On the Adaptive Integrand Decomposition of Two-loop Scattering Amplitudes

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Loops & Legs in Quantum Field Theory

01.05.2018, St. Goar, Germany.

Introduction

- Standard Model (SM) of Particle Physics —> best Quantum Field Theory
- SM leaves to much physics without descriptions —> Physics Beyond Standard Model (BSM)
- LHC results demand a refinement of our understanding of the SM physics
  High precision predictions in background processes —> New physics at the TeV scale
- Relevant observables
  —> computation of Quantum Chromodynamics (QCD) **Scattering Amplitudes**

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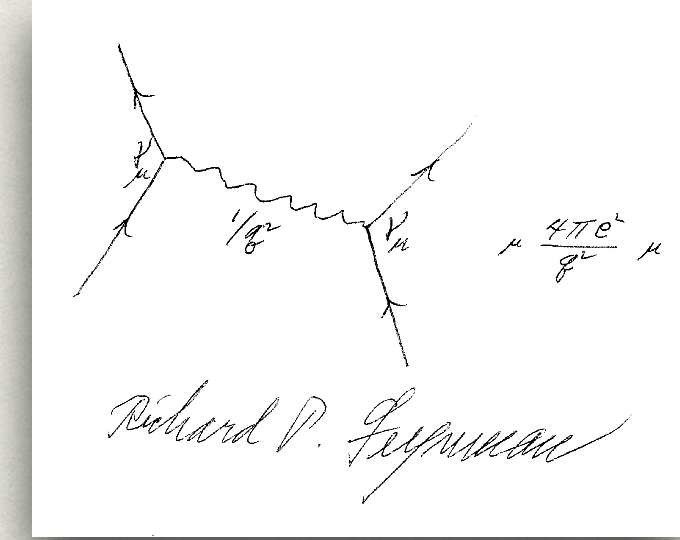
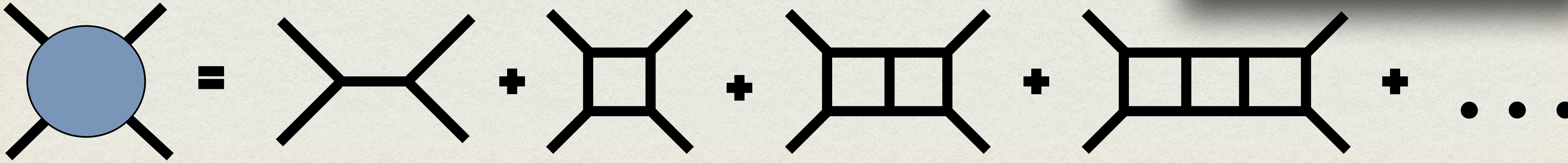
  — Practical applications in particle physics
  — Mathematical elegance
  — Gauge invariant objects



>> Henn

- Perturbative expansion



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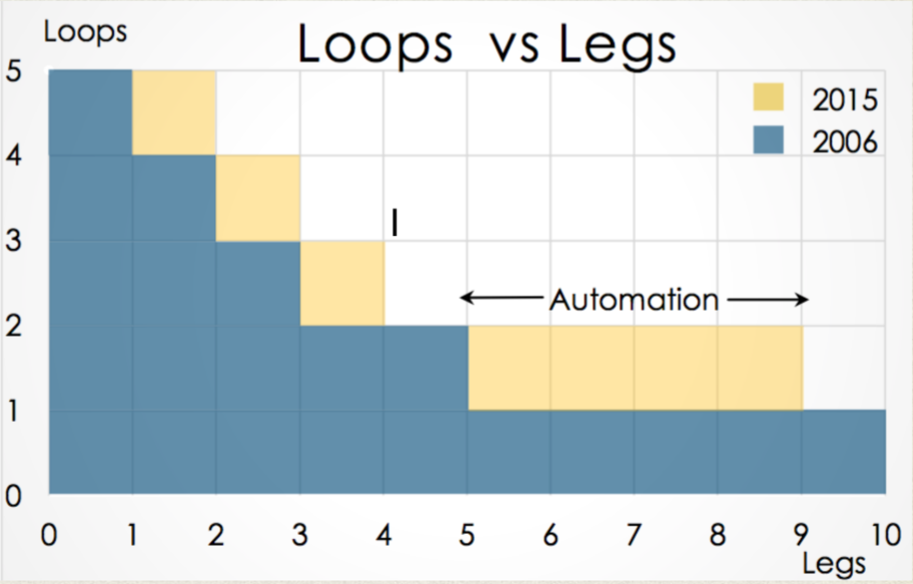


Motivation

- ☑ Simplify the calculations in High-Energy Physics.
- ☑ Discover hidden properties of Quantum Field Theories
- ☑ Towards NNLO is the **Next Frontier.**

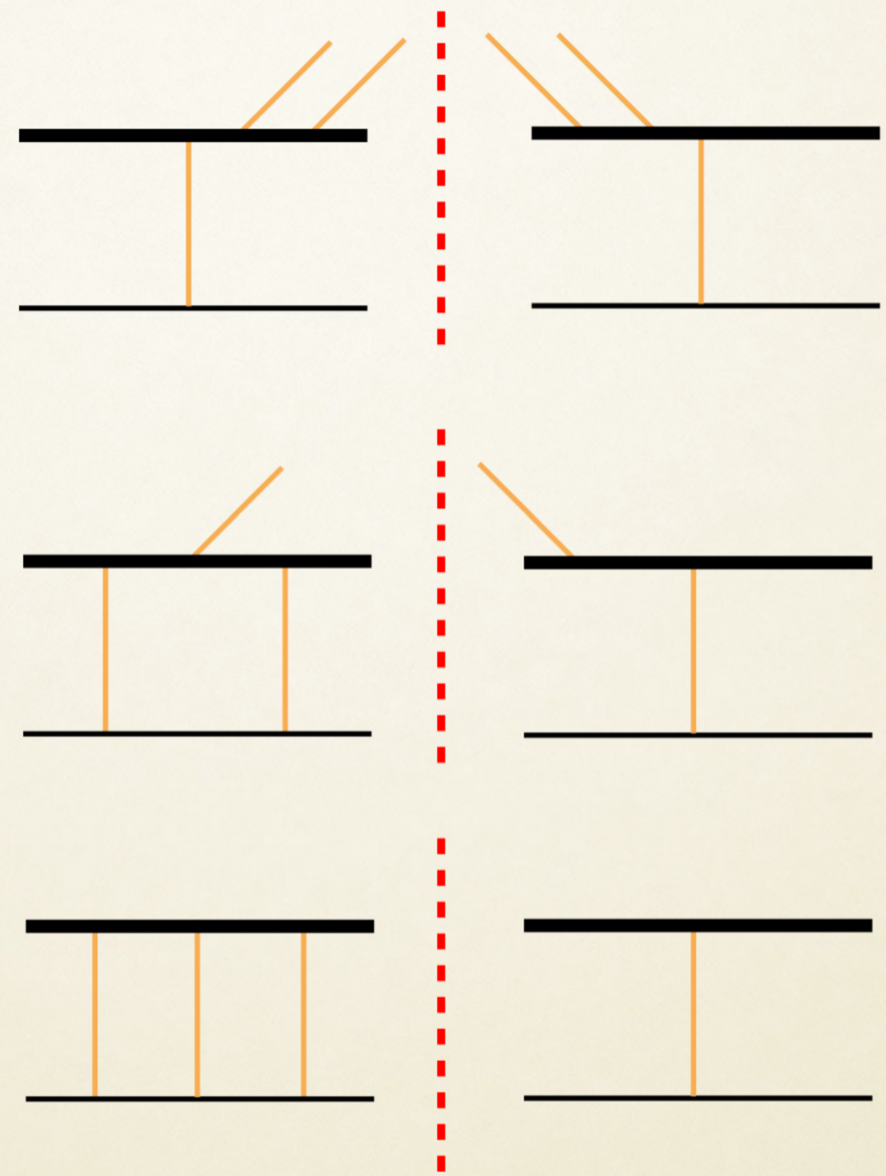
Introduction

What do we need for NNLO?

- **Double-real Radiation tree-level matrix elements**
  $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$

- **Single-real Radiation 1-loop matrix elements**
  $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$

- **Virtual 2-loop matrix element**
  $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$



>> Gerhmann-De Ridder
>> Cruz Martinez
>> Magnea
+ many more!

$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

+ Subtractions and MC integration

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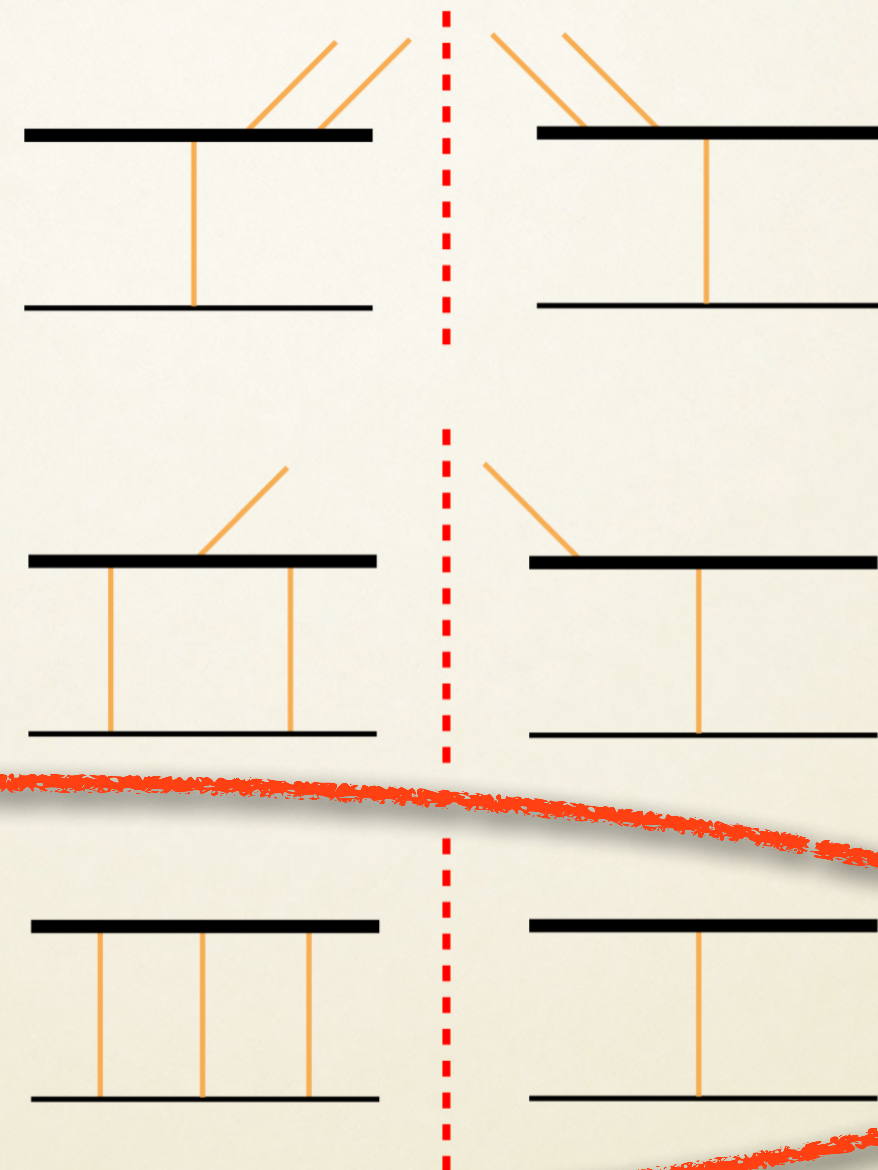

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+ Subtractions and MC integration

Outline

- Calculation of multi-loop scattering amplitudes
  — Integrand reduction methods
  — Automated 1- and 2-loop reduction for any generic process
- Applications
  — Einstein-Yang-Mills amplitudes @1-loop
  — eμ scattering @2-loops
- Conclusions & Outlook

Dimensional regularisation schemes

For all dimensional schemes

$$\int \frac{\mathrm{d}^4 l}{(2\pi)^4} \to \mu_{\mathrm{DS}}^{4-d} \int \frac{\mathrm{d}^d \bar{l}}{(2\pi)^d}$$

with the unified framework

$S_{[4]}$	$\subset$	$QS_{[d]}$	$\subset$	$QS_{[d_s]}$	$\equiv$	$QS_{[d]}$	$\oplus$	$QS_{[n_\epsilon]}$
strictly four-dim.		quasi d-dim.		quasi d_s-dim.				'evanescent'
(unregularized)		(PS integration)		(usually $d_s = 4$)				space

[Gnendiger, *et al* (W.J.T.) (2017)]

In this talk

>> Gnendiger

	tHV	FDH
singular vector fields	$g^{\mu\nu}_{[d]}$	$g^{\mu\nu}_{[d_s]}$
regular vector fields	4	4

$$n_\epsilon = d_s - d$$

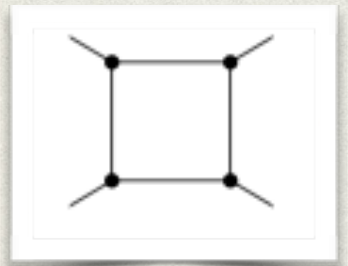
$$\mathrm{tHV} : 0$$

$$\mathrm{FDH} : 2\epsilon$$

[Signer, Stöckinger (2008)]

One-loop scattering amplitudes

Deal with with integrals of the form

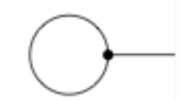


$$I_{i_1 \cdots i_k}\left[\mathcal{N}(\bar{l}, p_i)\right] = \int d^d\bar{l}\, \frac{\mathcal{N}_{i_1 \cdots i_k}(\bar{l}, p_i)}{D_{i_1} \cdots D_{i_k}}$$

$\bar{l}^2,\ \bar{l}\cdot p_i,\ \bar{l}\cdot \varepsilon_i$

Numerator and denominators are polynomials in the integration variable

Tensor reduction

$$A_n^{(1),D=4}(\{p_i\}) = \sum_{K_4} C^{[0]}_{4;K4}$$  $$+ \sum_{K_3} C^{[0]}_{3;K3}$$  $$+ \sum_{K_2} C^{[0]}_{2;K2}$$  $$+ \sum_{K_1} C^{[0]}_{1;K1}$$ 

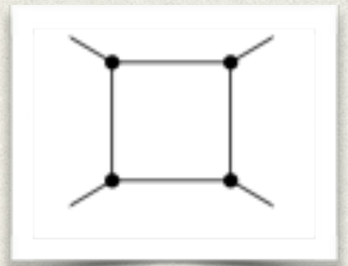
[Passarino - Veltman (1979)]

- ☑ Cut-constructible amplitude -> determined by its branch cuts
- ☑ All one-loop amplitudes are cut-constructible in dimensional regularisation.
- ☑ Master integrals are known

One-loop scattering amplitudes

Deal with with integrals of the form

$\bar{l}^2,\, \bar{l}\cdot p_i,\, \bar{l}\cdot \varepsilon_i$



$$I_{i_1\cdots i_k}\left[\mathcal{N}(\bar{l},p_i)\right] = \int d^d\bar{l}\, \frac{\mathcal{N}_{i_1\cdots i_k}(\bar{l},p_i)}{D_{i_1}\cdots D_{i_k}}$$

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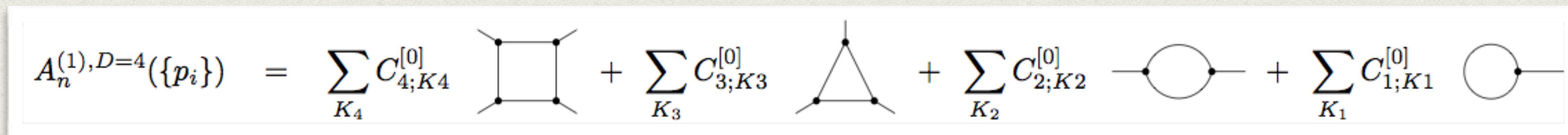
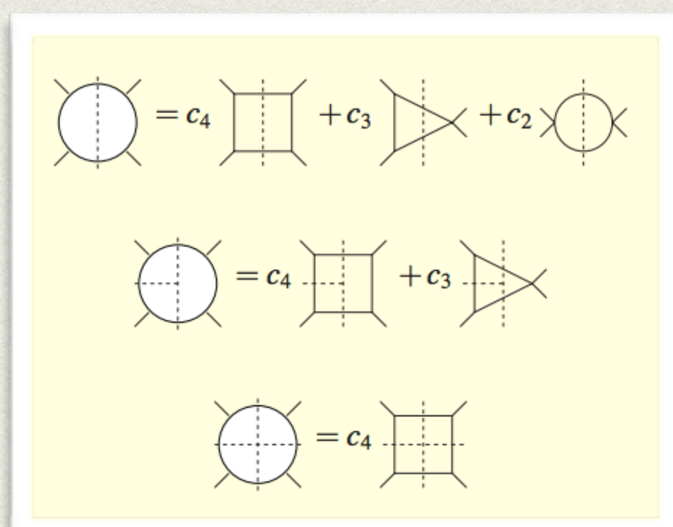


$$A_n^{(1),D=4}(\{p_i\}) = \sum_{K_4} C^{[0]}_{4;K4} \;[\text{box}]\; + \sum_{K_3} C^{[0]}_{3;K3} \;[\text{triangle}]\; + \sum_{K_2} C^{[0]}_{2;K2} \;[\text{bubble}]\; + \sum_{K_1} C^{[0]}_{1;K1} \;[\text{tadpole}]$$

[Passarino - Veltman (1979)]

Unitarity based methods $\quad \frac{i}{q_i^2 - m^2 - i\epsilon} \to 2\pi\,\delta^{(+)}\left(q_i^2 - m_i^2\right)$



cut-4 :: Britto Cachazo Feng

cut-3 :: Forde
Bjerrum-Bohr, Dunbar, Ita, Perkins
Mastrolia

cut-2 :: Bern, Dixon, Dunbar, Kosower.
Britto, Buchbinder, Cachazo, Feng.
Britto, Feng, Mastrolia.

Isolate the leading discontinuity!

>> Page
>> Badger
>> Febres Cordero
>> Zeng

One-loop scattering amplitudes

$$\bar{\ell}^\nu = \ell^\nu + \tilde{\ell}^\nu$$

D=4 D=-2ε

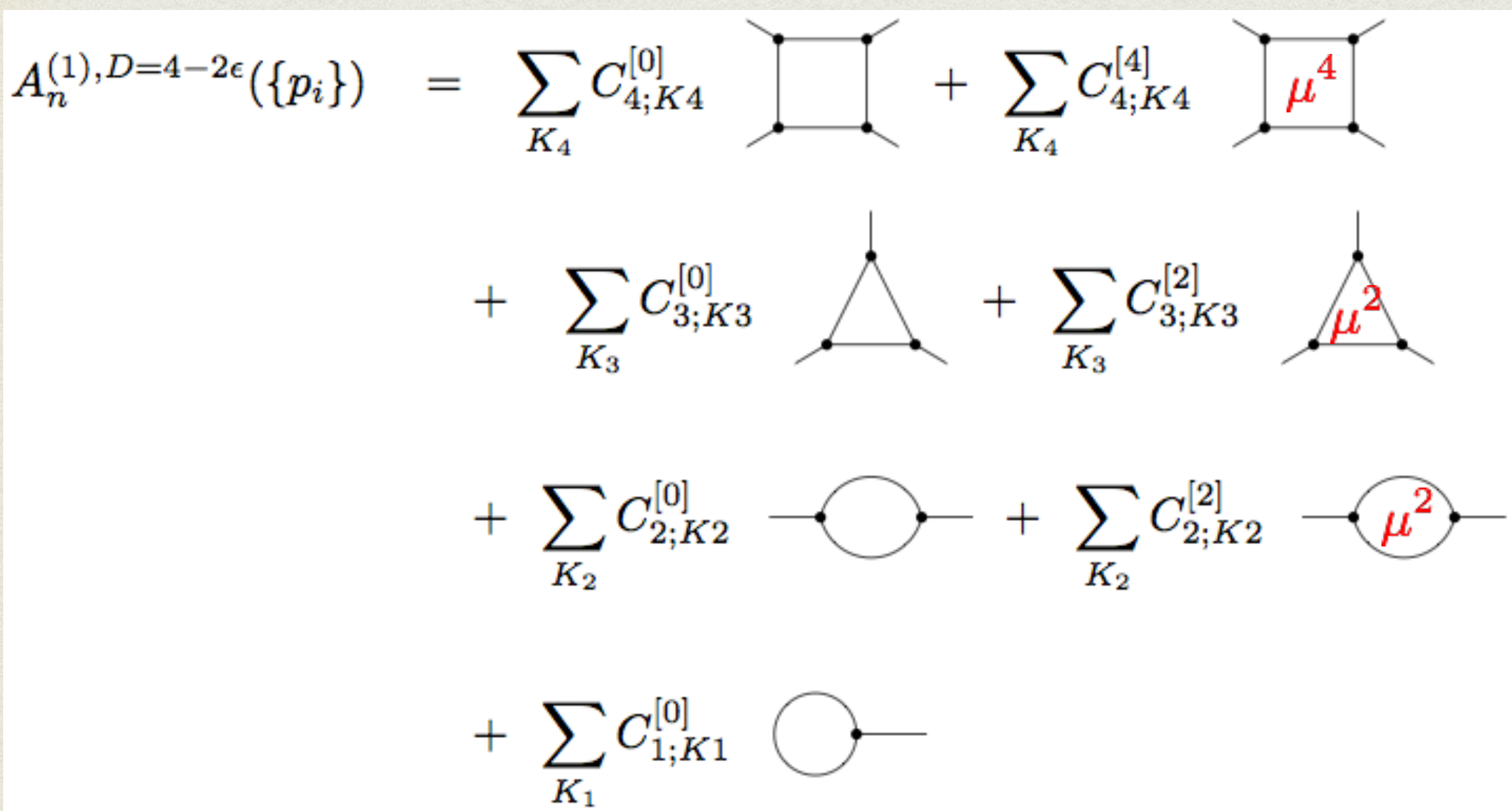
In D=4-2ε we can do the decomposition

At integral level $$\int \frac{d^d \bar{l}}{(2\pi)^d} \equiv \int \frac{d^4 l}{(2\pi)^4} \frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}}$$

The on-shell condition $$\bar{\ell}^2 = \ell^2 - \mu^2 = 0 \rightarrow \ell^2 = \mu^2$$

Mass term

Any one-loop amplitude becomes



[Ossola,Papadopoulos,Pittau (2006)]
[Giele,Kunszt,Melnikov (2008)]
[Badger (2008)]
[Mastrolia, Mirabella, Peraro (2012)]

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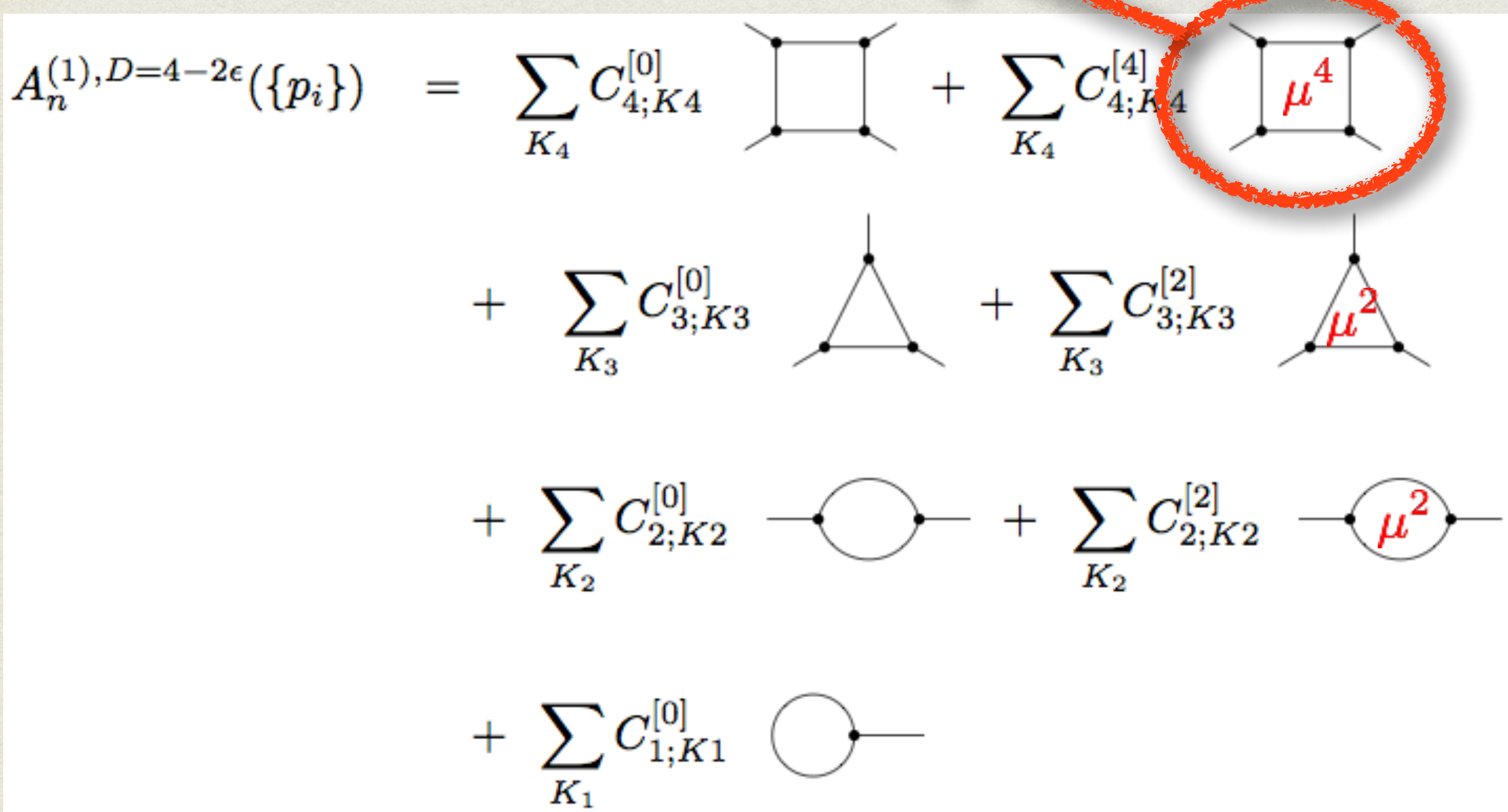
Th $\int d^4 l_1 d^{-2\epsilon}\mu \, \frac{\mu^4}{l^2 (l-K_1)^2 (l-K_1-K_2)^2 (l+K_4)^2} = -\frac{1}{6}$ $= \mu^2$

Mass term

Any one-loop amplitude becomes



$$I_n^{(1)\,d}[\mu^2] = -\epsilon I_n^{(1)\,d+2}[1]$$
$$I_n^{(1)\,d}[\mu^4] = -\epsilon(1-\epsilon) I_n^{(1)\,d+4}[1]$$

[Bern, Morgan (1995)]

[Ossola,Papadopoulos,Pittau (2006)]
[Giele,Kunszt,Melnikov (2008)]
[Badger (2008)]
[Mastrolia, Mirabella, Peraro (2012)]

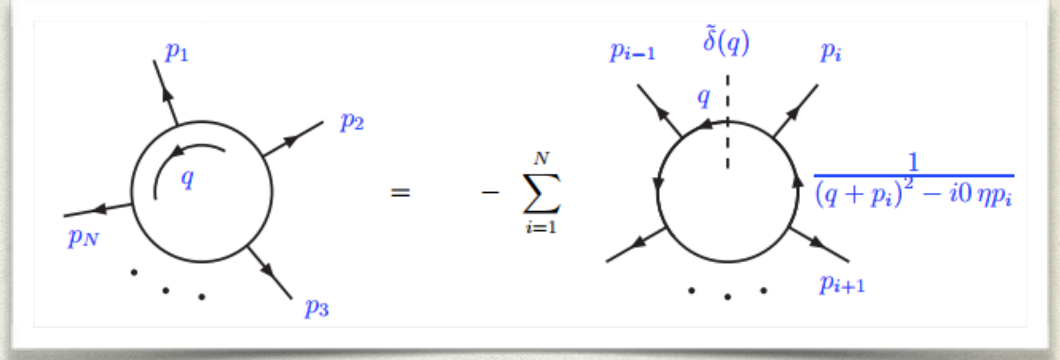
The loop-tree duality theorem

[Catani, Gleisberg, Krauss, Rodrigo, Winter (2008)]

One-loop integrals decomposes as a linear combination of N single-cut phase-space integrals

$$\int_\ell \prod G_F(q_i) = -\sum \int_\ell \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j)$$

$$\tilde{\delta}(q_i) \equiv 2\pi\imath\,\delta^{(+)}(q_i^2 - m_i^2) \qquad G_D(q_i;q_j) = \frac{1}{q_j^2 - m_j^2 - i0\,\eta\,k_{ji}}$$



- Modify +i0 prescription of the Feynman props.
  It compensates for the absence of multiple-cut contributions that appear in the Feynman Tree Theorem
- Lorentz-covariant dual prescription —> η a **future-like** vector

>> Rodrigo

- Number of single cut dual contributions = the number of legs.
- Singularities of the loop diagram —> singularities of the dual integrals.
- Loop-Tree Duality works only on propagators.
  Same procedure for Tensor loop integrals and scattering amplitudes.

One-loop integrand decomposition

[Ossola, Papadopoulos, Pittau (2006)]
[Ellis, Giele, Kunszt, Melnikov (2007)]
[Mastrolia, Ossola, Papadopoulos, Pittau (2008)]

Recall

$$\int d^4\bar{l}\,\frac{\mathcal{N}(l)}{D_1\cdots D_n} = \sum_{i\ll m} c_{ijkm}\int d^4\bar{l}\,\frac{1}{D_iD_jD_kD_m} + \sum_{i\ll k} c_{ijk}\int d^4\bar{l}\,\frac{1}{D_iD_jD_k} + \sum_{i<j} c_{ij}\int d^4\bar{l}\,\frac{1}{D_iD_j} + \sum_{i} c_{i}\int d^4\bar{l}\,\frac{1}{D_i}$$

Find an identity between integrands. Moreover,

$$\frac{\mathcal{N}(l)}{D_1\cdots D_n} \neq \sum_{i\ll m} c_{ijkm}\,\frac{1}{D_iD_jD_kD_m} + \sum_{i\ll k} c_{ijk}\,\frac{1}{D_iD_jD_k} + \sum_{i<j} c_{ij}\,\frac{1}{D_iD_j} + \sum_{i} c_{i}\,\frac{1}{D_i}$$

Suppose a multipole decomposition

$$\frac{\mathcal{N}(l)}{D_1\cdots D_n} = \sum_{i\ll m} \tilde{c}_{ijkm}\,\frac{\Delta_{ijkm}(l)}{D_iD_jD_kD_m} + \sum_{i\ll k} \tilde{c}_{ijk}\,\frac{\Delta_{ijk}(l)}{D_iD_jD_k} + \sum_{i<j} \tilde{c}_{ij}\,\frac{\Delta_{ij}(l)}{D_iD_j} + \sum_{i} \tilde{c}_{i}\,\frac{\Delta_{i}(l)}{D_i}$$

- ☑ Residues Δ are made of **I**rreducible **S**calar **P**roducts
- ☑ Can we find parametric expressions for Δ's in 4- or d-dimensions?
- ☑ Parametric expressions Yes. General way —> Use multivariate polynomial division

coefficients are fixed by sampling the numerators on the cut

One-loop integrand decomposition

Loop parametrisation

$$l_i^\alpha = p_i^\alpha + x_1 e_1^\alpha + x_2 e_2^\alpha + x_3 e_3^\alpha + x_4 e_4^\alpha$$
$$\mathcal{N}\left(\bar{l}\right) = \mathcal{N}\left(l, \mu^2\right) = \mathcal{N}\left(\boldsymbol{z}\right) \qquad \boldsymbol{z} = \{x_1, x_2, x_3, x_4, \mu^2\}$$

[Mastrolia, Ossola (2011)]
[Badger, Frellesvig, Zhang (2012)]
[Zhang (2012)]
[Mastrolia, Mirabella, Ossola, Peraro (2012)]

Multivariate polynomial division

Write the numerator in terms of
Irreducible polynomials

$$\mathcal{I} \equiv \frac{\mathcal{N}}{D_0 \cdots D_{n-1}} = \sum_{k=1}^{5} \sum_{\{i_1, \cdots, i_k\}} \frac{\Delta_{i_1 \cdots i_k}}{D_{i_1} \cdots D_{i_k}}$$

sum of integrands with five or less denominators

$\Delta_{i_1 \cdots i_k}$ Made of **I**rreducible **S**calar **P**roducts
Cannot be expressed in terms of denominators

Generic structure of the residue

$$\begin{aligned}
\Delta_{i_1 i_2 i_3 i_4 i_5} &= c_0\, \mu^2, \\
\Delta_{i_1 i_2 i_3 i_4} &= c_0 + c_1\, x_{4,v} + \mu^2 \left(c_2 + c_3\, x_{4,v} + \mu^2\, c_4\right), \\
\Delta_{i_1 i_2 i_3} &= c_0 + c_1\, x_4 + c_2\, x_4^2 + c_3\, x_4^3 + c_4\, x_3 + c_5\, x_3^2 + c_6\, x_3^3 + + \mu^2 \left(c_7 + c_8\, x_4 + c_9\, x_3\right), \\
\Delta_{i_1 i_2} &= c_0 + c_1\, x_1 + c_2\, x_1^2 + c_3\, x_4 + c_4\, x_4^2 + c_5\, x_3 + c_6\, x_3^2 + c_7\, x_1 x_4 + c_8\, x_1 x_3 + c_9\, \mu^2, \\
\Delta_{i_1} &= c_0 + c_1\, x_1 + c_2\, x_2 + c_3\, x_3 + c_4\, x_4,
\end{aligned}$$

Adaptive integrand decomposition (AID)

$$d = d_{\parallel} + d_{\perp}$$

$$d_{\parallel} = n - 1$$

$$d_{\perp} = (5-n) - 2\epsilon$$

- Splits d=4-2ε into parallel and orthogonal directions
- Nice properties for less than 5 external legs

[Collins (1984)]
[van Neerven and Vermaseren (1984)]
[Kreimer (1992)]

Loop momenta

$$\bar{l}_i^{\alpha} = \bar{l}_{\parallel i}^{\alpha} + \lambda_i^{\alpha} \longrightarrow \bar{l}_i^{\alpha} = \sum_{j=1}^{d_{\parallel}} x_{ji}\, e_j^{\alpha}\,, \qquad \lambda_i^{\alpha} = \sum_{j=d_{\parallel}+1}^{4} x_{ji}\, e_j^{\alpha} + \mu_i^{\alpha}\,, \qquad \lambda_{ij} = \sum_{l=d_{\parallel}+1}^{4} x_{li} x_{lj} + \mu_{ij}$$

- Numerator and denominators depend on different variables

[Mastrolia, Peraro, Primo (2016)]

$$\int \prod_i d^{d_{\parallel}} \bar{l}_{\parallel i} \int \prod_{1 \le i \le j \le \ell} d\lambda_{ij}\, G(\lambda_{ij})^{\frac{d_{\perp}-1-\ell}{2}} \int d\Theta_{\perp} \frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij} \Theta_{\perp})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

Straightforward integration of transverse components

Expand in Gegenbauer polynomials

$$\int d\,\Theta_{\perp} = \int_{-1}^{1} \prod_{i=1}^{4-d_{\parallel}} \prod_{j=1}^{\ell} d\cos\theta_{i+j-1\,j} (\sin\theta_{i+j-1\,j})^{d_{\perp}-i-j-1}$$

$$\int_{-1}^{1} d\cos\theta (\sin\theta)^{2\alpha-1} C_n^{(\alpha)}(\cos\theta) C_m^{(\alpha)}(\cos\theta) = \delta_{mn} \frac{2^{1-2\alpha}\pi\Gamma(n+2\alpha)}{n!(n+\alpha)\Gamma^2(\alpha)}$$

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and identification of spurious terms

Adaptive integrand decomposition (AID)

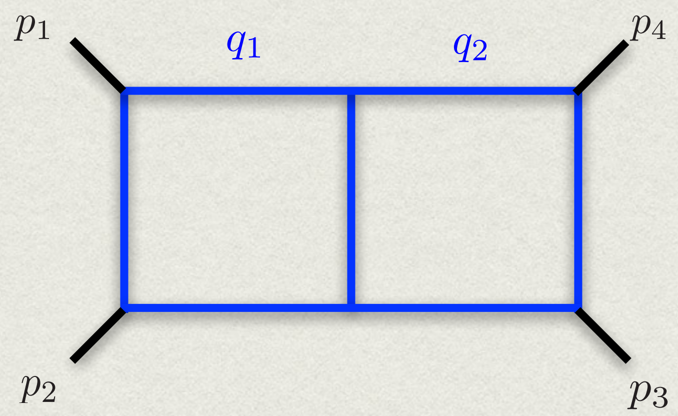
Example 1

- Decompose q_i^α in long/transv components:

$$q^\alpha_{i\,\|} = x_{i1}e^\alpha_1 + x_{i2}e^\alpha_2 + x_{i3}e^\alpha_3$$

$$\lambda^\alpha_i = x_{4i}e^\alpha_4 + \mu^\alpha_i$$

$$D_i = l^2_{\|\,i} + \sum_{j,k} \alpha_{ij}\alpha_{ik}\,\lambda_{jk} + m_i^2$$



$$d_\| = 3 \to e_4 \cdot p_i = 0$$

- Parametrise the integral as

$$I_4^{d\,(2)}[\mathcal{N}] = \frac{2^{d-6}}{\pi^5\Gamma(d-5)} \int d^3q_{1\,\|} \int d^3q_{2\,\|} \int d\lambda_{11}d\lambda_{22}d\lambda_{12}[G(\lambda_{ij})]^{\frac{d-6}{2}} \times \int_{-1}^{1} d\cos\theta_{11}d\cos\theta_{22}(\sin\theta_{11})^{d-6}(\sin\theta_{11})^{d-7}\frac{\mathcal{N}}{D_1\cdots D_7}$$

with

$$G(\lambda_{ij}) = \lambda_{11}\lambda_{22} - \lambda_{12}^2 \qquad \begin{cases} x_{41} = \sqrt{\lambda_{11}}\cos\theta_{11} \\ x_{42} = \sqrt{\lambda_{22}}(\cos\theta_{11}\cos\theta_{12} + \sin\theta_{12}\cos\theta_{22}) \end{cases}$$

- Integrate away transverse directions

$$I_4^{d\,(2)}[\,x_{41}^{\alpha_4}x_{42}^{\beta_4}\,] = 0 \qquad \alpha_4 + \beta_4 = 2n+1$$

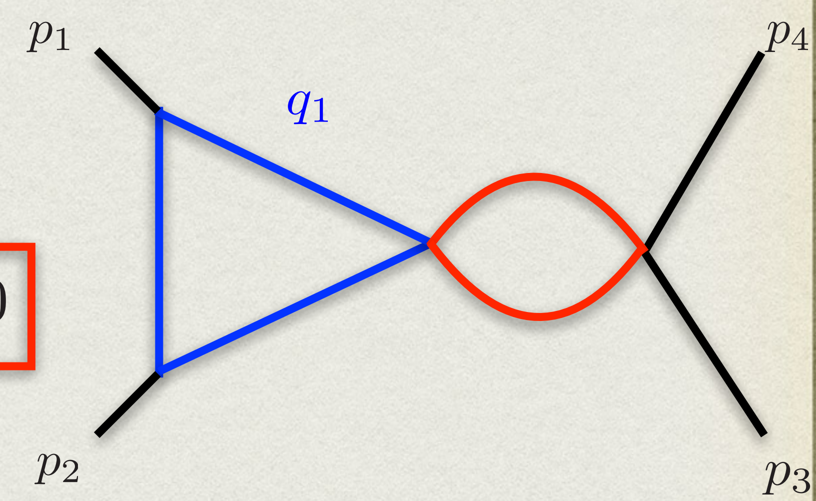
$$I_4^{d\,(2)}[\,x_{42}^3x_{41}^3\,] = \frac{3}{(d-3)(d-1)(d+1)} I_4^{d\,(2)}[\lambda_{12}(2\lambda_{12}^2 + 3\lambda_{11}\lambda_{22})]$$

$$I_4^{d\,(2)}[\,x_{41}^2x_{42}^2\,] = \frac{3}{(d-3)(d-1)} I_4^{d\,(2)}[\,2\lambda_{12}^2 + \lambda_{11}\lambda_{22}]$$

$$\ldots$$

Adaptive integrand decomposition (AID)

Example 2



- Decompose q_i^α in long/transv components:

$$d_{\parallel} = 2 \rightarrow e_{3,4} \cdot p_{1,2} = 0$$

$$q_{1\,\parallel}^{\alpha} = x_{11} e_1^{\alpha} + x_{12} e_2^{\alpha}$$
$$\lambda_1^{\alpha} = x_{13} e_3^{\alpha} + x_{14} e_4^{\alpha} + \mu_1^{\alpha}$$

$$d_{\parallel} = 1 \rightarrow e_{2,3,4} \cdot (p_3 + p_4) = 0$$

$$q_{2\,\parallel}^{\alpha} = x_{21} \hat{e}_1^{\alpha}$$
$$\lambda_2^{\alpha} = x_{22} \hat{e}_2^{\alpha} + x_{23} \hat{e}_3^{\alpha} + x_{24} \hat{e}_4^{\alpha} + \mu_2^{\alpha}$$

- Parametrise the integral as

$$\mathcal{N}(q_1, q_2) = (\mu_{12})^{\alpha}\, \mathcal{N}(q_{1\,[4]}, \mu_{11}) \mathcal{N}(q_{2\,[4]}, \mu_{22})$$

$$I_4^{d\,(2)}[\mathcal{N}] = \Omega_d \int d^2 q_{1\,\parallel} \int d\lambda_{11} [\lambda_{11}]^{\frac{d-4}{2}} \int dc_{\theta_{11}} dc_{\theta_{12}} (s_{\theta_{11}})^{d-5} (s_{\theta_{12}})^{d-6} \frac{\mathcal{N}_1}{D_1 D_2 D_3}$$
$$\times \int dq_{2\,\parallel} \int d\lambda_{22} [\lambda_{22}]^{\frac{d-3}{2}} \int dc_{\theta_{21}} dc_{\theta_{22}} dc_{\theta_{23}} (s_{\theta_{21}})^{d-4} (s_{\theta_{22}})^{d-5} (s_{\theta_{23}})^{d-6} \frac{\mathcal{N}_2}{D_4 D_5}$$

with

$$\begin{cases} x_{13} = & \sqrt{\lambda_{11}} c_{\theta_{11}} \\ x_{14} = & \sqrt{\lambda_{11}} s_{\theta_{11}} c_{\theta_{12}} \end{cases} \qquad \begin{cases} x_{22} = & \sqrt{\lambda_{22}} c_{\theta_{21}} \\ x_{23} = & \sqrt{\lambda_{22}} s_{\theta_{21}} c_{\theta_{22}} \\ x_{24} = & \sqrt{\lambda_{22}} s_{\theta_{21}} s_{\theta_{22}} c_{\theta_{23}} \end{cases}$$

Adaptive integrand decomposition (AID)

[Mastrolia, Peraro, Primo (2016)]

[Mastrolia, Peraro, Primo, W.J.T. (2016)]

Algorithm

- For each integrand, adapt longitudinal and parallel components
- Denominators depend on the minimal set of variables
- Loop components expressed as linear combination of denominators
- Poly division and integration reduced to substitution rules
- Extra dimension variables are always reducible

Recipe in **3** steps

1) Divide and get $\Delta(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)$
2) Integrate out transverse variables $\Theta_\perp$
3) Divide again to get rid of λ_{ij}

Features

- Final decomposition in terms of ISPs
- No need for TID
- Output ready to apply IBPs
- @1L no need of any integral identity

$$\frac{\mathcal{N}(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

1)

$$\frac{\Delta(\bar{l}_{\parallel i}, \lambda_{ij}, \Theta_\perp)}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

2)

$$\frac{\Delta^{\text{int}}(\bar{l}_{\parallel i}, \lambda_{ij})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

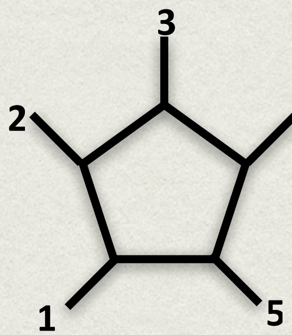
3)

$$\frac{\Delta'(\bar{l}_{\parallel i})}{D_1(\bar{l}_{\parallel i}, \lambda_{ij}) \cdots D_m(\bar{l}_{\parallel i}, \lambda_{ij})}$$

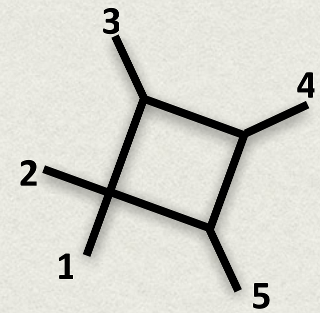
Adaptive integrand decomposition (AID)

Features with AID @ 1-loop

- Evaluation of pentagons

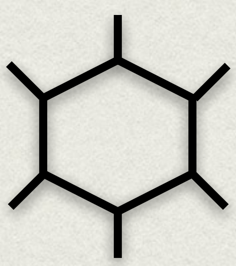


$$= \frac{s_{12}\left(s_{23}+s_{51}\right)+s_{34}\left(s_{45}-s_{23}\right)-s_{45}s_{51}}{4s_{12}s_{23}s_{51}}$$

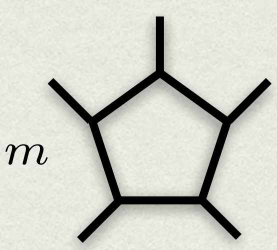


$$+ \text{ cyc. perm.}$$

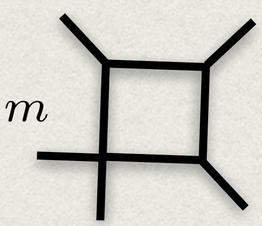
- Topologies with *n*>5 external legs *never computed*



$$\longrightarrow \sum_{ijklm} c_{ijklm}$$



$$\longrightarrow \sum_{ijklm} \tilde{c}_{ijklm}$$



- *No need* of Dimension shift relations $I_n^d[\mu_{11}^r] = I_n^{d+2r}[1]\prod_{j=0}^{r-1}(d-4+2j)$

[Bern & Morgan (1995)]

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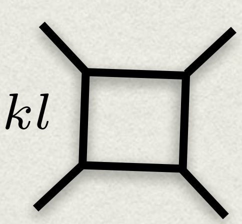
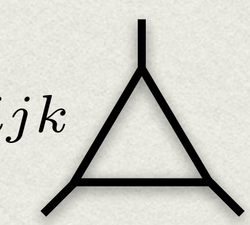


$$+\sum_{ijk} c_{ijk}$$



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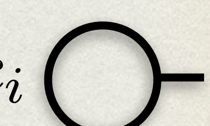
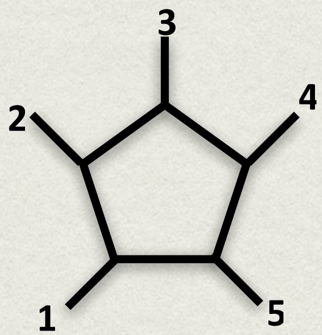


Adaptive integrand decomposition (AID)

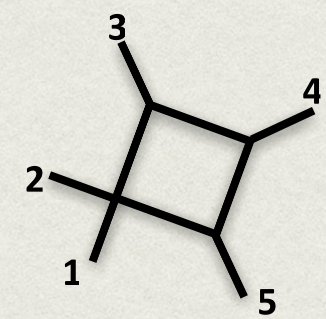
Features with AID @ 1-loop

$$\int \frac{d^d \ell}{(2\pi)^d} \frac{\mu_{11}}{\prod_{i=1}^{5} D_i} = 0 \,,$$

- Evaluation of pentagons



$$= \frac{s_{12}\,(s_{23}+s_{51}) + s_{34}\,(s_{45}-s_{23}) - s_{45}s_{51}}{4 s_{12} s_{23} s_{51}}$$



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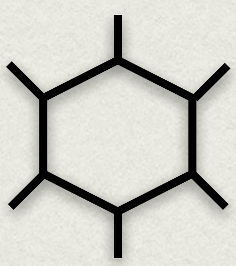
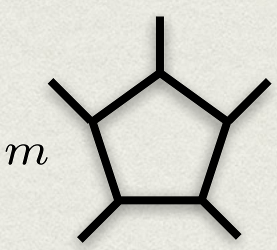


→ $\sum_{ijklm} c_{ijklm}$



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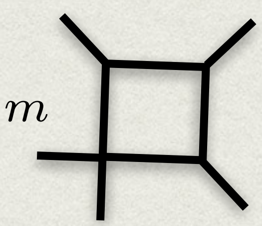

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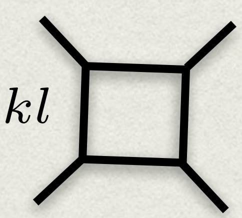

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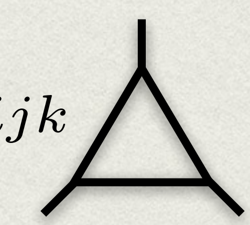

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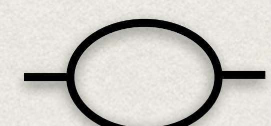
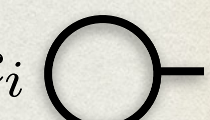


$+ \sum_{i} c_{i}$

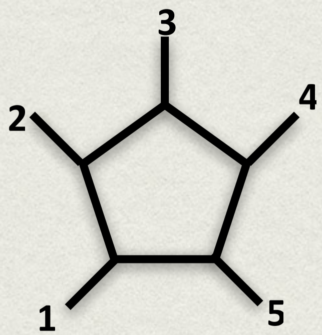


Adaptive integrand decomposition (AID)

Features with AID @ 1-loop

- Evaluation of pentagons

$$\int \frac{d^d \ell}{(2\pi)^d} \frac{\mu_{11}}{\prod_{i=1}^{5} D_i} = 0\,,$$



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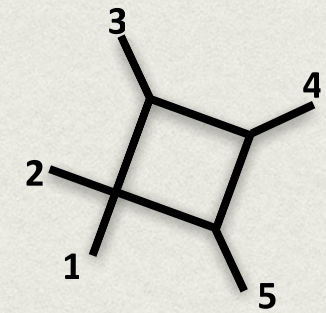

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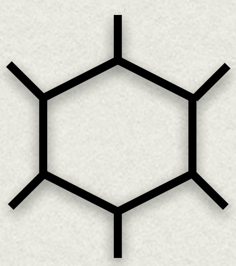
- Topologies with n>5 external legs *never computed*

$$d_k = c_0 + \sum_{i \neq k}^{n} c_i\, d_i$$



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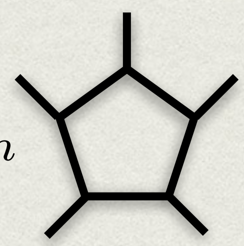

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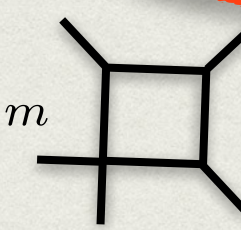

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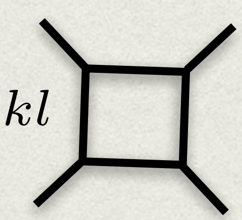

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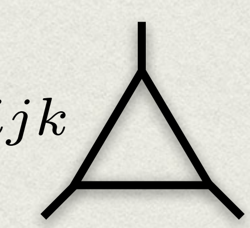

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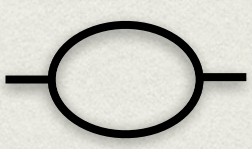

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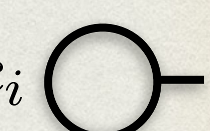


Adaptive integrand decomposition (AID)

Features with AID @ 1-loop

- Evaluation of pentagons

$$\int \frac{d^d\ell}{(2\pi)^d} \frac{\mu_{11}}{\prod_{i=1}^{5} D_i} = 0\,,$$

$$I_2\left[s;\mu_{11}\right] = \left(\frac{s}{4}\right)\frac{(D-4)}{(D-1)} I_2\left[s\right] = -\frac{s}{6} + \mathcal{O}\left(D-4\right)\,,$$

$$I_2\left[s;\mu_{11}^2\right] = \left(\frac{s}{4}\right)^2\frac{(D-4)(D-2)}{(D-1)(D+1)} I_2\left[s\right] = -\frac{s^2}{60} + \mathcal{O}\left(D-4\right)\,,$$

$$I_2\left[s;\mu_{11}^3\right] = \left(\frac{s}{4}\right)^3\frac{(D-4)(D-2)D}{(D-1)(D+1)(D+3)} I_2\left[s\right] = -\frac{s^3}{420} + \mathcal{O}\left(D-4\right)$$

$$I_3\left[s;\mu_{11}\right] = -\frac{(D-4)}{2(D-2)} I_2\left[s\right] = \frac{1}{2} + \mathcal{O}\left(D-4\right)\,,$$

$$I_3\left[s;\mu_{11}^2\right] = -\frac{(D-4)(D-2)}{2(D-1)D}\left(\frac{s}{4}\right) I_2\left[s\right] = \frac{s}{24} + \mathcal{O}\left(D-4\right)\,,$$

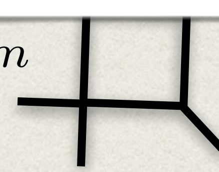
$$I_3\left[s;\mu_{11}^3\right] = -\frac{(D-4)(D-2)D}{2(D-1)(D+1)(D+2)}\left(\frac{s}{4}\right)^2 I_2\left[s\right] = \frac{s}{180} + \mathcal{O}\left(D-4\right)\,,$$

$$I_4\left[s,t;\mu_{11}\right] = \frac{(D-4)}{4(D-3)(s+t)}\left(stI_4\left[s,t\right] - 2sI_3\left[s\right] - 2tI_3\left[t\right]\right) = \mathcal{O}\left(D-4\right)\,,$$

$$I_4\left[s,t;\mu_{11}^2\right] = \frac{(D-4)(D-2)}{4(D-3)(D-1)(s+t)}\frac{st}{4(s+t)}\left(stI_4\left[s,t\right] - 2sI_3\left[s\right] - 2tI_3\left[t\right]\right) + \frac{(D-4)}{4(D-1)(s+t)}\left(sI_2\left[s\right] + tI_2\left[t\right]\right)$$
$$= -\frac{1}{6} + \mathcal{O}\left(D-4\right)\,,$$

$$I_4\left[s,t;\mu_{11}^3\right] = \frac{(D-4)}{16\left(D^2-1\right)\left(s+t\right)^2}\left(s^2((D-2)s + 2(D-1)t)I_2\left[s\right] + (s\leftrightarrow t)\right)$$
$$+ \frac{(D-4)(D-2)D}{4(D-3)(D-1)(D+1)(s+t)}\left(\frac{st}{4\left(s+t\right)}\right)^2\left(stI_4\left[s,t\right] - 2sI_3\left[s\right] - 2tI_3\left[t\right]\right) = -\frac{s+t}{60} + \mathcal{O}\left(D-4\right)\,,$$

$$I_4\left[s,t;\mu_{11}^4\right] = \frac{(D-4)}{64(D+3)\left(D^2-1\right)\left(s+t\right)^3}\left(t^3\left(\left(3D^2-4\right)s^2 + (D-2)(3D+2)st + (D-2)Dt^2\right)I_2\left[s\right] + (s\leftrightarrow t)\right)$$
$$+ \frac{(D-4)(D-2)D(D+2)}{4(D-3)(D-1)(D+1)(D+3)(s+t)}\left(\frac{st}{4\left(s+t\right)}\right)^3\left(stI_4\left[s,t\right] - 2sI_3\left[s\right] - 2tI_3\left[t\right]\right)$$
$$= -\frac{1}{840}\left(2s^2 + st + 2t^2\right) + \mathcal{O}\left(D-4\right)\,.$$

$\longrightarrow \sum_{ijklm} c_{ijklm}$ 

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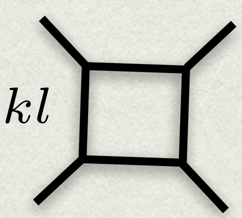
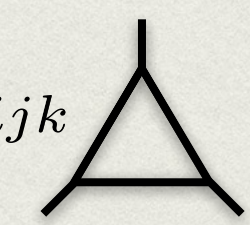
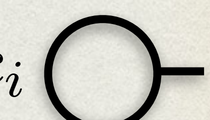
$I_n^d\left[\mu_{11}\right] = \sum_{ijkl} c_{ijkl}$  $+ \sum_{ijk} c_{ijk}$  $+ \sum_{ij} c_{ij}$  $+ \sum_{i} c_{i}$ 

AIDA: a Mathematica implementation

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

[W.J.T. (2018)]

AMPLITUDE GENERATOR
(FeynArts+FeynCalc, QGRAF+FORM...)

AIDA
(Adaptive Integrand Decomposition Algorithm)

IBPs REDUCTION CODE
(Reduze, FIRE, Kira, Azurite...)

COMPUTE MIs

Analytically
(Loopedia)

>> Hahn

Numerically
(SecDec, FIESTA...)

>> Jahn

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AMPLITUDE GENERATOR
(FeynArts+FeynCalc, QGRAF+FORM...)

Cut solution parametrisation

- Identify parent topology
- Analyse kinematics for all cuts
- Store adaptive parametrisation
- Organise cuts in Jobs

Read numerators

Divide
(apply substitution rules)

Numerators = $\Delta^{\text{int}}(\bar{l}_{\|i}, \lambda_{ij})$

Residues depend on $\Theta_{\perp}$?

No

Yes

Integrate
(apply substitution rules)

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The code is designed for general **one-** and **two-**loop **numerical** and **analytical** calculations

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(FeynArts+FeynCalc, QGRAF+FORM...)

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Adaptive integrand decomposition (AID)

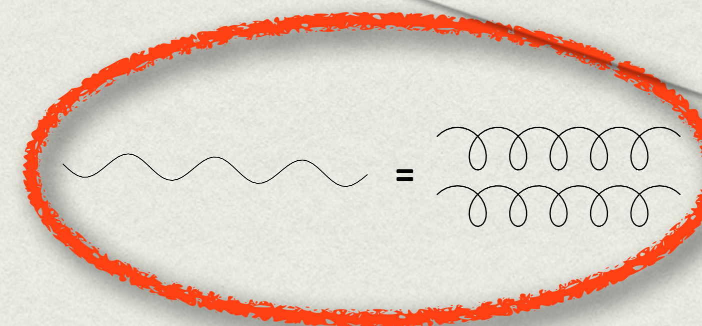
Einstein-Yang-Mills Amplitudes

$$\mathcal{L}_{\mathrm{EYM}} = \frac{2}{\kappa^2}\sqrt{-\boldsymbol{g}}\mathbf{R} - \frac{1}{4}\sqrt{-\boldsymbol{g}}\boldsymbol{g}^{\mu\rho}\boldsymbol{g}^{\nu\sigma}F^a_{\mu\nu}F^a_{\rho\sigma} + \mathcal{L}_{\mathrm{gf}}$$

$$\boldsymbol{R}_{\mu\nu} = \partial_\mu \Gamma^\rho_{\rho\nu} - \partial_\rho \Gamma^\rho_{\mu\nu} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\rho\nu} - \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\mu\nu},$$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2}\boldsymbol{g}^{\rho\sigma}\left(\partial_\mu \boldsymbol{g}_{\nu\sigma} + \partial_\nu \boldsymbol{g}_{\sigma\mu} - \partial_\sigma \boldsymbol{g}_{\mu\nu}\right),$$

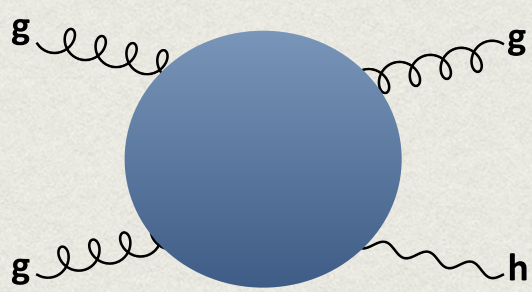
$$\boldsymbol{g}_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}.$$



$$h^{\mu\nu}(p_i) \to \varepsilon^\mu_{\lambda_i}(p_i)\varepsilon^\nu_{\lambda_i}(p_i)$$

- 4-point process depending on 2 scales + d

$$g(p_1) + g(p_2) \to g(-p_3) + h(-p_4)$$



$$s = (p_1 + p_2)^2 \qquad t = (p_2 + p_3)^2$$

Warming up exercise

More gravitons →

$$\{I_4[\mu_{11}], I_4[\mu_{11}^2], I_4[\mu_{11}^3], I_4[\mu_{11}^4], I_3[\mu_{11}], I_3[\mu_{11}^2], I_2[\mu_{11}], I_2[\mu_{11}^2]\}$$

AIDA for EYM amplitudes

Initialisation

- Identify parent topologies from Feynman graphs

e.g. 1-Loop

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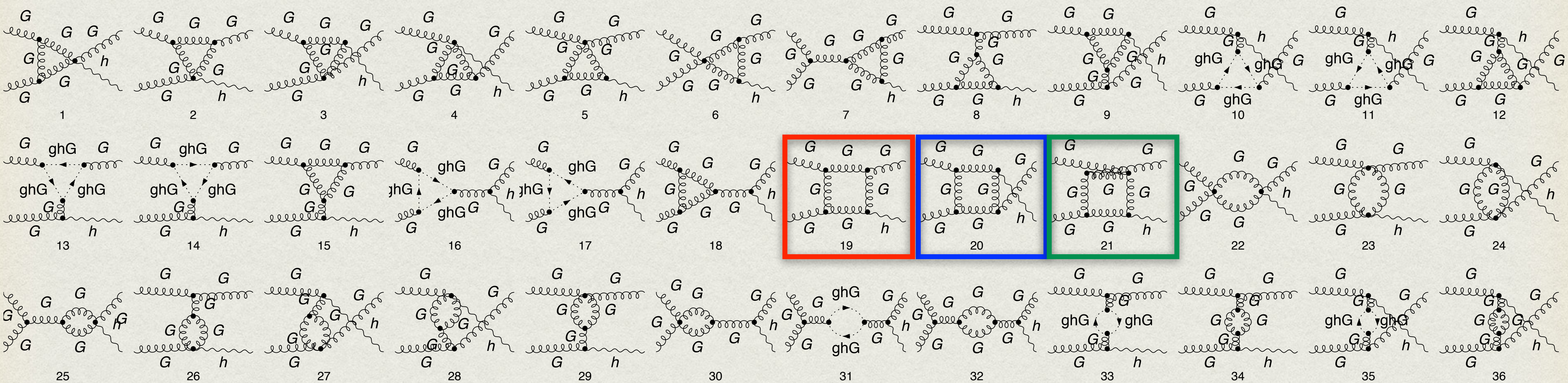

AIDA for EYM amplitudes

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- Identify parent topologies from Feynman graphs

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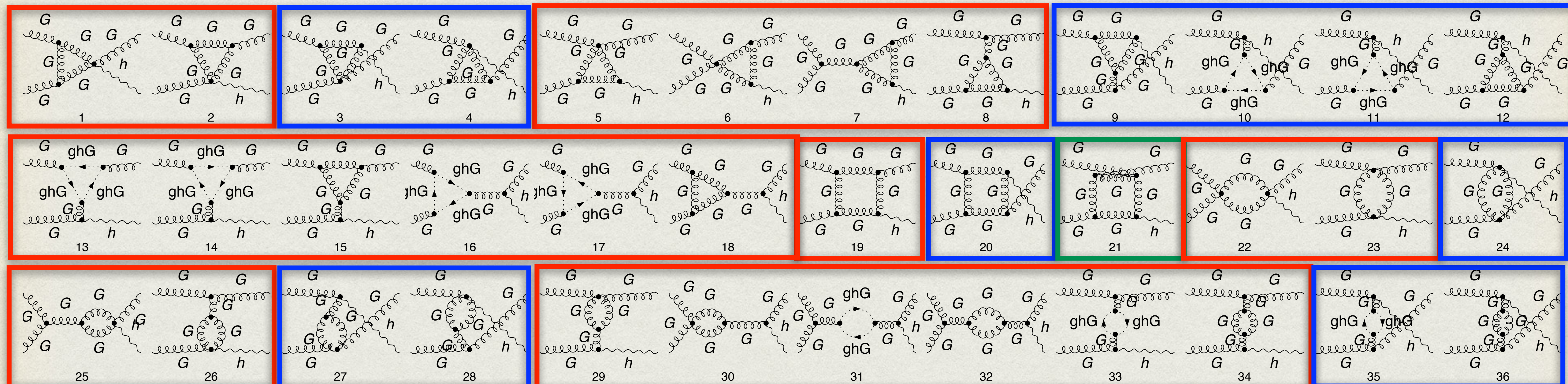


- Group diagrams

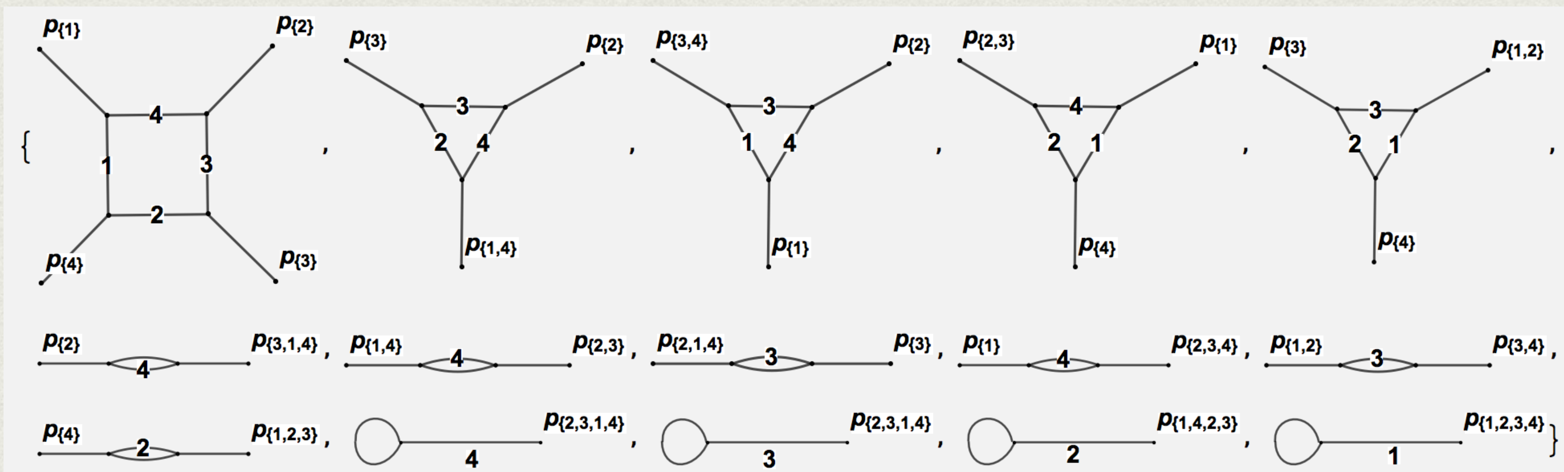
- Extract the leading colour contribution

$$\mathcal{A}\left(\{p_i, h_i\}_{i=1,3}\right)\Big|_{\text{leading colour}} = \sum_{\sigma \in S_3/Z_3} \mathrm{Tr}\left(T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} T^{a_{\sigma(3)}}\right) g_0^3 \left(A_4^{(0)} + \frac{\alpha_0 N_C}{4\pi} A_4^{(1)} + \left(\frac{\alpha_0 N_C}{4\pi}\right)^2 A_4^{(2)} + \mathcal{O}\left(\alpha_0^3\right)\right)$$

AIDA for EYM amplitudes

Initialisation

- Generate all cuts and analyse their kinematics



- Define adaptive variables and prepare substitution rules for all cuts

$$x_{1,(1\;\;2\;\;3)} \to -\frac{d_2}{t} + \frac{d_3}{t} - 1$$

$$x_{2,(1\;\;2\;\;3)} \to \frac{d_2}{t} - \frac{d_1}{t}$$

$$\lambda^2_{(1\;\;2\;\;3)} \to d_1 - \frac{-d_2\,t + d_1\,t - d_2^2 + d_1\,d_2 + d_3\,d_2 - d_1\,d_3}{t}$$

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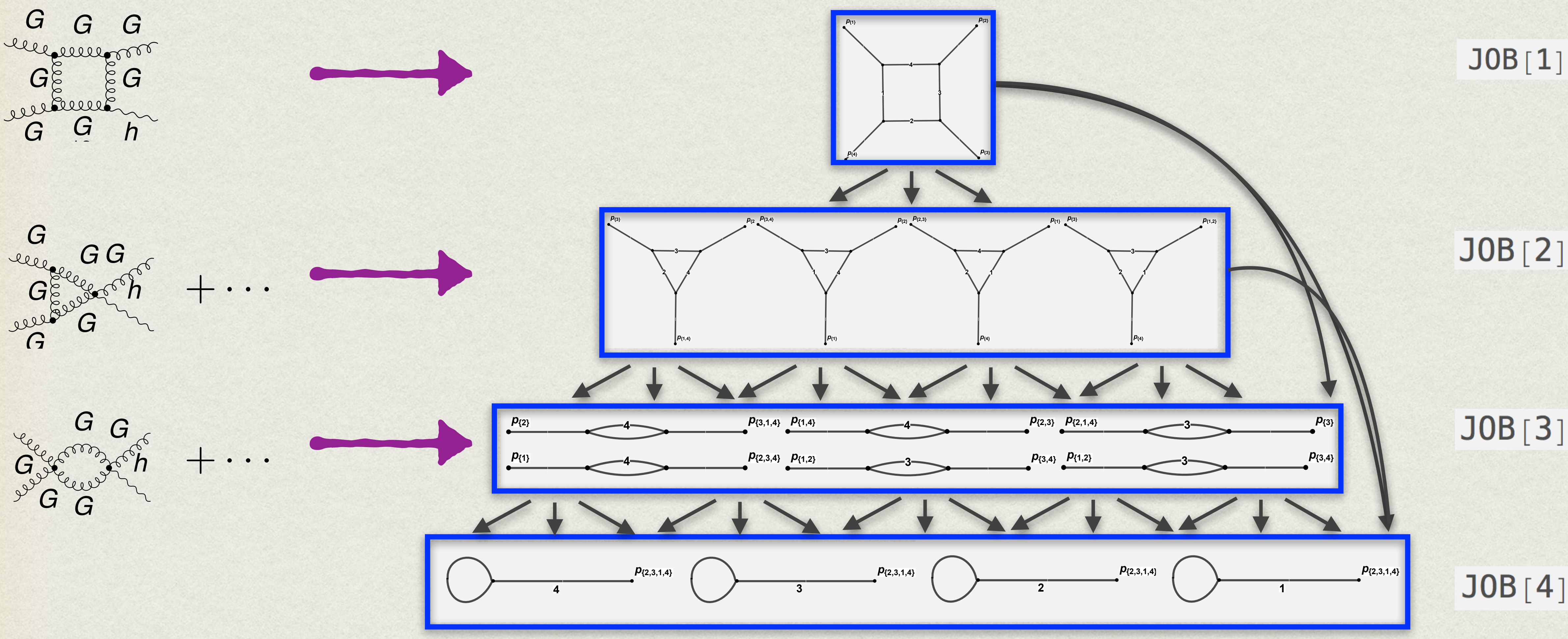
Job structure

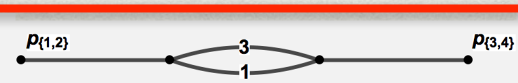
- Organise all cuts of the parent topology in **`Jobs`**



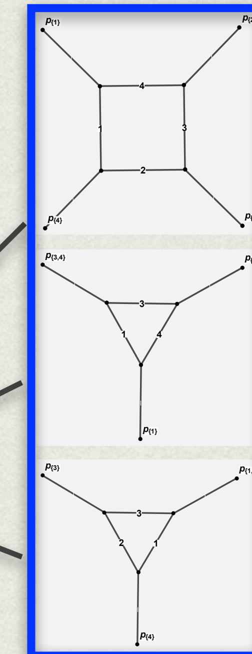
AIDA for EYM amplitudes

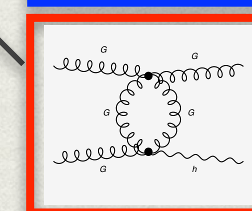
Divide - Integrate - Divide

- For every Job, build the numerators of the corresponding cuts



$$\text{Num}[\{\{1,3\}\},\{1,0,1,0\}] \to \begin{array}{l} \text{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,2,3,4\}\},\{1,1,1,1\}} \\ \text{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,3,4\}\},\{1,0,1,1\}} \\ \text{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,2,3\}\},\{1,1,1,0\}} \\ \text{Num}[\{\{1,3\}\},\{1,0,1,0\}]_{\{\{1,3\}\},\{1,0,1,0\}} \end{array}$$





- Apply substitution rules to the numerator

$$x_{1,\left(\begin{smallmatrix}1 & 3\end{smallmatrix}\right)} \to -\frac{d_1}{2t}+\frac{d_3}{2t}-\frac{1}{2}$$

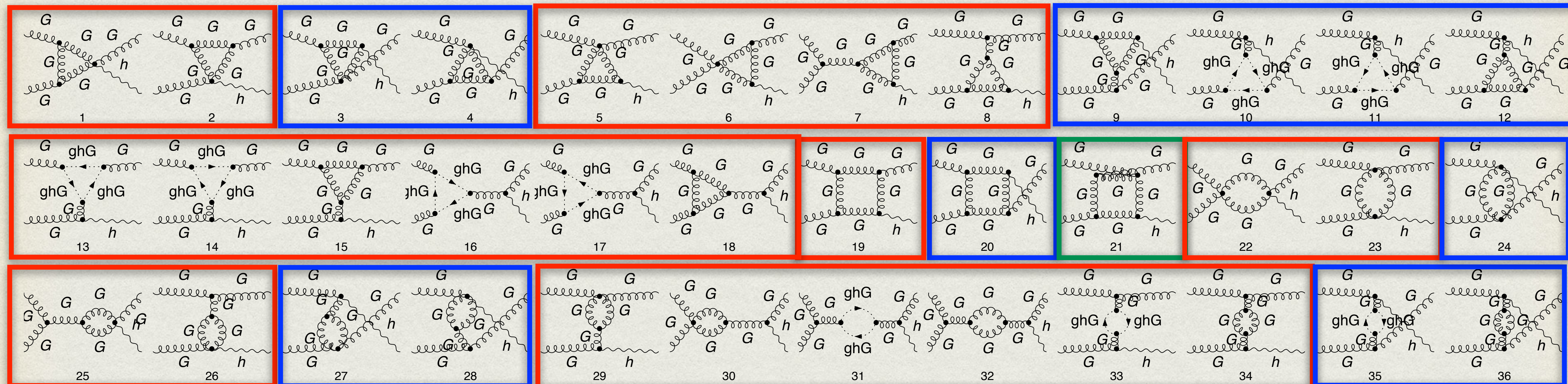
$$\lambda^2_{\left(\begin{smallmatrix}1 & 3\end{smallmatrix}\right)} \to d_1 - \frac{(d_1-d_3+t)^2}{4t}$$

μ-dependence in the numerator $\longrightarrow$ $\lambda_{ij} = \sum_{l=d_{\parallel}+1}^{4} x_{li}x_{lj} + \mu_{ij}$

- Collect powers of denominators to read off residue and numerators of lower cuts
- Integrate (substitute) transverse vars appearing in the residues
- Division again, using as input numerators the residues!

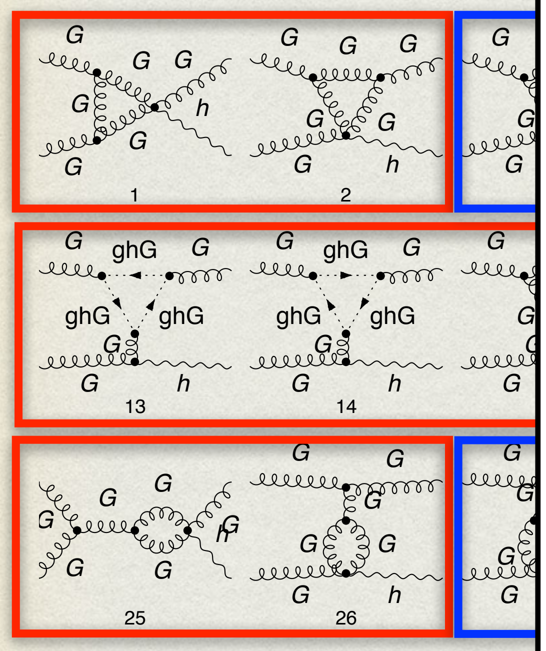
AIDA for EYM amplitudes

Input numerators



AIDA for EYM amplitudes

Input numerators

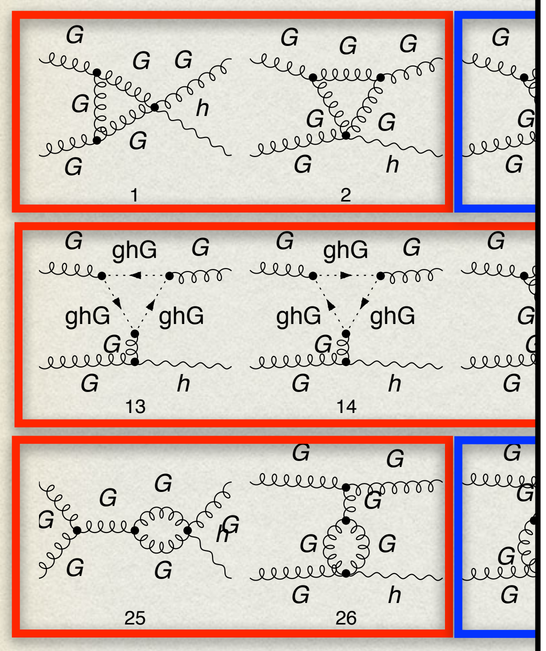


$$
\begin{aligned}
&\frac{1}{2}\Big(2\,(\mathrm{sp}(q,\varepsilon_4)+\mathrm{sp}(\varepsilon_4,p_1)+\mathrm{sp}(\varepsilon_4,p_4))\\
&\quad\Big(3\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)^2+\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,\varepsilon_2)\,\mathrm{sp}(q,p_1)-2\,\mathrm{sp}(q,\varepsilon_2)\,\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,\varepsilon_3)\,\mathrm{sp}(q,p_1)+\\
&\qquad 7\,\mathrm{sp}(q,\varepsilon_2)\,\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(q,p_1)-\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,\varepsilon_3)\,\mathrm{sp}(\varepsilon_2,p_1)\,\mathrm{sp}(q,p_1)+\\
&\qquad 5\,\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_1)\,\mathrm{sp}(q,p_1)-6\,\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,(\mathrm{sp}(q,\varepsilon_2)+\mathrm{sp}(\varepsilon_2,p_1))\,\mathrm{sp}(q,p_1)-\\
&\qquad \mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,\varepsilon_3)\,\mathrm{sp}(\varepsilon_2,p_3)\,\mathrm{sp}(q,p_1)+3\,\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_3)\,\mathrm{sp}(q,p_1)-\\
&\qquad \mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,\varepsilon_3)\,\mathrm{sp}(\varepsilon_2,p_4)\,\mathrm{sp}(q,p_1)+3\,\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_4)\,\mathrm{sp}(q,p_1)-\\
&\qquad 2\,\mathrm{sp}(q,\varepsilon_1)\,\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(\varepsilon_2,\varepsilon_4)\,\mathrm{sp}(q,p_1)-\mathrm{sp}(q,\varepsilon_3)\,\mathrm{sp}(\varepsilon_1,p_1)\,\mathrm{sp}(\varepsilon_2,\varepsilon_4)\,\mathrm{sp}(q,p_1)+\\
&\qquad 4\,\mathrm{sp}(q,\varepsilon_2)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,p_1)\,\mathrm{sp}(q,p_1)+2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_1)\,\mathrm{sp}(\varepsilon_3,p_1)\,\mathrm{sp}(q,p_1)-\\
&\qquad 6\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,(\mathrm{sp}(q,\varepsilon_2)+\mathrm{sp}(\varepsilon_2,p_1))\,\mathrm{sp}(\varepsilon_3,p_1)\,\mathrm{sp}(q,p_1)+2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_3)\,\mathrm{sp}(\varepsilon_3,p_1)\,\mathrm{sp}(q,p_1)+\\
&\qquad 2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_4)\,\mathrm{sp}(\varepsilon_3,p_1)\,\mathrm{sp}(q,p_1)+\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,\varepsilon_2)\,(\mathrm{sp}(q,\varepsilon_3)+\mathrm{sp}(\varepsilon_3,p_1))\,\mathrm{sp}(q,p_1)-\\
&\qquad 2\,\mathrm{sp}(q,\varepsilon_2)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,(\mathrm{sp}(q,\varepsilon_3)+\mathrm{sp}(\varepsilon_3,p_1))\,\mathrm{sp}(q,p_1)+2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_1)\,(\mathrm{sp}(q,\varepsilon_3)+\mathrm{sp}(\varepsilon_3,p_1))\,\mathrm{sp}(q,p_1)-\\
&\qquad 6\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_3)\,(\mathrm{sp}(q,\varepsilon_3)+\mathrm{sp}(\varepsilon_3,p_1))\,\mathrm{sp}(q,p_1)-2\,\mathrm{sp}(q,\varepsilon_1)\,\mathrm{sp}(\varepsilon_2,\varepsilon_4)\,(\mathrm{sp}(q,\varepsilon_3)+\mathrm{sp}(\varepsilon_3,p_1))\,\mathrm{sp}(q,p_1)-\\
&\qquad \mathrm{sp}(\varepsilon_1,p_1)\,\mathrm{sp}(\varepsilon_2,\varepsilon_4)\,(\mathrm{sp}(q,\varepsilon_3)+\mathrm{sp}(\varepsilon_3,p_1))\,\mathrm{sp}(q,p_1)+\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,\varepsilon_2)\,\mathrm{sp}(\varepsilon_3,p_2)\,\mathrm{sp}(q,p_1)+\\
&\qquad \mathrm{sp}(q,\varepsilon_2)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,p_2)\,\mathrm{sp}(q,p_1)+2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_1)\,\mathrm{sp}(\varepsilon_3,p_2)\,\mathrm{sp}(q,p_1)-\\
&\qquad 2\,\mathrm{sp}(q,\varepsilon_1)\,\mathrm{sp}(\varepsilon_2,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,p_2)\,\mathrm{sp}(q,p_1)-\mathrm{sp}(\varepsilon_1,p_1)\,\mathrm{sp}(\varepsilon_2,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,p_2)\,\mathrm{sp}(q,p_1)+\\
&\qquad 4\,\mathrm{sp}(q,\varepsilon_2)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,p_3)\,\mathrm{sp}(q,p_1)+2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_1)\,\mathrm{sp}(\varepsilon_3,p_3)\,\mathrm{sp}(q,p_1)-\\
&\qquad 3\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,(\mathrm{sp}(q,\varepsilon_2)+\mathrm{sp}(\varepsilon_2,p_1))\,\mathrm{sp}(\varepsilon_3,p_3)\,\mathrm{sp}(q,p_1)+2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_3)\,\mathrm{sp}(\varepsilon_3,p_3)\,\mathrm{sp}(q,p_1)+\\
&\qquad 2\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_4)\,\mathrm{sp}(\varepsilon_3,p_3)\,\mathrm{sp}(q,p_1)+8\,\mathrm{sp}(q,\varepsilon_2)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,p_4)\,\mathrm{sp}(q,p_1)+\\
&\qquad 4\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_1)\,\mathrm{sp}(\varepsilon_3,p_4)\,\mathrm{sp}(q,p_1)-6\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,(\mathrm{sp}(q,\varepsilon_2)+\mathrm{sp}(\varepsilon_2,p_1))\,\mathrm{sp}(\varepsilon_3,p_4)\,\mathrm{sp}(q,p_1)+\\
&\qquad 4\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_3)\,\mathrm{sp}(\varepsilon_3,p_4)\,\mathrm{sp}(q,p_1)+4\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_2,p_4)\,\mathrm{sp}(\varepsilon_3,p_4)\,\mathrm{sp}(q,p_1)-\\
&\qquad \mathrm{sp}(q,\varepsilon_1)\,\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)-\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,p_2)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)+\\
&\qquad \mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,p_3)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)+\mathrm{sp}(q,\varepsilon_4)\,\mathrm{sp}(\varepsilon_1,p_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)-2\,s\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)+\\
&\qquad (\mathrm{sp}(q,q)-\mu 11)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)+3\,(-\mu 11+\mathrm{sp}(q,q)+\mathrm{sp}(q,p_1))\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)-\\
&\qquad \mathrm{sp}(q,p_2)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)+\mathrm{sp}(q,p_3)\,\mathrm{sp}(\varepsilon_1,\varepsilon_4)\,\mathrm{sp}(\varepsilon_3,\varepsilon_2)\,\mathrm{sp}(q,p_1)+
\end{aligned}
$$

Input: rank 5 numerator

AIDA for EYM amplitudes

Input numerators



$$\frac{1}{2}\Big(2\,(\mathrm{sp}(q,\varepsilon_4)+\mathrm{sp}(\varepsilon_4,p_1)+\mathrm{sp}(\varepsilon_4,p_4))$$

$\mathrm{sp}(q,p_1)+$

$\mathrm{sp}(q,p_1)-$

$\mathrm{sp}(q,p_1)-$

$\varepsilon_2)\,\mathrm{sp}(q,p_1)+$

$(q,p_1)-$

$$\Delta((1\;2\;3\;4),\{1,1,1,1\})\to$$

$$\frac{320\,x^4_{4,(1\;2\;3\;4)}\left(\dim s^5 t^6-2 s^5 t^6\right)}{s+t}-\frac{8\,x^2_{4,(1\;2\;3\;4)}\left(-5\dim s^5 t^6+24 s^6 t^5+58 s^5 t^6+24 s^4 t^7\right)}{(s+t)^3}+\frac{\dim s^5 t^6+16 s^7 t^4+48 s^6 t^5+62 s^5 t^6+48 s^4 t^7+16 s^3 t^8}{4\,(s+t)^5}$$

$$\Delta((2\;3\;4),\{0,1,1,1\})\to$$

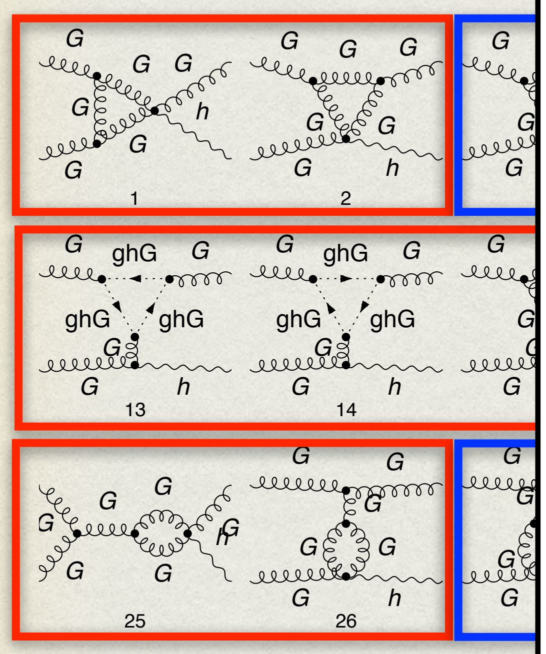
$$-\frac{1}{(s+t)^5}64\,(225\dim t^5 s^{13}-450 t^5 s^{13}+90\dim t^4 s^{13}-180 t^4 s^{13}+9\dim t^3 s^{13}-18 t^3 s^{13}+1200\dim t^6 s^{12}-2400 t^6 s^{12}+$$
$$780\dim t^5 s^{12}-1560 t^5 s^{12}+168\dim t^4 s^{12}-336 t^4 s^{12}+12\dim t^3 s^{12}-24 t^3 s^{12}+1030\dim t^7 s^{11}-$$
$$2060 t^7 s^{11}+1356\dim t^6 s^{11}-2712 t^6 s^{11}+535\dim t^5 s^{11}-1070 t^5 s^{11}+81\dim t^4 s^{11}-162 t^4 s^{11}+$$
$$4\dim t^3 s^{11}-8 t^3 s^{11}-1476\dim t^8 s^{10}+2952 t^8 s^{10}-1104\dim t^7 s^{10}+2208 t^7 s^{10}-160\dim t^6 s^{10}+$$
$$320 t^6 s^{10}+26\dim t^5 s^{10}-52 t^5 s^{10}+5\dim t^4 s^{10}-10 t^4 s^{10}+2241\dim t^9 s^9-4482 t^9 s^9+$$
$$570\dim t^8 s^9-1140 t^8 s^9-377\dim t^7 s^9+754 t^7 s^9-122\dim t^6 s^9+244 t^6 s^9-8\dim t^5 s^9+$$
$$16 t^5 s^9-324\dim t^{10} s^8+648 t^{10} s^8+1908\dim t^9 s^8-3816 t^9 s^8+864\dim t^8 s^8-1728 t^8 s^8+$$
$$60\dim t^7 s^8-120 t^7 s^8-6\dim t^6 s^8+12 t^6 s^8-432\dim t^{10} s^7+864 t^{10} s^7+489\dim t^9 s^7-978 t^9 s^7+$$
$$209\dim t^8 s^7-418 t^8 s^7+16\dim t^7 s^7-32 t^7 s^7-216\dim t^{10} s^6+432 t^{10} s^6+22\dim t^9 s^6-44 t^9 s^6+$$
$$13\dim t^8 s^6-26 t^8 s^6-48\dim t^{10} s^5+96 t^{10} s^5-4\dim t^9 s^5+8 t^9 s^5-4\dim t^{10} s^4+8 t^{10} s^4)\,x^4_{3,(2\;3\;4)}+$$

$$\Big(\frac{1}{(s+t)^5}512\,(225\dim t^5 s^{13}-450 t^5 s^{13}+90\dim t^4 s^{13}-180 t^4 s^{13}+9\dim t^3 s^{13}-18 t^3 s^{13}+180\dim t^6 s^{12}-$$
$$360 t^6 s^{12}+276\dim t^5 s^{12}-552 t^5 s^{12}+78\dim t^4 s^{12}-156 t^4 s^{12}+6\dim t^3 s^{12}-12 t^3 s^{12}-2352\dim t^7 s^{11}+$$
$$4704 t^7 s^{11}-1488\dim t^6 s^{11}+2976 t^6 s^{11}-350\dim t^5 s^{11}+700 t^5 s^{11}-38\dim t^4 s^{11}+76 t^4 s^{11}-2\dim t^3 s^{11}+$$
$$4 t^3 s^{11}+3420\dim t^8 s^{10}-6840 t^8 s^{10}+2040\dim t^7 s^{10}-4080 t^7 s^{10}+452\dim t^6 s^{10}-904 t^6 s^{10}+$$
$$44\dim t^5 s^{10}-88 t^5 s^{10}+2\dim t^4 s^{10}-4 t^4 s^{10}-5427\dim t^9 s^9+10\,854 t^9 s^9-1470\dim t^8 s^9+2940 t^8 s^9+$$
$$421\dim t^7 s^9-842 t^7 s^9+144\dim t^6 s^9-288 t^6 s^9+11\dim t^5 s^9-22 t^5 s^9+648\dim t^{10} s^8-1296 t^{10} s^8-$$
$$4212\dim t^9 s^8+8424 t^9 s^8-1710\dim t^8 s^8+3420 t^8 s^8-170\dim t^7 s^8+340 t^7 s^8-4\dim t^6 s^8+8 t^6 s^8-$$
$$486\dim t^{11} s^7+972 t^{11} s^7+324\dim t^{10} s^7-648 t^{10} s^7-1044\dim t^9 s^7+2088 t^9 s^7-310\dim t^8 s^7+$$
$$620 t^8 s^7-22\dim t^7 s^7+44 t^7 s^7-648\dim t^{11} s^6+1296 t^{11} s^6-108\dim t^{10} s^6+216 t^{10} s^6-96\dim t^9 s^6+$$
$$192 t^9 s^6-10\dim t^8 s^6+20 t^8 s^6-324\dim t^{11} s^5+648 t^{11} s^5-84\dim t^{10} s^5+168 t^{10} s^5-5\dim t^9 s^5+$$
$$10 t^9 s^5-72\dim t^{11} s^4+144 t^{11} s^4-12\dim t^{10} s^4+24 t^{10} s^4-6\dim t^{11} s^3+12 t^{11} s^3)\,x^2_{4,(2\;3\;4)}-\frac{1}{(s+t)^5}$$
$$(-720 t^4 s^{10}-144 t^3 s^{10}-104\dim t^5 s^9-3200 t^5 s^9-56\dim t^4 s^9-1088 t^4 s^9-11\dim t^3 s^9-86 t^3 s^9+$$
$$240\dim t^6 s^8-3584 t^6 s^8+144\dim t^5 s^8-1952 t^5 s^8+18\dim t^4 s^8-272 t^4 s^8-360\dim t^7 s^7+560 t^7 s^7-$$
$$40\dim t^6 s^7-16 t^6 s^7+29\dim t^5 s^7-66 t^5 s^7-1040 t^8 s^6-240\dim t^7 s^6+336 t^7 s^6-40\dim t^6 s^6+$$
$$248 t^6 s^6-1296 t^9 s^5-1520 t^8 s^5-40\dim t^7 s^5-108 t^7 s^5-864 t^9 s^4-380 t^8 s^4-144 t^9 s^3)\Big)\,x^2_{3,(2\;3\;4)}-$$

Input: rank 5 numerator

AIDA for EYM amplitudes

Input numerators



$$\frac{1}{2}\Big(2\,(\mathrm{sp}(q,\varepsilon_4)+\mathrm{sp}(\varepsilon_4,p_1)+\mathrm{sp}(\varepsilon_4,p_4))$$

$\mathrm{sp}(q,p_1)+$

$\Delta((1\ \ 2\ \ 3\ \ 4),\{1,1,1,1\})\to$

$$\frac{320\,x^4_{4,(1\ 2\ 3\ 4)}(\dim s^5 t^6 - 2 s^5 t^6)}{s+t} - \frac{8\,x^2_{4,(1\ 2\ 3\ 4)}(-5\dim s^5 t^6+24 s^6 t^5+58 s^5 t^6+24 s^4 t^7)}{(s+t)^3} + \frac{\dim s^5 t^6+16 s^7 t^4+48 s^6 t^5+62 s^5 t^6+48 s^4 t^7+16 s^3 t^8}{4\,(s+t)^5}$$

$\Delta((2\ \ 3\ \ 4),\{0,1,1,1\})\to$

$-\frac{1}{(s+t)^5}64\,(225\dim t^5 s^{13} - 450 t^5 s^{13} + 90\dim t^4 s^{13} - 180 t^4 s^{13} + 9\dim t^3 s^{13} - 18 t^3 s^{13} + 1200\dim t^6 s^{12} - 2400 t^6 s^{12} +$

$780\dim t^5 s^{12} - 1560 t^5 s^{12} + 168\dim t^4 s^{12} - 336 t^4 s^{12} + 12\dim t^3 s^{12} - 24 t^3 s^{12} + 1030\dim t^7 s^{11} -$

$2060\ t$ … $4\ \mathrm{dim}$ … $320\ t^6$ … $570\ \mathrm{di}$ … $16\ t^5\ s$ … $60\ \mathrm{dim}$ … $209\ \mathrm{di}$ … $13\ \mathrm{dim}$ …

$\Big(\frac{1}{(s+t)^5}512\,(225$ … 36 … 47 … 4 … 44 …

$\Delta^{\mathrm{int}}((1\ \ 2\ \ 3\ \ 4),\{1,1,1,1\})\to$

$$\frac{60(\dim-2)\,s^3\,t^4\,\lambda^4_{(1\ 2\ 3\ 4)}}{(\dim-3)(\dim-1)(s+t)^2} - \frac{2 s^3 t^4\,\lambda^2_{(1\ 2\ 3\ 4)}(5\dim s\,t-24 s^2-58 s\,t-24 t^2)}{(\dim-3)(s+t)^4} + \frac{\dim s^5 t^6+16 s^7 t^4+48 s^6 t^5+62 s^5 t^6+48 s^4 t^7+16 s^3 t^8}{4\,(s+t)^5}$$

$\Delta^{\mathrm{int}}((2\ \ 3\ \ 4),\{0,1,1,1\})\to$

$$-\frac{32 s^3 t^3\,\lambda^4_{(2\ 3\ 4)}}{\dim (s+t)^3} + \frac{\lambda^2_{(2\ 3\ 4)}(9\dim s^3 t^4-24 s^4 t^3-54 s^3 t^4+4 s^2 t^5+20 s\,t^6+4 t^7)}{(\dim-2)(s+t)^4} + \frac{-\dim s^5 t^5-16 s^7 t^3-48 s^6 t^4-62 s^5 t^5-48 s^4 t^6-16 s^3 t^7}{4\,(s+t)^5}$$

$\Delta^{\mathrm{int}}((1\ \ 3\ \ 4),\{1,0,1,1\})\to$

$$-\frac{32 s^2 t^4\,\lambda^4_{(1\ 3\ 4)}}{\dim (s+t)^3} + \frac{t^2\,\lambda^2_{(1\ 3\ 4)}(9\dim s^3 t^3+8 s^6+40 s^5 t+40 s^4 t^2-26 s^3 t^3-16 s^2 t^4)}{(\dim-2)(s+t)^4} + \frac{-\dim s^4 t^6-16 s^6 t^4-48 s^5 t^5-62 s^4 t^6-48 s^3 t^7-16 s^2 t^8}{4\,(s+t)^5}$$

$\Delta^{\mathrm{int}}((1\ \ 2\ \ 4),\{1,1,0,1\})\to$

$$-\frac{32 s^3 t^3\,\lambda^4_{(1\ 2\ 4)}}{\dim (s+t)^3} + \frac{\lambda^2_{(1\ 2\ 4)}(9\dim s^4 t^4-32 s^5 t^3-82 s^4 t^4-32 s^3 t^5)}{(\dim-2)(s+t)^4} + \frac{-\dim s^5 t^5-16 s^7 t^3-48 s^6 t^4-62 s^5 t^5-48 s^4 t^6-16 s^3 t^7}{4\,(s+t)^5}$$

$$\Delta^{\mathrm{int}}((1\ \ 2\ \ 3),\{1,1,1,0\})\to -\frac{32 s^2 t^4\,\lambda^4_{(1\ 2\ 3)}}{\dim (s+t)^3} + \frac{s^2 t^4\,\lambda^2_{(1\ 2\ 3)}(9\dim s\,t-32 s^2-82 s\,t-32 t^2)}{(\dim-2)(s+t)^4} + \frac{-\dim s^4 t^6-16 s^6 t^4-48 s^5 t^5-62 s^4 t^6-48 s^3 t^7-16 s^2 t^8}{4\,(s+t)^5}$$

$421\dim t^7 s^9 - 842 t^7 s^9 + 144\dim t^6 s^9 - 288 t^6 s^9 + 11\dim t^5 s^9 - 22 t^5 s^9 + 648\dim t^{10} s^8 - 1296 t^{10} s^8 -$

$4212\dim t^9 s^8 + 8424 t^9 s^8 - 1710\dim t^8 s^8 + 3420 t^8 s^8 - 170\dim t^7 s^8 + 340 t^7 s^8 - 4\dim t^6 s^8 + 8 t^6 s^8 -$

$486\dim t^{11} s^7 + 972 t^{11} s^7 + 324\dim t^{10} s^7 - 648 t^{10} s^7 - 1044\dim t^9 s^7 + 2088 t^9 s^7 - 310\dim t^8 s^7 +$

$620 t^8 s^7 - 22\dim t^7 s^7 + 44 t^7 s^7 - 648\dim t^{11} s^6 + 1296 t^{11} s^6 - 108\dim t^{10} s^6 + 216 t^{10} s^6 - 96\dim t^9 s^6 +$

$192 t^9 s^6 - 10\dim t^8 s^6 + 20 t^8 s^6 - 324\dim t^{11} s^5 + 648 t^{11} s^5 - 84\dim t^{10} s^5 + 168 t^{10} s^5 - 5\dim t^9 s^5 +$

$10 t^9 s^5 - 72\dim t^{11} s^4 + 144 t^{11} s^4 - 12\dim t^{10} s^4 + 24 t^{10} s^4 - 6\dim t^{11} s^3 + 12 t^{11} s^3\Big)\,x^4_{4,(2\ 3\ 4)} - \frac{1}{(s+t)^5}$

$(-720 t^4 s^{10} - 144 t^3 s^{10} - 104\dim t^5 s^9 - 3200 t^5 s^9 - 56\dim t^4 s^9 - 1088 t^4 s^9 - 11\dim t^3 s^9 - 86 t^3 s^9 +$

$240\dim t^6 s^8 - 3584 t^6 s^8 + 144\dim t^5 s^8 - 1952 t^5 s^8 + 18\dim t^4 s^8 - 272 t^4 s^8 - 360\dim t^7 s^7 + 560 t^7 s^7 -$

$40\dim t^6 s^7 - 16 t^6 s^7 + 29\dim t^5 s^7 - 66 t^5 s^7 - 1040 t^8 s^6 - 240\dim t^7 s^6 + 336 t^7 s^6 - 40\dim t^6 s^6 +$

$248 t^6 s^6 - 1296 t^9 s^5 - 1520 t^8 s^5 - 40\dim t^7 s^5 - 108 t^7 s^5 - 864 t^9 s^4 - 380 t^8 s^4 - 144 t^9 s^3)\,x^2_{3,(2\ 3\ 4)} -$

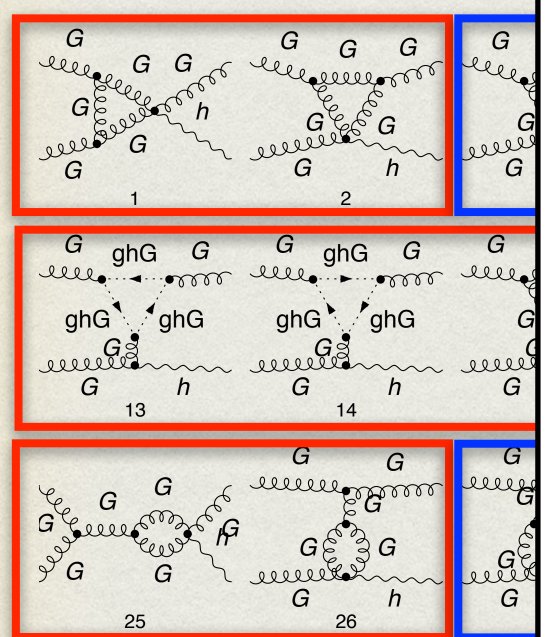
$\varepsilon_2)\,\mathrm{sp}(q,p_1)+$

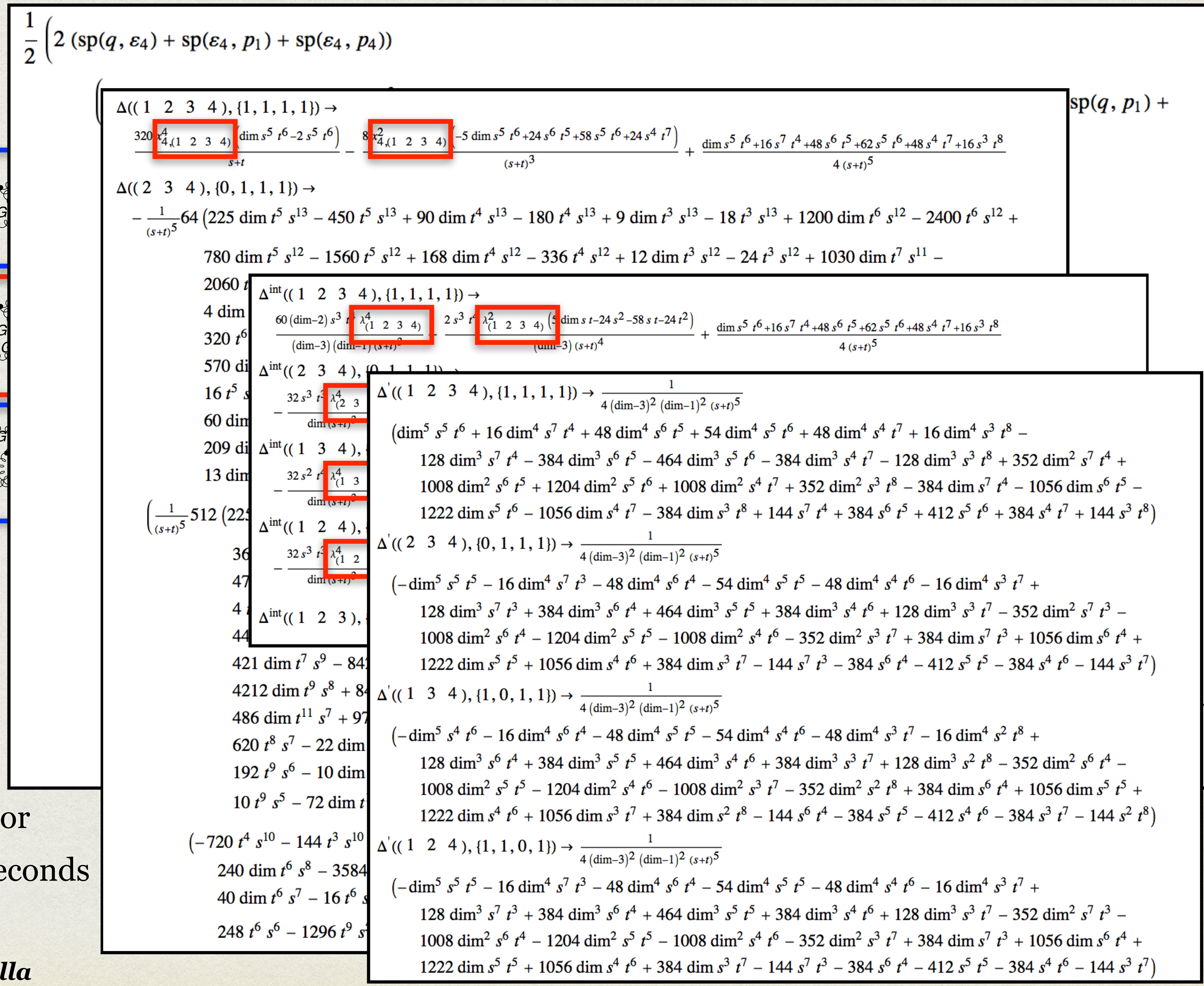
$(q,p_1)-$

Input: rank 5 numerator

AIDA for EYM amplitudes

Input numerators



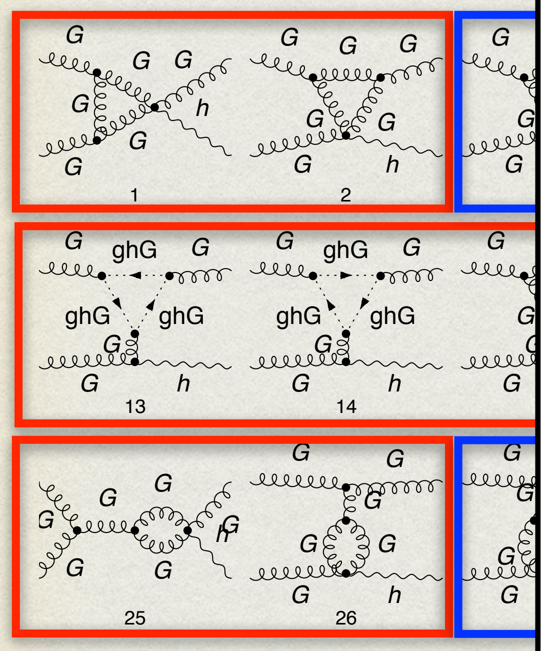


Input: rank 5 numerator

Reduction time ~ 30 seconds

AIDA for EYM amplitudes

Input numerators



$$\frac{1}{2}\left(2\,(\mathrm{sp}(q,\varepsilon_4)+\mathrm{sp}(\varepsilon_4,p_1)+\mathrm{sp}(\varepsilon_4,p_4))\right.$$

@ 1 Loop:: same result as TID

$$\Delta'((1\;2\;3\;4),\{1,1,1,1\})\to\frac{1}{4\,(\mathrm{dim}-3)^2\,(\mathrm{dim}-1)^2\,(s+t)^5}$$
$$(\mathrm{dim}^5 s^5 t^6+16\,\mathrm{dim}^4 s^7 t^4+48\,\mathrm{dim}^4 s^6 t^5+54\,\mathrm{dim}^4 s^5 t^6+48\,\mathrm{dim}^4 s^4 t^7+16\,\mathrm{dim}^4 s^3 t^8-128\,\mathrm{dim}^3 s^7 t^4-384\,\mathrm{dim}^3 s^6 t^5-464\,\mathrm{dim}^3 s^5 t^6-384\,\mathrm{dim}^3 s^4 t^7-128\,\mathrm{dim}^3 s^3 t^8+352\,\mathrm{dim}^2 s^7 t^4+1008\,\mathrm{dim}^2 s^6 t^5+1204\,\mathrm{dim}^2 s^5 t^6+1008\,\mathrm{dim}^2 s^4 t^7+352\,\mathrm{dim}^2 s^3 t^8-384\,\mathrm{dim}\, s^7 t^4-1056\,\mathrm{dim}\, s^6 t^5-1222\,\mathrm{dim}\, s^5 t^6-1056\,\mathrm{dim}\, s^4 t^7-384\,\mathrm{dim}\, s^3 t^8+144\, s^7 t^4+384\, s^6 t^5+412\, s^5 t^6+384\, s^4 t^7+144\, s^3 t^8)$$

$$\Delta'((2\;3\;4),\{0,1,1,1\})\to\frac{1}{4\,(\mathrm{dim}-3)^2\,(\mathrm{dim}-1)^2\,(s+t)^5}$$
$$(-\mathrm{dim}^5 s^5 t^5-16\,\mathrm{dim}^4 s^7 t^3-48\,\mathrm{dim}^4 s^6 t^4-54\,\mathrm{dim}^4 s^5 t^5-48\,\mathrm{dim}^4 s^4 t^6-16\,\mathrm{dim}^4 s^3 t^7+128\,\mathrm{dim}^3 s^7 t^3+384\,\mathrm{dim}^3 s^6 t^4+464\,\mathrm{dim}^3 s^5 t^5+384\,\mathrm{dim}^3 s^4 t^6+128\,\mathrm{dim}^3 s^3 t^7-352\,\mathrm{dim}^2 s^7 t^3-1008\,\mathrm{dim}^2 s^6 t^4-1204\,\mathrm{dim}^2 s^5 t^5-1008\,\mathrm{dim}^2 s^4 t^6-352\,\mathrm{dim}^2 s^3 t^7+384\,\mathrm{dim}\, s^7 t^3+1056\,\mathrm{dim}\, s^6 t^4+1222\,\mathrm{dim}\, s^5 t^5+1056\,\mathrm{dim}\, s^4 t^6+384\,\mathrm{dim}\, s^3 t^7-144\, s^7 t^3-384\, s^6 t^4-412\, s^5 t^5-384\, s^4 t^6-144\, s^3 t^7)$$

$$\Delta'((1\;3\;4),\{1,0,1,1\})\to\frac{1}{4\,(\mathrm{dim}-3)^2\,(\mathrm{dim}-1)^2\,(s+t)^5}$$
$$(-\mathrm{dim}^5 s^4 t^6-16\,\mathrm{dim}^4 s^6 t^4-48\,\mathrm{dim}^4 s^5 t^5-54\,\mathrm{dim}^4 s^4 t^6-48\,\mathrm{dim}^4 s^3 t^7-16\,\mathrm{dim}^4 s^2 t^8+128\,\mathrm{dim}^3 s^6 t^4+384\,\mathrm{dim}^3 s^5 t^5+464\,\mathrm{dim}^3 s^4 t^6+384\,\mathrm{dim}^3 s^3 t^7+128\,\mathrm{dim}^3 s^2 t^8-352\,\mathrm{dim}^2 s^6 t^4-1008\,\mathrm{dim}^2 s^5 t^5-1204\,\mathrm{dim}^2 s^4 t^6-1008\,\mathrm{dim}^2 s^3 t^7-352\,\mathrm{dim}^2 s^2 t^8+384\,\mathrm{dim}\, s^6 t^4+1056\,\mathrm{dim}\, s^5 t^5+1222\,\mathrm{dim}\, s^4 t^6+1056\,\mathrm{dim}\, s^3 t^7+384\,\mathrm{dim}\, s^2 t^8-144\, s^6 t^4-384\, s^5 t^5-412\, s^4 t^6-384\, s^3 t^7-144\, s^2 t^8)$$

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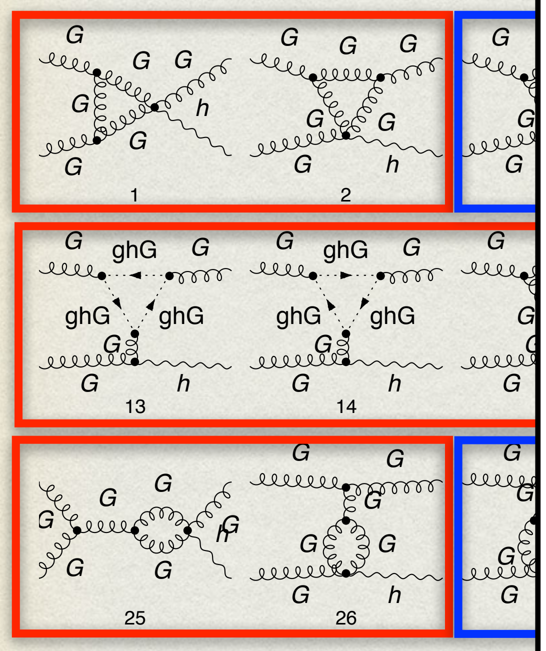
Input: rank 5 numerator

Reduction time ~ 30 seconds

AIDA for EYM amplitudes

Input numerators



$$\frac{1}{2}\Big(2\,(\mathrm{sp}(q,\varepsilon_4)+\mathrm{sp}(\varepsilon_4,p_1)+\mathrm{sp}(\varepsilon_4,p_4))$$

$\Delta((1\ 2\ 3\ 4),\{1,1,1,1\}) \to$

$\Delta((2\ 3\ 4),\{0,1,1,1\}) \to$

$\Delta^{\mathrm{int}}((1\ 2\ 3\ 4),\{1,1,1,1\}) \to$

$$\Delta'((1\ 2\ 3\ 4),\{1,1,1,1\}) \to \frac{1}{4\,(\mathrm{dim}-3)^2\,(\mathrm{dim}-1)^2\,(s+t)^5}$$
$$(\mathrm{dim}^5 s^5 t^6 + 16\,\mathrm{dim}^4 s^7 t^4 + 48\,\mathrm{dim}^4 s^6 t^5 + 54\,\mathrm{dim}^4 s^5 t^6 + 48\,\mathrm{dim}^4 s^4 t^7 + 16\,\mathrm{dim}^4 s^3 t^8 - 128\,\mathrm{dim}^3 s^7 t^4 - 384\,\mathrm{dim}^3 s^6 t^5 - 464\,\mathrm{dim}^3 s^5 t^6 - 384\,\mathrm{dim}^3 s^4 t^7 - 128\,\mathrm{dim}^3 s^3 t^8 + 352\,\mathrm{dim}^2 s^7 t^4 + 1008\,\mathrm{dim}^2 s^6 t^5 + 1204\,\mathrm{dim}^2 s^5 t^6 + 1008\,\mathrm{dim}^2 s^4 t^7 + 352\,\mathrm{dim}^2 s^3 t^8 - 384\,\mathrm{dim}\, s^7 t^4 - 1056\,\mathrm{dim}\, s^6 t^5 - 1222\,\mathrm{dim}\, s^5 t^6 - 1056\,\mathrm{dim}\, s^4 t^7 - 384\,\mathrm{dim}\, s^3 t^8 + 144\, s^7 t^4 + 384\, s^6 t^5 + 412\, s^5 t^6 + 384\, s^4 t^7 + 144\, s^3 t^8)$$

$$\Delta'((2\ 3\ 4),\{0,1,1,1\}) \to \frac{1}{4\,(\mathrm{dim}-3)^2\,(\mathrm{dim}-1)^2\,(s+t)^5}$$
$$(-\mathrm{dim}^5 s^5 t^5 - 16\,\mathrm{dim}^4 s^7 t^3 - 48\,\mathrm{dim}^4 s^6 t^4 - 54\,\mathrm{dim}^4 s^5 t^5 - 48\,\mathrm{dim}^4 s^4 t^6 - 16\,\mathrm{dim}^4 s^3 t^7 + 128\,\mathrm{dim}^3 s^7 t^3 + 384\,\mathrm{dim}^3 s^6 t^4 + 464\,\mathrm{dim}^3 s^5 t^5 + 384\,\mathrm{dim}^3 s^4 t^6 + 128\,\mathrm{dim}^3 s^3 t^7 - 352\,\mathrm{dim}^2 s^7 t^3 - 1008\,\mathrm{dim}^2 s^6 t^4 - 1204\,\mathrm{dim}^2 s^5 t^5 - 1008\,\mathrm{dim}^2 s^4 t^6 - 352\,\mathrm{dim}^2 s^3 t^7 + 384\,\mathrm{dim}\, s^7 t^3 + 1056\,\mathrm{dim}\, s^6 t^4 + 1222\,\mathrm{dim}\, s^5 t^5 + 1056\,\mathrm{dim}\, s^4 t^6 + 384\,\mathrm{dim}\, s^3 t^7 - 144\, s^7 t^3 - 384\, s^6 t^4 - 412\, s^5 t^5 - 384\, s^4 t^6 - 144\, s^3 t^7)$$

$$A_4^{(1)}(1^-,2^-,3^+;4^{++}) = A_4^{(0)} c_\Gamma \left(-\frac{\mu^2}{s}\right)^{\epsilon}\Bigg[-\frac{3}{\varepsilon^2}-\frac{11}{3\varepsilon}-\frac{1}{\varepsilon}\left(\log\left(\frac{-s}{-t}\right)+\log\left(\frac{-s}{s+t}\right)\right)$$
$$-\frac{11}{3}\log\left(\frac{-s}{-t}\right)+\frac{t\,(14s^2+9st+6t^2)}{3s^3}\log\left(\frac{-t}{s+t}\right)$$
$$+\left(\frac{t(s+t)\,(2s^2+st+t^2)}{s^4}+\frac{1}{2}\right)\left(\log^2\left(\frac{-t}{s+t}\right)+\pi^2\right)+\pi^2$$
$$+\frac{t(s+t)}{s^2}-\frac{64}{9}+\frac{\delta}{6}\Bigg].$$

$\delta=-2$ or $\delta=0$

tHV and FDH

@ 1 Loop:: same result as TID

Input: rank 5 numerator

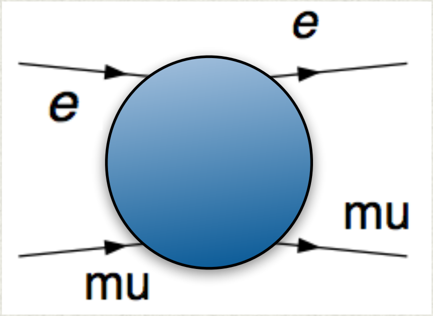
Reduction time ~ 30 seconds

AIDA for muon-electron scattering

- Recent proposal for the determination of the hadronic contribution to the muon from the measurement of muon-electron scattering $g-2$

[Carloni Calame, Passera, Trentadue, Venanzoni (2015)]
[Abbiendi, Carloni Calame, Marconi et al (2017)]

- In the massless electron limit, 4-point process depending on 3 scales



$$s = (p_1 + p_2)^2 \qquad t = (p_2 + p_3)^2$$
$$m_e^2 \simeq 0 \qquad u = -s - t + 2m^2$$

$$e(p_1) + \mu(p_4) \to e(-p_2) + \mu(-p_3)$$

- NNLO virtual contribution with adaptive integrand decomposition

[Ossola, Mastrolia, Peraro, Primo, Ronca, Schubert, W.J.T. (work in progress)]

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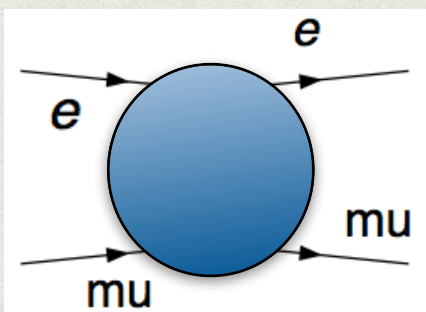

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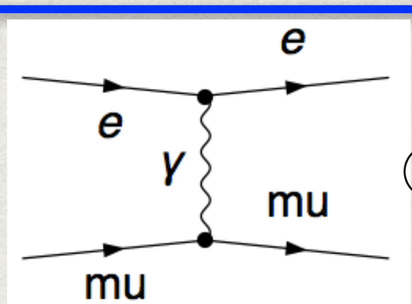

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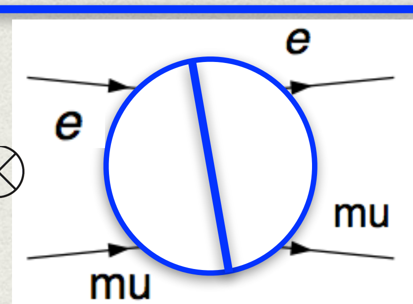


$$= \sum_{k=2}^{7} \sum_{i_1\cdots i_k} \frac{\Delta_{i_1\cdots i_k}(q_i)}{D_{i_1}\cdots D_{i_k}}$$

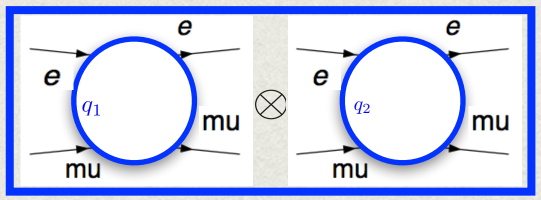
$$c_{i_1\cdots i_k}(s,t,m^2,d) \prod_{i,j} (q_i \cdot p_j)^{\alpha_{ij}}$$

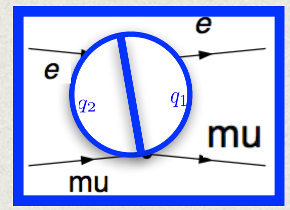
AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Two-loop features

- Different treatment for Factorised and non factorised topologies

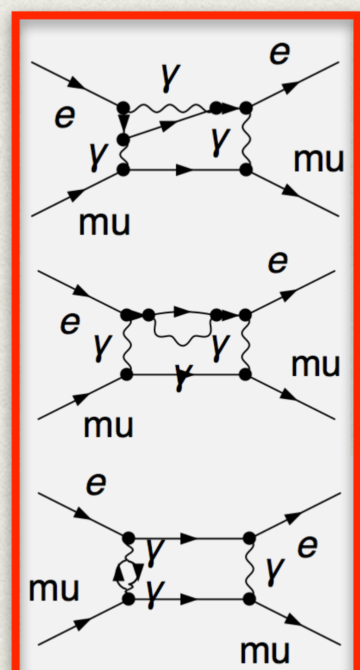




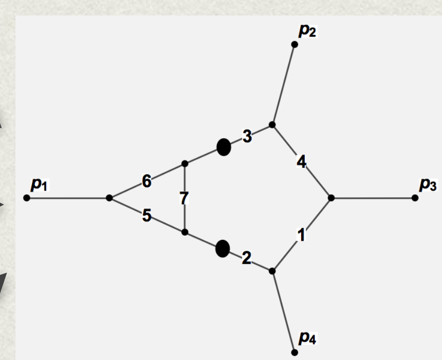
Independent   Same

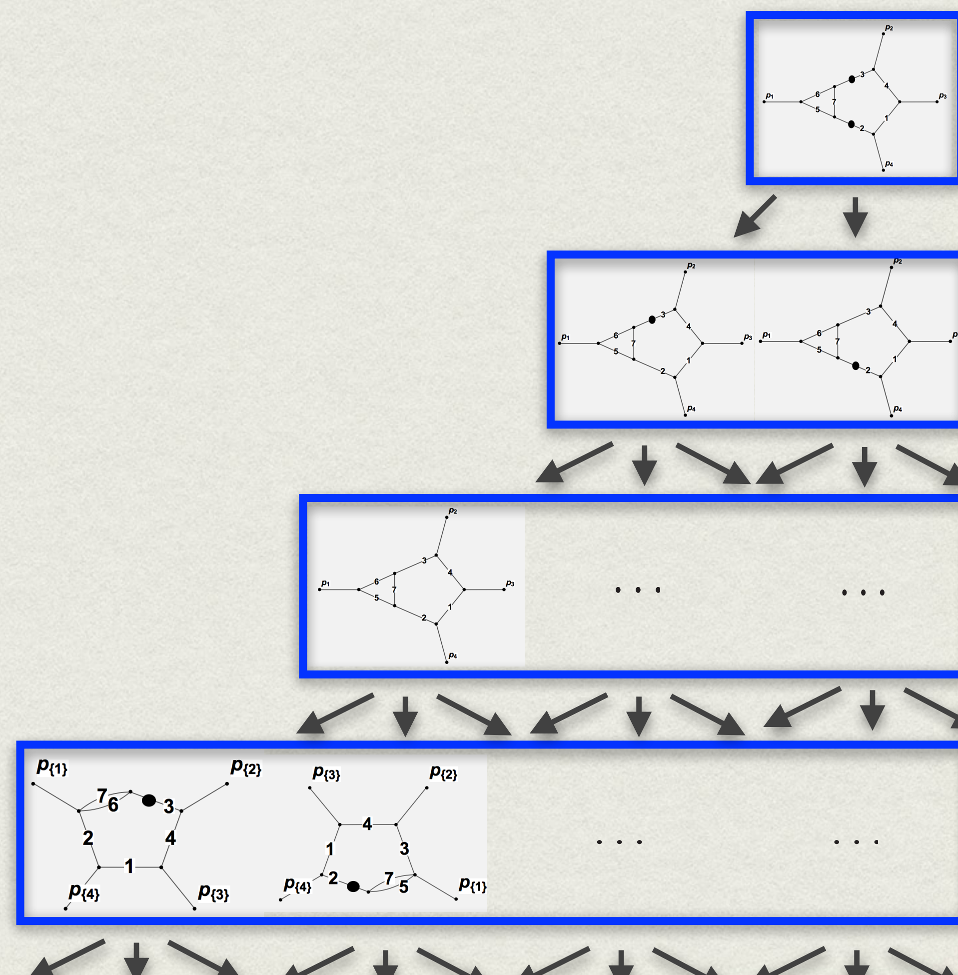
parametrisation for each loop

- Squared propagators affect Jobs organisation



Parent Topology

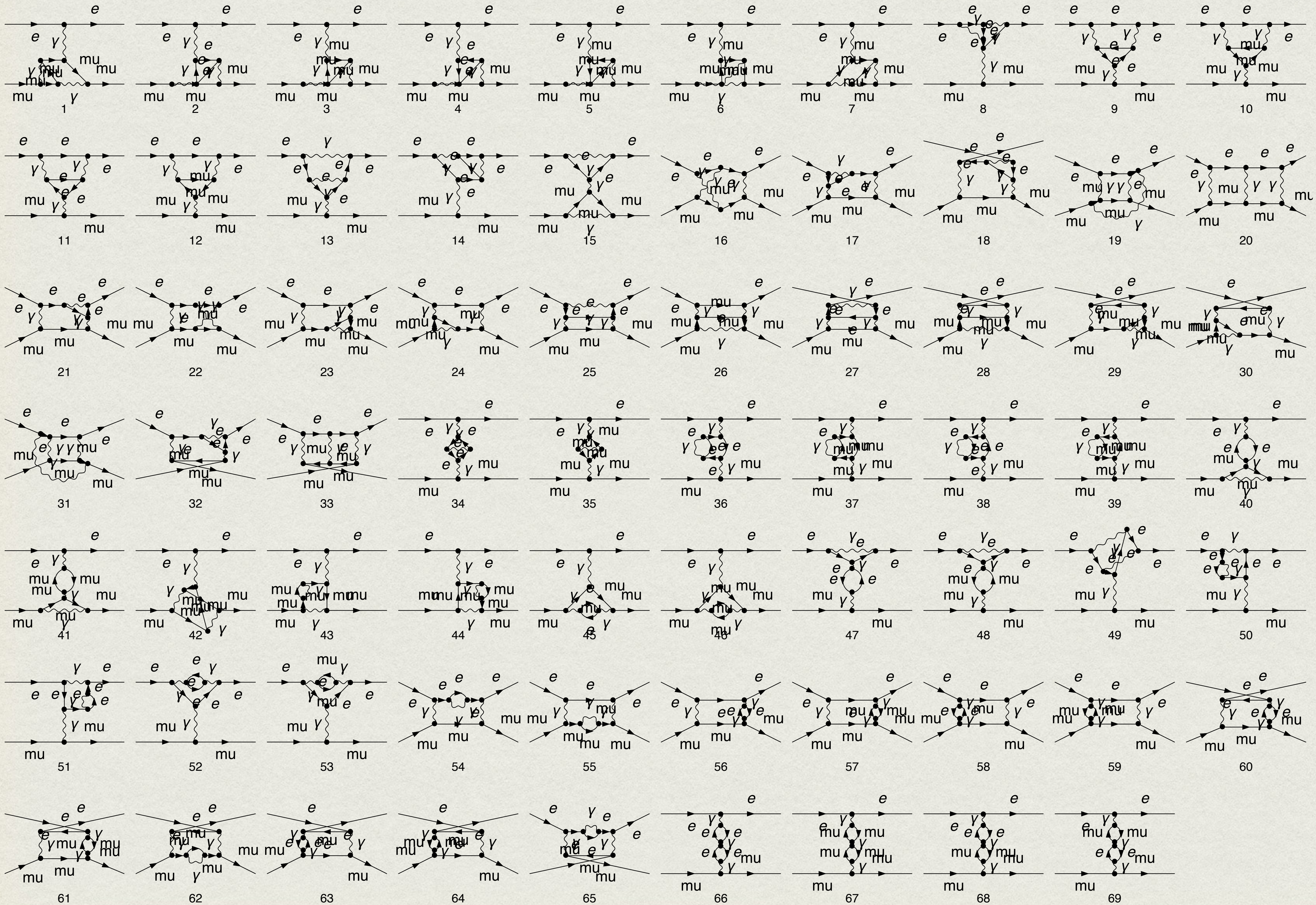




AIDA for muon-electron scattering

Two-loop preliminary results

- 69 Feynman diagrams identified
- 10 genuine 2 loop 4-point functions appear



William J. Torres Bobadilla

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

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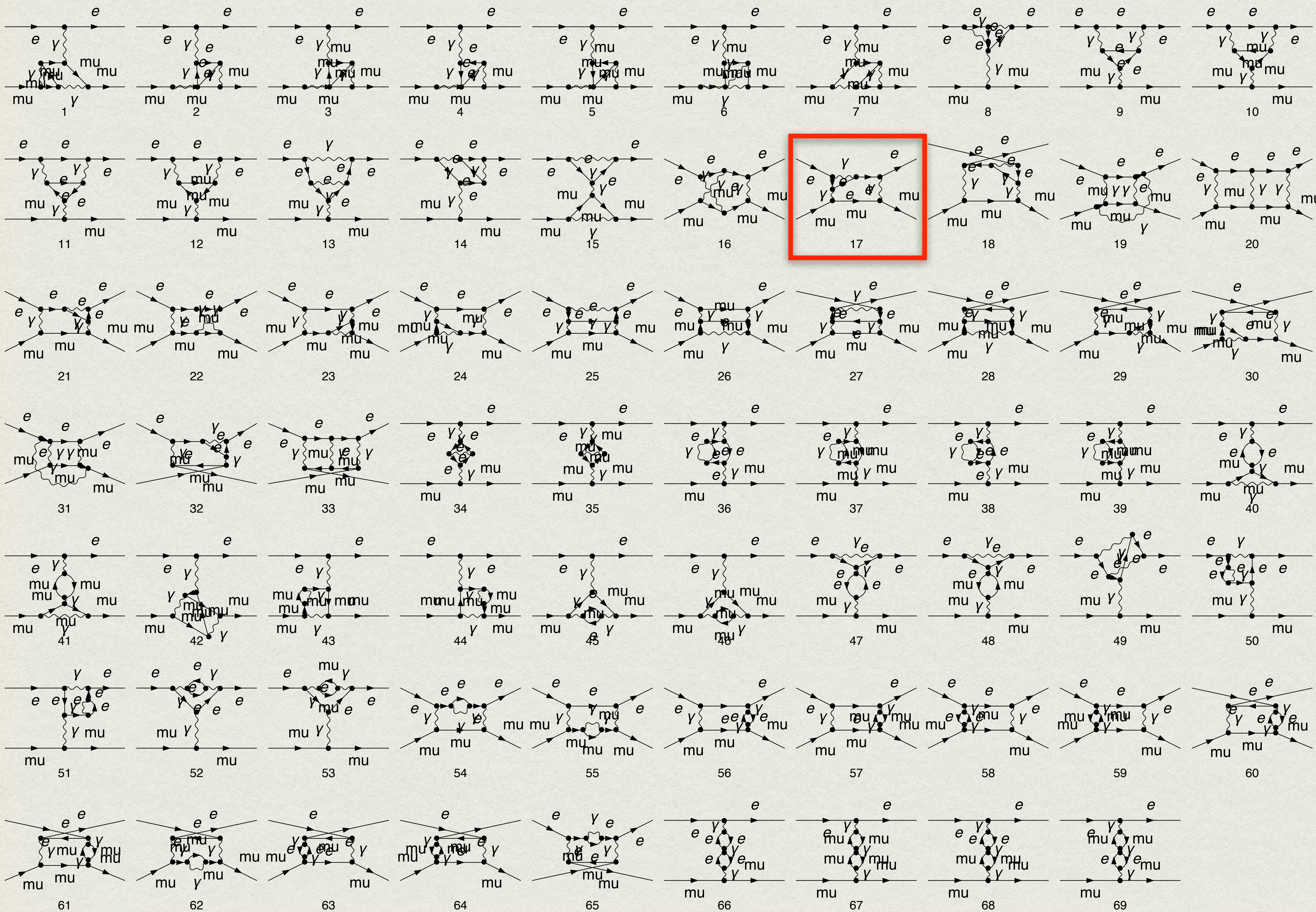

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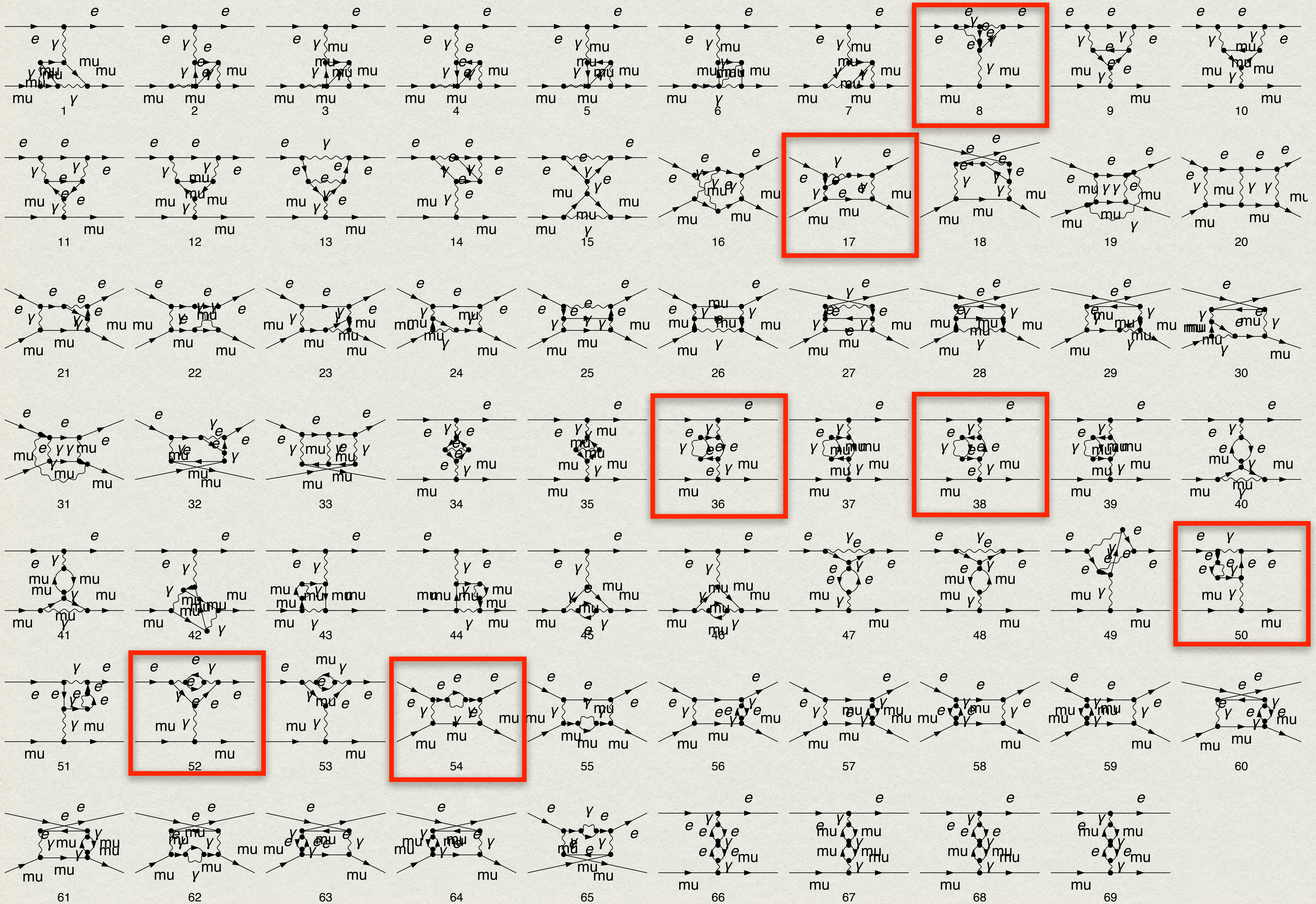

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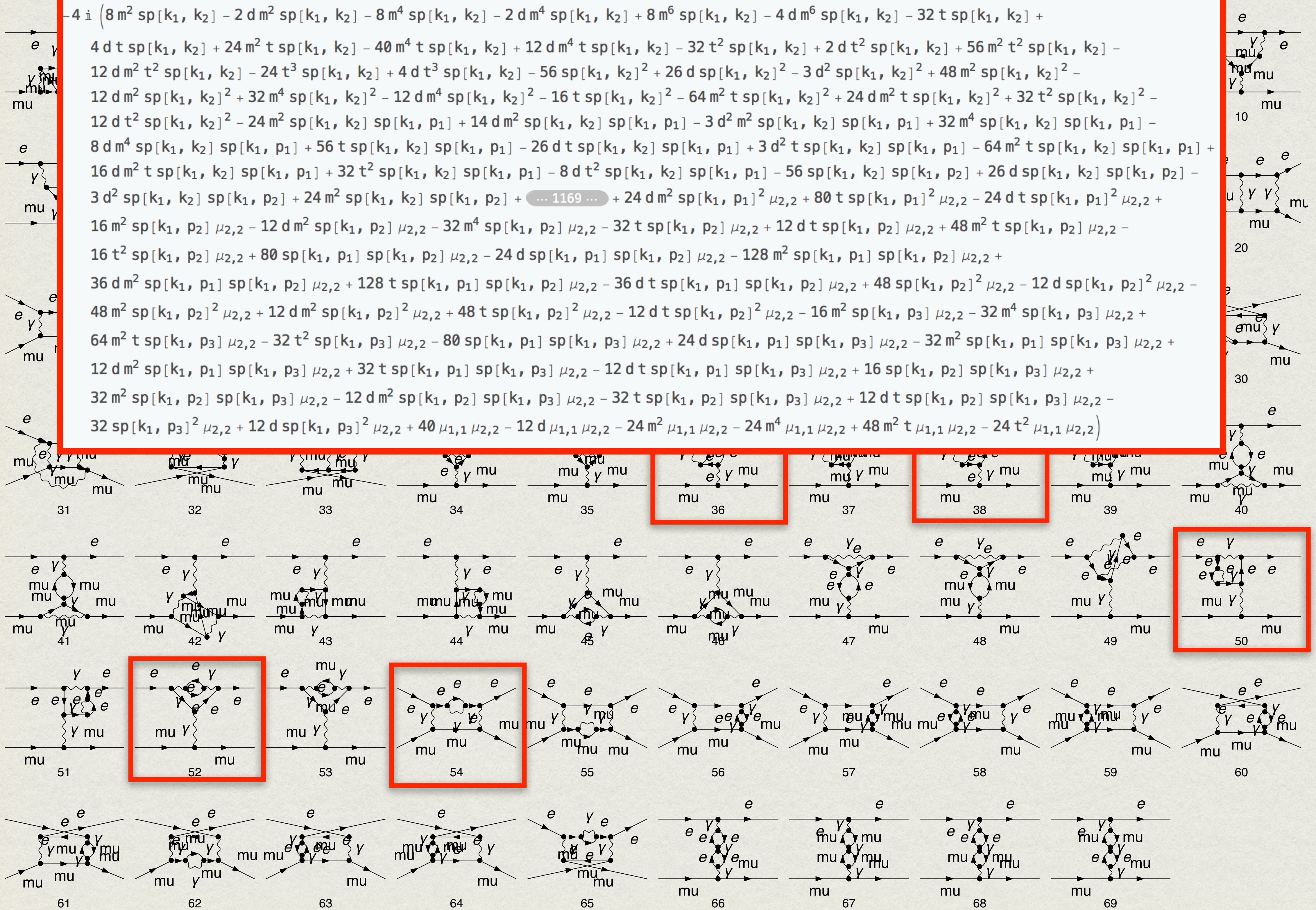

$$
\begin{aligned}
&-4\,i\,\big(8 m^2\,\mathrm{sp}[k_1,k_2] - 2 d m^2\,\mathrm{sp}[k_1,k_2] - 8 m^4\,\mathrm{sp}[k_1,k_2] - 2 d m^4\,\mathrm{sp}[k_1,k_2] + 8 m^6\,\mathrm{sp}[k_1,k_2] - 4 d m^6\,\mathrm{sp}[k_1,k_2] - 32 t\,\mathrm{sp}[k_1,k_2] + \\
&4 d t\,\mathrm{sp}[k_1,k_2] + 24 m^2 t\,\mathrm{sp}[k_1,k_2] - 40 m^4 t\,\mathrm{sp}[k_1,k_2] + 12 d m^4 t\,\mathrm{sp}[k_1,k_2] - 32 t^2\,\mathrm{sp}[k_1,k_2] + 2 d t^2\,\mathrm{sp}[k_1,k_2] + 56 m^2 t^2\,\mathrm{sp}[k_1,k_2] - \\
&12 d m^2 t^2\,\mathrm{sp}[k_1,k_2] - 24 t^3\,\mathrm{sp}[k_1,k_2] + 4 d t^3\,\mathrm{sp}[k_1,k_2] - 56\,\mathrm{sp}[k_1,k_2]^2 + 26 d\,\mathrm{sp}[k_1,k_2]^2 - 3 d^2\,\mathrm{sp}[k_1,k_2]^2 + 48 m^2\,\mathrm{sp}[k_1,k_2]^2 - \\
&12 d m^2\,\mathrm{sp}[k_1,k_2]^2 + 32 m^4\,\mathrm{sp}[k_1,k_2]^2 - 12 d m^4\,\mathrm{sp}[k_1,k_2]^2 - 16 t\,\mathrm{sp}[k_1,k_2]^2 - 64 m^2 t\,\mathrm{sp}[k_1,k_2]^2 + 24 d m^2 t\,\mathrm{sp}[k_1,k_2]^2 + 32 t^2\,\mathrm{sp}[k_1,k_2]^2 - \\
&12 d t^2\,\mathrm{sp}[k_1,k_2]^2 - 24 m^2\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] + 14 d m^2\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] - 3 d^2 m^2\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] + 32 m^4\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] - \\
&8 d m^4\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] + 56 t\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] - 26 d t\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] + 3 d^2 t\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] - 64 m^2 t\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] + \\
&16 d m^2 t\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] + 32 t^2\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] - 8 d t^2\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_1] - 56\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_2] + 26 d\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_2] - \\
&3 d^2\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_2] + 24 m^2\,\mathrm{sp}[k_1,k_2]\,\mathrm{sp}[k_1,p_2] + \ldots 1169 \ldots + 24 d m^2\,\mathrm{sp}[k_1,p_1]^2\,\mu_{2,2} + 80 t\,\mathrm{sp}[k_1,p_1]^2\,\mu_{2,2} - 24 d t\,\mathrm{sp}[k_1,p_1]^2\,\mu_{2,2} + \\
&16 m^2\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} - 12 d m^2\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} - 32 m^4\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} - 32 t\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} + 12 d t\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} + 48 m^2 t\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} - \\
&16 t^2\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} + 80\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} - 24 d\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} - 128 m^2\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} + \\
&36 d m^2\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} + 128 t\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} - 36 d t\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_2]\,\mu_{2,2} + 48\,\mathrm{sp}[k_1,p_2]^2\,\mu_{2,2} - 12 d\,\mathrm{sp}[k_1,p_2]^2\,\mu_{2,2} - \\
&48 m^2\,\mathrm{sp}[k_1,p_2]^2\,\mu_{2,2} + 12 d m^2\,\mathrm{sp}[k_1,p_2]^2\,\mu_{2,2} + 48 t\,\mathrm{sp}[k_1,p_2]^2\,\mu_{2,2} - 12 d t\,\mathrm{sp}[k_1,p_2]^2\,\mu_{2,2} - 16 m^2\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - 32 m^4\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} + \\
&64 m^2 t\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - 32 t^2\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - 80\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} + 24 d\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - 32 m^2\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} + \\
&12 d m^2\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} + 32 t\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - 12 d t\,\mathrm{sp}[k_1,p_1]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} + 16\,\mathrm{sp}[k_1,p_2]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} + \\
&32 m^2\,\mathrm{sp}[k_1,p_2]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - 12 d m^2\,\mathrm{sp}[k_1,p_2]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - 32 t\,\mathrm{sp}[k_1,p_2]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} + 12 d t\,\mathrm{sp}[k_1,p_2]\,\mathrm{sp}[k_1,p_3]\,\mu_{2,2} - \\
&32\,\mathrm{sp}[k_1,p_3]^2\,\mu_{2,2} + 12 d\,\mathrm{sp}[k_1,p_3]^2\,\mu_{2,2} + 40\,\mu_{1,1}\,\mu_{2,2} - 12 d\,\mu_{1,1}\,\mu_{2,2} - 24 m^2\,\mu_{1,1}\,\mu_{2,2} - 24 m^4\,\mu_{1,1}\,\mu_{2,2} + 48 m^2 t\,\mu_{1,1}\,\mu_{2,2} - 24 t^2\,\mu_{1,1}\,\mu_{2,2}\big)
\end{aligned}
$$

William J. Torres Bobadilla

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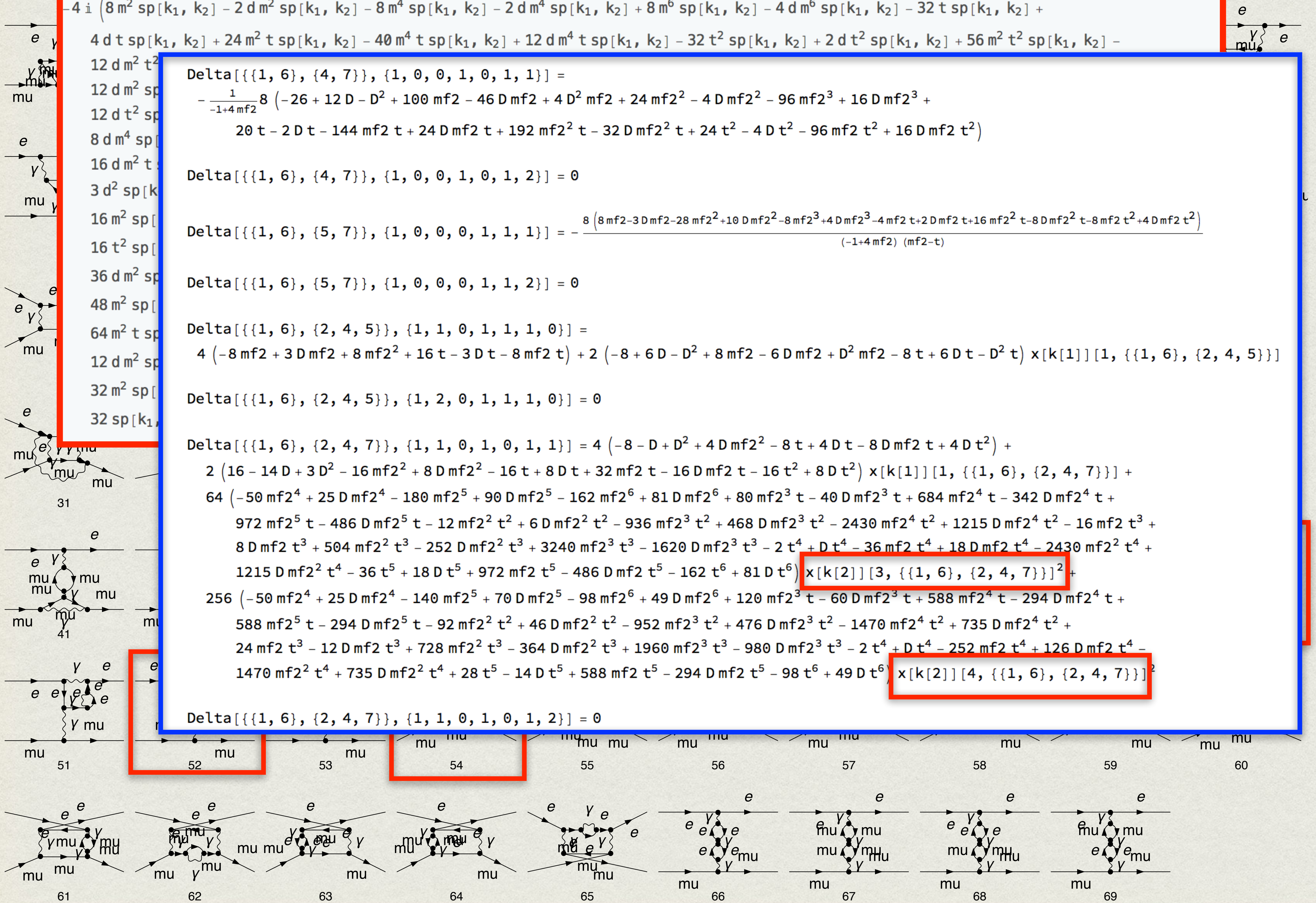


```
-4 i (8 m^2 sp[k1, k2] - 2 d m^2 sp[k1, k2] - 8 m^4 sp[k1, k2] - 2 d m^4 sp[k1, k2] + 8 m^6 sp[k1, k2] - 4 d m^6 sp[k1, k2] - 32 t sp[k1, k2] +
  4 d t sp[k1, k2] + 24 m^2 t sp[k1, k2] - 40 m^4 t sp[k1, k2] + 12 d m^4 t sp[k1, k2] - 32 t^2 sp[k1, k2] + 2 d t^2 sp[k1, k2] + 56 m^2 t^2 sp[k1, k2] -
  12 d m^2 t^2
  12 d m^2 sp
  12 d t^2 sp
  8 d m^4 sp[
  16 d m^2 t
  3 d^2 sp[k
  16 m^2 sp[
  16 t^2 sp[
  36 d m^2 sp
  48 m^2 sp[
  64 m^2 t sp
  12 d m^2 sp
  32 m^2 sp[
  32 sp[k1,
```

```
Delta[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 1}] =
 - 1/(-1+4 mf2) 8 (-26 + 12 D - D^2 + 100 mf2 - 46 D mf2 + 4 D^2 mf2 + 24 mf2^2 - 4 D mf2^2 - 96 mf2^3 + 16 D mf2^3 +
     20 t - 2 D t - 144 mf2 t + 24 D mf2 t + 192 mf2^2 t - 32 D mf2^2 t + 24 t^2 - 4 D t^2 - 96 mf2 t^2 + 16 D mf2 t^2)

Delta[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 2}] = 0

Delta[{{1, 6}, {5, 7}}, {1, 0, 0, 0, 1, 1, 1}] = - 8 (8 mf2-3 D mf2-28 mf2^2+10 D mf2^2-8 mf2^3+4 D mf2^3-4 mf2 t+2 D mf2 t+16 mf2^2 t-8 D mf2^2 t-8 mf2 t^2+4 D mf2 t^2) / ((-1+4 mf2) (mf2-t))

Delta[{{1, 6}, {5, 7}}, {1, 0, 0, 0, 1, 1, 2}] = 0

Delta[{{1, 6}, {2, 4, 5}}, {1, 1, 0, 1, 1, 1, 0}] =
 4 (-8 mf2 + 3 D mf2 + 8 mf2^2 + 16 t - 3 D t - 8 mf2 t) + 2 (-8 + 6 D - D^2 + 8 mf2 - 6 D mf2 + D^2 mf2 - 8 t + 6 D t - D^2 t) x[k[1]][1, {{1, 6}, {2, 4, 5}}]

Delta[{{1, 6}, {2, 4, 5}}, {1, 2, 0, 1, 1, 1, 0}] = 0

Delta[{{1, 6}, {2, 4, 7}}, {1, 1, 0, 1, 0, 1, 1}] = 4 (-8 - D + D^2 + 4 D mf2^2 - 8 t + 4 D t - 8 D mf2 t + 4 D t^2) +
  2 (16 - 14 D + 3 D^2 - 16 mf2^2 + 8 D mf2^2 - 16 t + 8 D t + 32 mf2 t - 16 D mf2 t - 16 t^2 + 8 D t^2) x[k[1]][1, {{1, 6}, {2, 4, 7}}] +
  64 (-50 mf2^4 + 25 D mf2^4 - 180 mf2^5 + 90 D mf2^5 - 162 mf2^6 + 81 D mf2^6 + 80 mf2^3 t - 40 D mf2^3 t + 684 mf2^4 t - 342 D mf2^4 t +
     972 mf2^5 t - 486 D mf2^5 t - 12 mf2^2 t^2 + 6 D mf2^2 t^2 - 936 mf2^3 t^2 + 468 D mf2^3 t^2 - 2430 mf2^4 t^2 + 1215 D mf2^4 t^2 - 16 mf2 t^3 +
     8 D mf2 t^3 + 504 mf2^2 t^3 - 252 D mf2^2 t^3 + 3240 mf2^3 t^3 - 1620 D mf2^3 t^3 - 2 t^4 + D t^4 - 36 mf2 t^4 + 18 D mf2 t^4 - 2430 mf2^2 t^4 +
     1215 D mf2^2 t^4 - 36 t^5 + 18 D t^5 + 972 mf2 t^5 - 486 D mf2 t^5 - 162 t^6 + 81 D t^6) x[k[2]][3, {{1, 6}, {2, 4, 7}}]^2 +
  256 (-50 mf2^4 + 25 D mf2^4 - 140 mf2^5 + 70 D mf2^5 - 98 mf2^6 + 49 D mf2^6 + 120 mf2^3 t - 60 D mf2^3 t + 588 mf2^4 t - 294 D mf2^4 t +
     588 mf2^5 t - 294 D mf2^5 t - 92 mf2^2 t^2 + 46 D mf2^2 t^2 - 952 mf2^3 t^2 + 476 D mf2^3 t^2 - 1470 mf2^4 t^2 + 735 D mf2^4 t^2 +
     24 mf2 t^3 - 12 D mf2 t^3 + 728 mf2^2 t^3 - 364 D mf2^2 t^3 + 1960 mf2^3 t^3 - 980 D mf2^3 t^3 - 2 t^4 + D t^4 - 252 mf2 t^4 + 126 D mf2 t^4 -
     1470 mf2^2 t^4 + 735 D mf2^2 t^4 + 28 t^5 - 14 D t^5 + 588 mf2 t^5 - 294 D mf2 t^5 - 98 t^6 + 49 D t^6) x[k[2]][4, {{1, 6}, {2, 4, 7}}]^2

Delta[{{1, 6}, {2, 4, 7}}, {1, 1, 0, 1, 0, 1, 2}] = 0
```


[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

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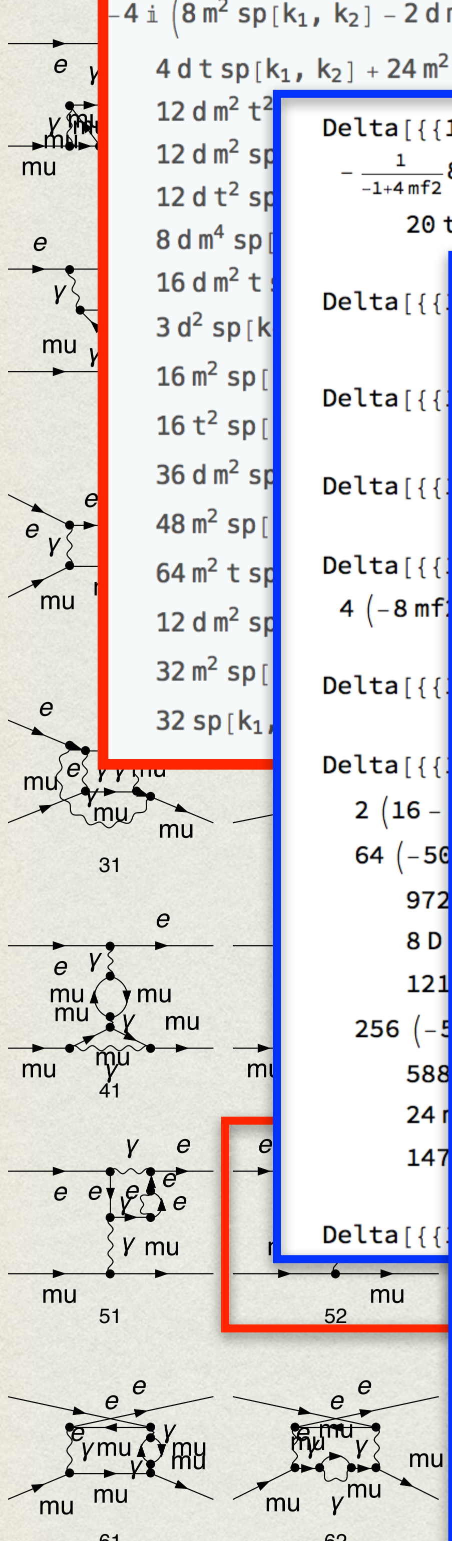

```
-4 i (8 m^2 sp[k1, k2] - 2 d m^2 sp[k1, k2] - 8 m^4 sp[k1, k2] - 2 d m^4 sp[k1, k2] + 8 m^6 sp[k1, k2] - 4 d m^6 sp[k1, k2] - 32 t sp[k1, k2] +
  4 d t sp[k1, k2] + 24 m^2 t sp[k1, k2] - 40 m^4 t sp[k1, k2] + 12 d m^4 t sp[k1, k2] - 32 t^2 sp[k1, k2] + 2 d t^2 sp[k1, k2] + 56 m^2 t^2 sp[k1, k2] -
```

```
Delta[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 1}] =
 - 1/(-1+4 mf2) 8 (-26 + 12 D - D^2 + 100 mf2 - 46 D mf2 + 4 D^2 mf2 + 24 mf2^2 - 4 D mf2^2 - 96 mf2^3 + 16 D mf2^3 +
    20 t - 2 D t - 144 mf2 t + 24 D mf2 t + 192 mf2^2 t - 32 D mf2^2 t + 24 t^2 - 4 D t^2 - 96 mf2 t^2 + 16 D mf2 t^2)
```

```
Deltaint[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 1}] =
 - 1/(-1+4 mf2) 8 (-26 + 12 D - D^2 + 100 mf2 - 46 D mf2 + 4 D^2 mf2 + 24 mf2^2 - 4 D mf2^2 - 96 mf2^3 + 16 D mf2^3 +
    20 t - 2 D t - 144 mf2 t + 24 D mf2 t + 192 mf2^2 t - 32 D mf2^2 t + 24 t^2 - 4 D t^2 - 96 mf2 t^2 + 16 D mf2 t^2)

Deltaint[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 2}] = 0

Deltaint[{{1, 6}, {5, 7}}, {1, 0, 0, 0, 1, 1, 1}] = - 8 (8 mf2 - 3 D mf2 - 28 mf2^2 + 10 D mf2^2 - 8 mf2^3 + 4 D mf2^3 - 4 mf2 t + 2 D mf2 t + 16 mf2^2 t - 8 D mf2^2 t - 8 mf2 t^2 + 4 D mf2 t^2) / ((-1+4 mf2) (mf2-t))

Deltaint[{{1, 6}, {5, 7}}, {1, 0, 0, 0, 1, 1, 2}] = 0

Deltaint[{{1, 6}, {2, 4, 5}}, {1, 1, 0, 1, 1, 1, 0}] =
 4 (-8 mf2 + 3 D mf2 + 8 mf2^2 + 16 t - 3 D t - 8 mf2 t) + 2 (-8 + 6 D - D^2 + 8 mf2 - 6 D mf2 + D^2 mf2 - 8 t + 6 D t - D^2 t) x[k[1]][1, {{1, 6}, {2, 4, 5}}]

Deltaint[{{1, 6}, {2, 4, 5}}, {1, 2, 0, 1, 1, 1, 0}] = 0

Deltaint[{{1, 6}, {2, 4, 7}}, {1, 1, 0, 1, 0, 1, 1}] = 4 (-8 - D + D^2 + 4 D mf2^2 - 8 t + 4 D t - 8 D mf2 t + 4 D t^2) +
 (64 (-50 mf2^4 + 25 D mf2^4 - 180 mf2^5 + 90 D mf2^5 - 162 mf2^6 + 81 D mf2^6 + 80 mf2^3 t - 40 D mf2^3 t + 684 mf2^4 t - 342 D mf2^4 t + 972 mf2^5 t -
      486 D mf2^5 t - 12 mf2^2 t^2 + 6 D mf2^2 t^2 - 936 mf2^3 t^2 + 468 D mf2^3 t^2 - 2430 mf2^4 t^2 + 1215 D mf2^4 t^2 - 16 mf2 t^3 + 8 D mf2 t^3 +
      504 mf2^2 t^3 - 252 D mf2^2 t^3 + 3240 mf2^3 t^3 - 1620 D mf2^3 t^3 - 2 t^4 + D t^4 - 36 mf2 t^4 + 18 D mf2 t^4 - 2430 mf2^2 t^4 + 1215 D mf2^2 t^4 -
      36 t^5 + 18 D t^5 + 972 mf2 t^5 - 486 D mf2 t^5 - 162 t^6 + 81 D t^6) (d[2] + d[2]^2 - d[2] d[4] - d[2] d[7] + d[4] d[7])) /
  ((-2 + D) ((-5 mf2 - 9 mf2^2 + 18 mf2 t - 9 t^2)^2 - (-5 mf2 - 7 mf2^2 + 14 mf2 t - 7 t^2)^2)) +
 (256 (-50 mf2^4 + 25 D mf2^4 - 140 mf2^5 + 70 D mf2^5 - 98 mf2^6 + 49 D mf2^6 + 120 mf2^3 t - 60 D mf2^3 t + 588 mf2^4 t - 294 D mf2^4 t +
      588 mf2^5 t - 294 D mf2^5 t - 92 mf2^2 t^2 + 46 D mf2^2 t^2 - 952 mf2^3 t^2 + 476 D mf2^3 t^2 - 1470 mf2^4 t^2 + 735 D mf2^4 t^2 + 24 mf2 t^3 -
      12 D mf2 t^3 + 728 mf2^2 t^3 - 364 D mf2^2 t^3 + 1960 mf2^3 t^3 - 980 D mf2^3 t^3 - 2 t^4 + D t^4 - 252 mf2 t^4 + 126 D mf2 t^4 - 1470 mf2^2 t^4 +
      735 D mf2^2 t^4 + 28 t^5 - 14 D t^5 + 588 mf2 t^5 - 294 D mf2 t^5 - 98 t^6 + 49 D t^6) (d[2] + d[2]^2 - d[2] d[4] - d[2] d[7] + d[4] d[7])) /
  ((-2 + D) (4 (5 mf2 + 7 mf2^2 - 14 mf2 t + 7 t^2)^2 - 4 (5 mf2 + 9 mf2^2 - 18 mf2 t + 9 t^2)^2)) +
 2 (16 - 14 D + 3 D^2 - 16 mf2^2 + 8 D mf2^2 - 16 t + 8 D t + 32 mf2 t - 16 D mf2 t - 16 t^2 + 8 D t^2)
  x[k[1]][1, {{1, 6}, {2, 4, 7}}]
```


[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

AIDA for muon-electron scattering

Two-loop preliminary results

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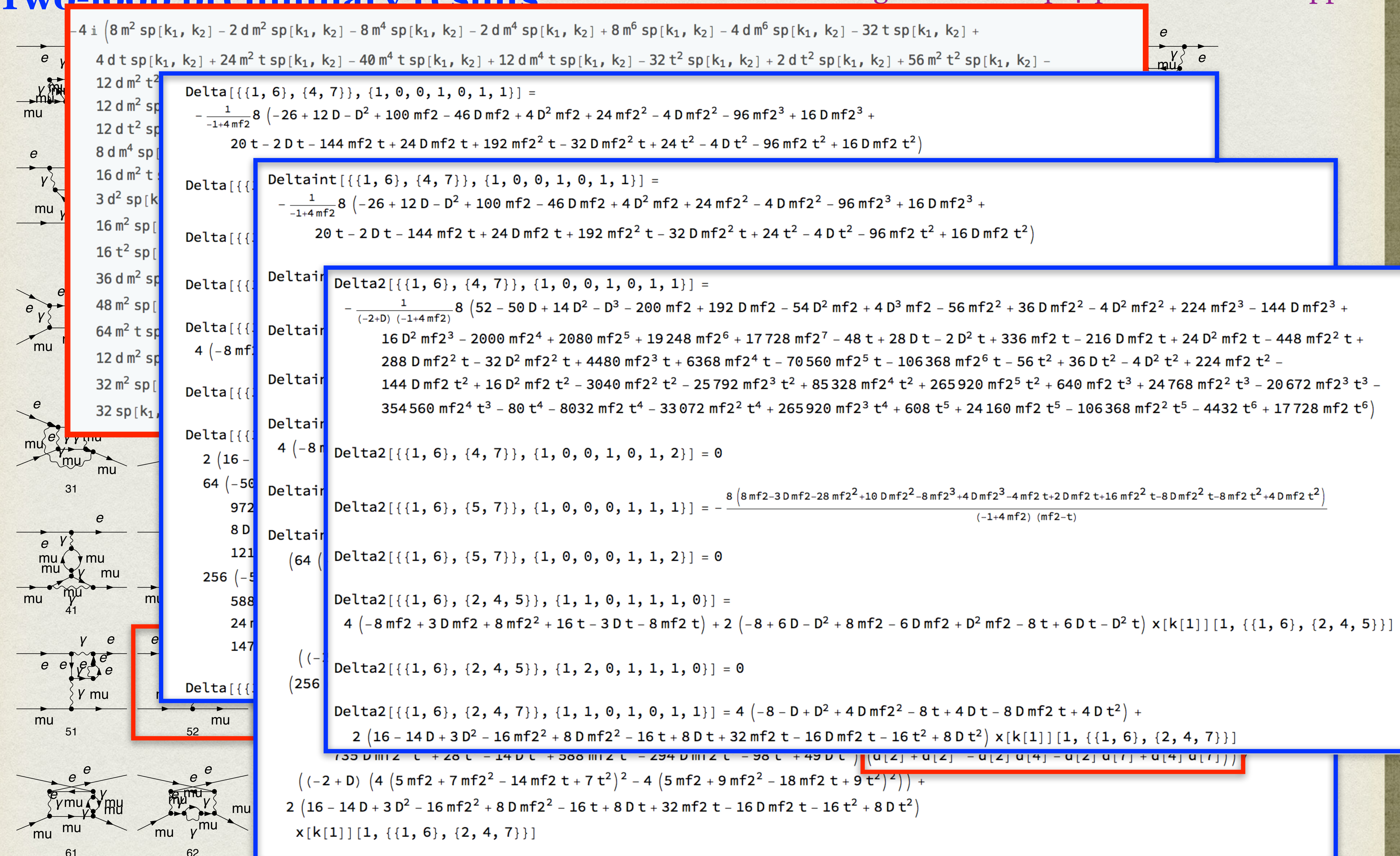

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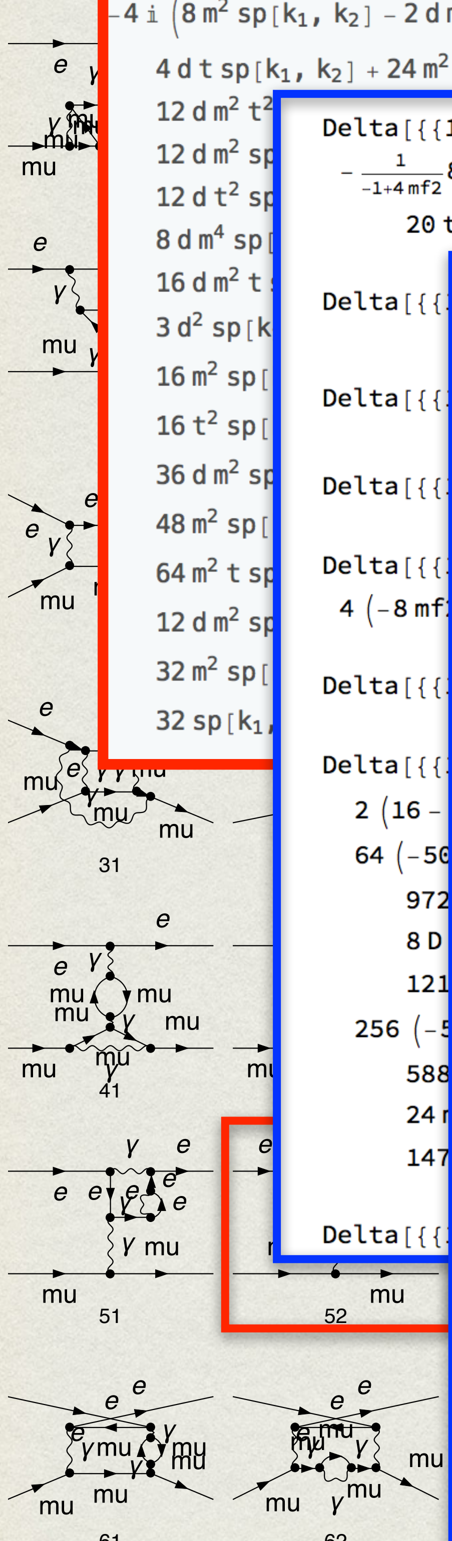


```
-4 i (8 m^2 sp[k1, k2] - 2 d m^2 sp[k1, k2] - 8 m^4 sp[k1, k2] - 2 d m^4 sp[k1, k2] + 8 m^6 sp[k1, k2] - 4 d m^6 sp[k1, k2] - 32 t sp[k1, k2] +
  4 d t sp[k1, k2] + 24 m^2 t sp[k1, k2] - 40 m^4 t sp[k1, k2] + 12 d m^4 t sp[k1, k2] - 32 t^2 sp[k1, k2] + 2 ...
```

```
Delta[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 1}] =
 - 1/(-1+4 mf2) 8 (-26 + 12 D - D^2 + 100 mf2 - 46 D mf2 + 4 D^2 mf2 + 24 mf2^2 - 4 D mf2^2 - 96 mf2^3 ...
   20 t - 2 D t - 144 mf2 t + 24 D mf2 t + 192 mf2^2 t - 32 D mf2^2 t + 24 t^2 - 4 D t^2 - 96 m...
```

Input: rank 4 numerator with 289 monomials

Reduction time ~ 1.5 mins

Output: 135 contributions

```
Deltaint[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 1}] =
 - 1/(-1+4 mf2) 8 (-26 + 12 D - D^2 + 100 mf2 - 46 D mf2 + 4 D^2 mf2 + 24 mf2^2 - 4 D mf2^2 - 96 mf2^3 + 16 D mf2^3 +
   20 t - 2 D t - 144 mf2 t + 24 D mf2 t + 192 mf2^2 t - 32 D mf2^2 t + 24 t^2 - 4 D t^2 - 96 mf2 t^2 + 16 D mf2 t^2)
```

```
Delta2[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 1}] =
 - 1/((-2+D) (-1+4 mf2)) 8 (52 - 50 D + 14 D^2 - D^3 - 200 mf2 + 192 D mf2 - 54 D^2 mf2 + 4 D^3 mf2 - 56 mf2^2 + 36 D mf2^2 - 4 D^2 mf2^2 + 224 mf2^3 - 144 D mf2^3 +
   16 D^2 mf2^3 - 2000 mf2^4 + 2080 mf2^5 + 19248 mf2^6 + 17728 mf2^7 - 48 t + 28 D t - 2 D^2 t + 336 mf2 t - 216 D mf2 t + 24 D^2 mf2 t - 448 mf2^2 t +
   288 D mf2^2 t - 32 D^2 mf2^2 t + 4480 mf2^3 t + 6368 mf2^4 t - 70560 mf2^5 t - 106368 mf2^6 t - 56 t^2 + 36 D t^2 - 4 D^2 t^2 + 224 mf2 t^2 -
   144 D mf2 t^2 + 16 D^2 mf2 t^2 - 3040 mf2^2 t^2 - 25792 mf2^3 t^2 + 85328 mf2^4 t^2 + 265920 mf2^5 t^2 + 640 mf2 t^3 + 24768 mf2^2 t^3 - 20672 mf2^3 t^3 -
   354560 mf2^4 t^3 - 80 t^4 - 8032 mf2 t^4 - 33072 mf2^2 t^4 + 265920 mf2^3 t^4 + 608 t^5 + 24160 mf2 t^5 - 106368 mf2^2 t^5 - 4432 t^6 + 17728 mf2 t^6)

Delta2[{{1, 6}, {4, 7}}, {1, 0, 0, 1, 0, 1, 2}] = 0

Delta2[{{1, 6}, {5, 7}}, {1, 0, 0, 0, 1, 1, 1}] = - 8 (8 mf2 - 3 D mf2 - 28 mf2^2 + 10 D mf2^2 - 8 mf2^3 + 4 D mf2^3 - 4 mf2 t + 2 D mf2 t + 16 mf2^2 t - 8 D mf2^2 t - 8 mf2 t^2 + 4 D mf2 t^2) / ((-1+4 mf2) (mf2-t))

Delta2[{{1, 6}, {5, 7}}, {1, 0, 0, 0, 1, 1, 2}] = 0

Delta2[{{1, 6}, {2, 4, 5}}, {1, 1, 0, 1, 1, 1, 0}] =
 4 (-8 mf2 + 3 D mf2 + 8 mf2^2 + 16 t - 3 D t - 8 mf2 t) + 2 (-8 + 6 D - D^2 + 8 mf2 - 6 D mf2 + D^2 mf2 - 8 t + 6 D t - D^2 t) x[k[1]][1, {{1, 6}, {2, 4, 5}}]

Delta2[{{1, 6}, {2, 4, 5}}, {1, 2, 0, 1, 1, 1, 0}] = 0

Delta2[{{1, 6}, {2, 4, 7}}, {1, 1, 0, 1, 0, 1, 1}] = 4 (-8 - D + D^2 + 4 D mf2^2 - 8 t + 4 D t - 8 D mf2 t + 4 D t^2) +
 2 (16 - 14 D + 3 D^2 - 16 mf2^2 + 8 D mf2^2 - 16 t + 8 D t + 32 mf2 t - 16 D mf2 t - 16 t^2 + 8 D t^2) x[k[1]][1, {{1, 6}, {2, 4, 7}}]
```

```
((-2 + D) (4 (5 mf2 + 7 mf2^2 - 14 mf2 t + 7 t^2)^2 - 4 (5 mf2 + 9 mf2^2 - 18 mf2 t + 9 t^2)^2)) +
 2 (16 - 14 D + 3 D^2 - 16 mf2^2 + 8 D mf2^2 - 16 t + 8 D t + 32 mf2 t - 16 D mf2 t - 16 t^2 + 8 D t^2)
  x[k[1]][1, {{1, 6}, {2, 4, 7}}]
```

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Interface with IBP generators + Eval of MIs

Recall AIDA output

AIDA for muon-electron scattering

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Interface with IBP generators + Eval of MIs

Recall AIDA output

```
ScalarProducts = {mp[p[1], p[1]] -> 0, mp[p[1], p[2]] -> ex[1]/2,
  mp[p[1], p[3]] -> (mf2 - t - ex[1])/2, mp[p[1], p[4]] -> (-mf2 + t)/2,
  mp[p[2], p[2]] -> 0, mp[p[2], p[3]] -> (-mf2 + t)/2,
  mp[p[2], p[4]] -> (mf2 - t - ex[1])/2, mp[p[3], p[3]] -> mf2,
  mp[p[3], p[4]] -> (-2*mf2 + ex[1])/2, mp[p[4], p[4]] -> mf2}

ParentGraph = {{p[2], e[4], 0, 0, 0}, {p[3], e[5], 0, 0, 0},
  {p[4], e[6], 0, 0, 0}, {p[2], e[1], 0, 0, 0},
  {e[1], e[2], "0", "k[1]", "1"}, {e[2], e[4], "0", "k[2]", "2"},
  {e[2], e[3], "0", "k[1] + k[2]", "3"}, {e[4], e[5], "0", "k[2] + p[2]",
  "4"}, {e[5], e[6], "mf", "k[2] + p[2] + p[3]", "5"},
  {e[1], e[3], "0", "k[1] - p[2] - p[3] - p[4]", "6"},
  {e[3], e[6], "0", "k[2] + p[2] + p[3] + p[4]", "7"}}

listISP = {mp[k[1], p[3]], mp[k[1], p[4]]}

integrals = {INT["emu_2L.g6", {-2, 1, 1, 0, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 0, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 0, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 1, 0, 0}],
```

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Interface with IBP generators + Eval of MIs

Recall AIDA output

```
ScalarProducts = {mp[p[1], p[1]] -> 0, mp[p[1], p[2]] ->
  mp[p[1], p[3]] -> (mf2 - t - ex[1]) / 2, mp[p[1], p[4]]
  mp[p[2], p[2]] -> 0, mp[p[2], p[3]] -> (-mf2 + t) / 2,
  mp[p[2], p[4]] -> (mf2 - t - ex[1]) / 2, mp[p[3], p[3]]
  mp[p[3], p[4]] -> (-2*mf2 + ex[1]) / 2, mp[p[4], p[4]]

ParentGraph = {{p[2], e[4], 0, 0, 0}, {p[3], e[5], 0, 0
  {p[4], e[6], 0, 0, 0}, {p[2], e[1], 0, 0, 0},
  {e[1], e[2], "0", "k[1]", "1"}, {e[2], e[4], "0", "k[
  {e[2], e[3], "0", "k[1] + k[2]", "3"}, {e[4], e[5], "
   "4"}, {e[5], e[6], "mf", "k[2] + p[2] + p[3]", "5"},
  {e[1], e[3], "0", "k[1] - p[2] - p[3] - p[4]", "6"},
  {e[3], e[6], "0", "k[2] + p[2] + p[3] + p[4]", "7"}}

listISP = {mp[k[1], p[3]], mp[k[1], p[4]]}

integrals = {INT["emu_2L.g6", {-2, 1, 1, 0, 0, 1, 2, 0,
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 0, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 0, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 1, 0, 0}],
```

```
 INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 2, 0, 0}], 0,
(-192/(2 - dim) + (96*dim)/(2 - dim) - (16*dim^2)/(2 - dim) +
  (128*mf2^2)/(2 - dim) - (64*dim*mf2^2)/(2 - dim) + (128*t)/(2 - dim) -
  (64*dim*t)/(2 - dim) - (256*mf2*t)/(2 - dim) +
  (128*dim*mf2*t)/(2 - dim) + (128*t^2)/(2 - dim) -
  (64*dim*t^2)/(2 - dim))*INT["emu_2L.g6", {0, 2, 1, 0, 0, 0, 0, 0, 0}],
0, (-32 + 16*dim + 64*mf2^2 + 64*t - 128*mf2*t + 64*t^2)*
 INT["emu_2L.g6", {1, 2, 0, 0, 0, 0, 0, 0, 0}],
(-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 0, 0, 0, 1, 1, 0, 0}],
0, (64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 0, 1, 0, 1, 0, 0, 0}],
(64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 1, 0, 0}],
0, 0, (-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim))*INT["emu_2L.g6", {0, 0, 1, 1, 0, 0, 0, 0, 0}],
```

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Interface with IBP generators + Eval of MIs

Recall AIDA output

```
ScalarProducts = {mp[p[1], p[1]] -> 0, mp[p[1], p[2]] ->
  mp[p[1], p[3]] -> (mf2 - t - ex[1])/2, mp[p[1], p[4]]
  mp[p[2], p[2]] -> 0, mp[p[2], p[3]] -> (-mf2 + t)/2,
  mp[p[2], p[4]] -> (mf2 - t - ex[1])/2, mp[p[3], p[3]]
  mp[p[3], p[4]] -> (-2*mf2 + ex[1])/2, mp[p[4], p[4]]

ParentGraph = {{p[2], e[4], 0, 0, 0}, {p[3], e[5], 0, 0
  {p[4], e[6], 0, 0, 0}, {p[2], e[1], 0, 0, 0},
  {e[1], e[2], "0", "k[1]", "1"}, {e[2], e[4], "0", "k[
  {e[2], e[3], "0", "k[1] + k[2]", "3"}, {e[4], e[5], "
   "4"}, {e[5], e[6], "mf", "k[2] + p[2] + p[3]", "5"},
  {e[1], e[3], "0", "k[1] - p[2] - p[3] - p[4]", "6"},
  {e[3], e[6], "0", "k[2] + p[2] + p[3] + p[4]", "7"}}

listISP = {mp[k[1], p[3]], mp[k[1], p[4]]}

integrals = {INT["emu_2L.g6", {-2, 1, 1, 0, 0, 1, 2, 0,
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 0, 1, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 0, 1, 2, 0, 0}],
  INT["emu_2L.g6", {-2, 1, 1, 1, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 0, 1, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 0, 1, 1, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 0, 0, 0}],
  INT["emu_2L.g6", {-1, 0, 1, 1, 1, 1, 1, 0, 0}],
```

```
INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 2, 0, 0}]  0,
(-192/(2 - dim) + (96*dim)/(2 - dim) - (16*dim^2)/(2 - dim) +
  (128*mf2^2)/(2 - dim) - (64*dim*mf2^2)/(2 - dim) + (128*t)/(2 - dim) -
  (64*dim*t)/(2 - dim) - (256*mf2*t)/(2 - dim) +
  (128*dim*mf2*t)/(2 - dim) + (128*t^2)/(2 - dim) -
  (64*dim*t^2)/(2 - dim)) INT["emu_2L.g6", {0, 2, 1, 0, 0, 0, 0, 0, 0}],
0, (-32 + 16*dim + 64*mf2^2 + 64*t - 128*mf2*t + 64*t^2)*
INT["emu_2L.g6", {1, 2, 0, 0, 0, 0, 0, 0, 0}]
(-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 0, 0, 0, 1, 1, 0, 0}],
0, (64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 0, 1, 0, 1, 0, 0, 0}],
(64/(1 - dim) - (64*dim)/(1 - dim) + (16*dim^2)/(1 - dim) -
  (128*mf2^2)/(1 - dim) + (64*dim*mf2^2)/(1 - dim) - (128*t)/(1 - dim) +
  (64*dim*t)/(1 - dim) + (256*mf2*t)/(1 - dim) -
  (128*dim*mf2*t)/(1 - dim) - (128*t^2)/(1 - dim) +
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 1, 0, 0, 0, 1, 0, 0}]
0, 0, (-64/(1 - dim) + (64*dim)/(1 - dim) - (16*dim^2)/(1 - dim) +
  (128*mf2^2)/(1 - dim) - (64*dim*mf2^2)/(1 - dim) + (128*t)/(1 - dim) -
  (64*dim*t)/(1 - dim) - (256*mf2*t)/(1 - dim) +
  (128*dim*mf2*t)/(1 - dim) + (128*t^2)/(1 - dim) -
  (64*dim*t^2)/(1 - dim)) INT["emu_2L.g6", {0, 0, 1, 1, 0, 0, 0, 0, 0}],
```

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Interface with IBP generators + Eval of MIs

Generate IBPs with **REDUZE**

```
INT["emu_2L.g6",4,15,5,2,{1,2,1,1,0,0,0,0,-2}] -> 0,

INT["emu_2L.g6",4,23,4,0,{1,1,1,0,1,0,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (3*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(-2+d)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (2*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(5*t+5*mf2-2*d*(t+mf2))),

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,-1,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (-3/4*(4*t-t*d)^(-1)*(-2+d)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (-1/4*(4*t-t*d)^(-1)*(14*t-d*(5*t+3*mf2)+6*mf2)),

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,0,0,-1,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (-1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,0,0,0,-1}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (-3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)+
```

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Interface with IBP generators + Eval of MIs

Generate IBPs with **REDUZE**

```
INT["emu_2L.g6",4,15,5,2,{1,2,1,1,0,0,0,0,-2}] -> 0,

INT["emu_2L.g6",4,23,4,0,{1,1,1,0,1,0,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (3*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(-2+d)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (2*(4*mf2^2-8*t*mf2-d*(mf2^2-2*t*mf2+t^2)+4*t^2)^(-1)*(5*t+5*mf2-2*d*(t+mf2))),

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,-1,0,0,0}] ->
  INT["emu_2L.g6",3,21,3,1,
      (-3/4*(4*t-t*d)^(-1)*
  INT["emu_2L.g6",3,21,3,0,
      (-1/4*(4*t-t*d)^(-1)*

INT["emu_2L.g6",4,23,4,1,{1
  INT["emu_2L.g6",3,21,3,1,
      (3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (-1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)

INT["emu_2L.g6",4,23,4,1,{1,1,1,0,1,0,0,0,-1}] ->
  INT["emu_2L.g6",3,21,3,1,{1,-1,1,0,1,0,0,0,0}] *
      (-3/8*(2*t+2*mf2-d*(t+mf2))*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)) +
  INT["emu_2L.g6",3,21,3,0,{1,0,1,0,1,0,0,0,0}] *
      (1/8*(4*t*mf2-4*t^2-d*(t*mf2-t^2))^(-1)*(6*mf2^2+20*t*mf2-d*(3*mf2^2+8*t*mf2+5*t^2)+
```

Apply IBPS to the integrals

$$
\begin{aligned}
&c(0)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,0,1,0,1,1,0,0,0\}) + c(1)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,0,1,1,0,1,0,0,0\}) +\\
&c(2)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,0,1,1,1,1,0,0,0\}) + c(3)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,1,1,0,1,1,0,0,0\}) +\\
&c(4)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{0,1,1,1,1,1,0,0,0\}) + c(5)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,-1,1,0,1,0,0,0,0\}) +\\
&c(6)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,-1,1,1,1,0,1,0,0\}) + c(7)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,-1,1,1,1,1,0,0,0\}) +\\
&c(8)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,0,1,0,0,0,0\}) + c(9)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,1,0,0,1,0,0\}) +\\
&c(10)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,1,1,0,1,0,0\}) + c(11)\,\mathrm{INT}(\mathrm{emu\_2L.g6}, \{1,0,1,1,1,1,0,0,0\})
\end{aligned}
$$

>> Primo' talk

AIDA for muon-electron scattering

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Interface with IBP generators + Eval of MIs

Evaluate integrals with **SecDec**

$c(0)$ INT(emu_2L.g6, {0, 0, 1, 0, 1, 1, 0, 0, 0}) + $c(1)$ INT(emu_2L.g6, {0, 0, 1, 1, 0, 1, 0, 0, 0}) +
$c(2)$ INT(emu_2L.g6, {0, 0, 1, 1, 1, 1, 0, 0, 0}) + $c(3)$ INT(emu_2L.g6, {0, 1, 1, 0, 1, 1, 0, 0, 0}) +
$c(4)$ INT(emu_2L.g6, {0, 1, 1, 1, 1, 1, 0, 0, 0}) + $c(5)$ INT(emu_2L.g6, {1, −1, 1, 0, 1, 0, 0, 0, 0}) +
$c(6)$ INT(emu_2L.g6, {1, −1, 1, 1, 1, 0, 1, 0, 0}) + $c(7)$ INT(emu_2L.g6, {1, −1, 1, 1, 1, 1, 0, 0, 0}) +
$c(8)$ INT(emu_2L.g6, {1, 0, 1, 0, 1, 0, 0, 0, 0}) + $c(9)$ INT(emu_2L.g6, {1, 0, 1, 1, 0, 0, 1, 0, 0}) +
$c(10)$ INT(emu_2L.g6, {1, 0, 1, 1, 1, 0, 1, 0, 0}) + $c(11)$ INT(emu_2L.g6, {1, 0, 1, 1, 1, 1, 0, 0, 0})

INT[emu_2L.g6, {0, 0, 1, 0, 1, 1, 0, 0, 0}] $\to 4.37747 + \frac{0.5}{\epsilon^2} + \frac{0.672784}{\epsilon} + 5.62086\,\epsilon + 24.0491\,\epsilon^2$

INT[emu_2L.g6, {0, 0, 1, 0, 1, 1, 1, 0, 0}] $\to 11.0594 + \frac{0.5}{\epsilon^2} + \frac{1.92278}{\epsilon} + 43.2924\,\epsilon + 188.787\,\epsilon^2$

INT[emu_2L.g6, {0, 0, 1, 1, 0, 1, 0, 0, 0}] $\to 1.33639 + \frac{0.25}{\epsilon} + 5.0669\,\epsilon + 14.3487\,\epsilon^2$

INT[emu_2L.g6, {0, 0, 1, 1, 1, 1, 0, 0, 0}] $\to 7.52838 + \frac{0.5}{\epsilon^2} + \frac{1.92278}{\epsilon} + 17.0981\,\epsilon + 43.2469\,\epsilon^2$

INT[emu_2L.g6, {0, 1, 1, 0, 1, 1, 0, 0, 0}] $\to 4.67763 + \frac{0.5}{\epsilon^2} + \frac{0.706389}{\epsilon} + 6.86664\,\epsilon + 28.7621\,\epsilon^2$

INT[emu_2L.g6, {0, 1, 1, 1, 1, 1, 0, 0, 0}] $\to$
$5.50306 - \frac{0.333333}{\epsilon^3} - \frac{0.0115462}{\epsilon^2} + \frac{0.728702}{\epsilon} + 7.23089\,\epsilon + 15.3931\,\epsilon^2$

INT[emu_2L.g6, {1, -1, 1, 0, 1, 0, 0, 0, 0}] $\to 5.88647 + \frac{0.5}{\epsilon^2} + \frac{1.63112}{\epsilon} + 13.6127\,\epsilon + 34.8251\,\epsilon^2$

INT[emu_2L.g6, {1, -1, 1, 1, 1, 0, 1, 0, 0}] $\to 4.94128 + \frac{0.5}{\epsilon^2} + \frac{1.56968}{\epsilon} + 9.3714\,\epsilon + 20.032\,\epsilon^2$

INT[emu_2L.g6, {1, 0, 1, 0, 1, 0, 0, 0, 0}] $\to 4.59281 + \frac{0.5}{\epsilon^2} + \frac{1.04778}{\epsilon} + 7.95572\,\epsilon + 24.5317\,\epsilon^2$

INT[emu_2L.g6, {1, 0, 1, 1, 0, 0, 1, 0, 0}] $\to 7.76956 + \frac{0.5}{\epsilon^2} + \frac{1.92278}{\epsilon} + 21.0245\,\epsilon + 59.5566\,\epsilon^2$

INT[emu_2L.g6, {1, 0, 1, 1, 1, 0, 1, 0, 0}] $\to -16.1718 - \frac{3.53106}{\epsilon} - 69.0777\,\epsilon - 222.764\,\epsilon^2$

INT[emu_2L.g6, {1, 0, 1, 1, 1, 1, 0, 0, 0}] $\to 14.1287 + \frac{4.41144}{\epsilon} + 54.4079\,\epsilon + 137.07\,\epsilon^2$

Applications to two-loop amplitudes

[Mastrolia, Peraro, Primo, Ronca, W.J.T. (work in progress)]

Process	**Scales + d**	**Analytic**	**Numeric**
$gg \to H$	$s, (m_t)$	✔	✔
$\gamma^* \to e^+e^-$	s, m_e	✔	✔
$gg \to gg$	s, t	✔	✔
$gg \to gh$	s, t	✔	✔
$gg \to gH$	$s, t, m_H, (m_t)$	✔	✔
$gg \to HH$	s, t, m_H, m_t	✔	✔
$e^-\mu^+ \to e^-\mu^+$	$s, t, m_\mu, (m_e)$	✔	✔
$gg \to ggg$	$s_{12}, s_{23}, s_{34}, s_{45}, s_{51}$	✖	✔
$gg \to ggH$	$s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_H, m_t$	✖	✔

Conclusions/Outlook

Multi-loop scattering amplitudes

- ☑ Integrand decomposition methods —> @1 and 2 Loops Automated (AIDA)
- ☑ Analytic decomposition for all 2—>2 processes—> Under control
- ☑ AIDA's output —> Apply IBPs + evaluation of MIs
- ☑ Muon-electron scattering at NNLO is at hand

- ☐ Deal with analytic expressions for 2—>3,4 processes
- ☐ More processes to come in the near future

Conclusions/Outlook

Multi-loop scattering amplitudes

- [x] Integrand decomposition methods —> @1 and 2 Loops Automated (AIDA)
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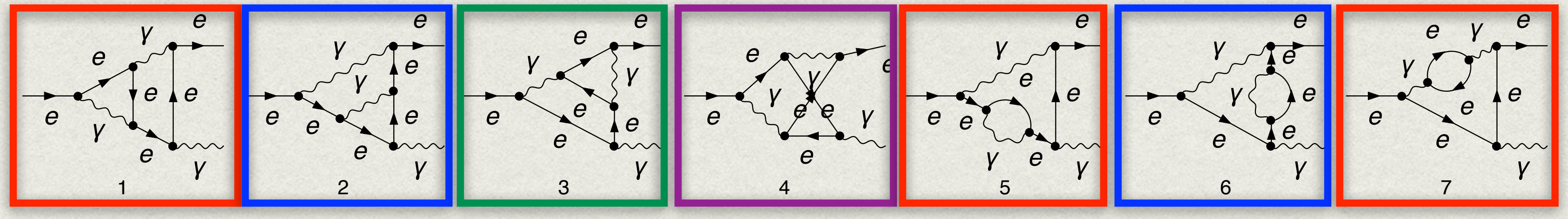
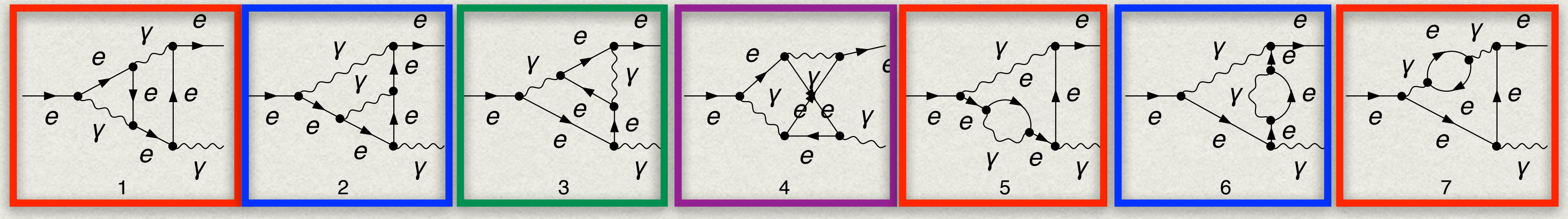
Thanks

Extra slides

AIDA for gamma-electron vertex



AIDA for gamma-electron vertex



Input: rank 4 numerator

$$4\left(\mathrm{mf}^2\,\mathrm{dim}^3 - 6\,\mathrm{mf}^2\,\mathrm{dim}^2 + 4\,\mathrm{mf}^2\,\mathrm{dim}\right)\mathrm{sp}(p_1,p_1)^2 - 4\,\mathrm{sp}(p_1,p_2)\left(8\,\mathrm{dim}\,\mathrm{mf}^4 + 4\,\mathrm{dim}^2\,s\,\mathrm{mf}^2 - 8\,\mathrm{dim}\,s\,\mathrm{mf}^2 + 8\,\mathrm{dim}\,\mu_{1,2}\,\mathrm{mf}^2 - 16\,\mu_{1,2}\,\mathrm{mf}^2 - \mathrm{dim}^3\,\mu_{2,2}\,\mathrm{mf}^2 +\right.$$

$$\left.4\,\mathrm{dim}^2\,\mu_{2,2}\,\mathrm{mf}^2 + 4\,\mathrm{dim}\,\mu_{2,2}\,\mathrm{mf}^2 - 16\,\mu_{2,2}\,\mathrm{mf}^2 + 4\,\mathrm{dim}^2\,s\,\mu_{1,2} - 16\,\mathrm{dim}\,s\,\mu_{1,2} + 16\,s\,\mu_{1,2} + 4\,\mathrm{dim}^2\,s\,\mu_{2,2} - 16\,\mathrm{dim}\,s\,\mu_{2,2} + 16\,s\,\mu_{2,2}\right) +$$

$$2\left(-32\,\mathrm{dim}\,\mathrm{mf}^6 - 8\,\mathrm{dim}^2\,s\,\mathrm{mf}^4 + 32\,\mathrm{dim}\,s\,\mathrm{mf}^4 - 32\,\mathrm{dim}\,\mu_{1,2}\,\mathrm{mf}^4 + 64\,\mu_{1,2}\,\mathrm{mf}^4 + 8\,\mathrm{dim}^2\,\mu_{2,2}\,\mathrm{mf}^4 - 40\,\mathrm{dim}\,\mu_{2,2}\,\mathrm{mf}^4 + 64\,\mu_{2,2}\,\mathrm{mf}^4 + 4\,\mathrm{dim}^2\,s^2\,\mathrm{mf}^2 - 8\,\mathrm{dim}\,s^2\,\mathrm{mf}^2 +\right.$$

$$4\,\mathrm{dim}^2\,\mu_{2,2}^2\,\mathrm{mf}^2 - 16\,\mathrm{dim}\,\mu_{2,2}^2\,\mathrm{mf}^2 + 16\,\mu_{2,2}^2\,\mathrm{mf}^2 - 8\,\mathrm{dim}^2\,s\,\mu_{1,2}\,\mathrm{mf}^2 + 48\,\mathrm{dim}\,s\,\mu_{1,2}\,\mathrm{mf}^2 - 64\,s\,\mu_{1,2}\,\mathrm{mf}^2 + 2\,\mathrm{dim}^3\,s\,\mu_{2,2}\,\mathrm{mf}^2 - 22\,\mathrm{dim}^2\,s\,\mu_{2,2}\,\mathrm{mf}^2 + 64\,\mathrm{dim}\,s\,\mu_{2,2}\,\mathrm{mf}^2 -$$

$$64\,s\,\mu_{2,2}\,\mathrm{mf}^2 + 4\,\mathrm{dim}^2\,\mu_{1,2}\,\mu_{2,2}\,\mathrm{mf}^2 - 16\,\mathrm{dim}\,\mu_{1,2}\,\mu_{2,2}\,\mathrm{mf}^2 + 16\,\mu_{1,2}\,\mu_{2,2}\,\mathrm{mf}^2 + \mathrm{dim}^3\,s\,\mu_{2,2}^2 - 8\,\mathrm{dim}^2\,s\,\mu_{2,2}^2 + 20\,\mathrm{dim}\,s\,\mu_{2,2}^2 - 16\,s\,\mu_{2,2}^2 + 4\,\mathrm{dim}^2\,s^2\,\mu_{1,2} - 16\,\mathrm{dim}\,s^2\,\mu_{1,2} + 16\,s^2\,\mu_{1,2} +$$

$$\left.4\,\mathrm{dim}^2\,s^2\,\mu_{2,2} - 16\,\mathrm{dim}\,s^2\,\mu_{2,2} + 16\,s^2\,\mu_{2,2} + \mathrm{dim}^3\,s\,\mu_{1,2}\,\mu_{2,2} - 8\,\mathrm{dim}^2\,s\,\mu_{1,2}\,\mu_{2,2} + 20\,\mathrm{dim}\,s\,\mu_{1,2}\,\mu_{2,2} - 16\,s\,\mu_{1,2}\,\mu_{2,2}\right) + \mathrm{sp}(k_2,p_2)\left(4\left(\mathrm{dim}^3 - 12\,\mathrm{dim}^2 + 36\,\mathrm{dim} - 32\right)\mathrm{sp}(p_1,p_1)^2 -\right.$$

$$4\left(\mathrm{mf}^2\,\mathrm{dim}^3 - \mu_{2,2}\,\mathrm{dim}^3 - 14\,\mathrm{mf}^2\,\mathrm{dim}^2 + 2\,s\,\mathrm{dim}^2 - 8\,\mu_{1,2}\,\mathrm{dim}^2 + 4\,\mu_{2,2}\,\mathrm{dim}^2 + 32\,\mathrm{mf}^2\,\mathrm{dim} - 8\,s\,\mathrm{dim} + 32\,\mu_{1,2}\,\mathrm{dim} - 4\,\mu_{2,2}\,\mathrm{dim} - 16\,\mathrm{mf}^2 + 8\,s - 32\,\mu_{1,2}\right)\mathrm{sp}(p_1,p_1) +$$

$$4\left(8\,\mathrm{dim}\,\mathrm{mf}^4 + 4\,\mathrm{dim}^2\,s\,\mathrm{mf}^2 - 8\,\mathrm{dim}\,s\,\mathrm{mf}^2 + 8\,\mathrm{dim}\,\mu_{1,2}\,\mathrm{mf}^2 - 16\,\mu_{1,2}\,\mathrm{mf}^2 - \mathrm{dim}^3\,\mu_{2,2}\,\mathrm{mf}^2 + 6\,\mathrm{dim}^2\,\mu_{2,2}\,\mathrm{mf}^2 - 8\,\mathrm{dim}\,\mu_{2,2}\,\mathrm{mf}^2 + 4\,\mathrm{dim}^2\,s\,\mu_{1,2} - 16\,\mathrm{dim}\,s\,\mu_{1,2} +\right.$$

$$\left.\left.16\,s\,\mu_{1,2} + 2\,\mathrm{dim}^2\,s\,\mu_{2,2} - 8\,\mathrm{dim}\,s\,\mu_{2,2} + 8\,s\,\mu_{2,2}\right)\right) + \mathrm{sp}(k_1,k_2)\left(2\left(s\,\mathrm{dim}^3 + 4\,\mathrm{mf}^2\,\mathrm{dim}^2 - 8\,s\,\mathrm{dim}^2 - 16\,\mathrm{mf}^2\,\mathrm{dim} + 20\,s\,\mathrm{dim} + 16\,\mathrm{mf}^2 - 16\,s\right)\mathrm{sp}(k_2,k_2) +\right.$$

$$\mathrm{sp}(k_2,p_2)\left(-16\left(s\,\mathrm{dim}^2 + 2\,\mathrm{mf}^2\,\mathrm{dim} - 4\,s\,\mathrm{dim} - 4\,\mathrm{mf}^2 + 4\,s\right) - 32\left(\mathrm{dim}^2 - 4\,\mathrm{dim} + 4\right)\mathrm{sp}(p_1,p_1)\right) + 16\left(s\,\mathrm{dim}^2 + 2\,\mathrm{mf}^2\,\mathrm{dim} - 4\,s\,\mathrm{dim} - 4\,\mathrm{mf}^2 + 4\,s\right)\mathrm{sp}(p_1,p_2) +$$

$$\mathrm{sp}(k_2,p_1)\left(-8\left(s\,\mathrm{dim}^3 + 4\,\mathrm{mf}^2\,\mathrm{dim}^2 - 10\,s\,\mathrm{dim}^2 - 20\,\mathrm{mf}^2\,\mathrm{dim} + 28\,s\,\mathrm{dim} + 24\,\mathrm{mf}^2 - 24\,s\right) + 32\left(\mathrm{dim}^2 - 4\,\mathrm{dim} + 4\right)\mathrm{sp}(k_2,p_2) - 32\left(\mathrm{dim}^2 - 4\,\mathrm{dim} + 4\right)\mathrm{sp}(p_1,p_2)\right) +$$

$$\mathrm{sp}(p_1,p_1)\left(2\left(3\,s\,\mathrm{dim}^3 + 12\,\mathrm{mf}^2\,\mathrm{dim}^2 - 32\,s\,\mathrm{dim}^2 - 64\,\mathrm{mf}^2\,\mathrm{dim} + 92\,s\,\mathrm{dim} + 80\,\mathrm{mf}^2 - 80\,s\right) + 32\left(\mathrm{dim}^2 - 4\,\mathrm{dim} + 4\right)\mathrm{sp}(p_1,p_2)\right) - 2\left(-32\,\mathrm{dim}\,\mathrm{mf}^4 + 64\,\mathrm{mf}^4 - 8\,\mathrm{dim}^2\,s\,\mathrm{mf}^2 +\right.$$

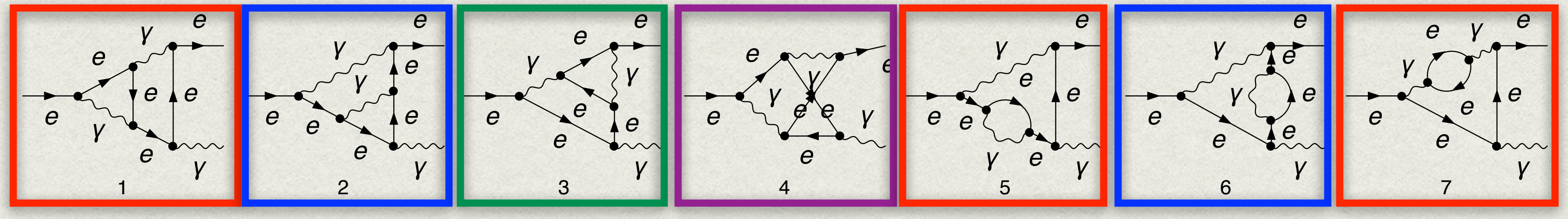
$$\left.\left.48\,\mathrm{dim}\,s\,\mathrm{mf}^2 - 64\,s\,\mathrm{mf}^2 + 4\,\mathrm{dim}^2\,\mu_{2,2}\,\mathrm{mf}^2 - 16\,\mathrm{dim}\,\mu_{2,2}\,\mathrm{mf}^2 + 16\,\mu_{2,2}\,\mathrm{mf}^2 + 4\,\mathrm{dim}^2\,s^2 - 16\,\mathrm{dim}\,s^2 + 16\,s^2 + \mathrm{dim}^3\,s\,\mu_{2,2} - 8\,\mathrm{dim}^2\,s\,\mu_{2,2} + 20\,\mathrm{dim}\,s\,\mu_{2,2} - 16\,s\,\mu_{2,2}\right)\right) +$$

$$\mathrm{sp}(p_1,p_1)\left(4\,\mathrm{sp}(p_1,p_2)\left(\mathrm{mf}^2\,\mathrm{dim}^3 - 12\,\mathrm{mf}^2\,\mathrm{dim}^2 - 8\,\mu_{1,2}\,\mathrm{dim}^2 - 8\,\mu_{2,2}\,\mathrm{dim}^2 + 20\,\mathrm{mf}^2\,\mathrm{dim} + 32\,\mu_{1,2}\,\mathrm{dim} + 32\,\mu_{2,2}\,\mathrm{dim} - 32\,\mu_{1,2} - 32\,\mu_{2,2}\right) -\right.$$

$$2\left(8\,\mathrm{dim}^2\,\mathrm{mf}^4 - 40\,\mathrm{dim}\,\mathrm{mf}^4 + 2\,\mathrm{dim}^3\,s\,\mathrm{mf}^2 - 18\,\mathrm{dim}^2\,s\,\mathrm{mf}^2 + 32\,\mathrm{dim}\,s\,\mathrm{mf}^2 + 12\,\mathrm{dim}^2\,\mu_{1,2}\,\mathrm{mf}^2 - 64\,\mathrm{dim}\,\mu_{1,2}\,\mathrm{mf}^2 + 80\,\mu_{1,2}\,\mathrm{mf}^2 - 2\,\mathrm{dim}^3\,\mu_{2,2}\,\mathrm{mf}^2 + 24\,\mathrm{dim}^2\,\mu_{2,2}\,\mathrm{mf}^2 -\right.$$

$$\left.\left.72\,\mathrm{dim}\,\mu_{2,2}\,\mathrm{mf}^2 + 80\,\mu_{2,2}\,\mathrm{mf}^2 + 3\,\mathrm{dim}^3\,s\,\mu_{1,2} - 32\,\mathrm{dim}^2\,s\,\mu_{1,2} + 92\,\mathrm{dim}\,s\,\mu_{1,2} - 80\,s\,\mu_{1,2} + 3\,\mathrm{dim}^3\,s\,\mu_{2,2} - 32\,\mathrm{dim}^2\,s\,\mu_{2,2} + 92\,\mathrm{dim}\,s\,\mu_{2,2} - 80\,s\,\mu_{2,2}\right)\right) +$$

AIDA for gamma-electron vertex

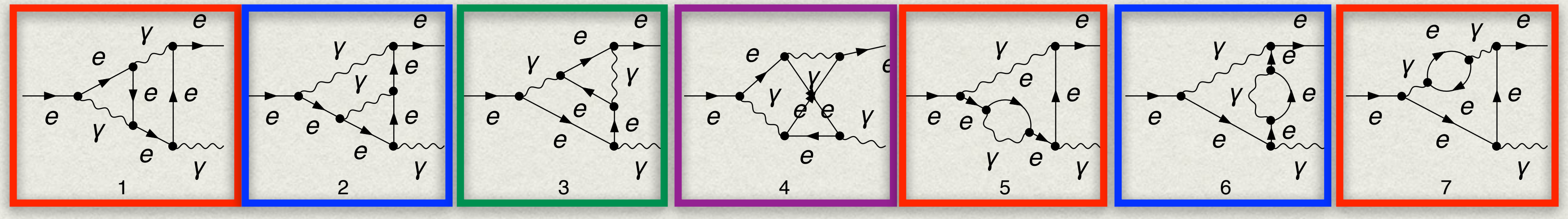


Input: rank 4 numerator

$$4\left(mf^2\, dim^3 - 6\, mf^2\, dim^2 + 4\, mf^2\, dim\right) sp(p_1, p_1)^2 - 4\, sp(p_1, p_2)\left(8\, dim\, mf^4 + 4\, dim^2\, s\, mf^2 - 8\, dim\, s\, mf^2 + 8\, dim\, \mu_{1,2}\, mf^2 - 16\, \mu_{1,2}\, mf^2 - dim^3\, \mu_{2,2}\, mf^2 + 4\, dim^2\, \mu_{2,2}\, mf^2 + 4\, dim\, \mu_{2,2}\, mf^2 - 16\, \mu_{2,2}\, mf^2 + 4\, dim^2\, s\, \mu_{1,2} - 16\, dim\, s\, \mu_{1,2} + 16\, s\, \mu_{1,2} + 4\, dim^2\, s\, \mu_{2,2} - 16\, dim\, s\, \mu_{2,2} + 16\, s\, \mu_{2,2}\right) +$$

```
Δ[{{1}, {2, 5}, {3}}, {1, 2, 1, 0, 2, 0}] → 0
Δ[{{6}, {4, 5}, {3}}, {0, 0, 1, 1, 2, 1}] → -16 mfe2 (4 mfe2 + (-2 + dim) s) x_{1,1,{{6},{4,5},{3}}} + 16 (-2 + dim) mfe2 s x^2_{1,1,{{6},{4,5},{3}}}
Δ[{{6}, {2, 4, 5}}, {0, 2, 0, 1, 1, 1}] → 0
Δ[{{6}, {2, 4, 5}}, {0, 1, 0, 1, 2, 1}] → -8 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s)
Δ[{{6}, {2, 5}, {3}}, {0, 2, 1, 0, 1, 1}] → 0
Δ[{{6}, {2, 5}, {3}}, {0, 1, 1, 0, 2, 1}] → 16 mfe2 (4 mfe2 + (-2 + dim) s) x_{1,1,{{6},{2,5},{3}}} + 16 (-2 + dim) mfe2 s x^2_{1,1,{{6},{2,5},{3}}}
Δ[{{6}, {2, 4}, {3}}, {0, 2, 1, 1, 0, 1}] → 0
Δ[{{2, 4, 5}, {3}}, {0, 2, 1, 1, 1, 0}] → -4 (-2 + dim) (2 mfe2 - s) (4 mfe2 + (-2 + dim) s)
Δ[{{2, 4, 5}, {3}}, {0, 1, 1, 1, 2, 0}] →
 -8 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s) - 64 s (mfe2^3 - 2 s^4 (1 + 3 s) + 2 mfe2 s^2 (2 + 5 s (2 + 3 s)) + mfe2^2 s (4 + 5 s (3 + 5 s))) x^2_{2,3,{{2,4,5},{3}}} -
  256 (4 mfe2 - s) (mfe2^3 - 2 s^4 (1 + 3 s) + 2 mfe2 s^2 (2 + 5 s (2 + 3 s)) + mfe2^2 s (4 + 5 s (3 + 5 s))) x^2_{2,4,{{2,4,5},{3}}}
Δ[{{1, 6}, {4, 5}}, {1, 0, 0, 1, 2, 1}] → 0
Δ[{{1, 6}, {5}, {3}}, {1, 0, 1, 0, 2, 1}] → 0
Δ[{{1}, {4, 5}, {3}}, {1, 0, 1, 1, 2, 0}] → 0
Δ[{{1, 6}, {2, 5}}, {1, 2, 0, 0, 1, 1}] → 0
Δ[{{1, 6}, {2, 5}}, {1, 1, 0, 0, 2, 1}] → 0
Δ[{{1, 6}, {2, 4}}, {1, 2, 0, 1, 0, 1}] → 0
Δ[{{1}, {2, 4, 5}}, {1, 2, 0, 1, 1, 0}] → 4 (-2 + dim) (2 mfe2 - s) (4 mfe2 + (-2 + dim) s)
Δ[{{1}, {2, 4, 5}}, {1, 1, 0, 1, 2, 0}] → 0
Δ[{{1, 6}, {2}, {3}}, {1, 2, 1, 0, 0, 1}] → 0
Δ[{{1}, {2, 5}, {3}}, {1, 2, 1, 0, 1, 0}] → -16 mfe2 (2 mfe2 + (-2 + dim) s)
```

AIDA for gamma-electron vertex



Input: rank 4 numerator

$$4\left(\mathrm{mf}^2\,\mathrm{dim}^3 - 6\,\mathrm{mf}^2\,\mathrm{dim}^2 + 4\,\mathrm{mf}^2\,\mathrm{dim}\right)\mathrm{sp}(p_1, p_1)^2 - 4\,\mathrm{sp}(p_1, p_2)\left(8\,\mathrm{dim}\,\mathrm{mf}^4 + 4\,\mathrm{dim}^2\,s\,\mathrm{mf}^2 - 8\,\mathrm{dim}\,s\,\mathrm{mf}^2 + 8\,\mathrm{dim}\,\mu_{1,2}\,\mathrm{mf}^2 - 16\,\mu_{1,2}\,\mathrm{mf}^2 - \mathrm{dim}^3\,\mu_{2,2}\,\mathrm{mf}^2 + \right.$$
$$\left. 4\,\mathrm{dim}^2\,\mu_{2,2}\,\mathrm{mf}^2 + 4\,\mathrm{dim}\,\mu_{2,2}\,\mathrm{mf}^2 - 16\,\mu_{2,2}\,\mathrm{mf}^2 + 4\,\mathrm{dim}^2\,s\,\mu_{1,2} - 16\,\mathrm{dim}\,s\,\mu_{1,2} + 16\,s\,\mu_{1,2} + 4\,\mathrm{dim}^2\,s\,\mu_{2,2} - 16\,\mathrm{dim}\,s\,\mu_{2,2} + 16\,s\,\mu_{2,2}\right) +$$

$\Delta[\{\{1\}, \{2, 5\}, \{3\}\}, \{1, 2, 1, 0, 2, 0\}] \to 0$

$\Delta[\{\{6\}, \{4, 5\}, \{3\}\}, \{0, 0, 1, 1, 2, 1\}] \to -16\,\mathrm{mfe2}\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s)\,x_{1,1,\{\{6\},\{4,5\},\{3\}\}} + 16\,(-2 + \mathrm{dim})\,\mathrm{mfe2}\,s\,x^2_{1,1,\{\{6\},\{4,5\},\{3\}\}}$

$\Delta[\{\{6\}, \{2, 4, 5\}\}, \{0, 2, 0, 1, 1, 1\}] \to 0$

$\Delta[\{\{6\}, \{2, 4, 5\}\}, \{0, 1, 0, 1, 2, 1\}] \to -8\,(2\,\mathrm{mfe2} - s)\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s)$

$\Delta^{\mathrm{int}}[\{\{1\}, \{2, 5\}, \{3\}\}, \{1, 2, 1, 0, 2, 0\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{6\}, \{4, 5\}, \{3\}\}, \{0, 0, 1, 1, 2, 1\}] \to -16\,\mathrm{mfe2}\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s)\,x_{1,1,\{\{6\},\{4,5\},\{3\}\}} + 16\,(-2 + \mathrm{dim})\,\mathrm{mfe2}\,s\,x^2_{1,1,\{\{6\},\{4,5\},\{3\}\}}$

$\Delta^{\mathrm{int}}[\{\{6\}, \{2, 4, 5\}\}, \{0, 2, 0, 1, 1, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{6\}, \{2, 4, 5\}\}, \{0, 1, 0, 1, 2, 1\}] \to -8\,(2\,\mathrm{mfe2} - s)\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s)$

$\Delta^{\mathrm{int}}[\{\{6\}, \{2, 5\}, \{3\}\}, \{0, 2, 1, 0, 1, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{6\}, \{2, 5\}, \{3\}\}, \{0, 1, 1, 0, 2, 1\}] \to 16\,\mathrm{mfe2}\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s)\,x_{1,1,\{\{6\},\{2,5\},\{3\}\}} + 16\,(-2 + \mathrm{dim})\,\mathrm{mfe2}\,s\,x^2_{1,1,\{\{6\},\{2,5\},\{3\}\}}$

$\Delta^{\mathrm{int}}[\{\{6\}, \{2, 4\}, \{3\}\}, \{0, 2, 1, 1, 0, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{2, 4, 5\}, \{3\}\}, \{0, 2, 1, 1, 1, 0\}] \to -4\,(-2 + \mathrm{dim})\,(2\,\mathrm{mfe2} - s)\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s)$

$\Delta^{\mathrm{int}}[\{\{2, 4, 5\}, \{3\}\}, \{0, 1, 1, 1, 2, 0\}] \to -8\,(2\,\mathrm{mfe2} - s)\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s) + \frac{32\,s\,\lambda^2_{2,2,\{\{2,4,5\},\{3\}\}}}{-2+\mathrm{dim}}$

$\Delta^{\mathrm{int}}[\{\{1, 6\}, \{4, 5\}\}, \{1, 0, 0, 1, 2, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{1, 6\}, \{5\}, \{3\}\}, \{1, 0, 1, 0, 2, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{1\}, \{4, 5\}, \{3\}\}, \{1, 0, 1, 1, 2, 0\}] \to 0$

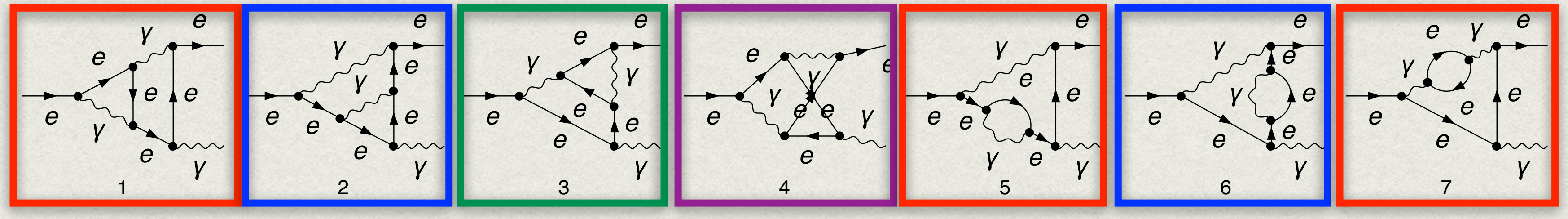
$\Delta^{\mathrm{int}}[\{\{1, 6\}, \{2, 5\}\}, \{1, 2, 0, 0, 1, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{1, 6\}, \{2, 5\}\}, \{1, 1, 0, 0, 2, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{1, 6\}, \{2, 4\}\}, \{1, 2, 0, 1, 0, 1\}] \to 0$

$\Delta^{\mathrm{int}}[\{\{1\}, \{2, 4, 5\}\}, \{1, 2, 0, 1, 1, 0\}] \to 4\,(-2 + \mathrm{dim})\,(2\,\mathrm{mfe2} - s)\,(4\,\mathrm{mfe2} + (-2 + \mathrm{dim})\,s)$

AIDA for gamma-electron vertex



Input: rank 4 numerator

Reduction time ~ 130 seconds

$4\left(\text{mf}^2\,\text{dim}^3 - 6\,\text{mf}^2\,\text{dim}^2 + 4\,\text{mf}^2\,\text{dim}\right)\text{sp}(p_1,p_1)^2 - 4\,\text{sp}(p_1,p_2)\left(8\,\text{dim}\,\text{mf}^4 + 4\,\text{dim}^2\,s\,\text{mf}^2 - 8\,\text{dim}\,s\,\text{mf}^2 + 8\,\text{dim}\,\mu_{1,2}\,\text{mf}^2 - 16\,\mu_{1,2}\,\text{mf}^2 - \text{dim}^3\,\mu_{2,2}\,\text{mf}^2 +\right.$

$\left.4\,\text{dim}^2\,\mu_{2,2}\,\text{mf}^2 + 4\,\text{dim}\,\mu_{2,2}\,\text{mf}^2 - 16\,\mu_{2,2}\,\text{mf}^2 + 4\,\text{dim}^2\,s\,\mu_{1,2} - 16\,\text{dim}\,s\,\mu_{1,2} + 16\,s\,\mu_{1,2} + 4\,\text{dim}^2\,s\,\mu_{2,2} - 16\,\text{dim}\,s\,\mu_{2,2} + 16\,s\,\mu_{2,2}\right) +$

$\Delta[\{\{1\},\{2,5\},\{3\}\},\{1,2,1,0,2,0\}] \to 0$

$\Delta[\{\{6\},\{4,5\},\{3\}\},\{0,0,1,1,2,1\}] \to -16\,\text{mfe2}\,(4\,\text{mfe2} + (-2+\text{dim})\,s)\,x_{1,1,\{\{6\},\{4,5\},\{3\}\}} + 16\,(-2+\text{dim})\,\text{mfe2}\,s\,x^2_{1,1,\{\{6\},\{4,5\},\{3\}\}}$

$\Delta[\{\{6\},\{2,4,5\}\},\{0,2,0,1,1,1\}] \to 0$

$\Delta[\{\{6\},\{2,4,5\}\},\{0,1,0,1,2,1\}] \to -8\,(2\,\text{mfe2} - s)\,(4\,\text{mfe2} + (-2+\text{dim})\,s)$

$\Delta^{\text{int}}[\{\{1\},\{2,5\},\{3\}\},\{1,2,1,0,2,0\}] \to 0$

$\Delta^{\text{int}}[\{\{6\},\{4,5\},\{3\}\},\{0,0,1,1,2,1\}] \to -16\,\text{mfe2}\,(4\,\text{mfe2} + (-2+\text{dim})\,s)\,x_{1,1,\{\{6\},\{4,5\},\{3\}\}} + 16\,(-2+\text{dim})\,\text{mfe2}\,s\,x^2_{1,1,\{\{6\},\{4,5\},\{3\}\}}$

$\Delta'[\{\{1\},\{2,5\},\{3\}\},\{1,2,1,0,2,0\}] \to 0$

$\Delta'[\{\{6\},\{4,5\},\{3\}\},\{0,0,1,1,2,1\}] \to -16\,\text{mfe2}\,(4\,\text{mfe2} + (-2+\text{dim})\,s)\,x_{1,1,\{\{6\},\{4,5\},\{3\}\}} + 16\,(-2+\text{dim})\,\text{mfe2}\,s\,x^2_{1,1,\{\{6\},\{4,5\},\{3\}\}}$

$\Delta'[\{\{6\},\{2,4,5\}\},\{0,2,0,1,1,1\}] \to 0$

$\Delta'[\{\{6\},\{2,4,5\}\},\{0,1,0,1,2,1\}] \to -8\,(2\,\text{mfe2} - s)\,(4\,\text{mfe2} + (-2+\text{dim})\,s)$

$\Delta'[\{\{6\},\{2,5\},\{3\}\},\{0,2,1,0,1,1\}] \to 0$

$\Delta'[\{\{6\},\{2,5\},\{3\}\},\{0,1,1,0,2,1\}] \to 16\,\text{mfe2}\,(4\,\text{mfe2} + (-2+\text{dim})\,s)\,x_{1,1,\{\{6\},\{2,5\},\{3\}\}} + 16\,(-2+\text{dim})\,\text{mfe2}\,s\,x^2_{1,1,\{\{6\},\{2,5\},\{3\}\}}$

$\Delta'[\{\{6\},\{2,4\},\{3\}\},\{0,2,1,1,0,1\}] \to 0$

$\Delta'[\{\{2,4,5\},\{3\}\},\{0,2,1,1,1,0\}] \to -4\,(-2+\text{dim})\,(2\,\text{mfe2} - s)\,(4\,\text{mfe2} + (-2+\text{dim})\,s)$

$\Delta'[\{\{2,4,5\},\{3\}\},\{0,1,1,1,2,0\}] \to -8\,(2\,\text{mfe2} - s)\,(4\,\text{mfe2} + (-2+\text{dim})\,s)$

$\Delta'[\{\{1,6\},\{4,5\}\},\{1,0,0,1,2,1\}] \to 0$

$\Delta'[\{\{1,6\},\{5\},\{3\}\},\{1,0,1,0,2,1\}] \to 0$

$\Delta'[\{\{1\},\{4,5\},\{3\}\},\{1,0,1,1,2,0\}] \to 0$

$\Delta'[\{\{1,6\},\{2,5\}\},\{1,2,0,0,1,1\}] \to 0$

$\Delta'[\{\{1,6\},\{2,5\}\},\{1,1,0,0,2,1\}] \to 0$

$\Delta'[\{\{1,6\},\{2,4\}\},\{1,2,0,1,0,1\}] \to 0$

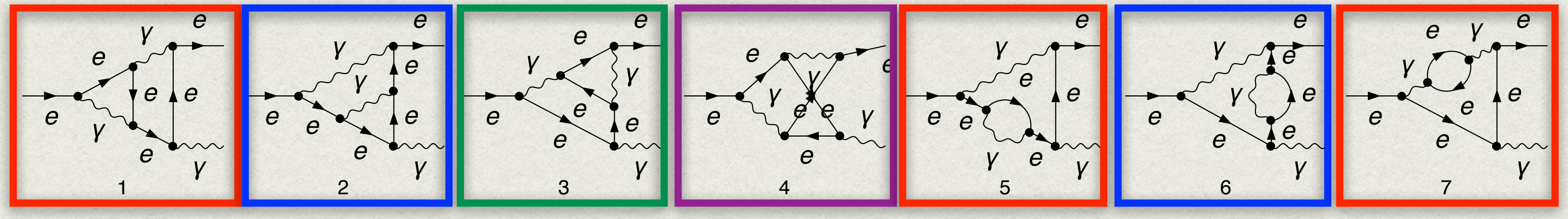
$\Delta'[\{\{1\},\{2,4,5\}\},\{1,2,0,1,1,0\}] \to 4\,(-2+\text{dim})\,(2\,\text{mfe2} - s)\,(4\,\text{mfe2} + (-2+\text{dim})\,s)$

$\Delta'[\{\{1\},\{2,4,5\}\},\{1,1,0,1,2,0\}] \to 0$

$\Delta'[\{\{1,6\},\{2\},\{3\}\},\{1,2,1,0,0,1\}] \to 0$

$\Delta'[\{\{1\},\{2,5\},\{3\}\},\{1,2,1,0,1,0\}] \to -16\,\text{mfe2}\,(2\,\text{mfe2} + (-2+\text{dim})\,s)$

AIDA for gamma-electron vertex

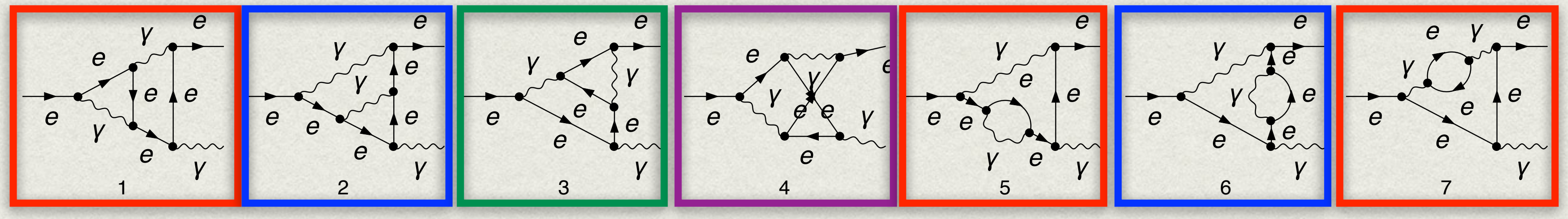


Interface with IBP generators

```
ScalarProducts = {mp[p[1], p[1]] -> mfe2, mp[p[1], p[2]] -> (-2*mfe2 + s)/2,
     mp[p[1], p[3]] -> -s/2, mp[p[2], p[2]] -> mfe2, mp[p[2], p[3]] -> -s/2,
     mp[p[3], p[3]] -> s}

ParentGraph = {{p[1], e[3], 0, 0, 0}, {p[2], e[1], 0, 0, 0},
     {p[100], e[5], 0, 0, 0}, {e[2], e[3], "0", "k[1]", "1"},
     {e[2], e[5], "Sqrt[mfe2]", "k[2]", "2"}, {e[2], e[4], "Sqrt[mfe2]",
      "k[1] + k[2]", "3"}, {e[1], e[5], "Sqrt[mfe2]", "k[2] - p[1] - p[2]",
      "4"}, {e[1], e[4], "0", "k[2] - p[1]", "5"}, {e[3], e[4], "Sqrt[mfe2]",
      "k[1] + p[1]", "6"}}

listISP = {mp[k[1], p[2]]}

integrals = {INT["emu_emu-vertex.2L.g1", {-2, 1, 1, 0, 2, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {-2, 1, 1, 1, 1, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 0, 1, 1, 1, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 0, 1, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 0, 2, 0, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 0, 2, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 1, 0, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 1, 1, 0, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 1, 1, 1, -1}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 1, 1, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 1, 2, 1, -1}],
     INT["emu_emu-vertex.2L.g1", {-1, 1, 1, 1, 2, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {0, -1, 1, 1, 1, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {0, 0, 0, 0, 2, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {0, 0, 0, 1, 1, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {0, 0, 0, 1, 2, 1, 0}],
     INT["emu_emu-vertex.2L.g1", {0, 0, 1, 0, 1, 1, 0}],
```

```
{0, 0, 0,
 -8 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 1, 1, 0}] +
  8 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {1, 1, 1, 1, 1, 0, 0}] +
  32 mfe2 (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {1, 1, 1, 1, 1, 1, -1}] -
  16 mfe2 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {1, 1, 1, 1, 1, 1, 0}],
 0, 0, 0, 0, 0, 32 (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {-1, 1, 1, 1, 2, 1, -1}] -
  16 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {-1, 1, 1, 1, 2, 1, 0}] -
  32 (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 2, 0, -1}] +
  16 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 2, 0, 0}] -
  64 mfe2 (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 2, 1, -1}] +
  32 mfe2 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 2, 1, 0}],
 0, 0, 0, 0, 0, 0, -16 mfe2 (2 mfe2 - s) (4 mfe2 + (-2 + dim) s)
  INT[emu_emu-vertex.2L.g1, {1, 2, 1, 1, 1, 0, 0}], 0,
 -8 (-2 + dim) INT[emu_emu-vertex.2L.g1, {-2, 1, 1, 1, 1, 1, 0}] +
  16 (-2 + dim) INT[emu_emu-vertex.2L.g1, {-1, 1, 1, 1, 1, 0, 0}] +
  32 (-4 + dim) INT[emu_emu-vertex.2L.g1, {-1, 1, 1, 1, 1, 1, -1}] +
  (96 mfe2 + 8 (-10 + 3 dim) s) INT[emu_emu-vertex.2L.g1, {-1, 1, 1, 1, 1, 1, 0}] -
  8 (-2 + dim) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 1, -1, 0}] -
  32 (-4 + dim) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 1, 0, -1}] +
  (-96 mfe2 + 8 (10 - 3 dim) s) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 1, 0, 0}] -
  32 (-2 + dim) INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 1, 1, -2}] +
  (32 (4 + dim) mfe2 + 8 (-4 + dim) (-2 + dim) s)
   INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 1, 1, -1}] +
  (-32 dim mfe2^2 - 8 (-4 + dim)^2 mfe2 s + 4 (-6 + dim) (-2 + dim) s^2)
   INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 1, 1, 0}],
 8 (-2 + dim)^2 mfe2^2 INT[emu_emu-vertex.2L.g1, {-1, 0, 1, 1, 1, 1, 0}]
 ---------------------------------------------------------------------- +
                            (-2 mfe2 + s)^2
  2 (-104 + dim (68 + (-14 + dim) dim)) mfe2 INT[emu_emu-vertex.2L.g1, {0, -1, 1, 1, 1, 1, 0}]
  -------------------------------------------------------------------------------------------- +
                                           -2 mfe2 + s
```

AIDA for gamma-electron vertex



Interface with IBP generators

REDUZE files

get MIs + IBPs

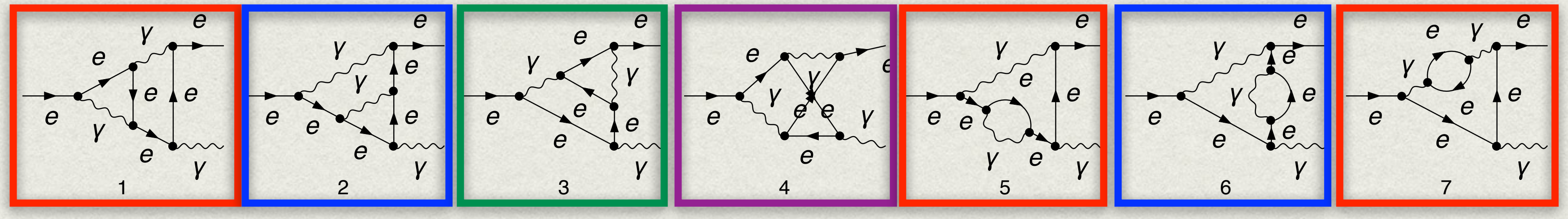
```
kinematics:
    incoming_momenta: [p3,p1,p2]
    outgoing_momenta: []
    momentum_conservation: [p3,-p1 - p2]
    kinemat
      - [mfe
      - [s,2
    scalarp
      - [[p1
      - [[p1
      - [[p2
```

```
integralfamilies:
  - name: emu_emu-vertex.2L.g1
    loop_momenta: [k1, k2]
    propagators:
     - [ k1 , 0 ]
     - [ k2 , mfe2 ]
     - [ k1 + k2 , mfe2 ]
     ...
     - [ k2 - p1 - p2 , mfe2 ]
     - [ k1 - p1 , mfe2 ]
     - [ k1 + k2 - p1 , 0 ]
     - {bilinear: [[k2,p2],0]}
```

```
  - name: emu_emu-vertex.2L.g3
    loop_momenta: [k1, k2]
    propagators:
     - [ k1 , mfe2 ]
     - [ k2 , mfe2 ]
     - [ k2 + p2 , 0 ]
     - [ k1 - p1 - p2 , mfe2 ]
     ...fe2 ]
     ...],0]}
  ...2L.g4
     ...]
     ...
     - [ k1 + k2 + p1 + p2 , mfe2 ]
     - [ k1 + p1 , 0 ]
     - {bilinear: [[k2,p1],0]}
```

```
{INT[emu_emu-vertex.2L.g1, {-1, 1, 1, 1, 0, 1, 0}],
 INT[emu_emu-vertex.2L.g1, {0, 1, 1, 0, 0, 0, 0}], INT[emu_emu-vertex.2L.g1, {0, 1, 1, 0, 0, 1, 0}],
 INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 0, 0, 0}], INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 0, 1, 0}],
 INT[emu_emu-vertex.2L.g1, {1, -1, 1, 1, 0, 0, 0}], INT[emu_emu-vertex.2L.g1, {1, -1, 1, 1, 0, 1, 0}],
 INT[emu_emu-vertex.2L.g1, {1, 0, 1, 0, 1, 0, 0}], INT[emu_emu-vertex.2L.g1, {1, 0, 1, 1, 0, 0, 0}],
 INT[emu_emu-vertex.2L.g1, {1, 0, 1, 1, 0, 1, 0}], INT[emu_emu-vertex.2L.g1, {1, 0, 1, 1, 1, 0, 0}],
 INT[emu_emu-vertex.2L.g2, {0, 1, 1, 1, 1, 0, 0}], INT[emu_emu-vertex.2L.g2, {1, 1, 0, 1, 0, 1, 0}],
 INT[emu_emu-vertex.2L.g2, {1, 1, 1, 1, -1, 1, 0}], INT[emu_emu-vertex.2L.g2, {1, 1, 1, 1, 0, 1, 0}],
 INT[emu_emu-vertex.2L.g4, {1, 1, 1, 1, 1, 1, -1}], INT[emu_emu-vertex.2L.g4, {1, 1, 1, 1, 1, 1, 0}]}
```

AIDA for gamma-electron vertex



Evaluate **MIs** **SecDec**

`INT[emu_emu-vertex.2L.g1, {-1, 1, 1, 1, 0, 1, 0}]` $\to 1.61388 + \frac{1.375}{\epsilon^2} - \frac{0.165921}{\epsilon} + 1.60481\,\epsilon - 3.7394\,\epsilon^2 + 14.4043\,\epsilon^3$

`INT[emu_emu-vertex.2L.g1, {0, 1, 1, 0, 0, 0, 0}]` $\to 3.00243 + \frac{1}{\epsilon^2} + \frac{0.845569}{\epsilon} + 2.20253\,\epsilon + 8432.68\,\epsilon^2 - 63958.5\,\epsilon^3$

`INT[emu_emu-vertex.2L.g1, {0, 1, 1, 0, 0, 1, 0}]` $\to 5.9356 + \frac{1.5}{\epsilon^2} + \frac{2.51835}{\epsilon} + 14.0959\,\epsilon + 20.1411\,\epsilon^2 + 62.3687\,\epsilon^3$

`INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 0, 0, 0}]` $\to 2.19404 + \frac{1}{\epsilon^2} - \frac{0.306476}{\epsilon} - 1.13085\,\epsilon + 3.17121\,\epsilon^2 + 1.07305\times 10^6\,\epsilon^3$

`INT[emu_emu-vertex.2L.g1, {0, 1, 1, 1, 0, 1, 0}]` $\to 0.370704 + \frac{0.5}{\epsilon^2} - \frac{0.22926}{\epsilon} + 0.577506\,\epsilon - 2.45766\,\epsilon^2 + 6.99908\,\epsilon^3$

`INT[emu_emu-vertex.2L.g1, {1, -1, 1, 1, 0, 0, 0}]` $\to -2.1375 - \frac{0.5}{\epsilon^2} - \frac{0.756118}{\epsilon} - 4.24832\,\epsilon - 7.65608\,\epsilon^2 - 61001$…

`INT[emu_emu-vertex.2L.g1, {1, -1, 1, 1, 0, 1, 0}]` $\to 6.39921 + \frac{0.875}{\epsilon^2} + \frac{1.42737}{\epsilon} + 8.52632\,\epsilon + 29.6557\,\epsilon^2$…

`INT[emu_emu-vertex.2L.g1, {1, 0, 1, 0, 1, 0, 0}]` $\to 4.37747 + \frac{0.5}{\epsilon^2} + \frac{0.672784}{\epsilon} + 5.62086\,\epsilon + 24.04$…

`INT[emu_emu-vertex.2L.g1, {1, 0, 1, 1, 0, 0, 0}]` $\to 5.40789 + \frac{1}{\epsilon^2} + \frac{2.09557}{\epsilon} + 12.3348\,\epsilon +$ … $573\,\epsilon^3$

`INT[emu_emu-vertex.2L.g1, {1, 0, 1, 1, 0, 1, 0}]` $\to 0.918812 + \frac{0.5}{\epsilon^2} + \frac{1.92278}{\epsilon} + 11$… $6\,\epsilon^2 + 49.1121\,\epsilon^3$

`INT[emu_emu-vertex.2L.g1, {1, 0, 1, 1, 1, 0, 0}]` $\to 4.20868 + \frac{0.5}{\epsilon^2} + \frac{1.9227}{}$… $6.6088\,\epsilon^2 + 84.8918\,\epsilon^3$

`INT[emu_emu-vertex.2L.g2, {0, 1, 1, 1, 1, 0, 0}]` $\to 2.71285 + \frac{1.}{}$… $83\,\epsilon + 5.01256\,\epsilon^2 - 6.26924\,\epsilon^3$

`INT[emu_emu-vertex.2L.g2, {1, 1, 0, 1, 0, 1, 0}]` $\to 3.41$… $- 1.97735\,\epsilon + 16.0908\,\epsilon^2 - 10.5393\,\epsilon^3$

`INT[emu_emu-vertex.2L.g2, {1, 1, 1, 1, -1, 1, 0}]` … $\frac{}{\epsilon^2} - \frac{0.229261}{\epsilon} - 2.4041\,\epsilon + 20.0453\,\epsilon^2 - 13.5935\,\epsilon^3$

`INT[emu_emu-vertex.2L.g2, {1, 1, 1, 1, 0` … $96 - 6.02761\,\epsilon - 44.469\,\epsilon^2 - 141.167\,\epsilon^3$

`INT[emu_emu-vertex.2L.g4, {1, 1, 1` … $0.921074 + 4.56461\,\epsilon + 21.4268\,\epsilon^2 + 90.1375\,\epsilon^3$

`INT[emu_emu-vertex.2L.g4, {1,` … $\to 0.125417 - \frac{0.19456}{\epsilon} + 1.01107\,\epsilon + 7.13851\,\epsilon^2 + 32.2325\,\epsilon^3$

$$(-63750.9 + 0.\,i) - \frac{8.64\times 10^{-9} + 0.\,i}{\epsilon^4} + \frac{1.16936\times 10^{-6} + 0.\,i}{\epsilon^3} + \frac{89.2 + 0.\,i}{\epsilon^2} + \frac{6.81889 + 0.\,i}{\epsilon}$$

Towards 2—>3 processes

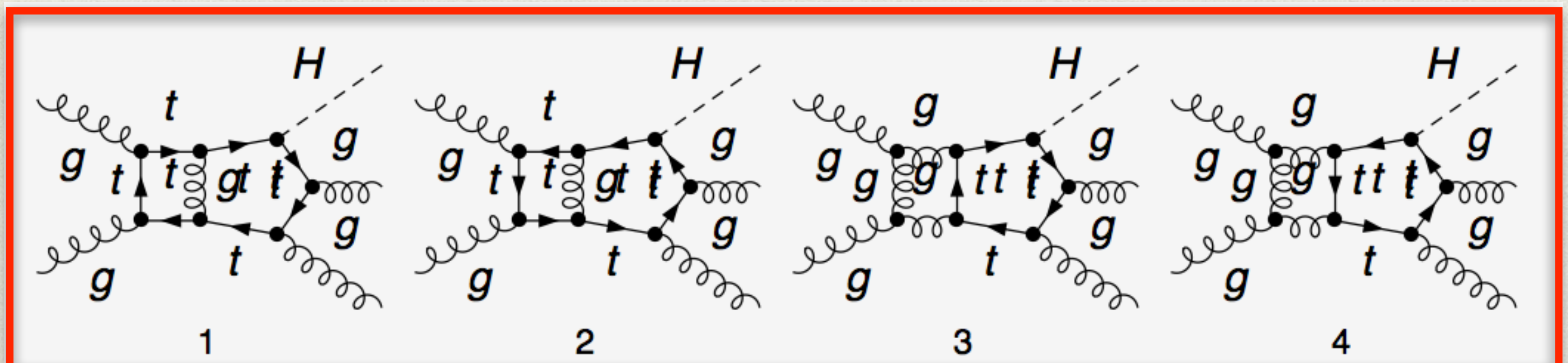
gg —> H g g (Numerical evaluation)

5-point process depending on 7 scales

$$s_{12} = (p_1 + p_2)^2 \qquad s_{23} = (p_2 + p_3)^2 \qquad s_{34} = (p_3 + p_4)^2$$
$$s_{45} = (p_4 + p_5)^2 \qquad s_{51} = (p_5 + p_1)^2 \qquad p_5^2 = m_H^2$$
$$m_t^2$$

+ dimension *d*

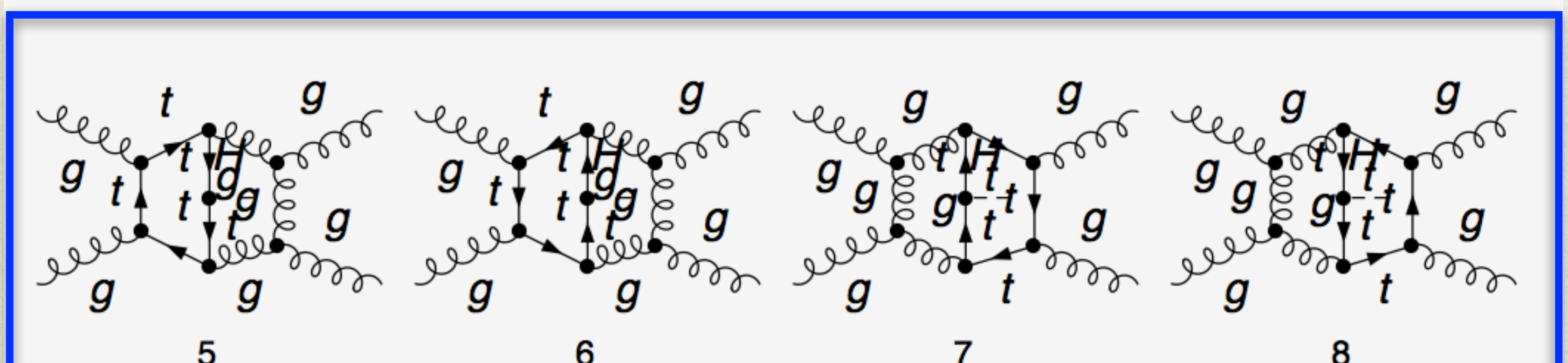
Numerical evaluation for all kin. vars —> Retain *d*-dependence



Input: rank 6 numerator with 1250 monomials

Reduction time ~ 1 min

Output: 1008 contributions



Input: rank 6 numerator with 2747 monomials

Reduction time ~ 2 min

Output: 1169 contributions

Still many things to improve...