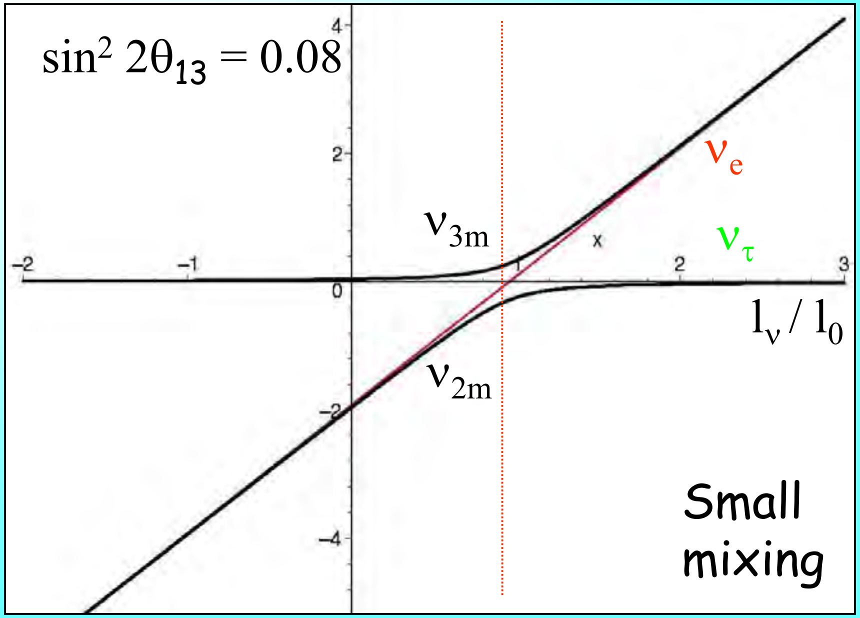
Dependence of the neutrino eigenvalues on the matter potential (density):

$$\frac{l_\nu}{l_0} = \frac{2E\,V}{\Delta m^2}$$

$$\boxed{\frac{l_\nu}{l_0} = \cos 2\theta}$$

Crossing point - resonance
- the level split is minimal
- the oscillation length is maximal





Oscillation length in matter

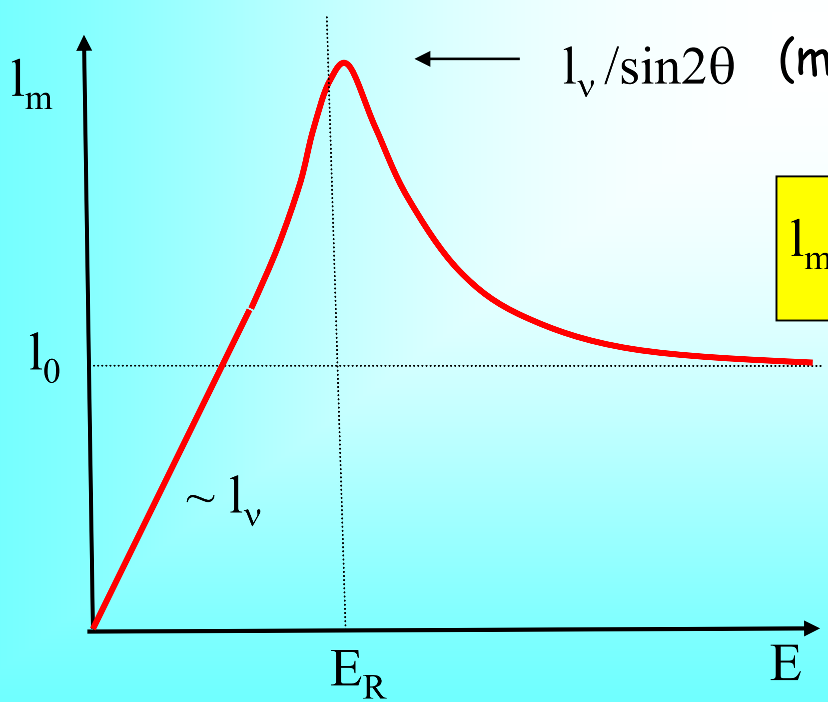
Oscillation length in vacuum

$$l_\nu = \frac{4\pi E}{\Delta m^2}$$

Refraction length

$$l_0 = \frac{2\pi}{\sqrt{2}\, G_F n_e}$$

- determines the phase produced by interaction with matter



$l_\nu / \sin 2\theta$ (maximum at $l_\nu = l_0 / \cos 2\theta$)

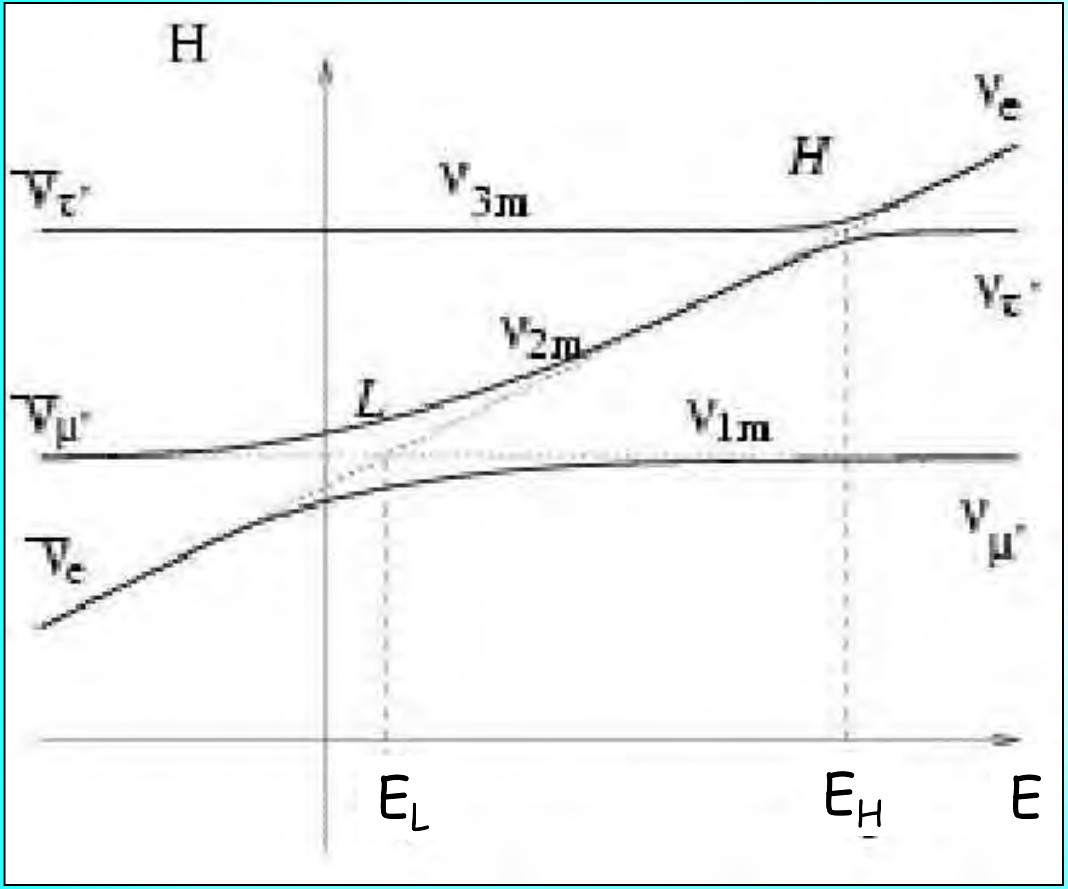
$$l_m = \frac{2\pi}{H_{2m} - H_{1m}}$$

shifts with respect resonance energy:

$$l_\nu (E_R) = l_0 \cos 2\theta$$

converges to the refraction length

Level crossings



Normal mass hierarchy

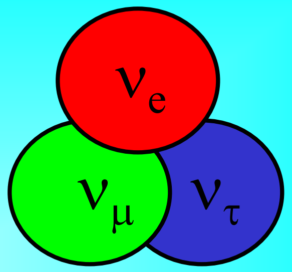
0.1 GeV

6 GeV

Resonance region

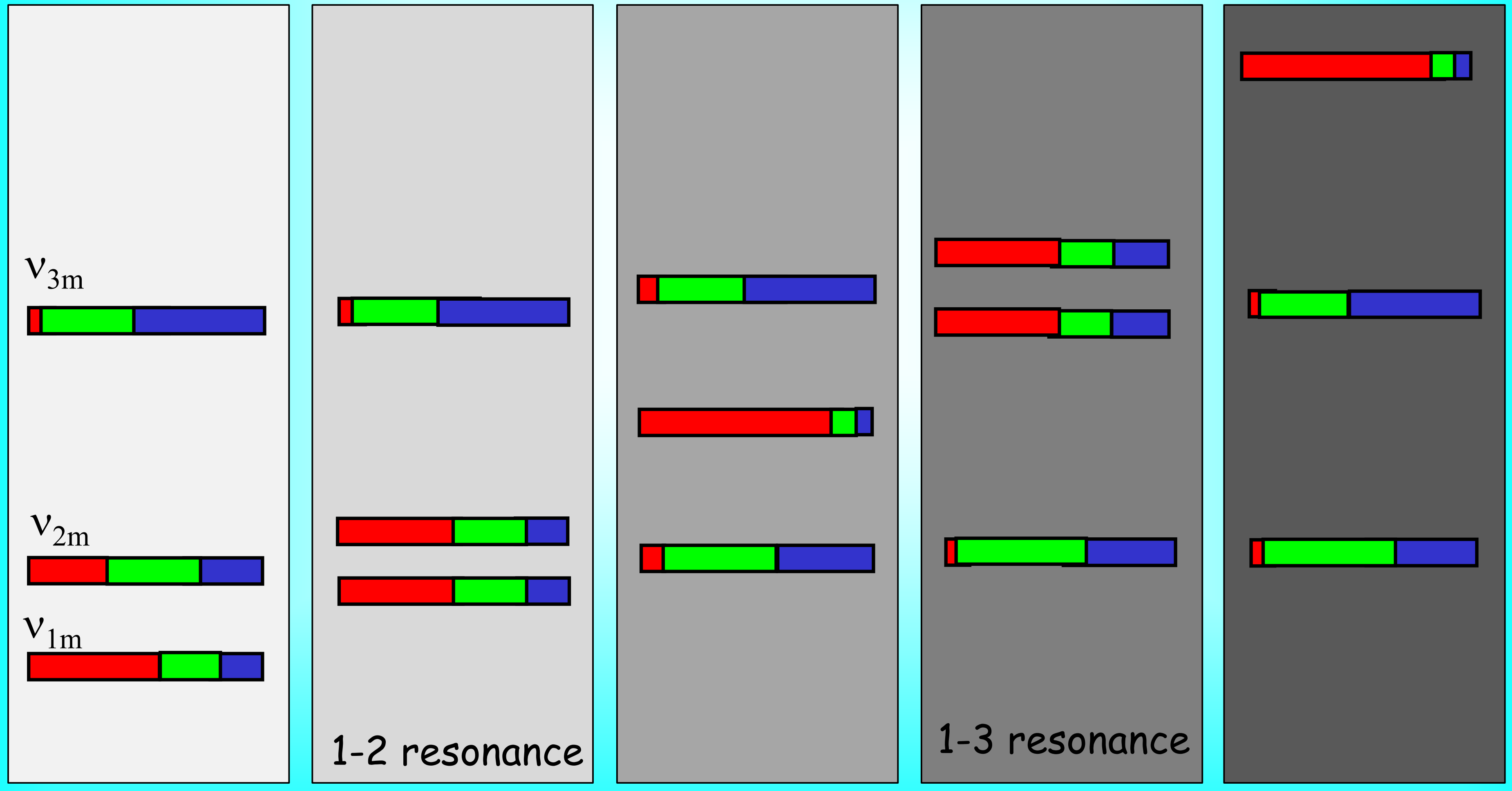
High energy range

Flavor in matter



Density increase →

Normal mass hierarchy, neutrinos



Propagation effects

Evolution equation

$$i\,\frac{d\nu_f}{dt} = H_{tot}\,\nu_f$$

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$H_{tot} = H_{vac} + V$ is the total Hamiltonian

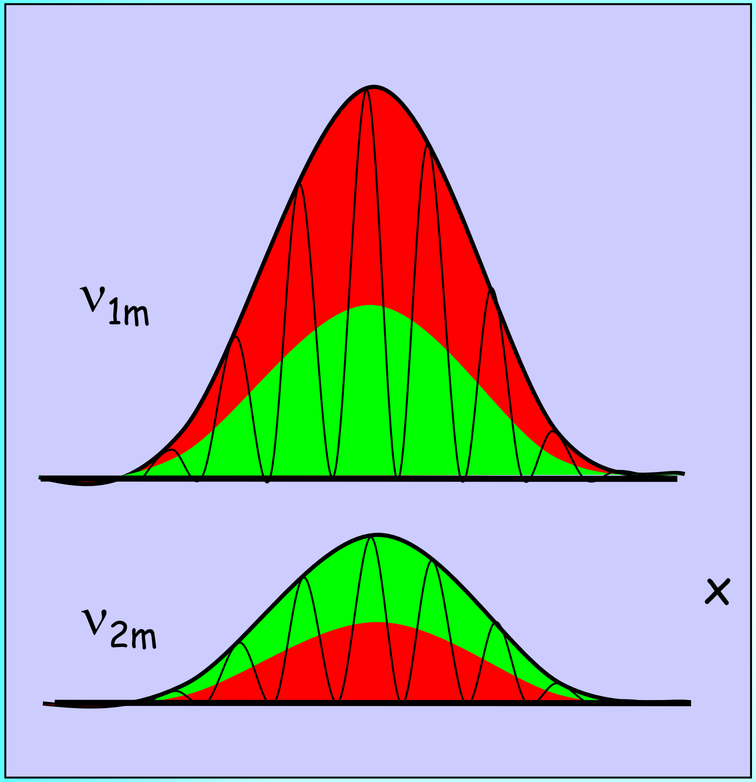
$H_{vac} = \dfrac{M^2}{2E}$ is the vacuum (kinetic) part

$V = \begin{pmatrix} V_e & 0 \\ 0 & 0 \end{pmatrix}$ matter part $\quad V_e = \sqrt{2}\,G_F n_e$

H_{tot}

$$i\,\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta m^2}{2E}\cos 2\theta + V_e & \dfrac{\Delta m^2}{4E}\sin 2\theta \\ \dfrac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Oscillations in matter



$H_0 \rightarrow H = H_0 + V$

$\nu_k \rightarrow \nu_{mk}$

eigenstates of H_0

eigenstates of H

$\theta \rightarrow \theta_m (n)$

Resonance - maximal mixing in matter – oscillations with maximal depth

$\theta_m = \pi/4$

Resonance condition:

$$V = \cos 2\theta \frac{\Delta m^2}{2E}$$

Constant density medium:
the same dynamics

Mixing changed
phase difference changed

Oscillations in matter

Probability in constant density

$$P(\nu_e \to \nu_a) = \sin^2 2\theta_m \sin^2\left(\frac{\pi L}{l_m}\right)$$

depth of oscillations

oscillatory factor

half-phase ϕ

$\theta_m(E, n)$ - mixing angle in matter

$l_m(E, n)$ - oscillation length in matter

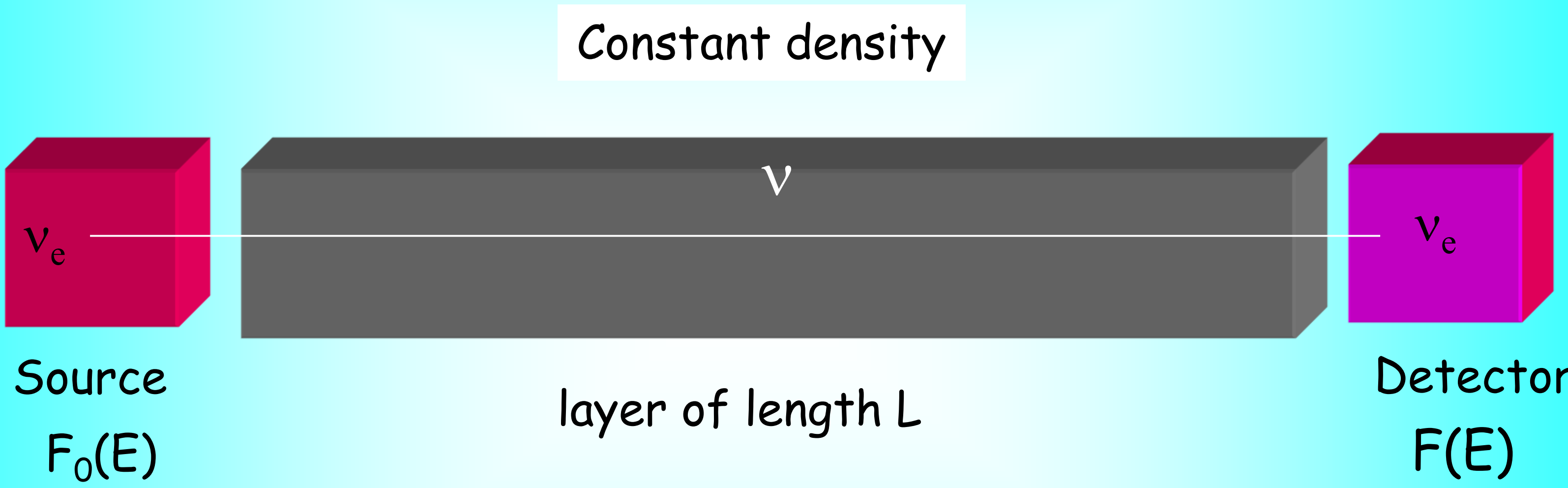
$$l_m = 2\pi/(H_{2m} - H_{1m})$$

In vacuum: $\theta_m \to \theta$, $l_m \to l_\nu$

Maximal effect: $\sin^2 2\theta_m = 1$ $\Rightarrow$ MSW resonance condition

$\phi = \pi/2 + \pi k$

Resonance enhancement



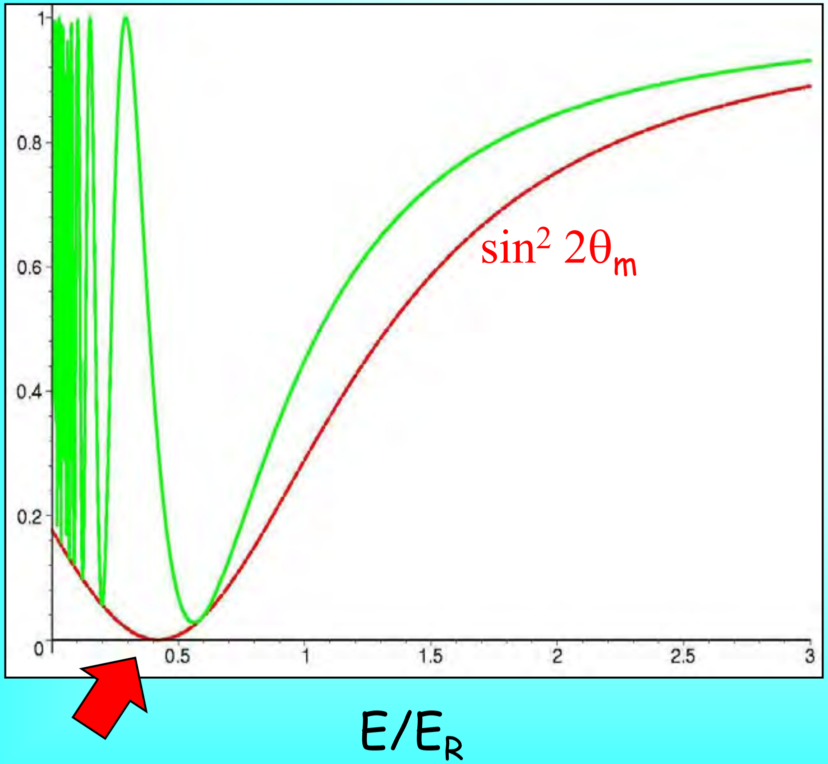
Depth of oscillations determined by $\sin^2 2\theta_m$
the oscillation length, l_m
depends on neutrino energy

For neutrinos propagating
in the mantle of the Earth

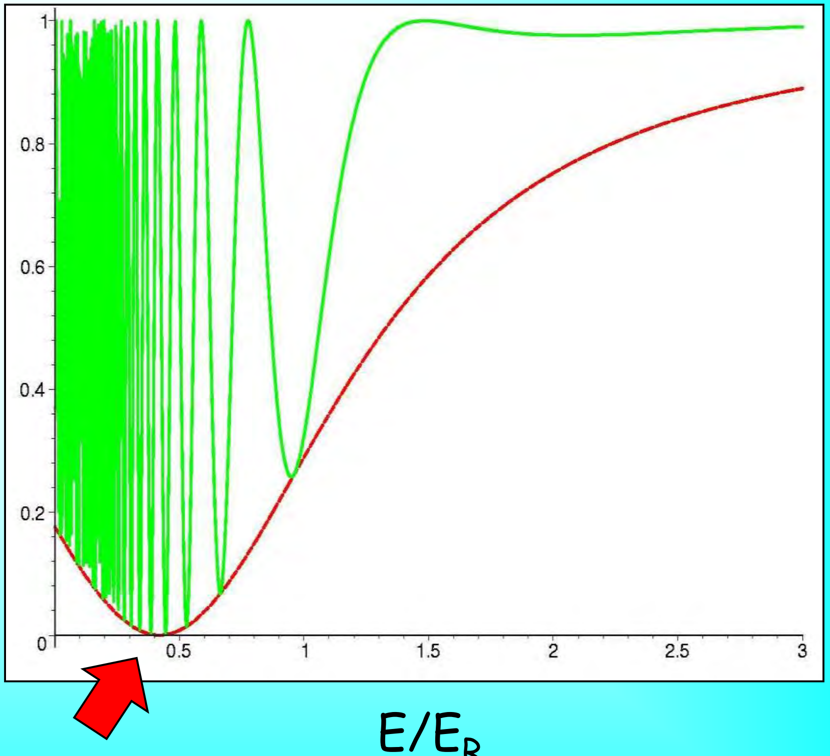
Large mixing $\sin^2 2\theta = 0.824$

Layer of length L ν $k = \pi L / l_0$

thin layer $k = 1$



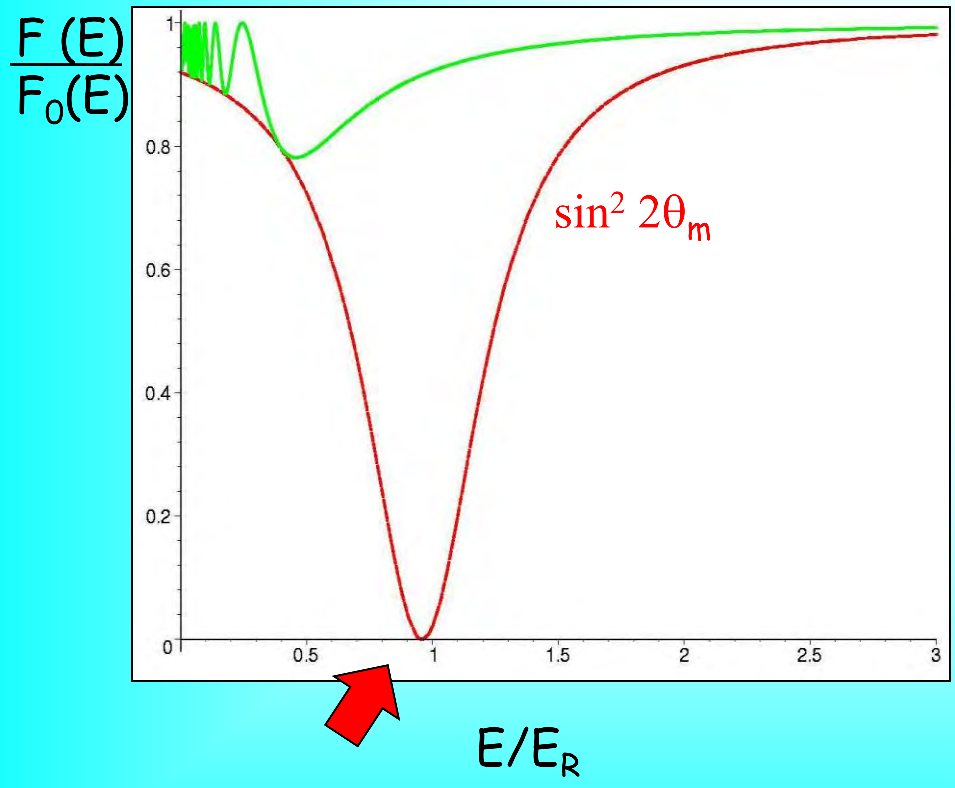
thick layer $k = 10$



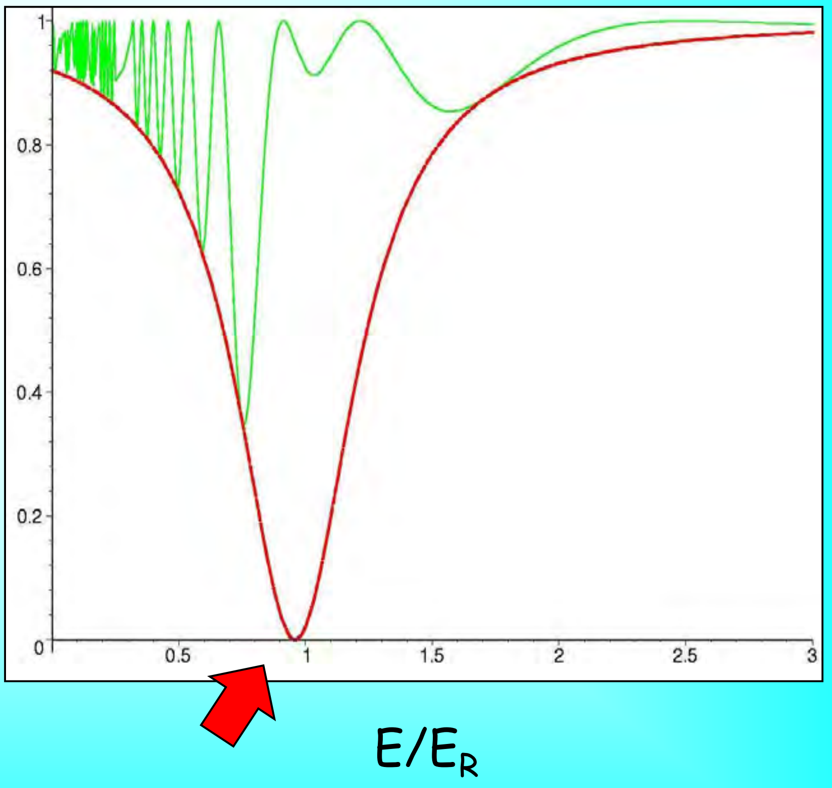
Oscillation in the Earth,
ORCA/PINGU

Small mixing $\sin^2 2\theta = 0.08$

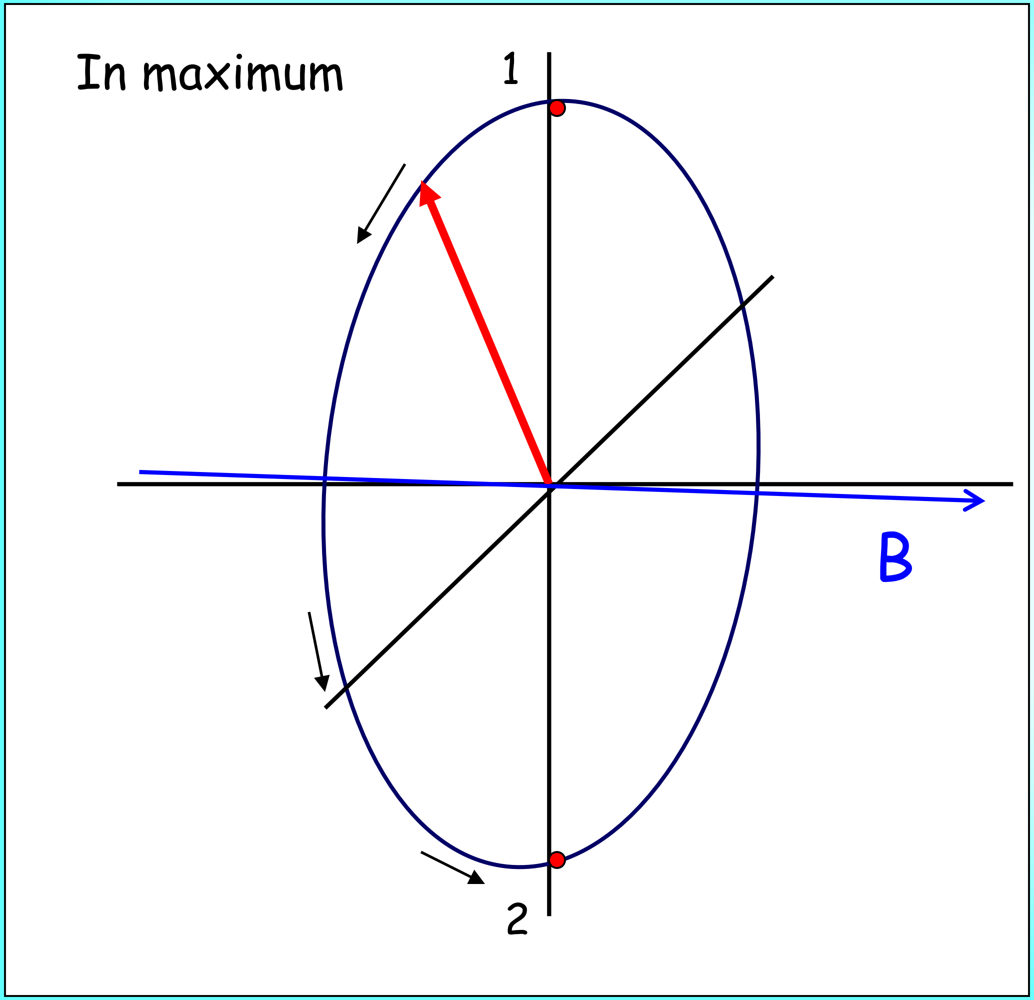
thin layer $k = 1$



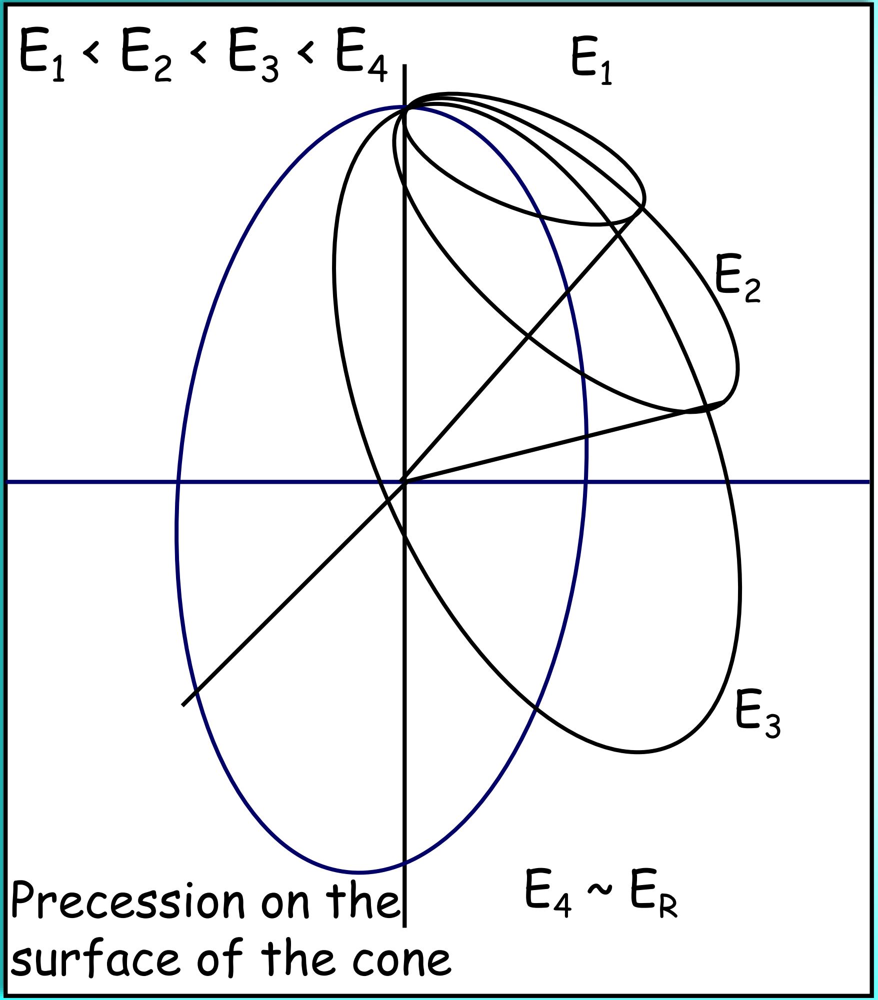
thick layer $k = 10$



Resonance enhancement



Graphic representation



Resonance enhancement

Adiabatic conversion

SNO
Varying density

Hamiltonian for flavor states in matter

In the flavor basis $\nu_f = (\nu_e , \nu_\mu)^T$

$$H_{tot} = \frac{M\,M^+}{2E} + V(t)$$

$$M\,M^+ = U\,M_{diag}^2\,U^+$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$M_{diag}^2 = \mathrm{diag}\,(m_1^2,\ m_2^2)$$

$$H_{tot} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta + \xi & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\xi = \frac{4V_e\,E}{\Delta m^2} \qquad V_e = \sqrt{2}\,G_F n_e$$

$$\nu_f = U(\theta_m)\,\nu_m$$

Mixing matrix in matter - diagonalizes H_{tot}

Evolution equation for eigenstates

In non-uniform medium the Hamiltonian depends on time: $H_{tot} = H_{tot}(n_e(t))$

$$\boxed{i\,\frac{d\nu_f}{dt} = H_{tot}\,\nu_f}$$

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \qquad \nu_m = \begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

Inserting $\nu_f = U(\theta_m)\,\nu_m$ $\qquad \theta_m = \theta_m(n_e(t))$

$$i\,\frac{d}{dt}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix} = \begin{pmatrix} 0 & i\,\frac{d\theta_m}{dt} \\ -i\,\frac{d\theta_m}{dt} & H_{2m} - H_{1m} \end{pmatrix}\begin{pmatrix} \nu_{1m} \\ \nu_{2m} \end{pmatrix}$$

off-diagonal terms imply transitios

$\nu_{1m} \Leftrightarrow \nu_{2m}$

However

if $\left|\frac{d\theta_m}{dt}\right| \ll H_{2m} - H_{1m}$

off-diagonal elements can be neglected
no transitions between eigenstates
propagate independently

Adiabaticity

Adiabaticity condition $\left|\frac{d\theta_m}{dt}\right| \ll H_{2m} - H_{1m}$

External conditions (density) change slowly the system has time to adjust them

transitions between the neutrino eigenstates can be neglected

$\nu_{1m} \not\leftrightarrow \nu_{2m}$ → The eigenstates propagate independently

Shape factors of the eigenstates do not change

Crucial in the resonance layer:
- the mixing changes fast
- level splitting is minimal

Adiabaticity condition $\Delta r_R > l_R$ if vacuum mixing is small

$\Delta r_R = n_R \tan 2\theta / (dn/dx)_R$

width of the resonance layer

$l_R = l_\nu / \sin 2\theta$

oscillation length in resonance

Adiabatic parameter

$$\kappa = \frac{H_{2m} - H_{1m}}{\left|\frac{d\theta_m}{dt}\right|}$$

Adiabaticity condition:
$\kappa > 1$

most crucial in the resonance where
the mixing angle in matter changes fast

$$\kappa_R = \frac{\Delta r_R}{l_R}$$

$\Delta r_R = h_n \tan 2\theta$ is the width of the resonance layer

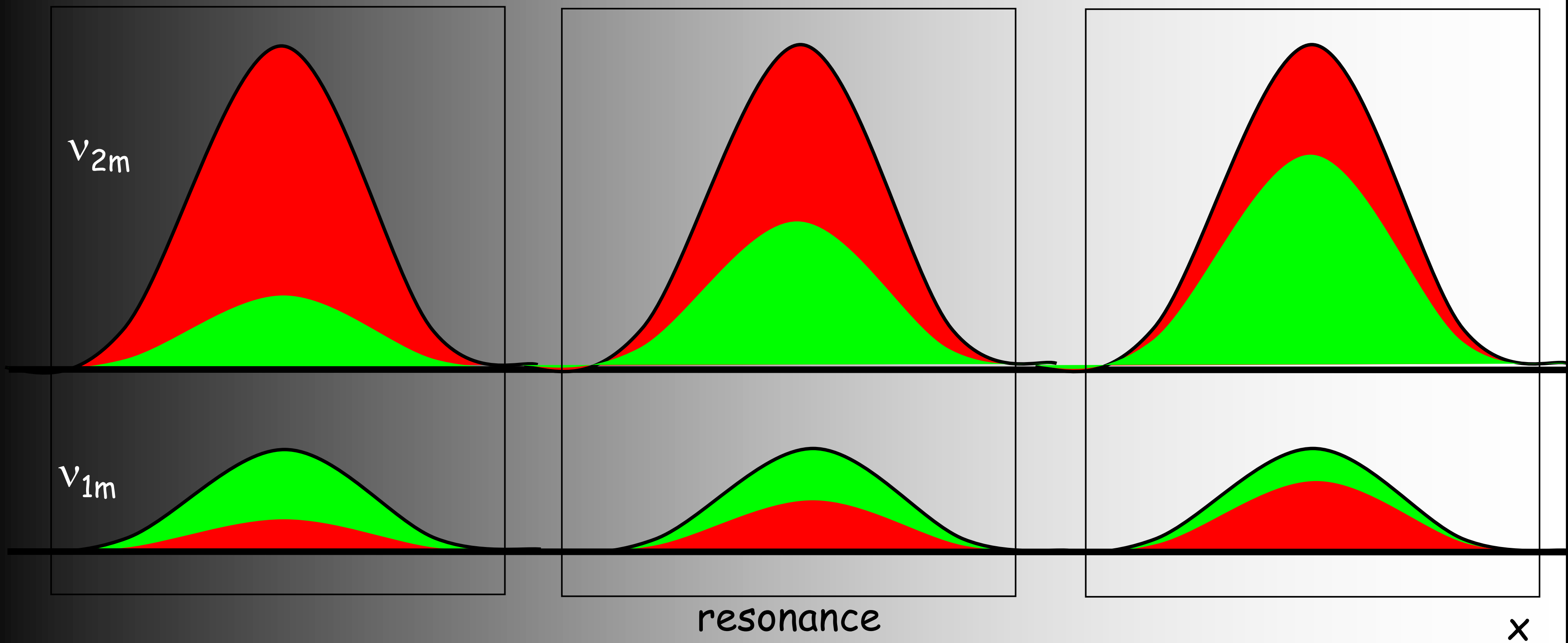
$h_n = \dfrac{n}{dn/dx}$ is the scale of density change

$l_R = l_\nu / \sin 2\theta$ is the oscillation length in resonance

Explicitly:

$$\kappa_R = \frac{\Delta m^2 \sin^2 2\theta \, h_n}{2E \cos 2\theta}$$

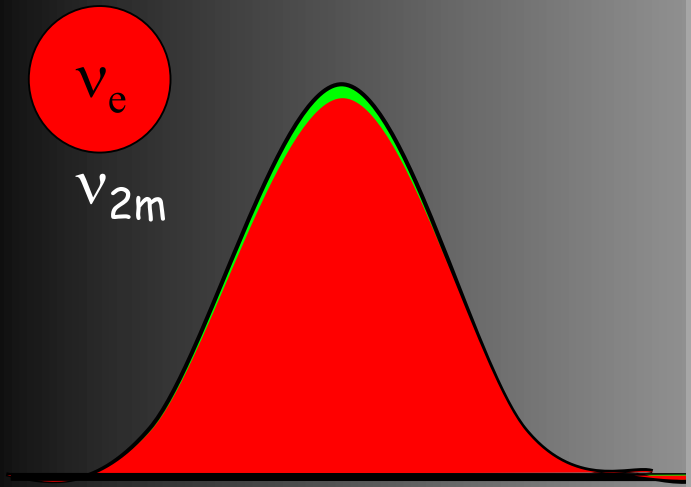
Adiabatic conversion



if density changes slowly

- the amplitudes of the wave packets do not change
-flavors of the eigenstates being determined by mixing angle follow the density change

Non-oscillatory transition



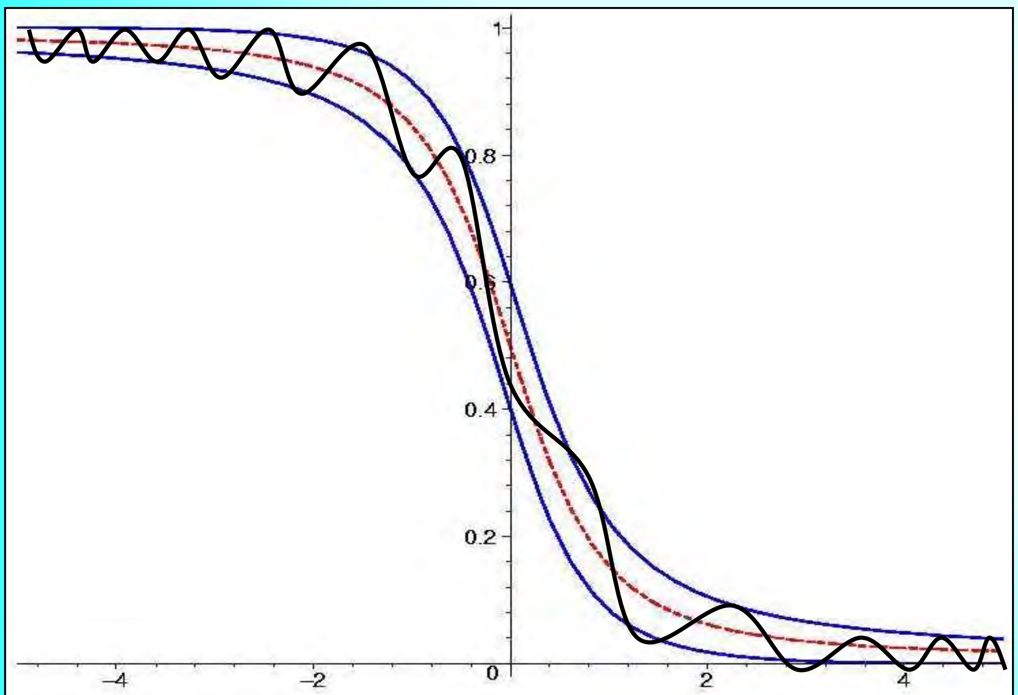
Single eigenstate:
→ no interference
→ no oscillations
→ phase is irrelevant

This happens when mixing is very small in matter with very high density

Spatial picture

Adiabatic conversion

survival probability



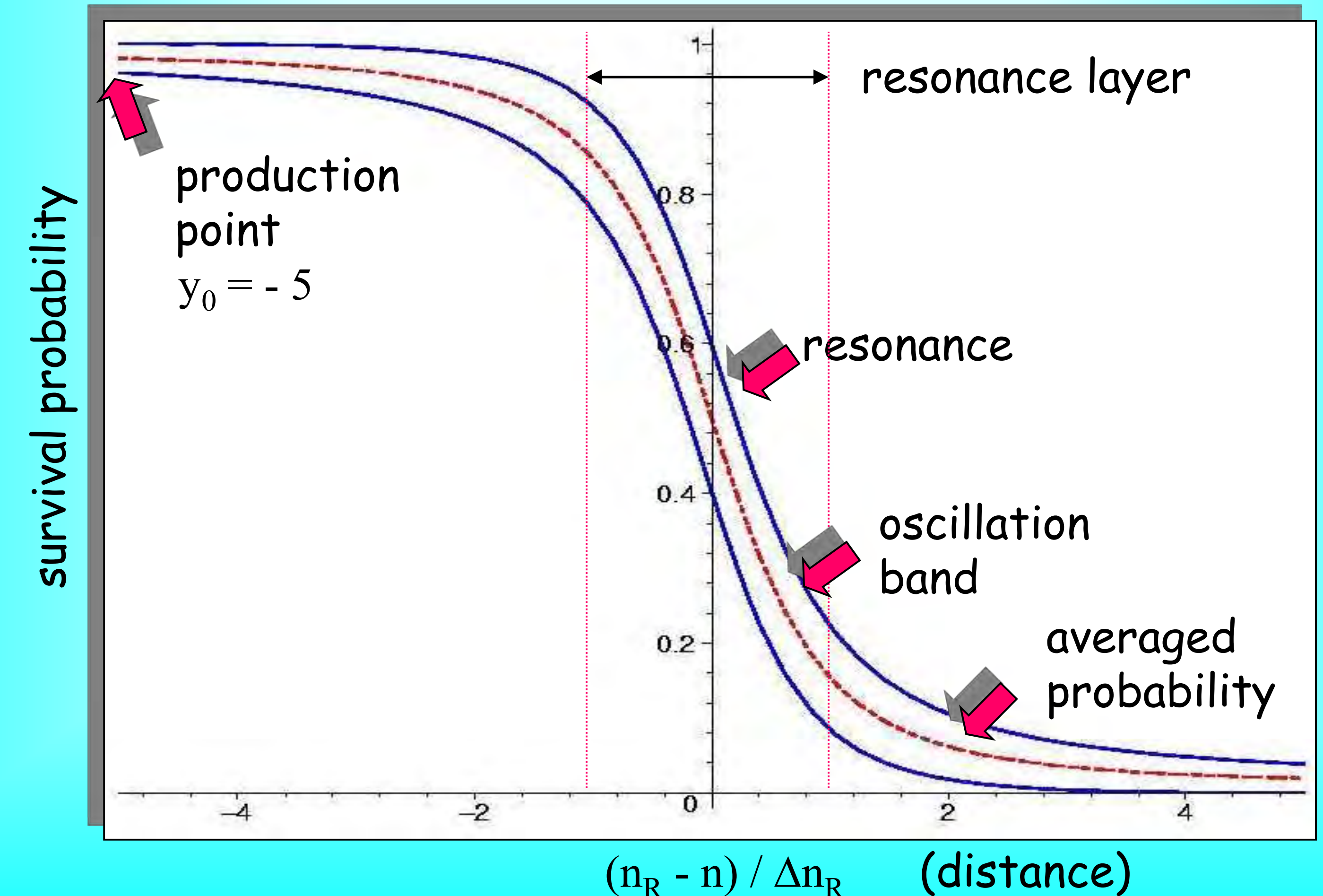
distance

interplay of adiabatic conversion and oscillations

Non-oscillatory transition is modulated by oscillations

Spatial picture

The picture is universal in terms of variable $y = (n_R - n) / \Delta n_R$
no explicit dependence on oscillation parameters, density distribution, etc.
only initial value y_0 matters



A Yu Smirnov

Adiabatic conversion probability

Sun, Supernova

Initial state: $\nu(0) = \nu_e = \cos\theta_m^0 \, \nu_{1m}(0) + \sin\theta_m^0 \, \nu_{2m}(0)$

Mixing angle in matter in production point

Adiabatic evolution to the surface of the Sun (zero density):

$$\nu_{1m}(0) \rightarrow \nu_1$$
$$\nu_{2m}(0) \rightarrow \nu_2$$

Final state: $\nu(f) = \cos\theta_m^0 \, \nu_1 + \sin\theta_m^0 \, \nu_2 \, e^{i\phi}$

Probability to find ν_e averaged over oscillations

$$P_{ee} = |\langle \nu_e | \nu(f) \rangle|^2 = (\cos\theta \cos\theta_m^0)^2 + (\sin\theta \sin\theta_m^0)^2$$
$$= 0.5[\, 1 + \cos 2\theta_m^0 \cos 2\theta \,]$$

or

$$P_{ee} = \sin^2\theta + \cos 2\theta \cos^2\theta_m^0$$

Adiabaticity violation

SN shock waves

If sterile neutrinos with small mixing exist

If density $n_e(t)$ changes fast

$$\left| \frac{d\theta_m}{dt} \right| \sim |H_{2m} - H_{1m}|$$

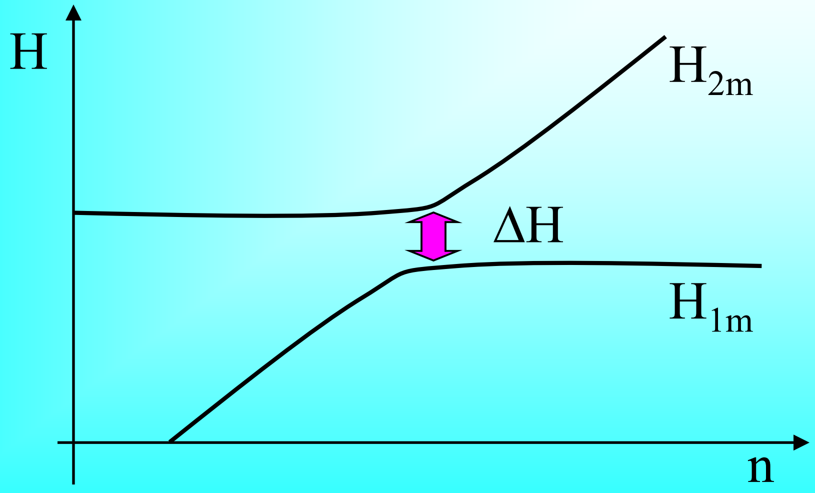
the off-diagonal terms in the Hamiltonian can not be neglected

transitions $\nu_{1m} \Leftrightarrow \nu_{2m}$

Admixtures of ν_{1m} ν_{2m} in a given propagating neutrino state change

``Jump probability'' = penetration under barrier:

$$P_{12} = e^{-\frac{\Delta H}{E_n}}$$



$E_n \sim 1/h_n$ is the energy associated to change of parameter (density)

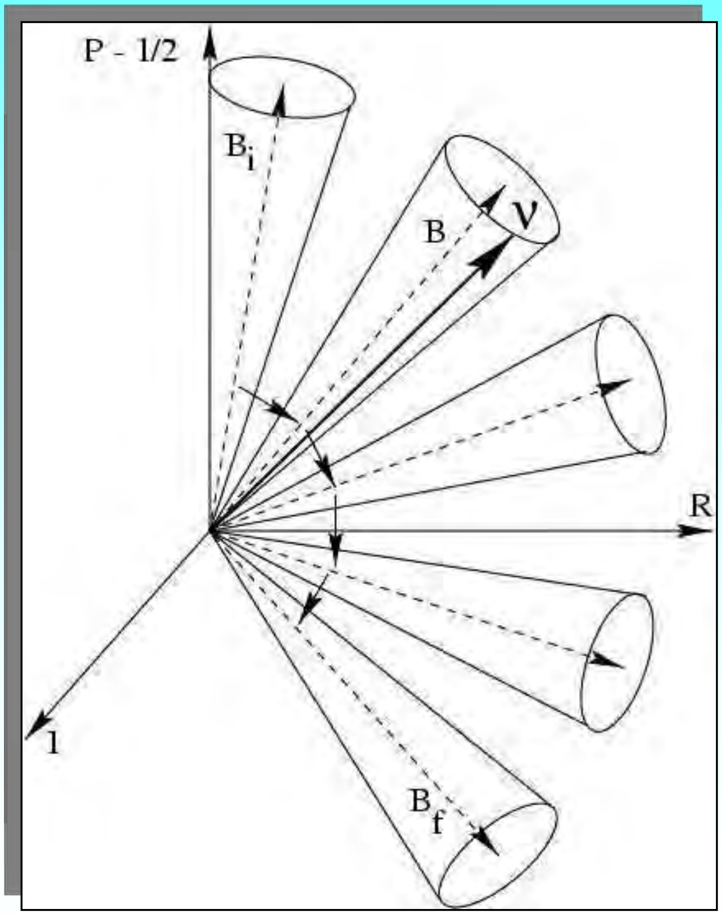
$$P_{12} = e^{-\pi\kappa_R/2}$$

Landau-Zenner

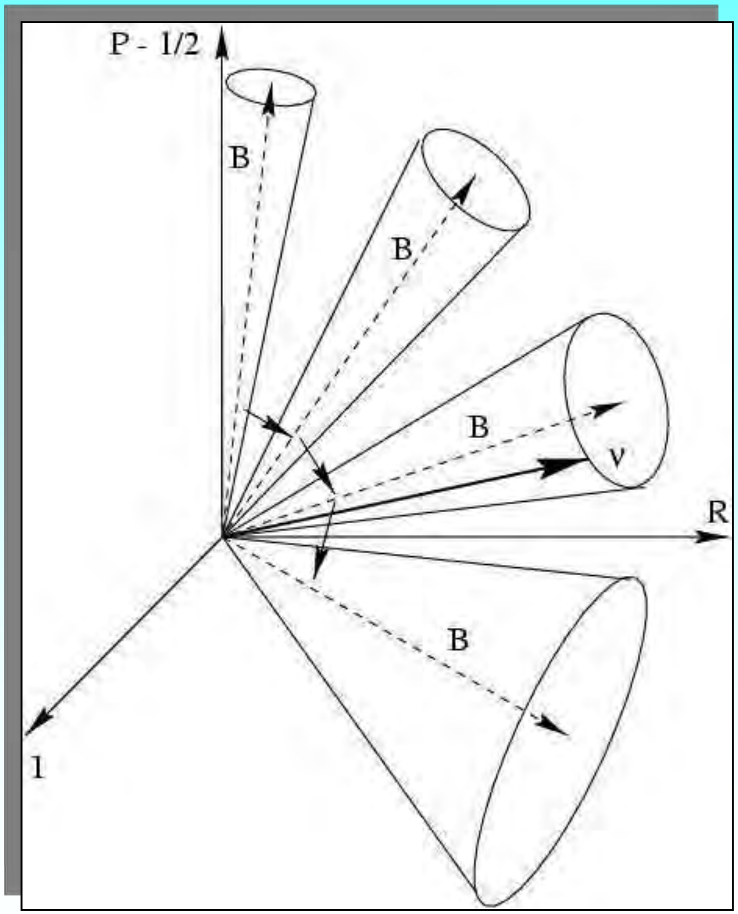
κ_R - adiabaticity parameter

Adiabatic conversion

Pure adiabatic conversion



Partialy adiabatic conversion



Oscillations versus adiabatic conversion

Different degrees of freedom

Oscillations

Vacuum or uniform medium with constant parameters

Phase difference increase between the eigenstates

$\phi(t)$

Mixing does not change

$\theta_m(E)$

Adiabatic conversion

Non-uniform medium or/and medium with varying in time parameters

Change of mixing in medium = change of flavor of the eigenstates

$\theta_m(t)$

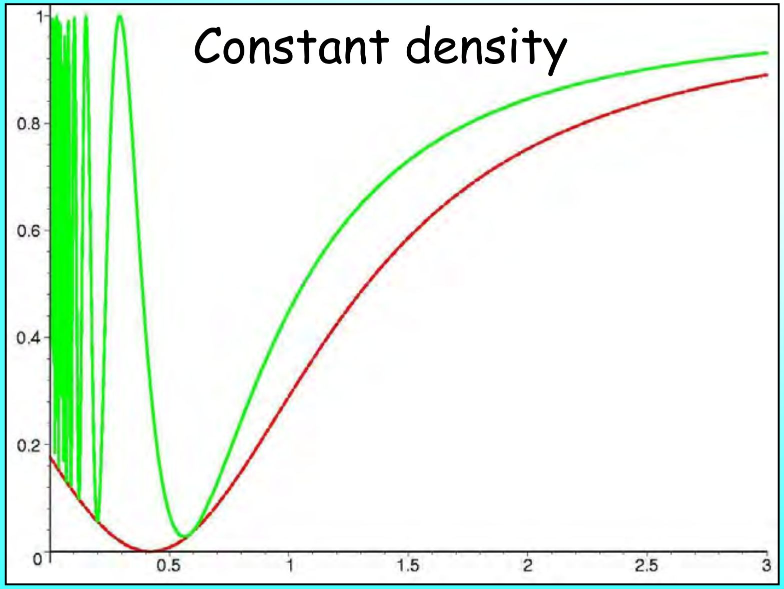
Phase is irrelevant

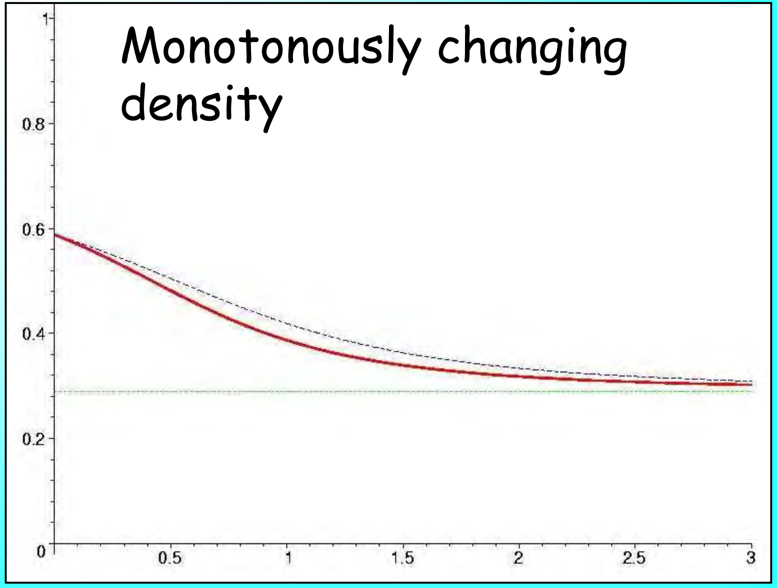
In non-uniform medium: interplay of both processes

Resonance oscillations vs. adiabatic conversion

Passing through the matter filter ν

$\frac{F(E)}{F_0(E)}$





E/E_R E/E_R

Parametric effects

Propagation on the Earth

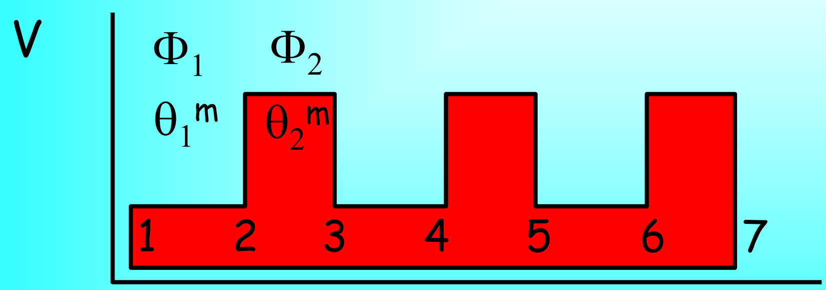
Parametric enhancement of oscillations

Enhancement associated to certain conditions for the phase of oscillations

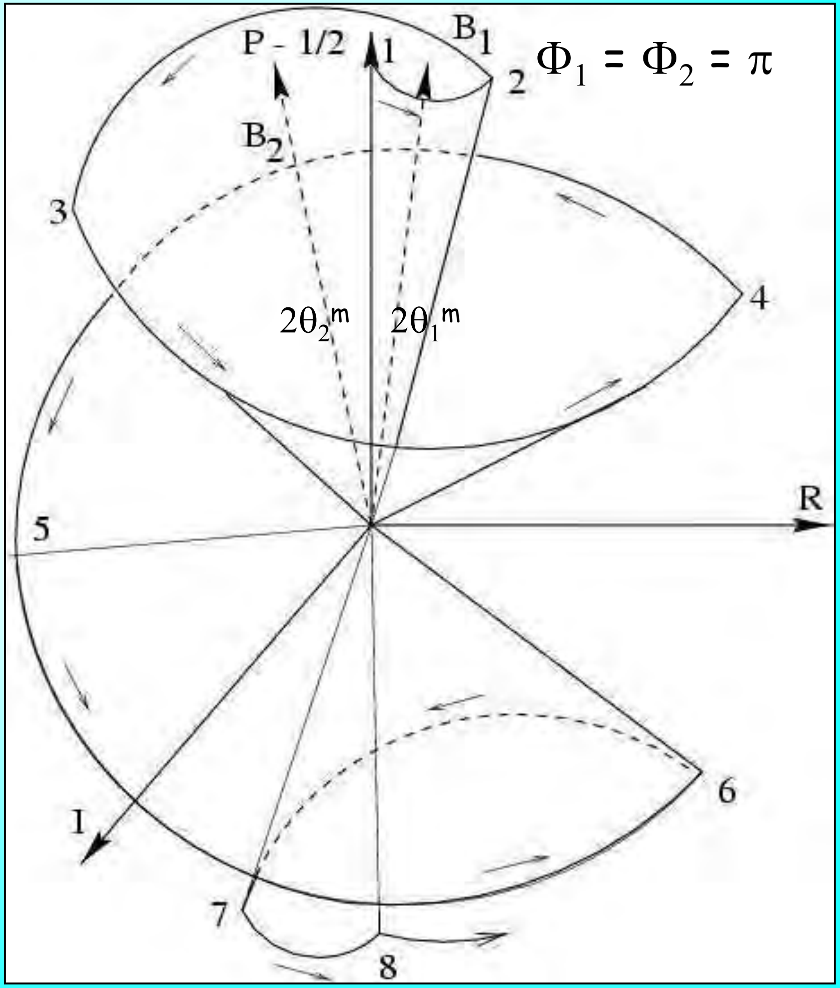
Another way of getting strong transition

No large vacuum mixing and no matter enhancement of mixing or resonance conversion

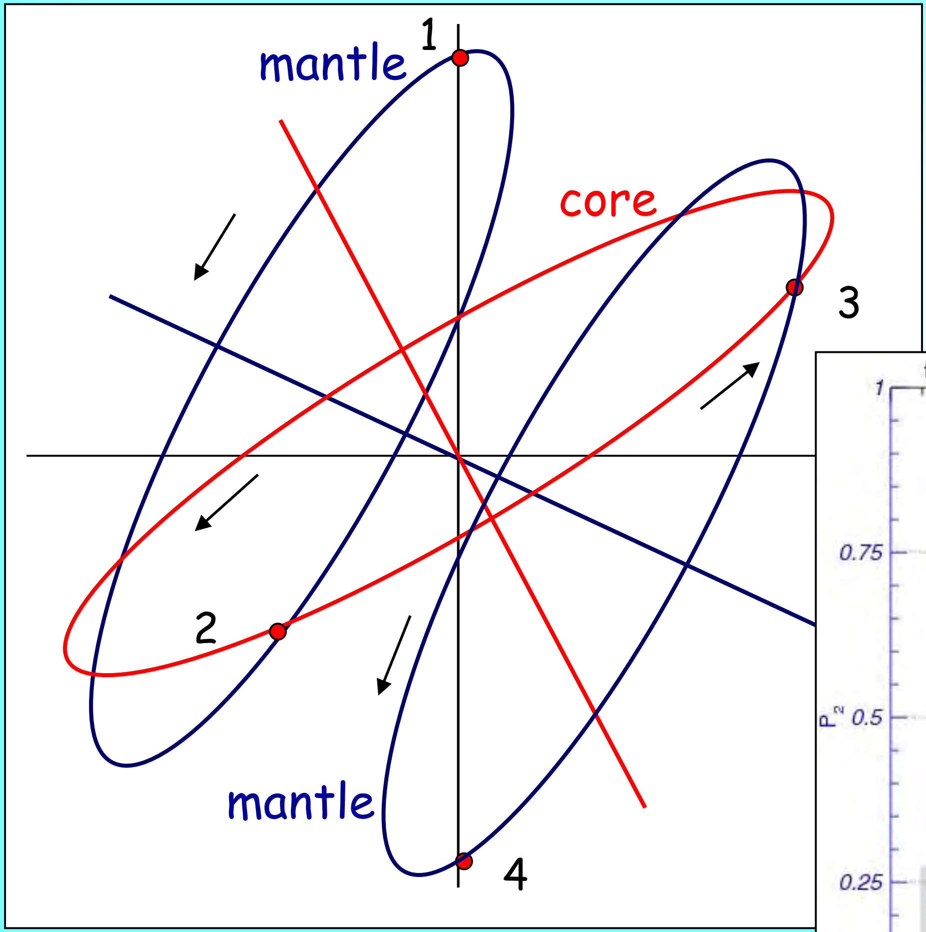
V. Ermilova V. Tsarev, V. Chechin
E. Akhmedov
P. Krastev, A.S., Q. Y. Liu,
S.T. Petcov, M. Chizhov

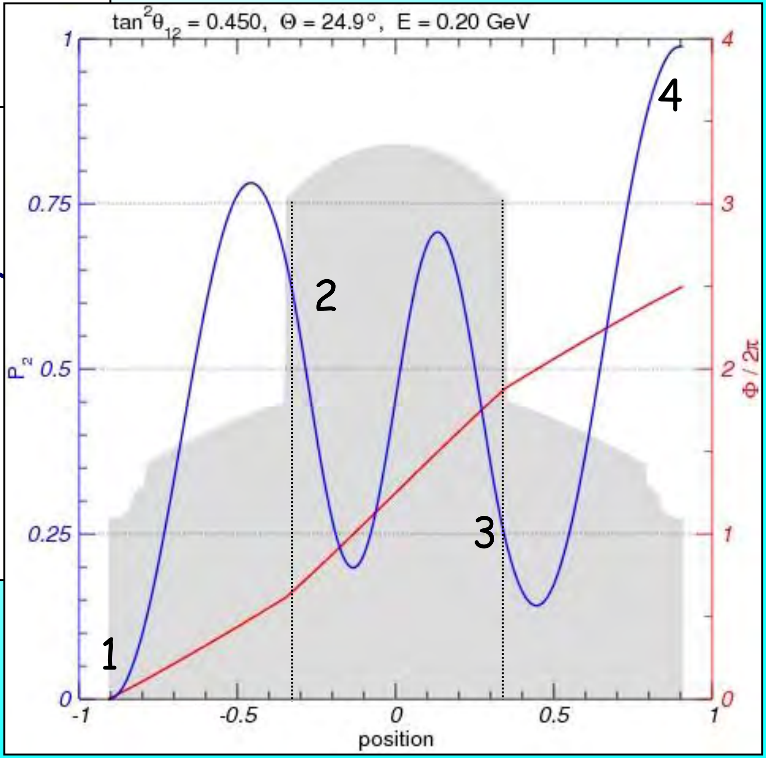


``Castle wall profile"

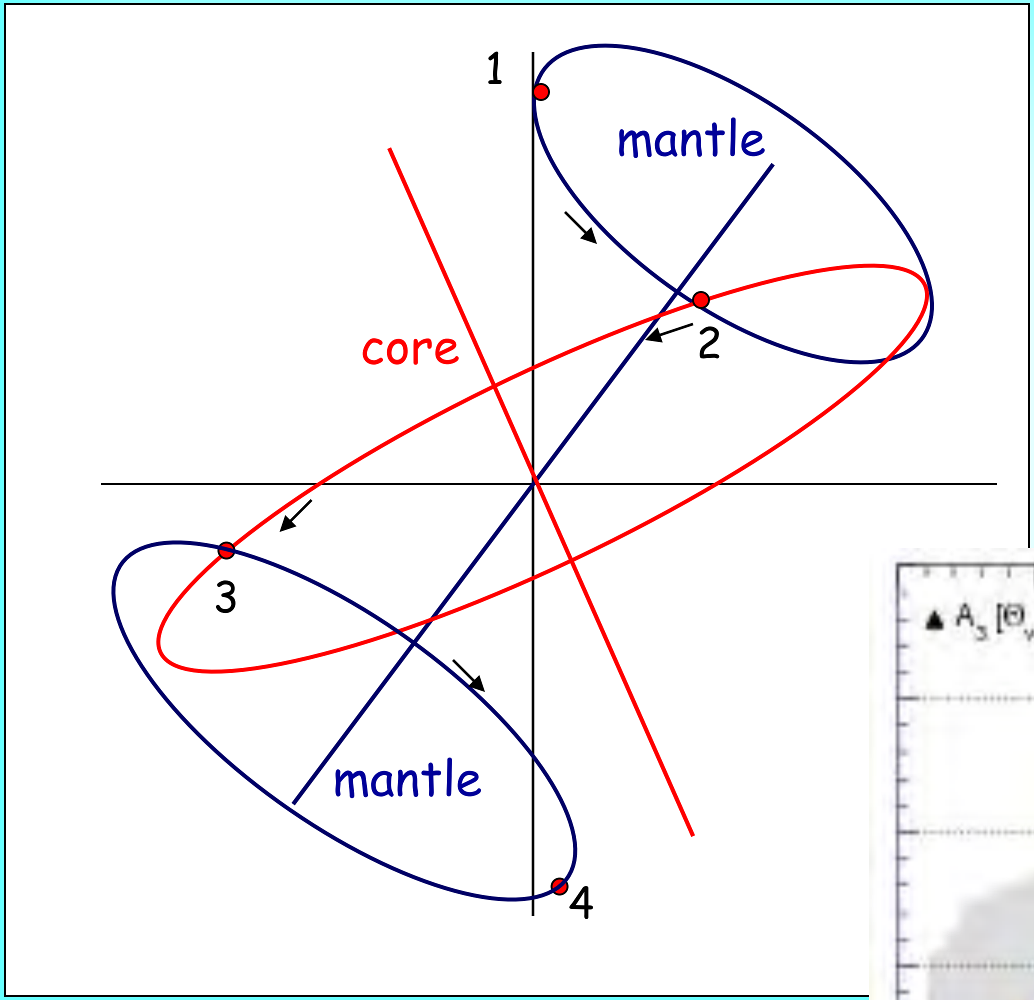


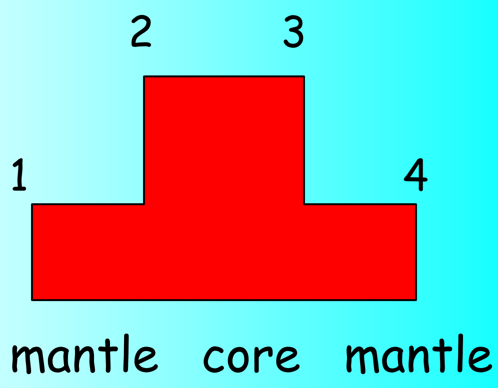
Parametric enhancement of 1-2 mode

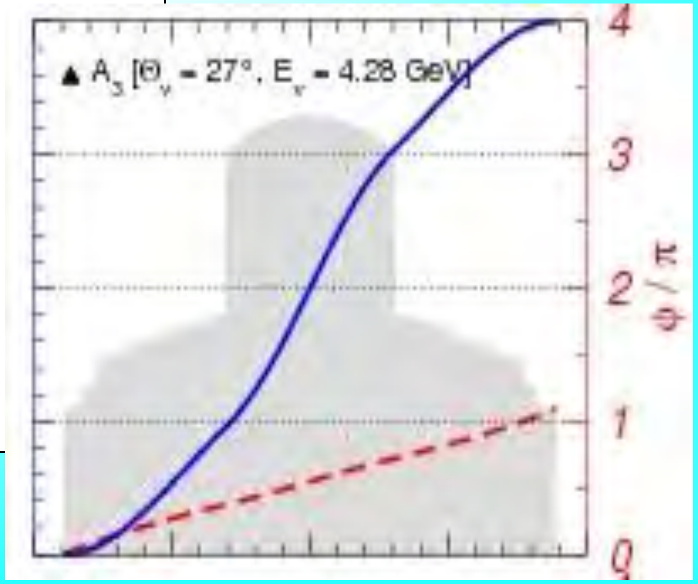




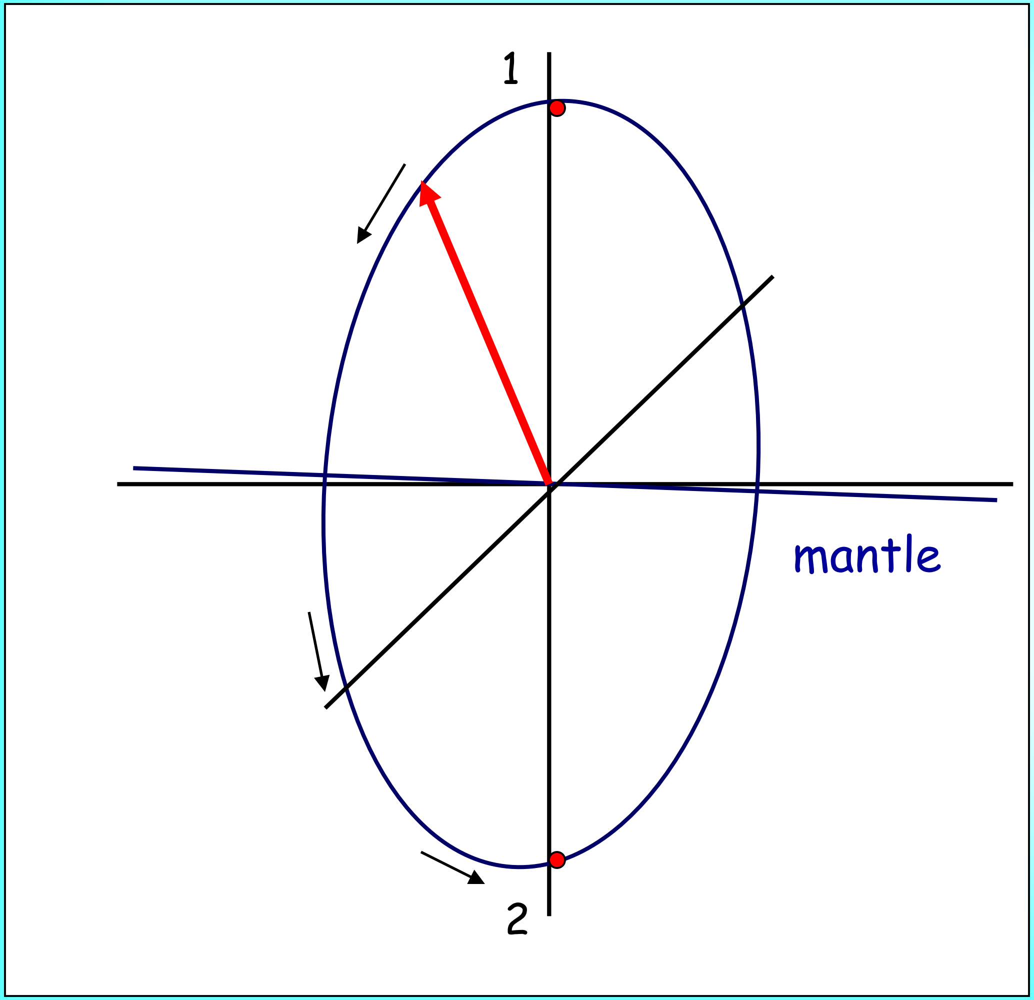
Parametric enhancement

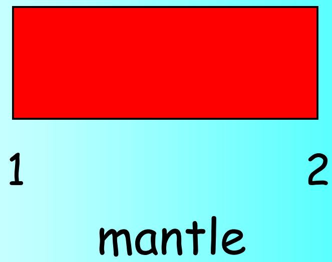




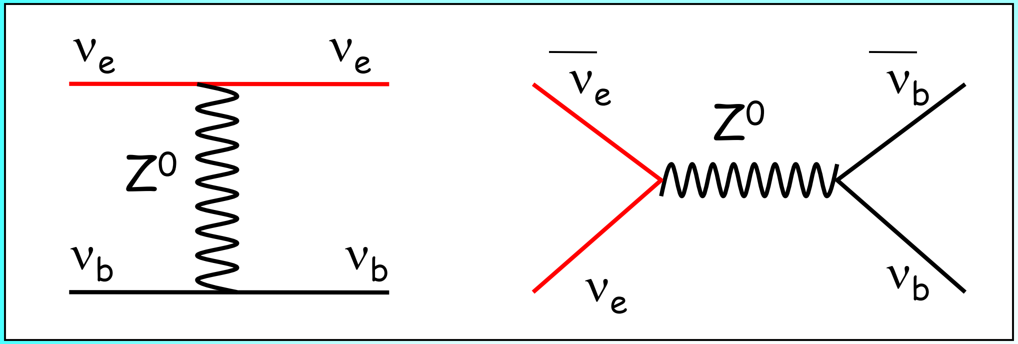


Resonance enhancement in mantle





$\nu\nu$-scattering

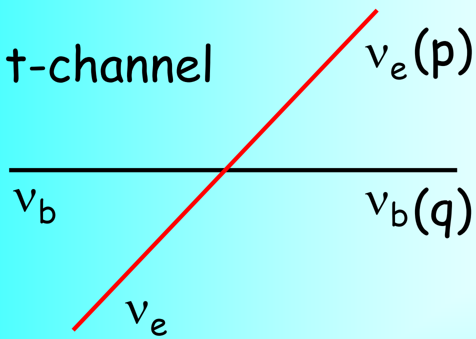


Refraction in neutrino gases

$A = \sqrt{2}\, G_F\, (1 - v_e\, v_b\,)$

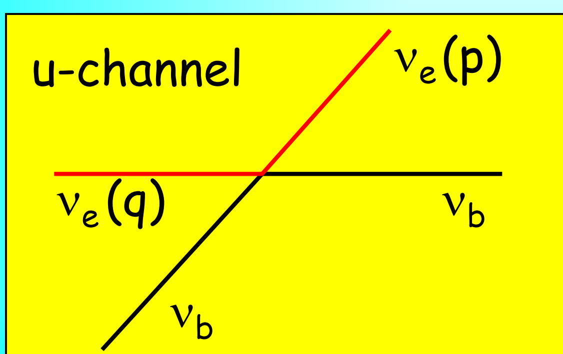
velocities

ν_e



elastic forward scattering

J. Pantaleone



can lead to the coherent effect

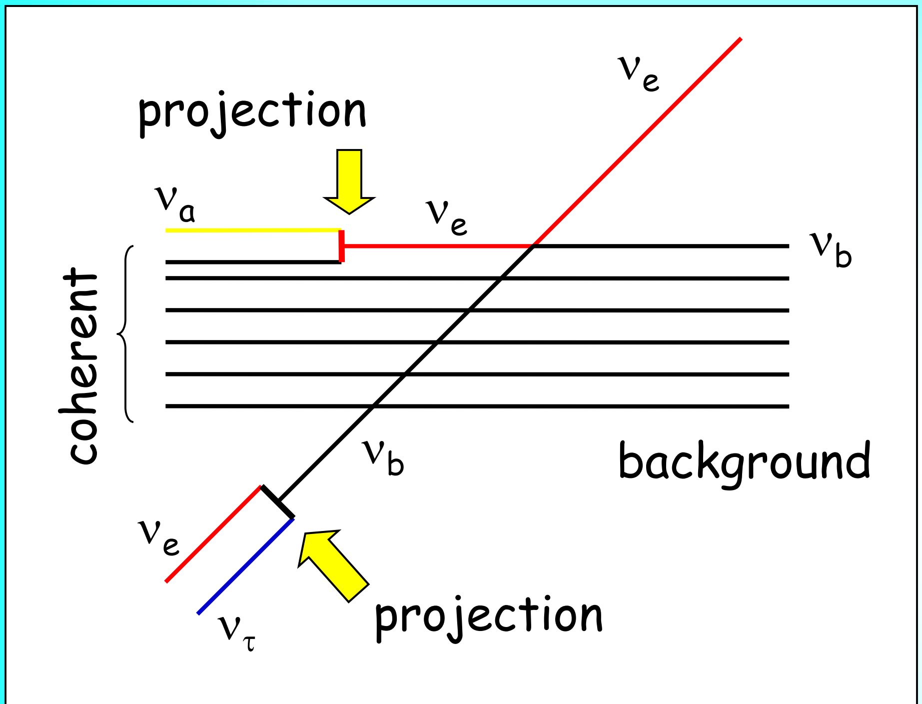
Momentum exchange → flavor exchange

→ flavor mixing

Collective flavor transformations

Flavor exchange

J. Pantaleone
S. Samuel
V.A. Kostelecky



$\nu\nu$ – scattering in u-channel due to Z^0 - exchange

1. Momentum exchange → flavor exchange

2. Coherence if the background is in mixed state:

$$|\nu_{ib}\rangle = \Phi_{ie}\,|\nu_e\rangle + \Phi_{i\tau}\,|\nu_\tau\rangle$$

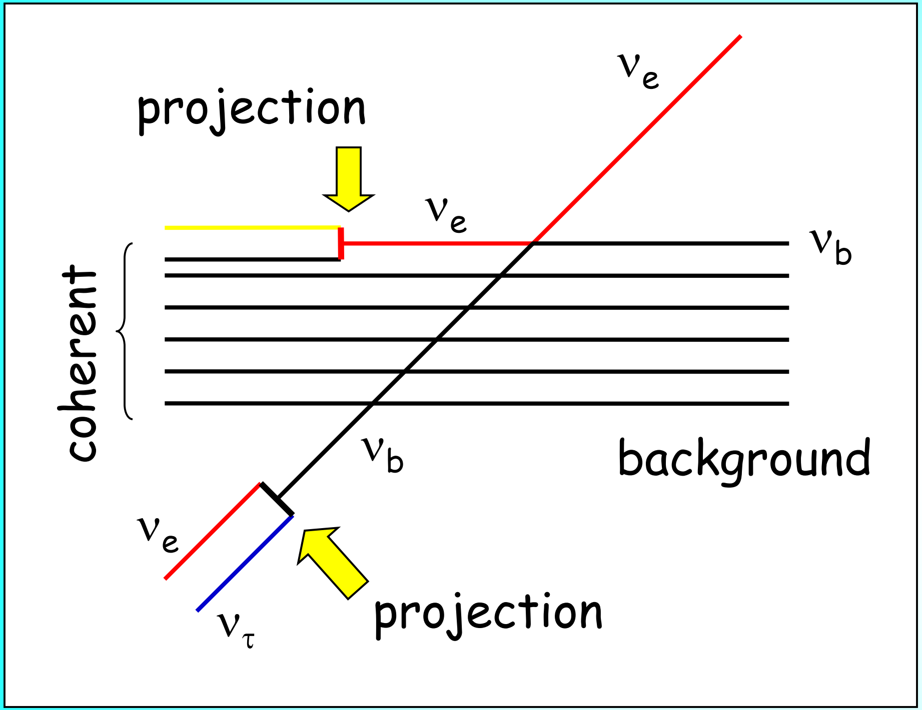
Coherent flavor changing transition

Probe neutrino = background neutrino

Potential depends on transition probability

Flavor exchange

J. Pantaleone
S. Samuel
V.A. Kostelecky



Flavor exchange between the beam (probe) and background neutrinos

If the background is in the mixed state:

$$|\nu_{ib}\rangle = \Phi_{ie}\,|\nu_e\rangle + \Phi_{i\tau}\,|\nu_\tau\rangle$$

$$B_{e\tau} \sim \Sigma_i\, \Phi_{ie}^{*}\Phi_{i\tau}$$

sum over particles of bg.
w.f. give projections

Contribution to the Hamiltonian in the flavor basis

$$H_{\nu\nu} = \sqrt{2}\, G_F\, \Sigma_i\, (1 - v_e\, v_{ib}) \begin{pmatrix} |\Phi_{ie}|^2 & \Phi_{ie}^{*}\Phi_{i\tau} \\ \Phi_{ie}\Phi_{i\tau}^{*} & |\Phi_{i\tau}|^2 \end{pmatrix}$$

Evolution equation

Ensemble of neutrino polarization vectors $\mathbf{P}_\omega$

$$d_t \mathbf{P}_\omega = (-\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{P}) \times \mathbf{P}_\omega$$

Negative frequencies for antineutrinos

Vacuum mixing term

$\mathbf{B} = (\sin 2\theta\ ,\ 0,\ \cos 2\theta)$

$\omega = \Delta m^2 / 2E$

Usual matter potential

$\mathbf{L} = (0,\ 0,\ 1)$

$\lambda = V = \sqrt{2}\, G_F\, n_e$

Collective vector

$$\mathbf{P} = \int_{-\mathrm{inf}}^{+\mathrm{inf}} d\omega\ \mathbf{P}_\omega$$

$\mu = \sqrt{2}\, G_F\, n_\nu\, (1 - \cos\theta_{\nu\nu})$

The term describes collective effects

Adiabaticity

Adiabaticity condition $\boxed{\left|\frac{d\theta_m}{dt}\right| \ll H_{2m} - H_{1m}}$

External conditions (density) change slowly the system has time to adjust them

transitions between the neutrino eigenstates can be neglected

$\nu_{1m} \not\leftrightarrow \nu_{2m}$ $\Rightarrow$ The eigenstates propagate independently

Shape factors of the eigenstates do not change

Crucial in the resonance layer:
- the mixing changes fast
- level splitting is minimal

$\Delta r_R > l_R$ if vacuum mixing is small

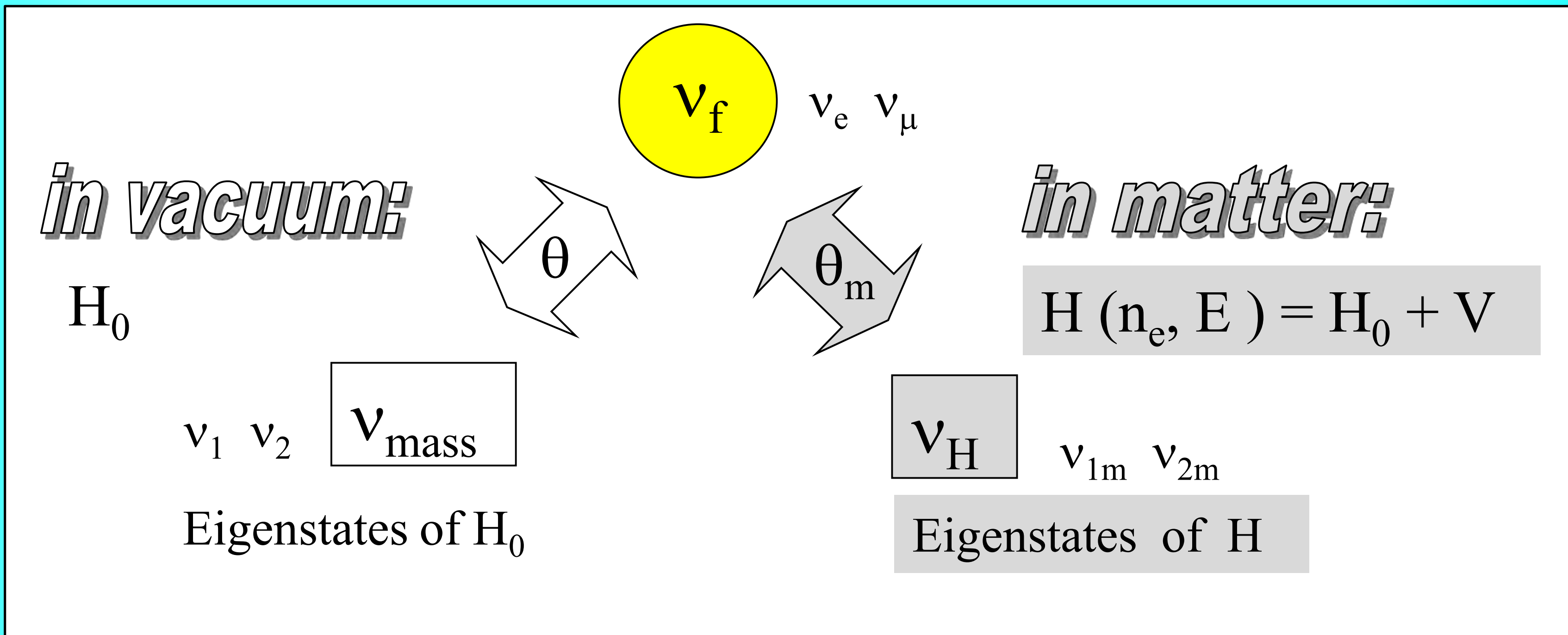
$l_R = l_\nu / \sin 2\theta$ oscillation length in resonance

$\Delta r_R = n_R / (dn/dx)_R \tan 2\theta$ width of the res. layer

If vacuum mixing is large, the point of maximal adiabaticity violation is shifted to larger densities

$n(a.v.) \to n_R^0 > n_R$

$n_R^0 = \Delta m^2 / 2\sqrt{2}\, G_F E$



Mixing in matter – dynamical variable