Threshold resummation in rapidity for colorless particle production at LHC

Goutam Das
Deutsches Elektronen-Synchroton

Loops and Legs in Quantum Field Theory
St. Goar, May 1, 2018

with V. Ravindran, P. Banerjee, P. K. Dhani

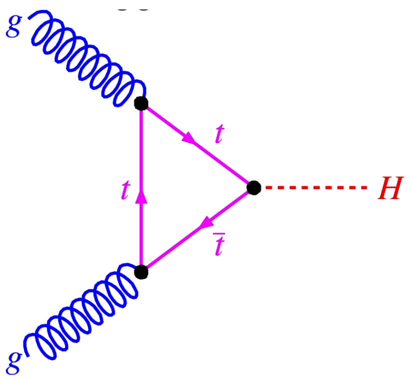
Based on arXiv: 1708.05706 and 1805.XXXX (Preprint: DESY 18-067)

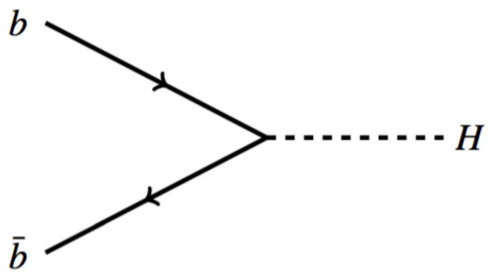
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

DESY

Prologue:Higgs & Drell-Yan Production

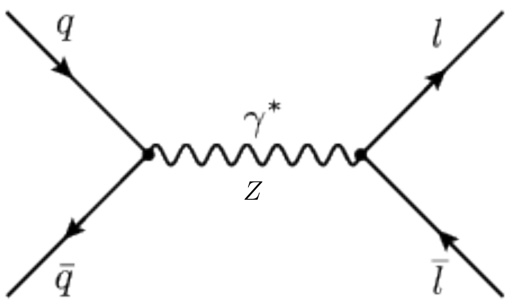
Higgs production:





- Precise prediction is needed to understand the Higgs properties.
- Higgs production is important in the context of BSM search.

DY production:



- Extraction of parton distribution functions (PDFs).
- Important tool to discovery of new physics (BSM).
- Important for electro-weak precision measurement.
  Measurement of W mass and properties.

Prologue:Higgs & Drell-Yan Rapidity

- Inclusive Higgs production in gluon fusion is known upto NNNLO.

  Dawson , Djouadi et al, Graudenz et al, Spira et al, Harlander, Catani et al,
  Harlander, Kilgore ('02), Anastasiou, Melnikov ('02), Ravindran, Smith, var Neerven ('03),
  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger ('14), Anastasiou, Duhr, Dulat, Herzog, Mistlberger ('15)

- Inclusive production of Drell-Yan known upto NNNLO SV.

  Kubar-Andre, Paige, Marseille , Altarelli et al, Humpert et al,
  Matsuura et al, Ahmed et al, Li et al, Catani et al

- Rapidity distribution@NNLO

  Anastasiou et al, Ravindran, Smith, van Neerven ('07),
  Ahmed, Mandal, Rana, Ravindran ('14),

- Going beyond 2-loop is extremely challenging!

  Dulat, Mistlberger, Pelloni ('17)

- Soft+Virtual correction to rapidity distribution

  Ravindran, Smith, Van Neerven('07),
  Ahmed, Mandal, Rana, Ravindran ('14)

- Resummation of soft-gluons improves the fixed order result.

  Catani, de Florian, Grazzini, Nason ('03)

Prologue:Resummation of Rapidity

- Catani and Trentadue approach for x_F distribution:
  → Threshold limit is taken by $z_1 \to 1$ and $z_2 \to 1$ simultaneously
  → Resums Delta and Distributions in both z_1 and z_2 .

  - Resummation for lepton-pair at NLL. Westmark & Owens ('17)

- Laenen and Sterman approach for rapidity:
  → Threshold limit is taken by $z \to 1$.
  → Resums Delta and Distributions in z ,
  only Delta piece in partonic y.

  - Resummation for $W^{\pm}$ production. Mukherjee & Vogelsang ('06)

  - Resummation for DY production. Bolzoni ('06), Bonvini, Forte & Ridolfi ('12)

Plan

- Formalism for the differential soft-virtual cross-section.
  - ✦ Form factor
  - ✦ Soft function
- Soft gluon resummation
- Numerical analysis: Higgs and DY
- Summary

Rapidity Distribution

- Rapidity distribution for colorless particle:

$$\frac{d\sigma^I}{dy} = \hat{\sigma}^I_B \sum_{ab=q,\bar{q},g} \int_{x_1^0}^1 \frac{dz_1}{z_1} \int_{x_2^0}^1 \frac{dz_2}{z_2} \hat{\mathcal{H}}^I_{ab}\left(\frac{x_1^0}{z_1}, \frac{x_2^0}{z_2}, \mu^2\right) \hat{\Delta}^I_{d,ab}\left(z_1, z_2, q^2, \mu^2\right)$$

Drell-Yan production: $\sigma^I = \frac{d\sigma^q\left(\tau, q^2, y\right)}{dq^2}\,.$

Higgs production (gluon/bottom): $\sigma^I = \sigma^{g(b)}\left(\tau, q^2, y\right)\,.$

- Hadronic rapidity $y = \frac{1}{2}\ln\left(\frac{p_2.q}{p_1.q}\right) = \frac{1}{2}\ln\left(\frac{x_1^0}{x_2^0}\right)$ and $\tau = \frac{q^2}{S} = x_1^0 x_2^0$
- Partonic scaling variable $z_i = \frac{x_i^0}{x_i}$

Soft-Virtual Terms

- Perturbative expansion of coefficient function:

$$\Delta_d^I = \delta(1-z_1)\delta(1-z_2) + a_s \Big\{ c_1^{(1)} \delta(1-z_1)\delta(1-z_2)$$
$$+ c_2^{(1)} \left(\frac{\ln(1-z_1)}{1-z_1} \right)_+ + R^{(1)}(z_1, z_2) + z_1 \leftrightarrow z_2 \Big\} + \mathcal{O}(a_s^2)$$

Virtual , Soft radiation → $\delta(1-z_i)$

Soft Radiation → $\left(\frac{\ln(1-z_i)}{(1-z_i)} \right)_+$

Regular terms (Virtual + Real) → $R(z_1, z_2, q^2)$

Soft-Virtual contribution

- Expansion around $z_j \to 1$

$$\Delta^I(z_i) = \Delta^{I,sing}(z_i) + \Delta^{I,hard}(z_i)$$

$\ln(1-z_1), \ln(1-z_2)$

Polynomials in z_1, z_2

$$\Delta^{I,sing}(z_j) \equiv \Delta^{SV}(z_j) = \Delta^{SV}_\delta \delta(1-z_j) + \sum_{i=0}^{2n-1} \Delta^{SV}_i \mathcal{D}_i$$

Soft & Virtual

$$\mathcal{D}_i = \left(\frac{\ln^i(1-z_j)}{1-z_j}\right)_+$$

- Significant contributions from singular terms when $z_j \to 1$

Soft-Virtual Exponentiation

- Form-Factor square normalised SV cross-section

$$\hat{\sigma}_d^{SV}(z_i, q^2) = Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon)^2 |\hat{F}^I(\hat{a}_s, Q^2, \mu^2)|^2 \, \mathcal{C}\exp(2\,\Phi_d^I(\hat{a}_s, q^2, \mu^2, z_i))$$

with $\mathcal{C}e^{f(z_i)} = \delta(1-z_i) + \frac{f(z_i)}{1!} + \frac{1}{2!} f(z_i) \otimes f(z_i) + \cdots$

- Mass factorisation:

DGLAP Kernel

$$\hat{\sigma}_{d,I}^{SV}(z_i, q^2, \mu_F^2) = \Gamma_{II}^T(z_i, \mu_F^2) \otimes \Delta_{d,I}^{SV}(z_i, \mu_F^2) \otimes \Gamma_{II}(z_i, \mu_F^2)$$

Collinear finite

SV Cross-section

- Finite Soft+Virtual cross-section:

$$\Delta^{\rm SV}_{d,I} = \mathcal{C} \exp\left(\Psi^I_d(q^2,\mu_R^2,\mu_F^2,\bar z_1,\bar z_2,\epsilon)\right)\Big|_{\epsilon=0}$$

[Ravindran]

with

$$\begin{aligned}\Psi^I_d = \Big(& \ln\big(Z^I(\hat a_s,\mu_R^2,\mu^2,\epsilon)\big)^2 \\ & + \ln\big|\hat F^I(\hat a_s,Q^2,\mu^2,\epsilon)\big|^2\Big)\delta(\bar z_1)\delta(\bar z_2) \\ & + 2\Phi^I_d(\hat a_s,q^2,\mu^2,\bar z_1,\bar z_2,\epsilon) \\ & - \mathcal{C}\Big(\ln\Gamma_{II}(\hat a_s,\mu^2,\mu_F^2,\bar z_1,\epsilon)\delta(\bar z_2) + (\bar z_1 \leftrightarrow \bar z_2)\Big)\end{aligned}$$

Overall operator renormalisation

Form factor

Soft distribution function

DGLAP kernel

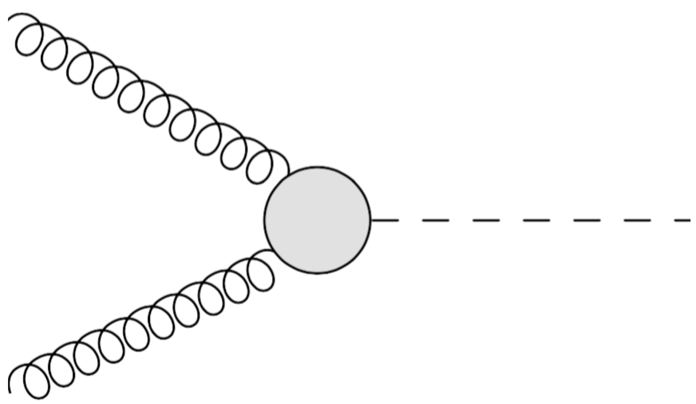
- Total sum has to be **UV and IR finite**!

Form Factor

- Onshell Matrix element of composite operators.

Gluon Form factor

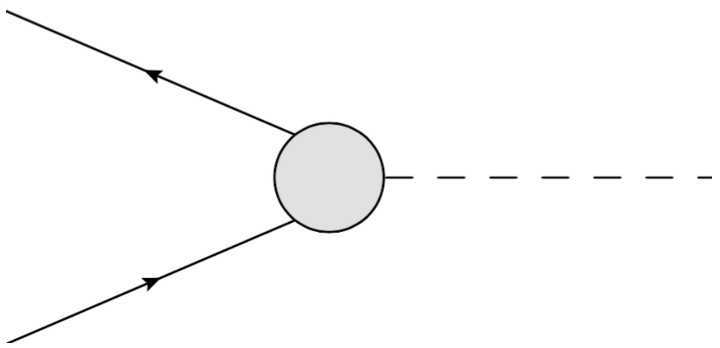
$$< g(p^{'})|G^{a}_{\mu\nu}G^{\mu\nu,a}|g(p) >$$



Higgs production

Quark Form factor

$$< q(p^{'})|\bar{\psi}\gamma^{\mu}\psi|q(p) >$$



DY production

Form Factor

- Form-factor satisfies K-G equation: **Gauge invariance, RG invariance**

$$\frac{d\ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)}{d\ln Q^2} = \frac{1}{2}\left[K^I(\hat{a}_s, \epsilon, \mu_R^2, \mu^2) + G^I(\hat{a}_s, \epsilon, Q^2, \mu_R^2, \mu^2)\right]$$

[Sen, Mueller, Collins, Magnea]

Poles in ϵ
Q-independent

Finite in $\epsilon \to 0$
Q-dependent

- RG invariance of the Form-factor:

$$\mu_R^2 \frac{d}{d\mu_R^2} K^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \epsilon\right) = -\mu_R^2 \frac{d}{d\mu_R^2} G^I\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \epsilon\right) = -A^I(a_s(\mu_R^2))$$

cusp anomalous dimensions

- Casimir duality

$$\frac{A_q}{A_g} = \frac{C_F}{C_A}$$

Valid upto 3-loops

Form Factor

- Solve K^I, G^I : Expand in powers in $a_s(\mu_R^2)$

$$K^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = \hat{a}_s \left(\frac{\mu_R^2}{\mu^2}\right)^{\epsilon/2} S_\epsilon \left(-\frac{2A_1^I}{\epsilon}\right) + \hat{a}_s^2 \left(\frac{\mu_R^2}{\mu^2}\right)^{\epsilon} S_\epsilon^2 \left(\frac{2\beta_0 A_1^I}{\epsilon^2} - \frac{A_2^I}{\epsilon}\right)$$
$$+ \hat{a}_s^3 \left(\frac{\mu_R^2}{\mu^2}\right)^{3\epsilon/2} S_\epsilon^3 \left(-\frac{8\beta_0^2 A_1^I}{3\epsilon^3} + \frac{2\beta_1 A_1^I + 8\beta_0 A_2^I}{3\epsilon^2} - \frac{2A_3^I}{3\epsilon}\right)$$

Found from lower order (RG)

New at this order

$$G^I(\hat{a}_s, Q^2, \mu_R^2, \mu^2, \epsilon) = \sum_i^\infty a_s^i(Q^2) G_i^I(\epsilon) + \sum_{i=1}^\infty \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2}\right)^{i\epsilon/2} \left[\left(\frac{Q^2}{\mu_R^2}\right)^{i\epsilon/2} - 1\right] S_\epsilon^i K^{I,(i)}(\epsilon)$$

Soft Ano. Dim.

$$G_1^I = 2(B_1^I - \delta_{I,g}\beta_0) + f_1^I + \sum_{j=1}^\infty \epsilon^j g_1^{I,j}$$
$$G_2^I = 2(B_2^I - 2\delta_{I,g}\beta_1) + f_2^I - 2\beta_0 g_1^{I,1} + \sum_{j=1}^\infty \epsilon^j g_2^{I,j}$$

$$\frac{f_q}{f_g} = \frac{C_F}{C_A}$$ upto 3-loop

Coll. Ano. Dim. | **UV Ano. Dim.** | **From $Z(a_s(\mu_R^2))$** | **Explicit computation**

Form Factor

- Solution upto 3-loops:

$$\ln \hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\epsilon}{2}} S_\epsilon^i \hat{\mathcal{L}}_F^{I,(i)}(\epsilon)$$

$$\hat{\mathcal{L}}_F^{I,(1)}(\epsilon) = \frac{1}{\epsilon^2}\left[-2A_1^I\right] + \frac{1}{\epsilon}\left[G_1^I(\epsilon)\right]$$

$$\hat{\mathcal{L}}_F^{I,(2)}(\epsilon) = \frac{1}{\epsilon^3}\left[\beta_0 A_1^I\right] + \frac{1}{\epsilon^2}\left[-\frac{1}{2}A_2^I - \beta_0 G_1^I(\epsilon)\right] + \frac{1}{\epsilon}\left[\frac{1}{2}G_2^I(\epsilon)\right]$$

$$\hat{\mathcal{L}}_F^{I,(3)}(\epsilon) = \frac{1}{\epsilon^4}\left[-\frac{8}{9}\beta_0^2 A_1^I\right] + \frac{1}{\epsilon^3}\left[\frac{2}{9}\beta_1 A_1^I + \frac{8}{9}\beta_0 A_2^I + \frac{4}{3}\beta_0^2 G_1^I(\epsilon)\right]$$
$$+ \frac{1}{\epsilon^2}\left[-\frac{2}{9}A_3^I - \frac{1}{3}\beta_1 G_1^I(\epsilon) - \frac{4}{3}\beta_0 G_2^I(\epsilon)\right] + \frac{1}{\epsilon}\left[\frac{1}{3}G_3^I(\epsilon)\right]$$

- All terms can be found, only computation of finite piece is needed.

SV Cross-section

- Soft+Virtual cross-section:

$$\Delta^{SV}_{d,I} = \mathcal{C} \exp\left(\Psi^I_d(q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon)\right)\Big|_{\epsilon=0}$$

with

$$\Psi^I_d = \Big(\ln(Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon))^2 + \ln\big|\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)\big|^2\Big)\delta(\bar{z}_1)\delta(\bar{z}_2) + 2\Phi^I_d(\hat{a}_s, q^2, \mu^2, \bar{z}_1, \bar{z}_2, \epsilon) - \mathcal{C}\Big(\ln\Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, \bar{z}_1, \epsilon)\delta(\bar{z}_2) + (\bar{z}_1 \leftrightarrow \bar{z}_2)\Big)$$

Overall operator renormalisation

Form factor

Soft distribution function

DGLAP kernel

Collinear RGE

- RGE for DGLAP kernel Γ :

$$\mu_F^2 \frac{d}{d\mu_F^2}\Gamma(z_i,\mu_F^2,\epsilon) = \frac{1}{2}P(z_i,\mu_F^2)\otimes\Gamma(z_i,\mu_F^2,\epsilon)$$

Altarelli-Parisi Splitting functions

- In SV cross-section, diagonal part contributes

$$P_{II}^i(z_j) = 2\left[B_{i+1}^I \delta(1-z_j) + A_{i+1}^I \mathcal{D}_0(z_j)\right] + P_{\mathrm{reg},II}^i(z_j)$$

Coll. Ano. Dim.

Cusp. Ano. Dim.

Γ is known upto $\mathcal{O}(a_s^4)$

[Moch, Vogt, Vermaseren]

UV RGE

- RGE for overall operator renormalisation constant Z:

$$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = \sum_{i=1}^{\infty} a_s^i(\mu_R^2) \gamma_{i-1}^I$$

UV Ano. Dim.

- γ^g, γ^b are known upto $\mathcal{O}(a_s^3)$

- For Drell-Yan: $\gamma^q = 0$ ⇒ $Z^q(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = 1$

SV Cross-section

- Soft+Virtual cross-section:

$$\Delta^{\rm SV}_{d,I} = \mathcal{C} \exp\left(\Psi^I_d(q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon)\right)\big|_{\epsilon=0}$$

with

$$\Psi^I_d = \Big(\ln(Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon))^2 + \ln |\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)|^2 \Big) \delta(\bar{z}_1)\delta(\bar{z}_2) + 2\Phi^I_d(\hat{a}_s, q^2, \mu^2, \bar{z}_1, \bar{z}_2, \epsilon) - \mathcal{C}\Big(\ln\Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, \bar{z}_1, \epsilon)\delta(\bar{z}_2) + (\bar{z}_1 \leftrightarrow \bar{z}_2) \Big)$$

Overall operator renormalisation

Form factor

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DGLAP kernel

Soft Function

- RGE for the Soft function:

$$\mu_R^2 \frac{d}{d\mu_R^2} \Phi_d^I(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = 0$$

- Demand finiteness in $\lim_{\epsilon \to 0} \Psi_d^I(z_i, q^2, \epsilon)$

K-G type equation for Soft function

[Ravindran ('06,'07)]

$$q^2 \frac{d}{dq^2} \Phi_d^I = \frac{1}{2} \left[\overline{K}_d^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \epsilon \right) + \overline{G}_d^I \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z_1, z_2, \epsilon \right) \right]$$

Poles in ϵ

Finite in $\epsilon \to 0$

Soft Function

- One possible ansatz:

$$\Phi^I_d(\hat{a}_s, q^2, \mu^2, z_1, z_2, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z_1)(1-z_2)}{\mu^2} \right)^{i\frac{\epsilon}{2}} S^i_\epsilon \left(\frac{(i\epsilon)^2}{4(1-z_1)(1-z_2)} \right) \hat{\phi}^{I,(i)}_d(\epsilon),$$

unknown

[Ravindran, Smith, van Neerven]

- Solution: $\hat{\phi}^{I,(i)}_d(\epsilon) = \hat{\mathcal{L}}^{I,(i)}_F(\epsilon) \left(A^I \to -A^I, G^I(\epsilon) \to \overline{\mathcal{G}}^I_d(\epsilon) \right)$

$$\overline{\mathcal{G}}^I_{d,1}(\epsilon) = -f^I_1 + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}^{I,k}_{d,1}$$

$$\overline{\mathcal{G}}^I_{d,2}(\epsilon) = -f^I_2 - 2\beta_0 \overline{\mathcal{G}}^{I,1}_{d,1} + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}^{I,k}_{d,2}$$

$$\overline{\mathcal{G}}^I_{d,3}(\epsilon) = -f^I_3 - 2\beta_1 \overline{\mathcal{G}}^{I,1}_{d,1} - 2\beta_0 \left(\overline{\mathcal{G}}^{I,1}_{d,2} + 2\beta_0 \overline{\mathcal{G}}^{I,2}_{d,1} \right) + \sum_{k=1}^{\infty} \epsilon^k \overline{\mathcal{G}}^{I,k}_{d,3}$$

Explicitly compute

Soft Function

- Find differential soft function from inclusive one:

$$\int dy \;\; \frac{d\sigma^I}{dy}(y,\tau) = \sigma^I(\tau)$$

Easy to work in Mellin space

$$\int_0^1 dx_1^0 \int_0^1 dx_2^0 \left(x_1^0 x_2^0\right)^{N-1} \frac{d\sigma^I}{dy} = \int_0^1 d\tau \; \tau^{N-1} \; \sigma^I$$

Need Inclusive soft function !!

known upto $\mathcal{O}(a_s^3)$

Inclusive $\Phi^I(q^2, z, \epsilon)$ → **Rapidity** $\Phi^I_d(q^2, z_1, z_2, \epsilon)$

$$\frac{\bar{\mathcal{G}}^{q,k}_{d,i}}{\bar{\mathcal{G}}^{g,k}_{d,i}} = \frac{C_F}{C_A} \implies \frac{\phi^q_d}{\phi^g_d} = \frac{C_F}{C_A}$$ upto $\mathcal{O}(a_s^3)$

Soft distributions are universal

[Ravindran, Smith, van Neerven, Ahmed, Mandal, Rana, Ravindran]

Soft-gluon resummation

- Soft-virtual cross-section:

$$\Delta^{\mathrm{SV}}_{d,I} = \mathcal{C} \exp\left(\Psi^I_d(q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon)\right)\Big|_{\epsilon=0}$$

$$\begin{aligned}\Psi^I_d = & \Big(\ln\big(Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon)\big)^2 \\ & + \ln \big|\hat{F}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)\big|^2 \Big)\delta(\bar{z}_1)\delta(\bar{z}_2) \\ & + 2\Phi^I_d(\hat{a}_s, q^2, \mu^2, \bar{z}_1, \bar{z}_2, \epsilon) \\ & - \mathcal{C}\Big(\ln \Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, \bar{z}_1, \epsilon)\delta(\bar{z}_2) + (\bar{z}_1 \leftrightarrow \bar{z}_2)\Big)\end{aligned}$$

Overall operator renormalisation

Form factor

Soft distribution function

DGLAP kernel

Soft-gluon resummation

$$\Delta_{d,I}^{\mathrm{SV}} = \mathcal{C} \exp\left(\Psi_d^I(q^2, \mu_R^2, \mu_F^2, \bar{z}_1, \bar{z}_2, \epsilon)\right)\Big|_{\epsilon=0}$$

$$\Psi_d^I = \delta(\bar{z}_2)\left(\frac{1}{\bar{z}_1}\left\{\int_{\mu_F^2}^{q^2\bar{z}_1}\frac{d\lambda^2}{\lambda^2}A_I(a_s(\lambda^2)) + D_d^I(a_s(q^2\bar{z}_1))\right\}\right)_+$$
$$+\frac{1}{2}\left(\frac{1}{\bar{z}_1\bar{z}_2}\left\{A^I(a_s(z_{12})) + \frac{dD_d^I(a_s(z_{12}))}{d\ln z_{12}}\right\}\right)_+$$
$$+\frac{1}{2}\delta(\bar{z}_1)\delta(\bar{z}_2)\ln\left(g_{d,0}^I(a_s(\mu_F^2))\right) + \bar{z}_1 \leftrightarrow \bar{z}_2$$

Related to $\bar{\mathcal{G}}_d^I$

$a_s(z_{12}) = a_s(q^2\bar{z}_1\bar{z}_2)$

Soft-gluon resummation

- $\Delta_d^{I,SV}$ ➡ Distributions $\left(\frac{\ln(1-z_i)}{(1-z_i)}\right)_+$ and Delta $\delta(1-z_i)$
- Threshold limits: $z_1 \to 1, z_2 \to 1$

Large logarithms !!

Fixed order becomes unreliable!!

- Sum all threshold large logs to all orders in perturbation theory

Threshold resummation

Soft-gluon resummation

- Sudakov type exponentiation of soft-function.
- Resummation in conjugate space!
  Amplitude factorises both in z-space and N-space.
  Phase space factorises only in N-space.
- Logarithmic enhanced contribution in N-space also contains subleading terms when transformed into z-space.

Soft-gluon resummation

- Double Mellin transformation:

$$\tilde{\Delta}_d^{I,\mathrm{SV}}(\omega) = \int_0^1 dz_1 z_1^{N_1-1} \int_0^1 dz_2 z_2^{N_2-1} \Delta_d^{I,\mathrm{SV}}(z_1, z_2)$$

$$\omega = a_s \beta_0 \ln\left(\overline{N}_1 \overline{N}_2\right)$$

$$\overline{N}_i = e^{\gamma_E} N_i$$

$$\delta(1-z_i) \to 1 \qquad \frac{\ln^i(1-z_j)}{1-z_j} \to \ln^{i+1} \overline{N}_j$$

- SV limits: $z_1 \to 1$, $z_2 \to 1$ $\Rightarrow$ $\overline{N}_1 \to \infty$, $\overline{N}_2 \to \infty$

- Resummed rapidity distribution:

$$\tilde{\Delta}_d^{\mathrm{SV},I}(\omega) = \tilde{g}_{d,0}^I(a_s) \exp\left(g_d^I(a_s, \omega)\right)$$

$\overline{N}_i$ **independent**

Logarithmically enhanced

Soft-gluon resummation

For any colorless particle,

$$\overline{g}^I_{d,1} = \overline{A}^I_1 \frac{1}{\omega}\Big(\omega + (1-\omega)\ln(1-\omega)\Big),$$

$$\overline{g}^I_{d,2} = \omega\left(\overline{A}^I_1\overline{\beta}_1 - \overline{A}^I_2\right) + \ln(1-\omega)\left(\overline{A}^I_1\overline{\beta}_1 + \overline{D}^I_{d,1} - \overline{A}^I_2\right) + \frac{1}{2}\ln^2(1-\omega)\overline{A}^I_1\overline{\beta}_1 + L_{qr}\ln(1-\omega)\overline{A}^I_1 + L_{fr}\omega\overline{A}^I_1,$$

$$\overline{g}^I_{d,3} = -\frac{\omega}{2}\overline{A}^I_3 - \frac{\omega}{2(1-\omega)}\left(-\overline{A}^I_3 + (2+\omega)\overline{\beta}_1\overline{A}^I_2 + \left((\omega-2)\overline{\beta}_2 - \omega\overline{\beta}^2_1 - 2\zeta_2\right)\overline{A}^I_1 + 2\overline{D}^I_{d,2} - 2\overline{\beta}_1\overline{D}^I_{d,1}\right)$$
$$-\ln(1-\omega)\left(\frac{\overline{\beta}_1}{(1-\omega)}\left(\overline{A}^I_2 - \overline{D}^I_{d,1} - \overline{A}^I_1\overline{\beta}_1\omega\right) - \overline{A}^I_1\overline{\beta}_2\right) + \frac{\ln^2(1-\omega)}{2(1-\omega)}\overline{A}^I_1\overline{\beta}^2_1 + L_{fr}\overline{A}^I_2\omega - \frac{1}{2}L^2_{fr}\overline{A}^I_1\omega$$
$$-L_{qr}\frac{1}{(1-\omega)}\left(\left(\overline{A}^I_2 - \overline{D}^I_{d,1}\right)\omega - \overline{A}^I_1\overline{\beta}_1\left(\omega + \ln(1-\omega)\right)\right) + \frac{1}{2}L^2_{qr}\frac{\omega}{(1-\omega)}\overline{A}^I_1.$$

where $\overline{g}^I_{d,1} = g^I_{d,1},\ \overline{g}^I_{d,i+2} = g^I_{d,i+2}/\beta^i_0,\ \overline{A}^I_i = A^I_i/\beta^i_0,\ \overline{D}^I_{d,i} = D^I_{d,i}/\beta^i_0,$

$$\overline{\beta}_i = \beta_i/\beta^{i+1}_0, L_{fr} = \ln\left(\mu^2_F/\mu^2_R\right), L_{qr} = \ln\left(q^2/\mu^2_R\right).$$

Also, $$\ln(g^I_{d,0}) = \sum_{i=0}^{\infty} a^i_s l^{I,i}_{g_0},$$

$$l^{I,0}_{g_0} = 1,$$

$$l^{I,(1)}_{g_0} = 2G^{I,1}_1 + 2\overline{\mathcal{G}}^{I,1}_{d,1} + 4A^I_1\zeta_2 - 2L_{fr}B^I_1 + 2L_{qr}\left(B^I_1 - \gamma^I_0\right),$$

$$l^{I,(2)}_{g_0} = G^{I,1}_2 + \overline{\mathcal{G}}^{I,1}_{d,2} + 2\beta_0\left(G^{I,2}_1 + \overline{\mathcal{G}}^{I,2}_{d,1}\right) + 2\zeta_2\left(2A^I_2 + \beta_0\left(3B^I_1 + 2f^I_1 - 3\gamma^I_0\right)\right) + \frac{2}{3}A^I_1\beta_0\zeta_3 - 2L_{fr}B^I_2 + L^2_{fr}B^I_1\beta_0$$
$$+L_{qr}\left(2B^I_2 - 2\gamma^I_1 - \beta_0\left(2G^{I,1}_1 + 2\overline{\mathcal{G}}^{I,1}_{d,1} + 4A^I_1\zeta_2\right)\right) + L^2_{qr}\beta_0\left(-B^I_1 + \gamma^I_0\right).$$

Soft-gluon resummation

- Resummed coefficient:

$$g_d^I(a_s,\omega) = g_{d,1}^I(\omega)\ln(\overline{N}_1\overline{N}_2) + \sum_{i=0}^{\infty} a_s^i g_{d,i+2}^I(\omega)$$

	LL	NLL	NNLL
$\mathcal{O}(a_s)$	$\ln^2(\bar{N}_1\bar{N}_2)$	$\ln(\bar{N}_1\bar{N}_2)$	
$\mathcal{O}(a_s^2)$	$\ln^3(\bar{N}_1\bar{N}_2)$	$\ln^2(\bar{N}_1\bar{N}_2)$	$\ln(\bar{N}_1\bar{N}_2)$
$\mathcal{O}(a_s^3)$	$\ln^4(\bar{N}_1\bar{N}_2)$	$\ln^3(\bar{N}_1\bar{N}_2)$	$\ln^2(\bar{N}_1\bar{N}_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Resummed logs:	$a_s^m \ln^{m+1}(\bar{N}_1\bar{N}_2)$	$a_s^m \ln^m(\bar{N}_1\bar{N}_2)$	$a_s^{m+1} \ln^m(\bar{N}_1\bar{N}_2)$
Resummed coeff.:	$g_{d,1}^I \ln(\bar{N}_1\bar{N}_2)$	$g_{d,2}^I$	$a_s g_{d,3}^I$

Matching : DY rapidity

- Matching Fixed order (NNLO) with resum (NNLL) for DY:

$$\frac{d^2\sigma^{q,\mathrm{res}}}{dq^2dy} = \frac{d^2\sigma^{q,\mathrm{f.o}}}{dq^2dy} + \sigma_B^q \int_{c_1-i\infty}^{c_1+i\infty} \frac{dN_1}{2\pi i} \int_{c_2-i\infty}^{c_2+i\infty} \frac{dN_2}{2\pi i} e^{y(N_2-N_1)} \left(\sqrt{\tau}\right)^{2-N_1-N_2} \tilde{f}_q(N_1)\tilde{f}_q(N_2) \left[\tilde{\Delta}_{d,q}^{\mathrm{SV}} - \tilde{\Delta}_{d,q}^{\mathrm{SV}}\Big|_{\mathrm{trunc}}\right]$$

Fixed order

Resummed contribution

Resum truncated to FO

$$\tilde{\Delta}_{d,q}^{\mathrm{SV}} - \tilde{\Delta}_{d,q}^{\mathrm{SV}}\Big|_{\mathrm{trunc}} = \tilde{\Delta}_{d,q}^{\mathrm{SV}}\Big|_{\geq \mathcal{O}(a_s^{n+1})}$$

Truncated at $\mathcal{O}(a_s^n)$

- Inverse Mellin transformation by Minimal Prescription scheme.

[Catani, Mangano, Nason, Trentadue ('96)]

- For NNLO+NNLL prediction:

$\frac{d^2\sigma^{q,f.o}}{dq^2dy}$ **: Upto NNLO** $\tilde{\Delta}_{d,q}^{SV}$ **: Upto NNLL**

Phenomenology: Higgs

$$\sqrt{S} = 13\ TeV$$

$$M_H = 125\ GeV$$

$$m_t = 173\ GeV$$

$$n_f = 5$$

PDF = MMHT 2014

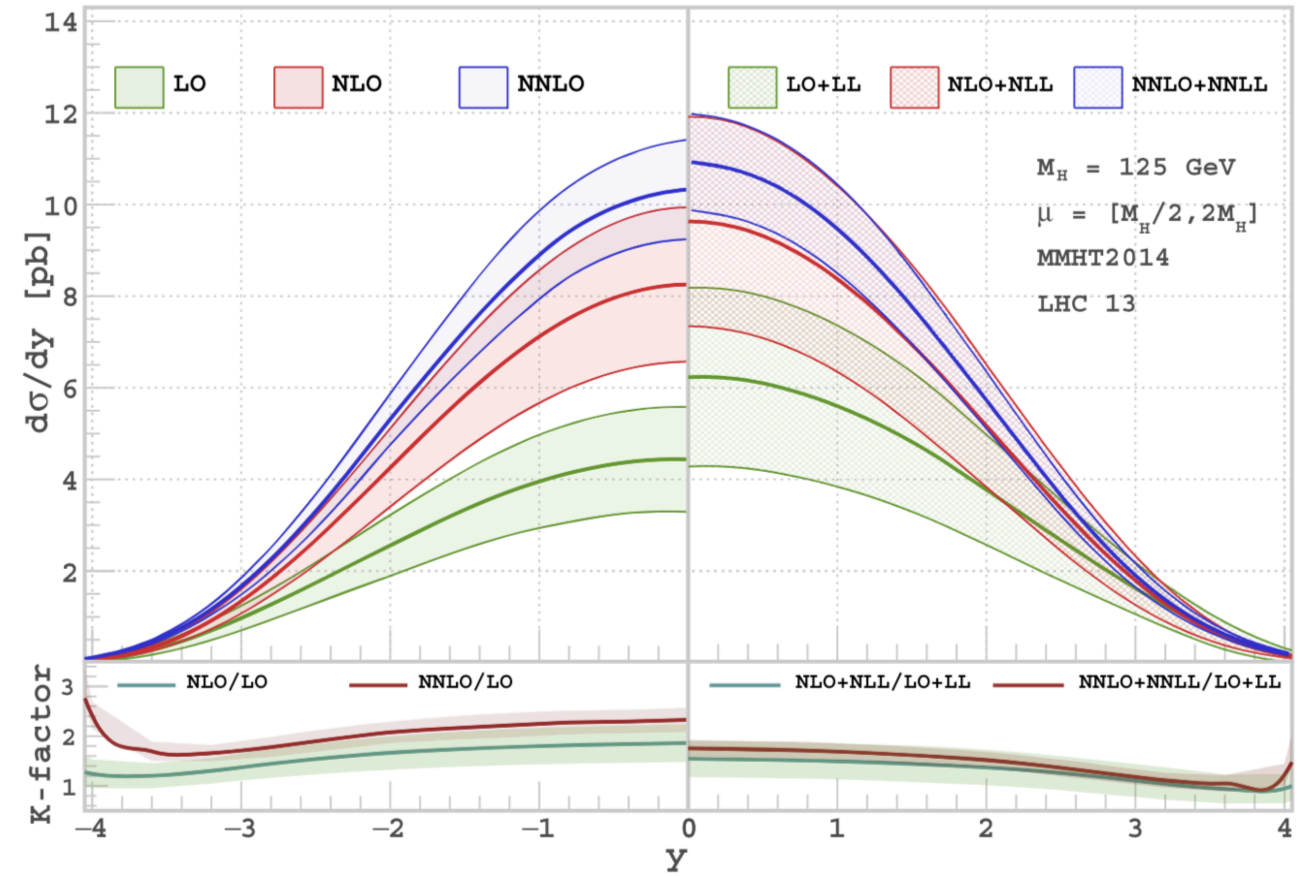
$\dfrac{d\sigma^{g,fo}}{dy}$ from FEHIP

[Anastasiou, Melnikov, Petriello ('05)]

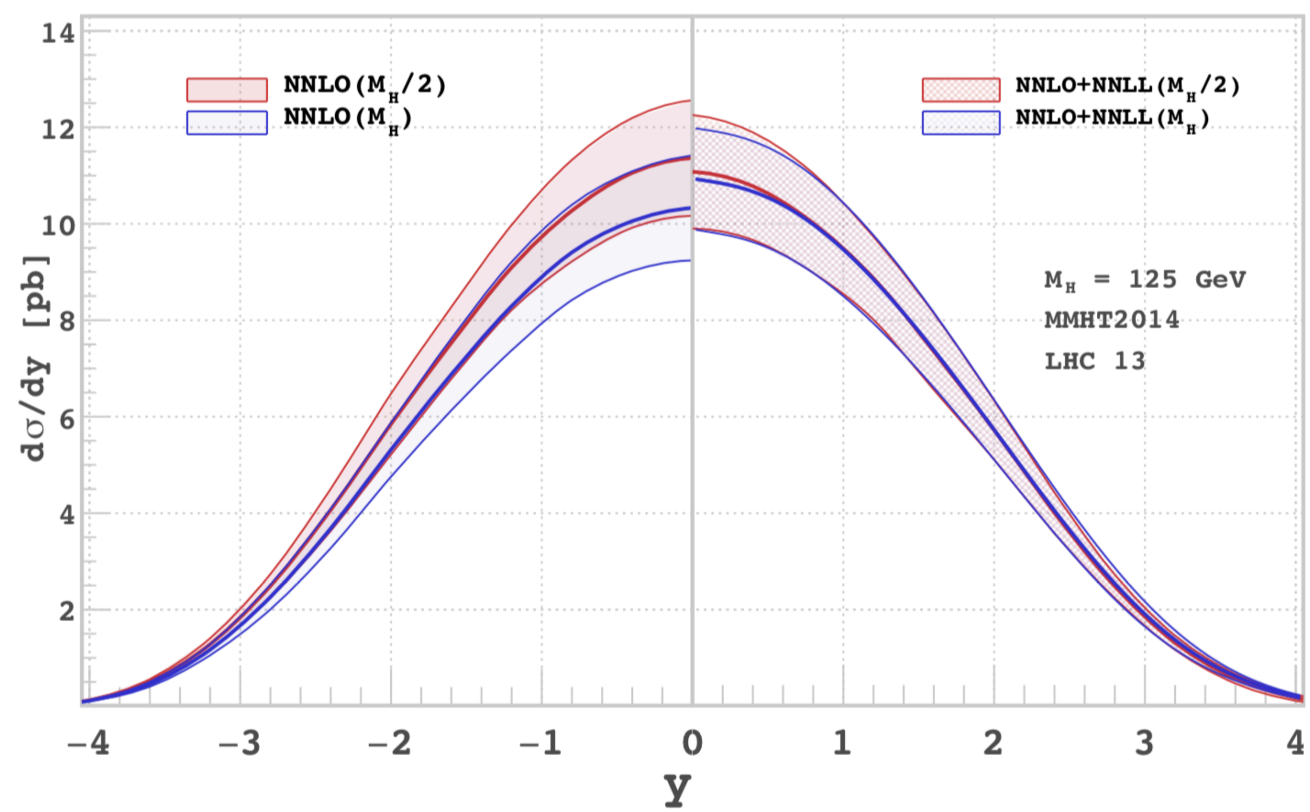
Higgs rapidity

- Better perturbative convergence.
- Magnitude and sign of resummed contribution varies depending on the order, y and scale.

Banerjee, GD, Dhani, Ravindran ('17)



- Resummed prediction is stable with choice of central scale.



Higgs rapidity

- F.O. and resummed results for few benchmark values of y

y	LO	LO + LL	NLO	NLO + NLL	NNLO	NNLO + NNLL	NNLO + NNNLL
0.0	4.435 ± 1.145	6.231 ± 1.950	8.255 ± 1.684	9.632 ± 2.286	10.329 ± 1.088	10.938 ± 1.050	10.517 ± 0.820
0.8	4.134 ± 1.067	5.833 ± 1.831	7.517 ± 1.530	8.820 ± 2.124	9.407 ± 0.988	9.992 ± 1.025	9.641 ± 0.718
1.6	3.189 ± 0.819	4.630 ± 1.468	5.522 ± 1.117	6.611 ± 1.676	6.877 ± 0.744	7.380 ± 0.849	7.045 ± 0.563
2.4	1.904 ± 0.492	2.887 ± 0.942	2.985 ± 0.597	$3.715 \pm .998$	3.683 ± 0.410	4.040 ± 0.501	3.821 ± 0.305

Banerjee, GD, Dhani, Ravindran ('17)

- Corrections from LL varies between **40%** to **50%.**
- At NLL it is **17%** to **24%**; at NNLL **6%** to **10%.**
- We also predict NNLO+NNNLL which is however within uncertainty band of NNLO+NNLL.

Phenomenology: DY

$$\sqrt{S} = 14 \text{ TeV}$$

$$Q = M_Z$$

$$\frac{d^2\sigma^{q,f.o}}{dq^2 dy}$$ From **Vrap** [Anastasiou, Dixon, Melnikov, Petriello]

Comparison Table

M-F : Mellin-Fourier

From **ReDY** [Bonvini]

y	$(\frac{\mu_R}{M_Z}, \frac{\mu_F}{M_Z})$	LO	LL_{M-F}	LL_{M-M}	NLO	NLL_{M-F}	NLL_{M-M}	NNLO	$NNLL_{M-F}$	$NNLL_{M-M}$
0.0	$(2,2)$	72.626	+0.988	+3.219	73.450	+1.639	+1.796	70.894	+ 0.630	+0.646
0.0	$(2,1)$	63.197	+0.768	+2.595	70.625	+0.761	+1.017	70.360	+0.292	+0.317
0.0	$(1,2)$	72.626	+1.095	+3.577	73.535	+1.912	+1.760	70.509	+0.510	+0.395
0.0	$(1,1)$	63.197	+0.851	+2.887	71.395	+0.858	+0.901	70.537	+0.248	+0.167
0.0	$(1,1/2)$	53.241	+0.621	+2.216	67.581	+ 0.156	+0.140	69.834	- 0.001	- 0.094
0.0	$(1/2,1)$	63.197	+0.953	+3.278	72.355	+0.945	+0.681	70.266	+0.091	- 0.015
0.0	$(1/2,1/2)$	53.241	+0.695	+2.504	69.259	+0.102	- 0.154	70.283	- 0.039	- 0.146

M-M : Mellin-Mellin

- Input: Same PDF at all orders.

- **Significant change** at LO+LL level.
  At NLO+NLL, NNLO+NNLL non-trivial change at coefficient level.

Banerjee, GD, Dhani, Ravindran (to be online)

Rapidity Distribution

- Central scale:

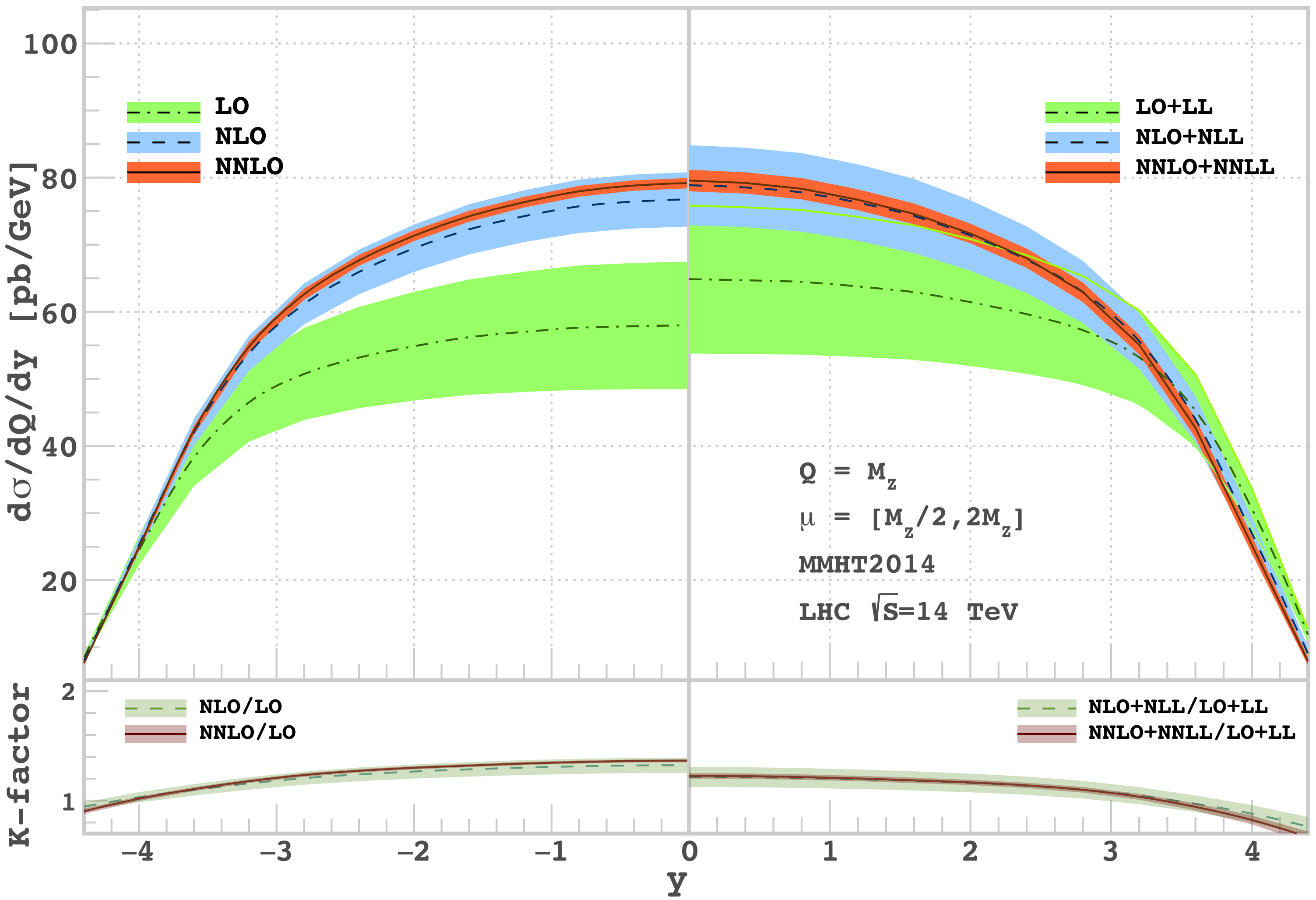
$$(\mu_r^C, \mu_f^C) = (1,1) m_Z$$

- Scale uncertainty:

$$(\mu_r, \mu_f) = (\kappa_1, \kappa_2) \otimes (\mu_r^C, \mu_f^C)$$

$$1/2 \leq \kappa_1/\kappa_2 \leq 2$$

- Scale uncertainty at NNLO: **1.96%**
- Scale uncertainty at NNLO+NNLL: **4.03%**



Better perturbative convergence!!

Expected better scale uncertainty?

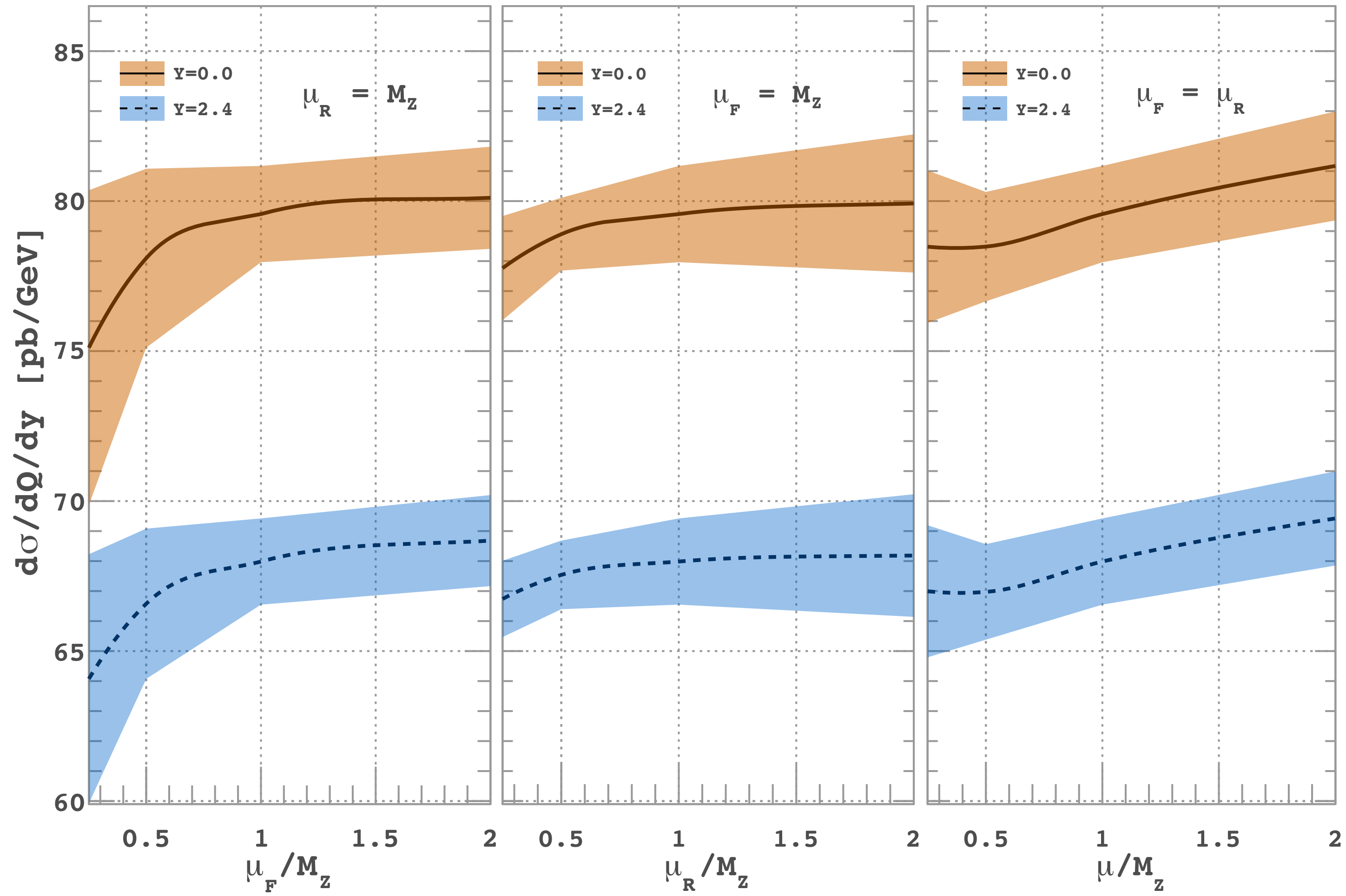
Central Scale Choice

- Scale variation around the central scale by:

$$(\mu_r, \mu_f) = (\kappa_1, \kappa_2) \otimes (\mu_r^C, \mu_f^C)$$

with

$$1/2 \le \kappa_1/\kappa_2 \le 2$$



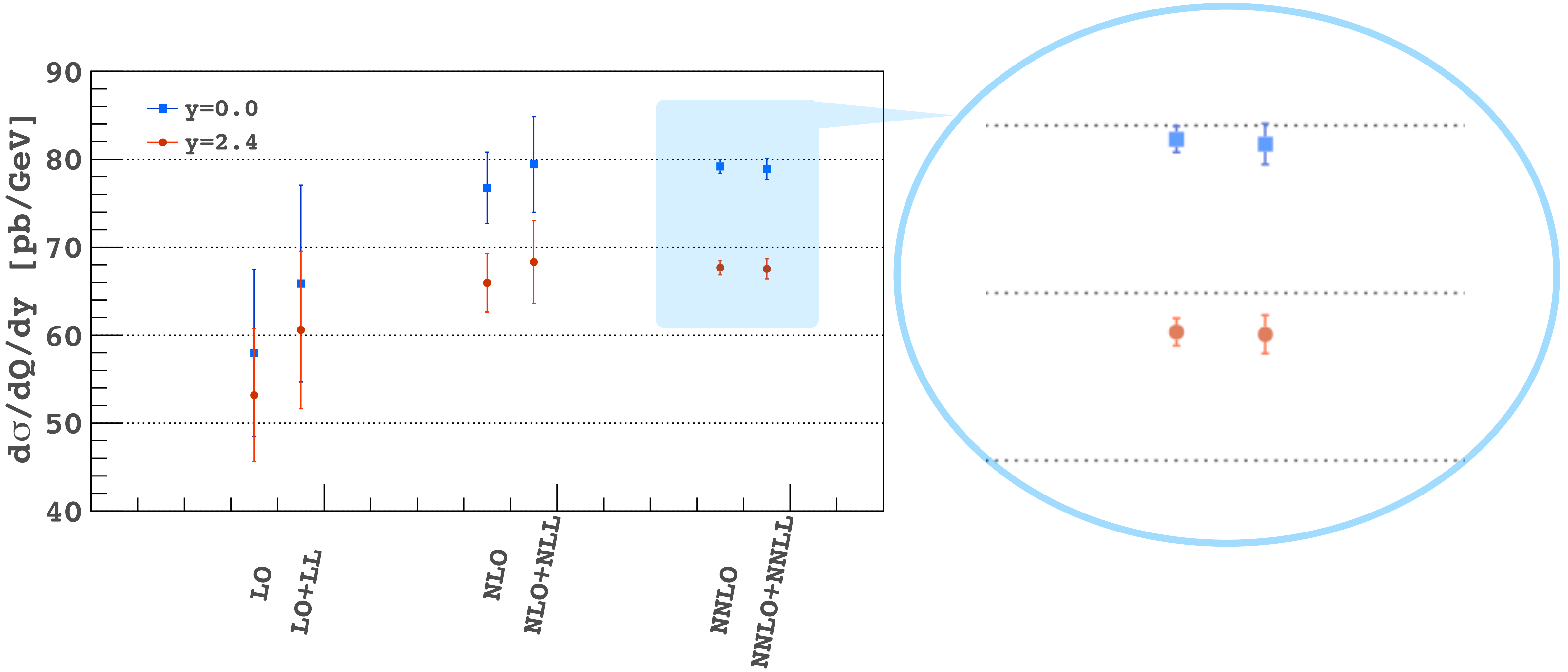
Best prediction
$(\mu_r^C, \mu_f^C) = (m_Z/2, m_Z)$

SCET resummation
$(\mu_r^C, \mu_f^C) = (1, 1/2)m_Z$

[Ebert, Michel, Tackmann]

Best Prediction: FO vs RESUM

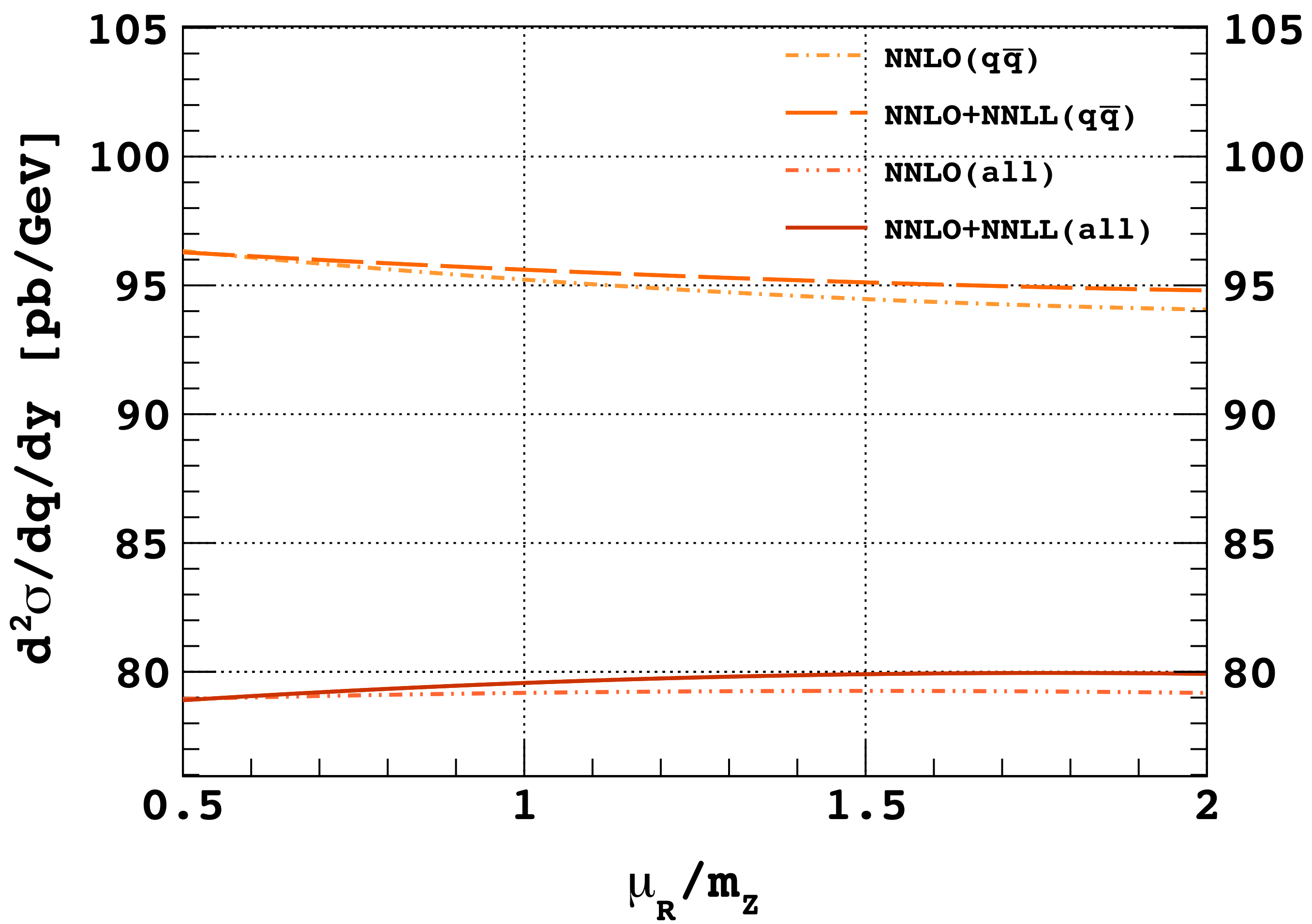
•



- Scale uncertainty at NNLO ($(\mu_r^C, \mu_f^C) = (1,1)m_Z$): **1.96%**
- Scale uncertainty at NNLO+NNLL ($(\mu_r^C, \mu_f^C) = (1/2,1)m_Z$): **3.06%**

Renorm. scale dependence

- Only resums large Distributions arising in $q\bar{q}$ channel.
  - NNLO ($q\bar{q}$): **2.36%**
  - NNLO+NNLL ($q\bar{q}$): **1.53%**



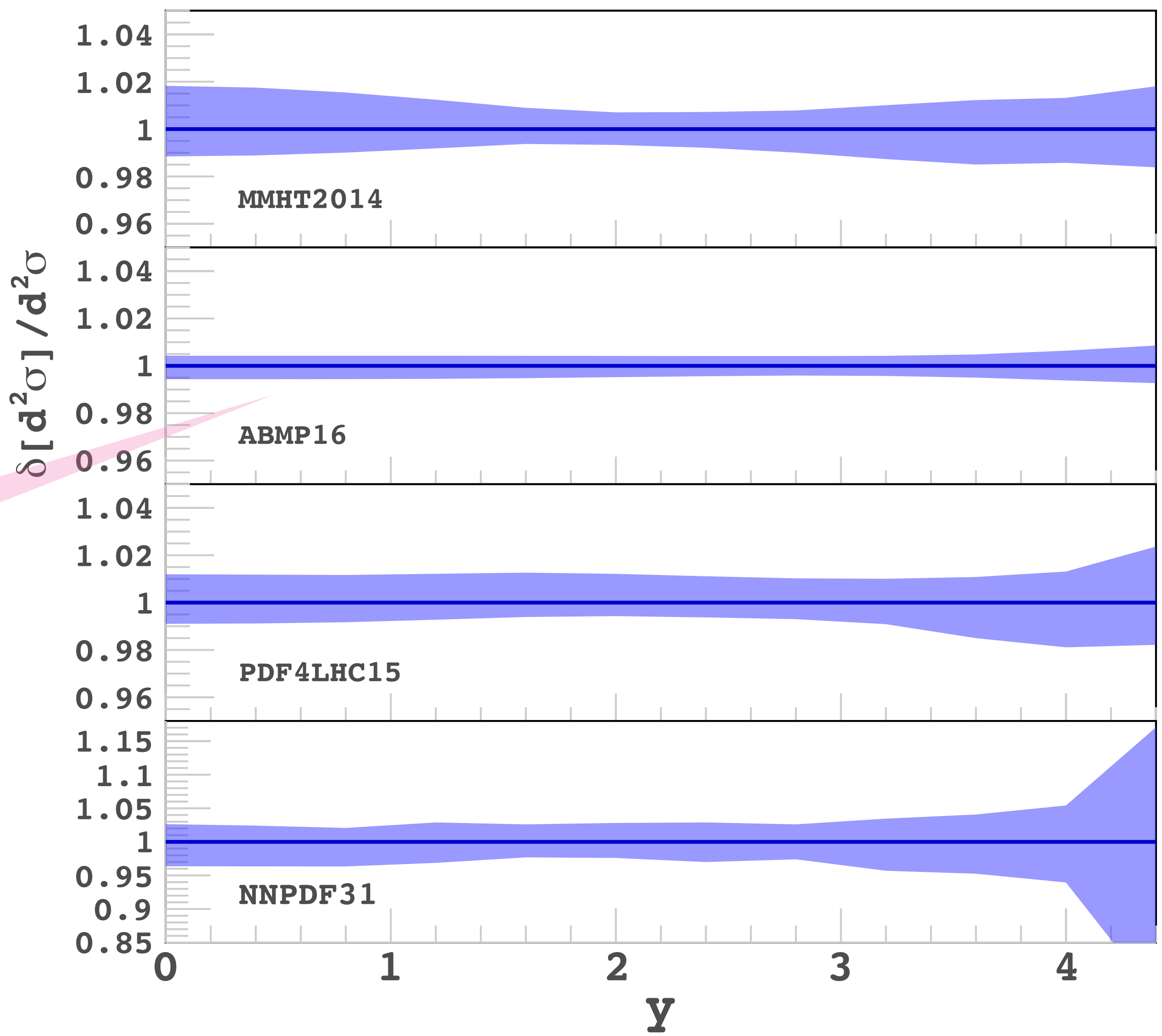
- Strong cancellation of scale dependence among different channels at NNLO.

Accidental?

PDF Uncertainty

- PDF uncertainties are within **2%**.

Includes latest NOMAD data



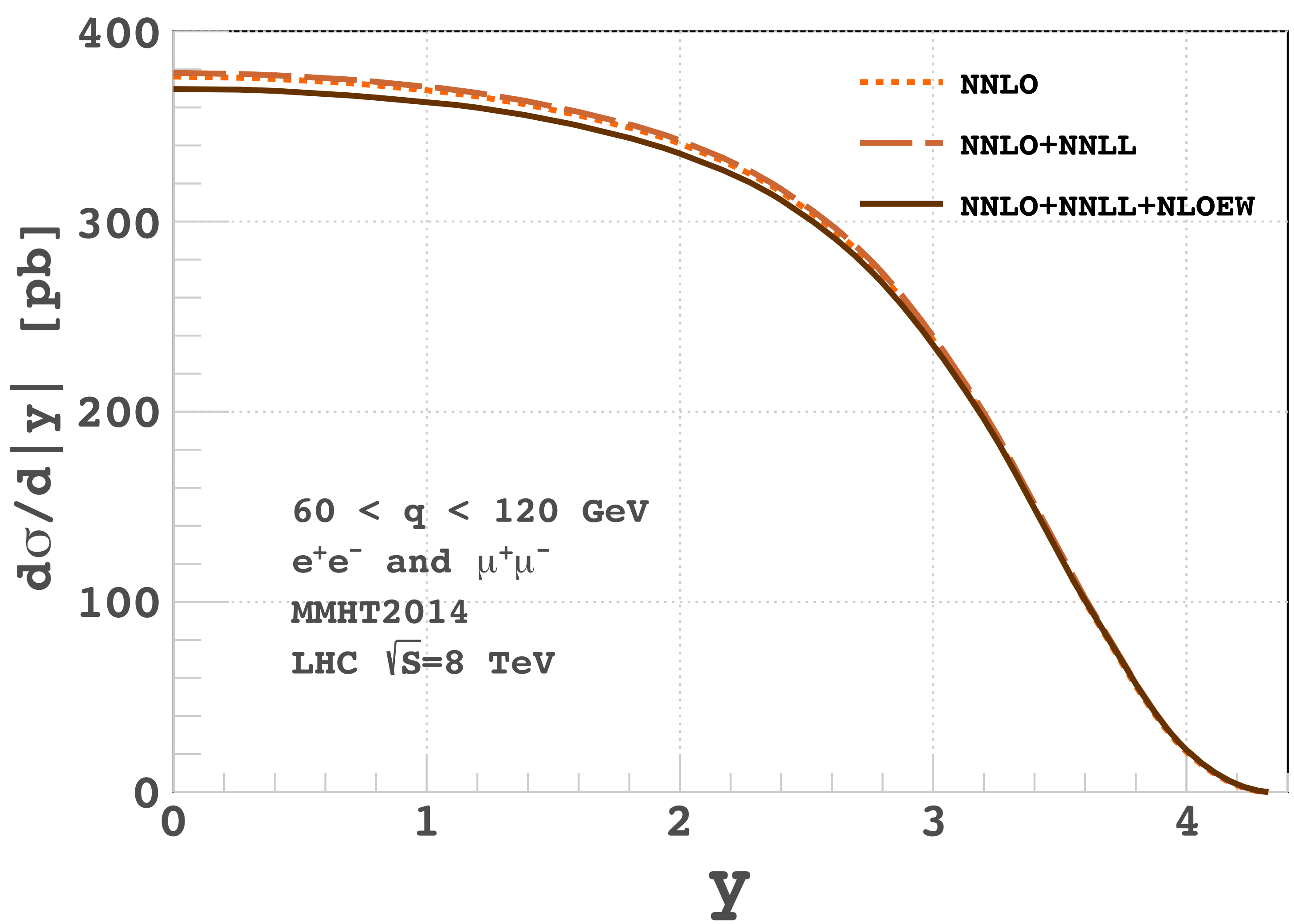
Rapidity Distribution at LHC

- Electro-weak corrections are important at this accuracy.
- Both e^+e^- and $\mu^+\mu^-$ give negative contribution at **NLO EW** level.
- Integrated Z-peak region

$$60 < q < 120 \quad \text{GeV}$$

- In G_μ scheme,
EW in e^+e^-: **-3.5%** (NLO)

EW in $\mu^+\mu^-$: **-1.8%** (NLO)



From **Horace**

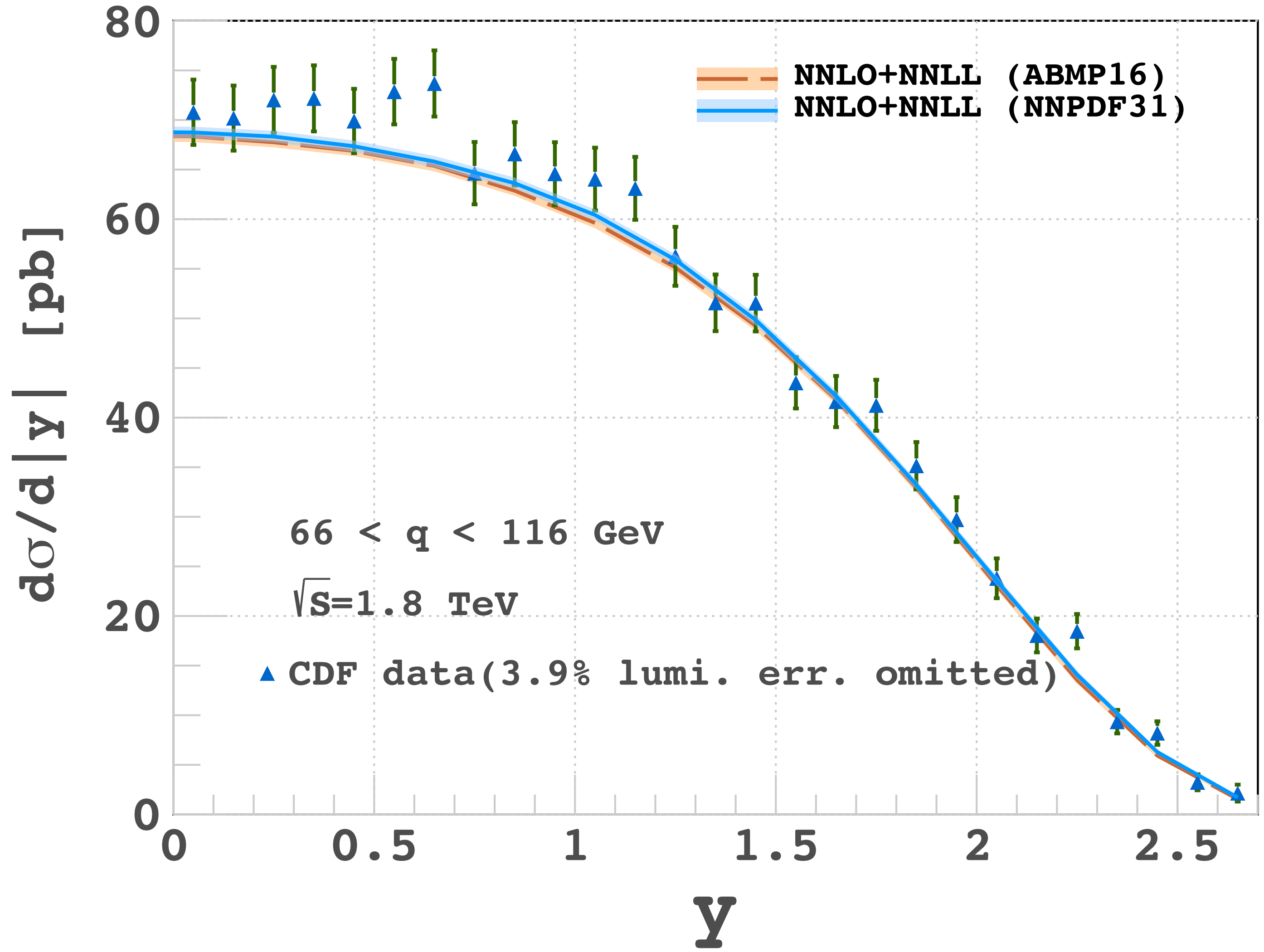
[Carloni Calame, Montagna, Nicrosini, Vicini]

Rapidity distribution at Tevatron

$$\sqrt{S} = 1.8 \quad \text{TeV}$$

$$66 < q < 116 \quad \text{GeV}$$

- Scale uncertainty
  ABMP16 ~ **1.64%**
  NNPDF31 ~ **1.68%**



Summary

- We have developed a systematic way of resuming **threshold logs** for rapidity distribution for colorless particle production.
- We exploited the **factorisation**, **RG invariance** and **K+G equations** to find different pieces.
- Following **Catani-Trentadue approach** we find an **all order resummed formula** in rapidity distribution for Higgs and DY production in **2-D Mellin space**.
- Rapidity distribution is presented with **NNLO+NNLL** accuracy.
  The resummed prediction changes the fixed order predictions significant way and **improves the reliability** of the perturbative prediction.
- We have presented a detailed study on **scale choice, scale variation, pdf variation** also along with available **EW results** which will be useful at the LHC in near future.

Acknowledgement

Thanks to S. Moch, S. Alekhin and J. Blumlein for many fruitful discussion.

Thanks to organisers of Loops and Legs 2018 for their endless effort behind such conference.

thank you!