Some 4-loop results for the cusp anomalous dimension

Andrey Grozin

Introduction

- HQET field h: γ_h (a straight Wilson line)
- $J = h^+_{v'} h_v$ $(v \cdot v' = \cosh\varphi)$: $\Gamma(\varphi)$
  (cusp on a Wilson line)

Non-abelian exponentiation $\Rightarrow$ some color structures are absent

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  (cusp on a Wilson line)

Non-abelian exponentiation $\Rightarrow$ some color structures are absent

3 loops

- γ_h: K. Melnikov, T. van Ritbergen (2000);
  K. Chetyrkin, A. Grozin (2003)
- Γ: A. Grozin, J. Henn, G. Korchemsky, P. Marquard (2015–2016)

Casimir scaling

Conjecture

$$\Gamma(\varphi) = C_R \frac{a}{\pi}\left[\Omega(\varphi) + C_A \Omega_A(\varphi)\frac{a}{\pi} + C_A^2 \Omega_{AA}(\varphi)\left(\frac{a}{\pi}\right)^2\right] + \mathcal{O}\big(a^4\big)$$

$$\frac{a}{\pi} = \frac{\alpha_s}{\pi} + (C_A B_A + T_F n_l B_l)\left(\frac{\alpha_s}{\pi}\right)^2$$

$$+ \left(C_A^2 B_{AA} + T_F n_l C_F B_{Fl} + T_F n_l C_A B_{Al} + (T_F n_l)^2 B_{ll}\right)\left(\frac{\alpha_s}{\pi}\right)^3$$

$$+ \mathcal{O}\big(\alpha_s^4\big)$$

Minkowski $\varphi \to \infty$: $\Gamma \sim \varphi \Rightarrow a$

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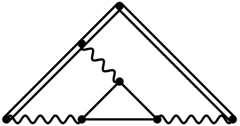

$$\Gamma = \cdots + C_R C_A T_F n_l \left[B_{Al} \Omega(\varphi) + 2 B_l \Omega_A(\varphi) \right] \left(\frac{\alpha_s}{\pi} \right)^3 + \cdots$$

4 loops

	γ_h	$\Gamma(\varphi)$	$\varphi \ll 1$	$\varphi \gg 1$
$C_F(T_F n_l)^3$	BG95	BB95		
$C_F^2(T_F n_l)^2$	G16	G16		
$C_F C_A (T_F n_l)^2$	MSSS18,		soon	HSSS16,
	soon			DVRUV17
$C_F^3 T_F n_l$	here	soon		MRUVV17
$d_{FF} n_l$	GHS17		GHS17	MRUVV17
$C_F^2 C_A T_F n_l$	MSSS18			MRUVV17
$C_F C_A^2 T_F n_l$	MSSS18			MRUVV17
$n_l,\ N_c \to \infty$				HSSS16
				MRUVV17
$C_F C_A^3$	MSSS18	$\varphi = \pi - \delta$		MRUVV17
d_{FA}	MSSS18			MRUVV17
$N_c \to \infty$				HLSSS17,
				MRUVV17

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$C_F^3 T_F n_l$	here	soon		MRUVV17
$d_{FF} n_l$	GHS17		GHS17	MRUVV17
$C_F^2 C_A T_F n_l$	MSSS18			MRUVV17
$C_F C_A^2 T_F n_l$	MSSS18			MRUVV17
$n_l,\ N_c \to \infty$				HSSS16
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$C_F C_A^3$	MSSS18	$\varphi = \pi - \delta$		MRUVV17
d_{FA}	MSSS18			MRUVV17
$N_c \to \infty$				HLSSS17,
				MRUVV17
QED	here		soon	(num)

n_l^3

$$b = -\frac{4}{3} T_F n_l \frac{\alpha_s}{4\pi} \qquad \text{Landau gauge}$$

$$\gamma_h = -6 C_F \frac{\alpha_s}{4\pi} \frac{\left(1+\frac{2}{3}b\right)^2 \Gamma(2+2b)}{(1+b)^2 \Gamma^3(1+b)\Gamma(1-b)}$$

$$= -6 C_F \frac{\alpha_s}{4\pi} \left[1 + \frac{4}{3} b - \frac{5}{9} b^2 - \left(2\zeta_3 - \frac{2}{3}\right) b^3 + \cdots \right]$$

$$\Gamma = 4 C_F \frac{\alpha_s}{4\pi} \frac{\left(1+\frac{2}{3}b\right) \Gamma(2+2b)}{(1+b) \Gamma^3(1+b)\Gamma(1-b)} f_1(\varphi)$$

$$= 4 C_F \frac{\alpha_s}{4\pi} \left[1 + \frac{5}{3} b - \frac{1}{3} b^2 - \left(2\zeta_3 - \frac{1}{3}\right) b^3 + \cdots \right] f_1(\varphi)$$

D. Broadhurst, A. Grozin (1995)
M. Beneke, V. Braun (1995)

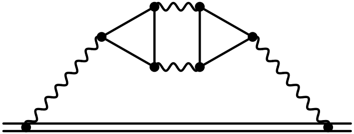
n_l^2 abelian

$$\gamma_h = 6C_F^2\left(-\frac{4}{3}T_F n_l\right)\left(\frac{\alpha_s}{4\pi}\right)^3\left[3\left(4\zeta_3-\frac{17}{4}\right)\right.$$
$$\left.-\left(\frac{\pi^4}{5}-36\zeta_3+\frac{103}{9}\right)b+\cdots\right]$$

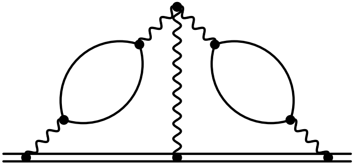
$$\Gamma = -4C_F^2\left(-\frac{4}{3}T_F n_l\right)\left(\frac{\alpha_s}{4\pi}\right)^3\left[12\zeta_3-\frac{55}{4}\right.$$
$$\left.-\left(\frac{\pi^4}{5}-40\zeta_3+\frac{299}{18}\right)b+\cdots\right]f_1(\varphi)$$

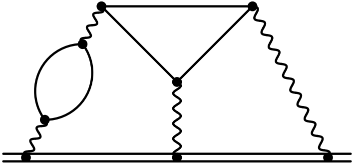
A. Grozin (2016)

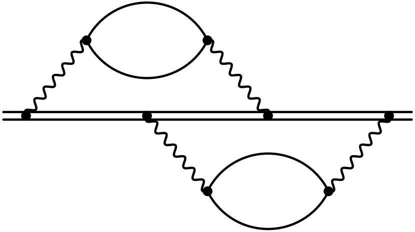
$C_F C_A (T_F n_l)^2$



3-loop gluon propagator $C_A (T_F n_l)^2$



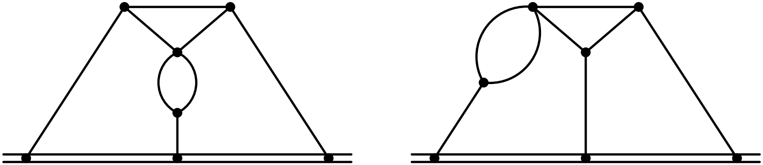




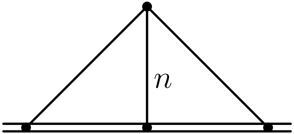
partial fractioning

$C_F C_A (T_F n_l)^2$

Topologies



Masters: 2 reduce to M. Beneke, V. Braun (1994)



2 from A. Grozin, J. Henn, M. Stahlhofen (2017)

γ_h

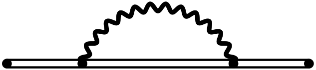
$$\gamma_h\Big|_{C_F C_A (T_F n_l)^2} = 2C_F C_A (T_F n_l)^2 \left[\frac{16}{15}\pi^4 - 192\zeta_3 - \frac{1027}{81} + \frac{4}{3}\left(4\zeta_3 - \frac{269}{81}\right) a\right] \left(\frac{\alpha_s}{4\pi}\right)^4$$

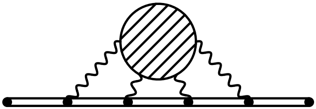
n_l abelian

QED problem
Wilson line of any shape: exponentiation

$$W = \exp \sum_i W_i$$

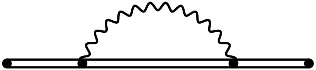
W_i — single-web
C-parity: webs with 2 or 4 legs

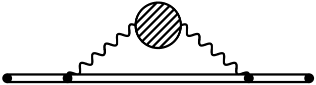




$C_F^3 T_F n_l$, $d_{FF} n_l$

$C_F^3 T_F n_l$





$$W_1 = L_1 A_0 \qquad \text{(Landau gauge)} \qquad A_0 = \frac{e_0^2 (\tau/2)^{2\varepsilon}}{(4\pi)^{d/2}} e^{\gamma\varepsilon}$$

$$W_n = n_l L_n \Pi_{n-1} A_0^n \qquad L_n = <\Gamma\text{-functions}>$$

Re-express via renormalized α (Z_α only needed in W_1)

γ_h

Landau gauge

$$\gamma_h = -6\frac{\alpha}{4\pi} + n_l\left[\frac{32}{3}\left(\frac{\alpha}{4\pi}\right)^2 - 6\,(16\zeta_3 - 17)\left(\frac{\alpha}{4\pi}\right)^3 + \frac{16}{3}(180\zeta_5 - 111\zeta_3 - 35)\left(\frac{\alpha}{4\pi}\right)^4\right] + \cdots$$

$n_l^{>1}$ omitted; 4-leg W omitted

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$n_l^{>1}$ omitted; 4-leg W omitted

All W_n ($n \neq 1$) are gauge invariant; the extra term in W_1

$$\Gamma(-\varepsilon)e^{-\gamma\varepsilon}a_0 A = \Gamma(-\varepsilon)e^{-\gamma\varepsilon}a(\mu)\frac{\alpha(\mu)}{4\pi}e^{2L\varepsilon}\,,$$

$$\frac{d\log\left(a(\mu)\alpha(\mu)\right)}{d\log\mu} = -2\varepsilon$$

exactly. The gauge dependent term in QED is *purely* 1-loop

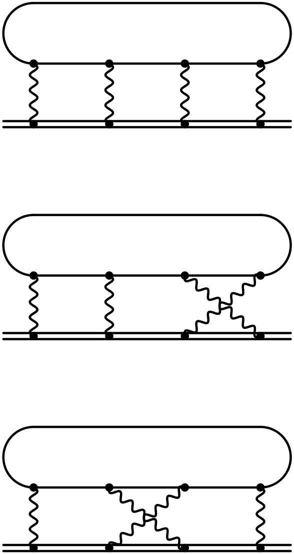
$$2a\frac{\alpha}{4\pi}$$

$d_{FF} n_l$

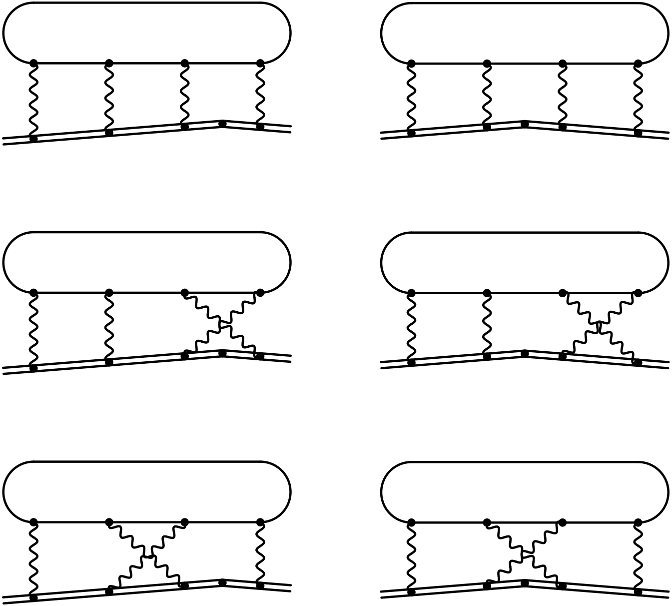
Casimir scaling breaking
A. Grozin, J. Henn, M. Stahlhofen (2017)

- γ_h
- $\Gamma(\varphi)$ up to φ^4

HQET self energy



Vertex function



Calculation

Residual energy $\omega = p \cdot v < 0$ — IR regulator
Vertex: $\omega_1 = \omega_2$ single scale
QED-like, Ward identity $\Rightarrow$ light-by-light off-shell amplitude is transverse $\Rightarrow$ gauge invariant

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IBP reduction
11 denominators + 3 numerators
FIRE (A. Smirnov), LiteRed (R. Lee)
Master integrals:

$$\left.\begin{array}{ll} a & 32 \\ b & 32 \\ c & 30 \end{array}\right\} 43$$

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Choose master integrals without subdivergences
Expand the integrand in ε:
HyperInt (E. Panzer) $\Rightarrow$ polylogarithms

HQET field anomalous dimension

$$\gamma_h\Big|_{d_{FF}n_l} = d_{FF}n_l\left(\frac{\alpha_s}{\pi}\right)^4\left(-\frac{5}{4}\zeta_5 + \frac{2}{3}\pi^2\zeta_3 + \zeta_3 - \frac{2}{3}\pi^2\right)$$

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On-shell heavy-quark field renormalization

$$Z_Q^{\text{os}}\big|_{d_{FF}n_l} = d_{FF}n_l\frac{g_0^8 m_{\text{os}}^{-8\epsilon}}{(4\pi)^{2d}}e^{4\gamma_E\epsilon}\left[-\frac{8}{\epsilon}\left(5\zeta_5-\frac{8}{3}\pi^2\zeta_3-4\zeta_3+\frac{8}{3}\pi^2+4\right)+\mathcal{O}\left(\epsilon^0\right)\right]$$

Confirmed numerically: P. Marquard, A. Smirnov, V. Smirnov, M. Steinhauser (2018)

Cusp anomalous dimension

$$\Gamma\Big|_{d_{FF}n_l} = d_{FF}n_l\left(\frac{\alpha_s}{\pi}\right)^4\frac{1}{9}\Bigg[\varphi^2\left(-4\pi^2\zeta_3+\frac{5}{12}\pi^4+\frac{5}{6}\pi^2\right)$$
$$+\varphi^4\left(-4\zeta_5-\frac{16}{75}\pi^2\zeta_3+\frac{71}{25}\zeta_3+\frac{49}{900}\pi^4-\frac{157}{900}\pi^2-\frac{23}{100}\right)$$
$$+\mathcal{O}\left(\varphi^6\right)\Bigg]$$
$$= d_{FF}n_l\left(\frac{\alpha_s}{\pi}\right)^4\left[0.150721\,\varphi^2+0.00965191\,\varphi^4+\mathcal{O}\left(\varphi^6\right)\right]$$

Conjecture

$$\Gamma(\varphi) = C_R \Omega(\varphi) \frac{a}{\pi} + \cdots$$

$$\frac{a}{\pi} = \cdots + \frac{d_{FF} n_l}{C_R} B \left(\frac{\alpha_s}{\pi} \right)^4 + \cdots$$

Conjecture

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$$\frac{a}{\pi} = \cdots + \frac{d_{FF} n_l}{C_R} B \left(\frac{\alpha_s}{\pi}\right)^4 + \cdots$$

$\varphi = \pi - \delta$: related to $V(q)$, known analytically:
R. Lee, A. Smirnov, V. Smirnov, M. Steinhauser (2016)

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$$\Gamma\Big|_{d_{FF} n_l} = d_{FF} n_l \left(\frac{\alpha_s}{\pi}\right)^4 \frac{1}{192} \left(\varphi^2 + \frac{\varphi^4}{15} + \mathcal{O}(\varphi^6)\right)$$

$$\left(16\pi^4 \log^2 2 - 336\pi^2 \zeta_3 \log 2 - \frac{16}{3}\pi^4 \log 2 - 32\pi^2 \log 2\right.$$

$$\left. + \frac{488}{3}\pi^2 \zeta_3 - \frac{5}{3}\pi^6 + \frac{92}{3}\pi^4 - \frac{632}{9}\pi^2\right)$$

$$= d_{FF} n_l \left(\frac{\alpha_s}{\pi}\right)^4 \left[0.14801\, \varphi^2 + 0.00986736\, \varphi^4 + \mathcal{O}(\varphi^6)\right]$$

QED

n_l light lepton flavors
Bloch–Nordsieck field

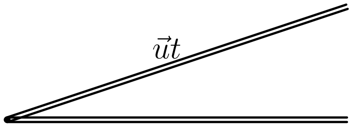
$$\gamma_h = 2(a-3)\frac{\alpha}{4\pi} + \frac{32}{3} n_l \left(\frac{\alpha}{4\pi}\right)^2 + \left[-6\left(16\zeta_3 - 17\right) + \frac{160}{27} n_l\right] n_l \left(\frac{\alpha}{4\pi}\right)^3 + \left[\frac{16}{3}\left(120\zeta_5 + 32\pi^2\zeta_3 - 63\zeta_3 - 32\pi^2 - 35\right) + \frac{32}{135}\left(-9\pi^4 + 1620\zeta_3 - 515\right) n_l - \frac{256}{27}\left(3\zeta_3 - 1\right) n_l^2\right] n_l \left(\frac{\alpha}{4\pi}\right)^4$$

Γ up to φ^4 will be known soon;
$\varphi \gg 1$ – known numerically

Euclidean angle $\varphi = \pi - \delta$, $\delta \to 0$

- 2 loops: $\Gamma(\pi - \delta) \sim \dfrac{1}{\delta}$
  the coefficient is related to the potential $V(r)$
  Conformal symmetry
- 3 loops: Conformal symmetry is broken
  an extra term $\sim \beta_0$ appears
- 4 loops: a $\dfrac{\log \delta}{\delta}$ term appears
  similar to the $\dfrac{\log(\mu r)}{r}$ term in $V(r)$
  N. Brambilla, A. Pineda, J. Soto, A. Vairo (1999-2000)

Wilson line

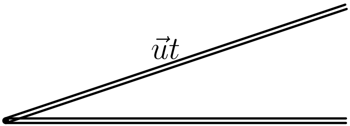


Minkowski space $u \ll 1$

Static quarks interact by exchanging instantaneous Coulomb gluons

$$V(\vec{q}) = -C_F \frac{g_0^2}{\vec{q}^2} \qquad V(\vec{r}) = -C_F G \frac{g_0^2}{(4\pi)^{1-\epsilon}} \frac{1}{r^{1-2\epsilon}}$$

Wilson line



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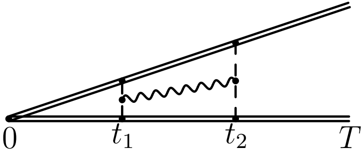

The first transverse-gluon contribution

Ultrasoft region

$$t_1 \sim t_2 \sim t_2 - t_1$$

$$\text{Coulomb gluons } q \sim \frac{1}{ut}$$

$$\text{transverse gluon } k \sim \frac{1}{t}$$

$$k \ll q$$

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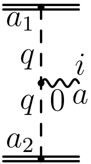
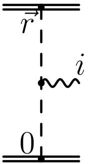


$$= f^{aa_1a_2} g_0^3 \frac{2q^i}{(\vec{q}^2)^2}$$



$$= i f^{aa_1a_2} G \frac{g_0^3}{(4\pi)^{1-\epsilon}} \frac{r^i}{r^{1-2\epsilon}}$$

Wilson line

Ratio to the one without transverse gluon

$$W = 1 + \int_0^T dt_2 \int_0^{t_2} dt_1 \, K(t_1, t_2)$$

$$K(t_1, t_2) = \frac{i}{4} C_F C_A^2 G^2 \frac{g_0^6}{(4\pi)^{2-2\epsilon}} \frac{r_1^i}{r_1^{1-2\epsilon}} \frac{r_2^j}{r_2^{1-2\epsilon}} D^{ij}(v(t_2 - t_1))$$

$$\times \exp\left[-i \int_{t_1}^{t_2} dt \, \Delta V(ut)\right]$$

$$\Delta V(r) \equiv V_A(r) - V(r) = \frac{1}{2} C_A G \frac{g_0^2}{(4\pi)^{1-\epsilon}} \frac{1}{r^{1-2\epsilon}}$$

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$$\Delta V(r) \equiv V_A(r) - V(r) = \frac{1}{2} C_A G \frac{g_0^2}{(4\pi)^{1-\epsilon}} \frac{1}{r^{1-2\epsilon}}$$

Neglect spatial sized of the regions of the transverse gluon emission and absorbtion, so that it propagates between the points vt_1 and vt_2

$$D^{ij}(vt) = 8i(i/2)^{2\epsilon} \frac{1-\epsilon}{3-2\epsilon} \Gamma(2-\epsilon) \frac{t^{-2+2\epsilon}}{(4\pi)^{2-\epsilon}} \delta^{ij}$$

Wilson line

$$K(t_1, t_2) = -\frac{2}{3} C_F C_A^2 \frac{g_0^6}{(4\pi)^{4-3\epsilon}} u^{4\epsilon} t_1^{2\epsilon} t_2^{2\epsilon} (t_2 - t_1)^{-2+2\epsilon} \kappa_1 \times \exp\left[-\frac{i}{4} C_A G \frac{g_0^2}{(4\pi)^{1-\epsilon}} \frac{t_2^{2\epsilon} - t_1^{2\epsilon}}{\epsilon u^{1-2\epsilon}} \right]$$

Wilson line

$$K(t_1,t_2) = -\frac{2}{3}C_F C_A^2 \frac{g_0^6}{(4\pi)^{4-3\epsilon}} u^{4\epsilon} t_1^{2\epsilon} t_2^{2\epsilon} (t_2 - t_1)^{-2+2\epsilon} \kappa_1$$
$$\times \exp\left[-\frac{i}{4} C_A G \frac{g_0^2}{(4\pi)^{1-\epsilon}} \frac{t_2^{2\epsilon} - t_1^{2\epsilon}}{\epsilon u^{1-2\epsilon}}\right]$$

Just 1 Coulomb gluon exchange between t_1 and t_2

$$K(t_1,t_2) = \frac{i}{6} C_F C_A^3 \frac{g_0^8}{(4\pi)^{5-4\epsilon}} \frac{t_1^{2\epsilon} t_2^{2\epsilon} (t_2^{2\epsilon} - t_1^{2\epsilon})(t_2 - t_1)^{-2+2\epsilon}}{\epsilon u^{1-6\epsilon}} \kappa_2$$

Wilson line

$$K(t_1,t_2) = -\frac{2}{3}C_F C_A^2 \frac{g_0^6}{(4\pi)^{4-3\epsilon}} u^{4\epsilon} t_1^{2\epsilon} t_2^{2\epsilon} (t_2 - t_1)^{-2+2\epsilon} \kappa_1$$
$$\times \exp\left[-\frac{i}{4} C_A G \frac{g_0^2}{(4\pi)^{1-\epsilon}} \frac{t_2^{2\epsilon} - t_1^{2\epsilon}}{\epsilon u^{1-2\epsilon}}\right]$$

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$t_2 = t$, $t_1 = xt$

$$\int_0^T dt\, t^{-1+8\epsilon} = \frac{T^{8\epsilon}}{8\epsilon}$$
$$\int_0^1 dx\, x^{2\epsilon} (1 - x^{2\epsilon})(1-x)^{-2+2\epsilon} = \frac{\Gamma(1+2\epsilon)}{1-2\epsilon}$$
$$\times \left[3\frac{\Gamma(1+4\epsilon)}{\Gamma(1+6\epsilon)} - 2\frac{\Gamma(1+2\epsilon)}{\Gamma(1+4\epsilon)}\right] \to 1$$

$\Gamma(\pi - \delta)$

$$W = 1 + \frac{i}{48} C_F C_A^3 \frac{g_0^8}{(4\pi)^{5-4\epsilon}} \frac{T^{8\epsilon}}{\epsilon^2 u^{1-6\epsilon}} \kappa_3$$

$$= 1 + \frac{i}{48} C_F C_A^3 \frac{\alpha_s(\mu)}{4\pi} \frac{(\mu T)^{8\epsilon}}{\epsilon^2 u^{1-6\epsilon}} \kappa_4$$

$$Z = 1 + \frac{i}{8} C_F C_A^3 \frac{\alpha_s^4}{4\pi\epsilon} \frac{\log u + \text{const}}{u}$$

$$\Gamma = -i C_F C_A^3 \frac{\alpha_s^4}{4\pi} \frac{\log u + \text{const}}{u}$$

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Analytical continuation $\varphi = \pi + i\varphi_M$

$$\Gamma(\pi - \delta) = -C_F C_A^3 \frac{\alpha_s^4}{4\pi} \frac{\log \delta + \text{const}}{\delta}$$

Conclusion

- $C_F C_A (T_F n_l)^2$
  - γ_h soon
  - $\Gamma(\varphi)$ $\varphi \ll 1$ soon
- $C_F^3 T_F n_l$
  - γ_h here
  - $\Gamma(\varphi)$ soon
- $d_{FF} n_l$
  - γ_h here
  - $\Gamma(\varphi)$ up to φ^4 here Conjecture disproved!
- QED
  - γ_h here
  - $\Gamma(\varphi)$ $\varphi \ll 1$ soon
- $C_F C_A^3$
  - $\Gamma(\pi - \delta)$ $\quad \dfrac{\log(\delta)}{\delta}$