k_T-dependent factorization at one loop

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Outline

- dijet azimuthal de-correlation
- kT-dependent factorization
- KaTie
- KaTie+Cascade
- amplitudes with off-shell initial states
- progress towards one-loop amplitudes

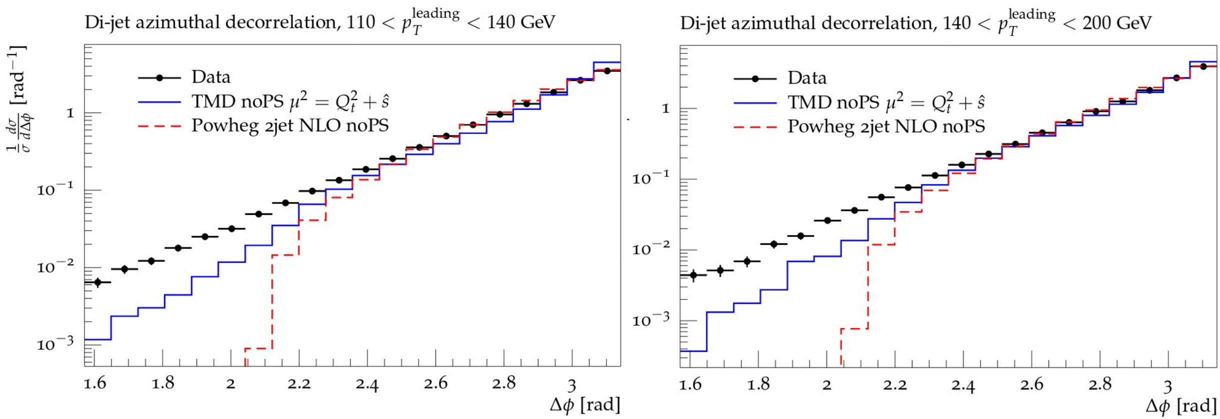
Dijet azimuthal de-correlation

Bury, AvH, Jung, Kutak, Sapeta, Serino 2017

The azimuthal de-correlations, that is the distribution of the angle in the transverse plane between the two hardest jets, for $pp \to jj$ at 7 TeV (data: CMS 2011).

This observable has no distribution at LO (tree-level) in collinear factorization.

Red prediction: collinear factorization at NLO
Blue prediction: k_T-dependent factorization at tree-level



High Energy Factorization a.k.a. k_T-factorization

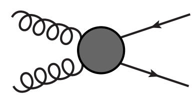
Catani, Ciafaloni, Hautmann 1991 Collins, Ellis 1991

$$\sigma_{h_1,h_2\to QQ} = \int d^2k_{1\perp}\,\frac{dx_1}{x_1}\,\mathcal{F}(x_1,k_{1\perp})\,d^2k_{2\perp}\,\frac{dx_2}{x_2}\,\mathcal{F}(x_2,k_{1\perp})\,\hat{\sigma}_{gg}\left(\frac{m^2}{x_1x_2s},\frac{k_{1\perp}}{m},\frac{k_{2\perp}}{m}\right)$$

- reduces to collinear factorization for $s \gg m^2 \gg k_\perp^2$, but holds al so for $s \gg m^2 \sim k_\perp^2$
- typically associated with small-x physics, forward physics, saturation ...
- $k_\perp$-dependent $\mathcal{F}$ may satisfy BFKL-eqn, CCFM-eqn, BK-eqn, KGBJS-eqn, ...
- allows for higher-order kinematical effects at leading order
- requires matrix elements with *off-shell* initial-state partons with $k_i^2 = k_{i\perp}^2 < 0$

$k_1 = x_1p_1 + k_{1\perp}$

$k_2 = x_2p_2 + k_{2\perp}$



- Can this factorization be generalized to other processes?
- This requires at least a formulation and calculation of off-shell matrix elements for these processes.

- parton level event generator, like ALPGEN, HELAC, MADGRAPH, etc.
- arbitrary processes within the standard model (including effective Hg) with several final-state particles.
- 0, 1, or 2 off-shell initial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids, or with TMDlib Hautmann, Jung, Krämer, Mulders, Nocera, Rogers, Signori 2014.
- a calculation is steered by a single input file.
- employs an optimization stage in which the pre-samplers for all channels are optimized.
- during the generation stage several event files can be created in parallel.
- can generate (naively factorized) MPI events.
- event files can be processed further by parton-shower program like CASCADE.

- has been used in several studies
  - *Four-jet production in single- and double-parton scattering within high-energy factorization*, Kutak, Maciuła, Serino, Szczurek, AvH 2016
  - *Associated production of D-mesons with jets at the LHC*, Maciuła, Szczurek 2017
  - *Towards tomography of quarkgluon plasma using double inclusive forward-central jets in PbPb collision*, Deák, Kutak, Tywoniuk 2017
  - *Single- and double-scattering production of four muons in ultraperipheral PbPb collisions at the Large Hadron Collider*, AvH, Kłusek-Gawenda, Szczurek 2017
  - *Double-parton scattering effects in D^0B^+ and B^+B^+ meson-meson pair production in proton-proton collisions at the LHC*, Maciuła, Szczurek 2018.

- covers complete parton-level phase space; no deformation of final-state momenta required when interfacing with initial-state parton shower
  - *Calculations with off-shell matrix elements, TMD parton densities and TMD Parton showers*, Bury, AvH, Jung, Kutak, Sapeta, Serino 2017

CASCADE

Jung, Baranov, Deak, Grebenyuk, Hautmann, Hentschinski, Knutsson, Kraemer, Kutak, Lipatov, Zotov, 2010

- full hadron level Monte Carlo event generator
- uses the CCFM evolution equation for the initial state parton shower following the backward evolution approach
- requires off-shell matrix elements for the hard scattering
- the transverse momentum of the initial partons of the hard scattering process is fixed by the TMD and the parton shower does not change the kinematics
- The transverse momenta during the cascade follow the behavior of the TMD

k_T-dependent pdfs from the prescription of Kimber, Martin, Ryskin, Watt 2001, 2010:

$$\mathcal{A}_a(x, k_T^2, \mu^2) = \frac{\partial}{\partial k_T^2}\left[x f_a(x, \mu^2)\Delta_a(k_T^2, \mu^2)\right]$$

$$\Delta_g(k_T^2, \mu^2) = \exp\left\{-\int_{k_T^2}^{\mu^2} \frac{d\kappa_T^2}{\kappa_T^2}\frac{\alpha_S(\kappa_T^2)}{2\pi}\left(\int_\eta^{1-\eta} d\xi\,\xi P_{gg}(\xi) + \int_0^1 n_f P_{qg}(\xi)\right)\right\} \quad \eta = \frac{k_T}{\mu + k_T}$$

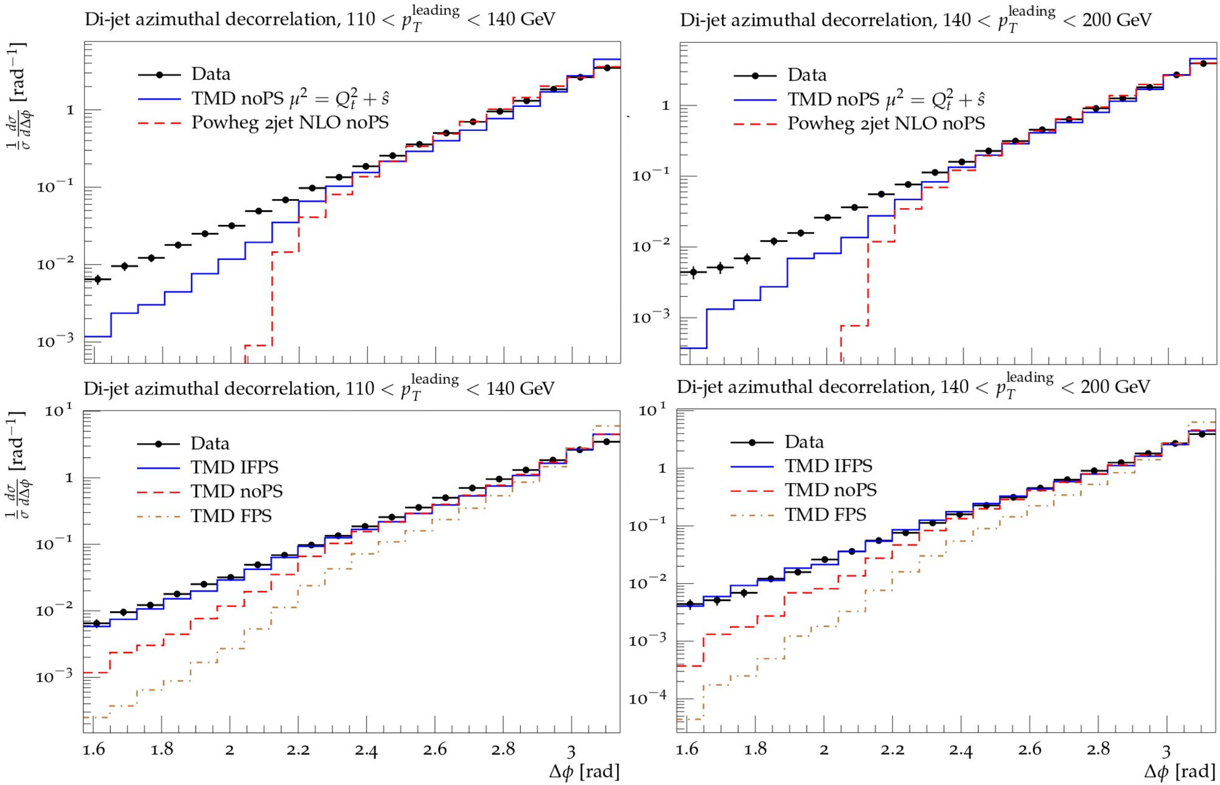
- also for quarks
- initial-state partons shower via backward-evolution and non-emission probability

$$\Delta(x, \mu_i, \mu_{i-1}) = \exp\left\{-\int_{\mu_{i-1}^2}^{\mu_i^2} \frac{dq^2}{q^2}\frac{\alpha_S(q^2)}{2\pi}\sum_a \int dz P_{ab}(z)\frac{\mathcal{A}_a\big(zx, k_T'(z), q\big)}{\mathcal{A}_b\big(x, k_T, q\big)}\right\}$$

- angular ordering with factorization scale $\mu^2 = (\vec{k}_{T\,1} + \vec{k}_{T\,2})^2 + \hat{s}$

Dijet azimuthal de-correlation

Bury, AvH, Jung, Kutak, Sapeta, Serino 2017



Amplitudes with off-shell initial states

Generalization of on-shellness

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$, satisfying the conditions

$$\begin{aligned} k_1^\mu + k_2^\mu + \cdots + k_n^\mu &= 0 && \text{momentum conservation} \\ p_1^2 = p_2^2 = \cdots = p_n^2 &= 0 && \text{light-likeness} \\ p_1\cdot k_1 = p_2\cdot k_2 = \cdots = p_n\cdot k_n &= 0 && \text{eikonal condition} \end{aligned}$$

With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q)p^\mu \quad \text{with} \quad x(q) \equiv \frac{q\cdot k}{q\cdot p}$$

Construct k_T^μ explicitly in terms of p^μ and q^μ:

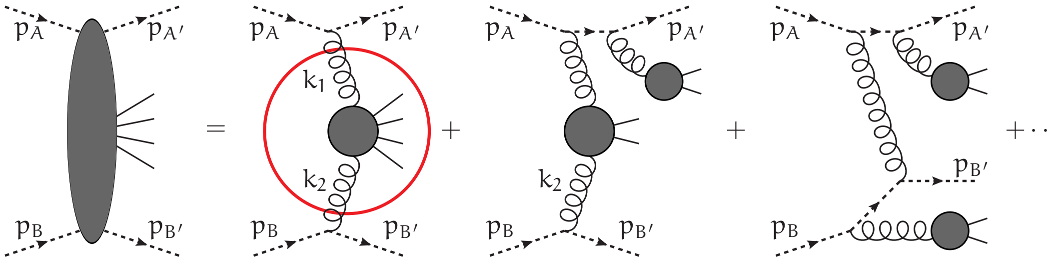
$$k_T^\mu(q) = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad \text{with} \quad \begin{cases} \varepsilon^\mu = \dfrac{\langle p|\gamma^\mu|q]}{[pq]} \quad , \quad \kappa = \dfrac{\langle q|\not{k}|p]}{\langle qp\rangle} \\ \varepsilon^{*\mu} = \dfrac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad , \quad \kappa^* = \dfrac{\langle p|\not{k}|q]}{[pq]} \end{cases}$$

$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

Amplitude as embedding

AvH, Kutak, Kotko 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} \qquad\qquad p_A^2 = p_{A'}^2 = 0$$

$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu \qquad\qquad p_A^\mu + p_{A'}^\mu = x_1 p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} = k_1^\mu$$

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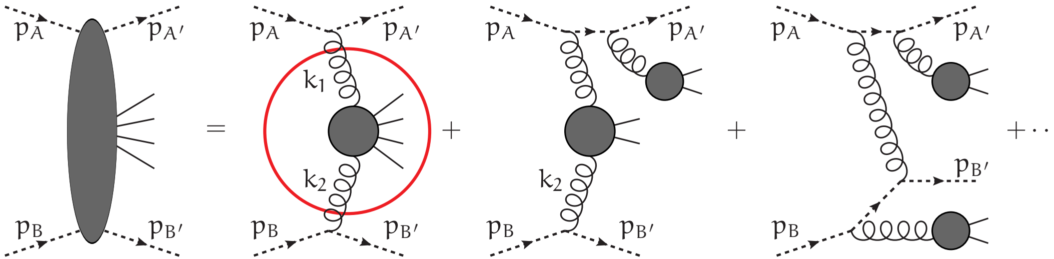
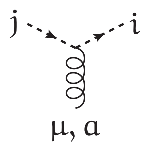
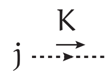


$$\left.\begin{aligned} p_A^\mu &= \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} \\ p_{A'}^\mu &= -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu \\ &\Lambda \to \infty \end{aligned}\right\} \Rightarrow$$



$$= -i\,T^a_{i,j}\,p_1^\mu$$

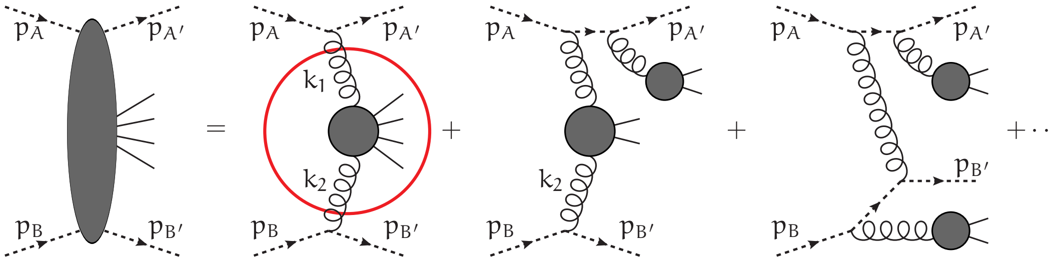


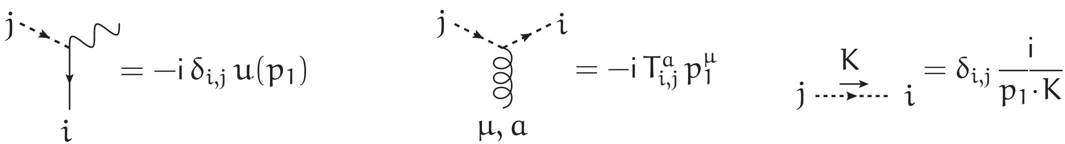
$$= \delta_{i,j}\,\frac{i}{p_1 \cdot K}$$

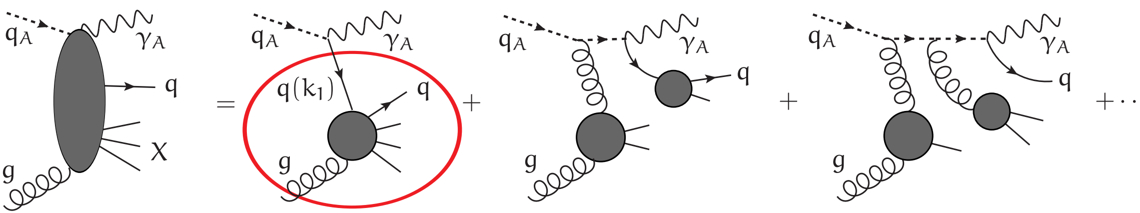
Amplitude as embedding

AvH, Kutak, Kotko 2013
AvH, Kutak, Salwa 2013

Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.







Amplitude as embedding

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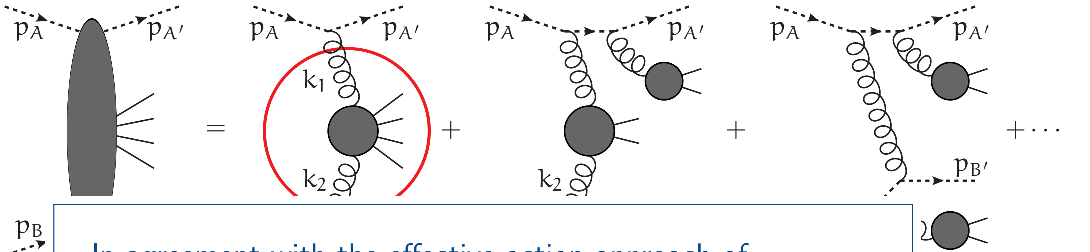
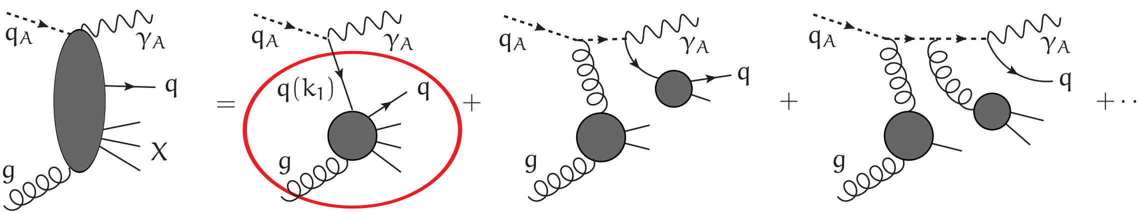
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



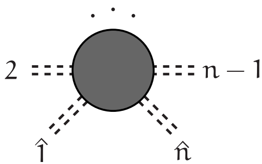
In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014

$j \qquad i = \delta_{i,j}\frac{i}{p_1\cdot K}$



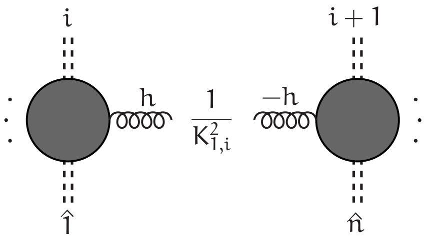
BCFW recursion for off-shell amplitudes

The BCFW recursion formula becomes



$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h}$$

$A_{i,h} =$



“On-shell condition” for “off-shell” gluons: $p_i \cdot k_i = 0$

BCFW recursion for off-shell amplitudes

AvH 2014
AvH, Serino 2015

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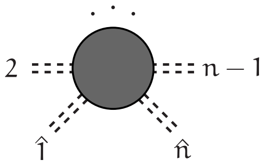
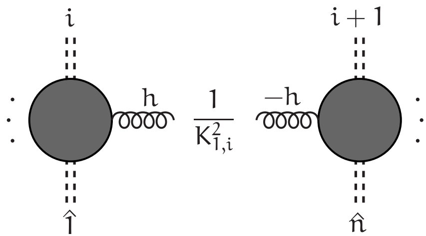


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i$$

$A_{i,h} =$


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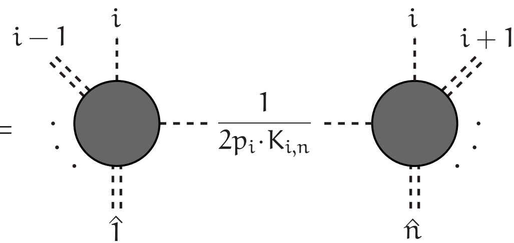

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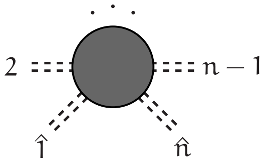
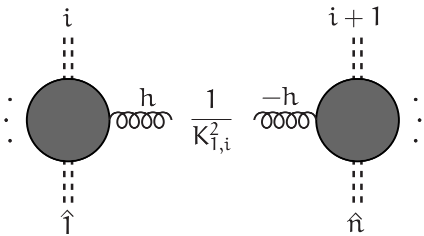
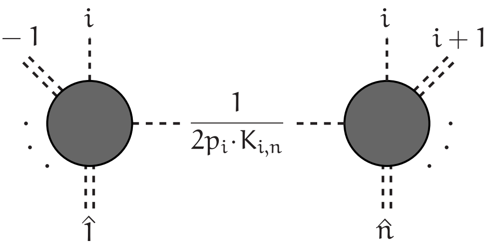


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D \,,$$

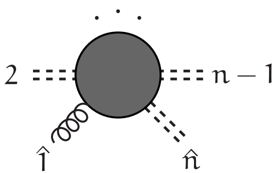
$A_{i,h} =$



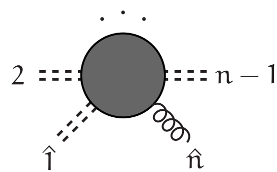
$B_i =$



$C = \frac{1}{\kappa_1}$



$D = \frac{1}{\kappa_1^*}$



Off-shell one-loop amplitudes

Initial steps have already been taken in the *parton reggeization approach* employing Lipatov's effective action.
Hentschinski, Sabio Vera 2012
Chachamis, Hentschinski, Madrigal, Sabio Vera 2012
Nefedov, Saleev 2017

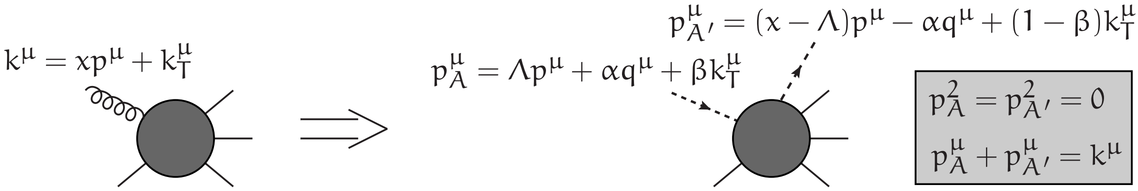
The main problem is caused by linear denominators in loop integrals and the divergecies they cause.

$$\int d^{4-2\varepsilon}\ell \, \frac{\mathcal{N}(\ell)}{p\cdot(\ell+K_0)\,(\ell+K_1)^2\,(\ell+K_3)^2\,(\ell+K_4)^2} = ?$$

In particular one would like to use a regularization that

- is manifestly Lorentz covariant
- manifestly preserves gauge invariance
- can be used incombination with dimensional regularization
- is practical

Off-shell one-loop amplitudes



where p, q are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \quad \Longrightarrow \quad \begin{cases} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu \end{cases}$$

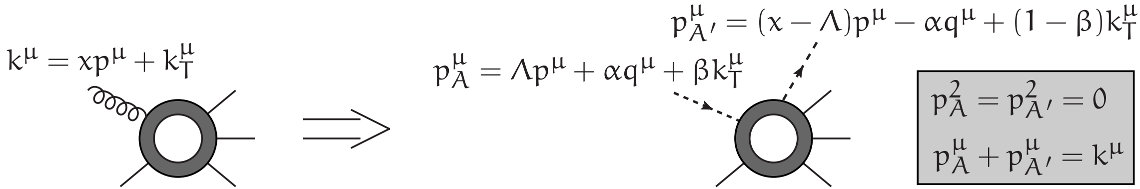
for any value of the parameter Λ. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$:

$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}(\Lambda^{-1})$$

Divide by Λ to get the desired amplitude

$$\langle p_A| \to \sqrt{\Lambda}\,\langle p| \quad , \quad |p_{A'}] \to -\sqrt{\Lambda}\,|p]$$

Off-shell one-loop amplitudes



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- Λ-parametrization provides natural regularization for linear denominators in loop integrals.
- Taking this limit after loop integration will lead to singularities $\log\Lambda$.

$\emptyset \to \mathrm{Hg\,g}^* $ from $\emptyset \to \mathrm{Hg\,q}\bar{\mathrm{q}}$

Schmidt 1997

$$m^1(g^+,q^-,\bar{q}^+) \;=\; m^0(g^+,q^-,\bar{q}^+)\,\frac{\alpha_s}{4\pi}\,r_\Gamma\left(\frac{4\pi\mu^2}{-M_H^2}\right)^\epsilon\left[N_cV_1+\frac{1}{N_c}V_2+n_fV_3\right]\,,$$

with

$$\begin{aligned}
V_1 \;=\;& \frac{1}{\epsilon^2}\left[-\left(\frac{-M_H^2}{-S_{gq}}\right)^\epsilon-\left(\frac{-M_H^2}{-S_{g\bar{q}}}\right)^\epsilon\right]+\frac{13}{6\epsilon}\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon\\
&-\ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right)-\ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right)\\
&-2\,\mathrm{Li}_2\left(1-\frac{S_{q\bar{q}}}{M_H^2}\right)-\mathrm{Li}_2\left(1-\frac{S_{gq}}{M_H^2}\right)-\mathrm{Li}_2\left(1-\frac{S_{g\bar{q}}}{M_H^2}\right)\\
&+\frac{83}{18}-\frac{\delta_R}{6}+\frac{\pi^2}{3}-\frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}}\,,\\
V_2 \;=\;& \left[\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right]\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon+\ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right)\\
&+\mathrm{Li}_2\left(1-\frac{S_{gq}}{M_H^2}\right)+\mathrm{Li}_2\left(1-\frac{S_{g\bar{q}}}{M_H^2}\right)\\
&+\frac{7}{2}+\frac{\delta_R}{2}-\frac{\pi^2}{6}-\frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}}\,,\\
V_3 \;=\;& -\frac{2}{3\epsilon}\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon-\frac{10}{9}\,.
\end{aligned}$$

$$\begin{aligned}
S_{gq} &= (p_A+k_g)^2 \longrightarrow 2\Lambda p\cdot k_g\\
S_{g\bar{q}} &= (p_{A'}+k_g) \longrightarrow -2\Lambda p\cdot k_g\\
S_{q\bar{q}} &= k_T^2
\end{aligned}$$

$\emptyset \to \mathrm{Hg\,g^*}$ from $\emptyset \to \mathrm{Hg\,q\bar{q}}$

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with

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&-\ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right)-\ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right)\ln\left(\frac{-S_{q\bar{q}}}{-M_H^2}\right)\\
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&+\frac{83}{18}-\frac{\delta_R}{6}+\frac{\pi^2}{3}-\frac{1}{2}\frac{S_{q\bar{q}}}{S_{g\bar{q}}},\\
V_2 \;=\;& \left[\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right]\left(\frac{-M_H^2}{-S_{q\bar{q}}}\right)^\epsilon+\ln\left(\frac{-S_{gq}}{-M_H^2}\right)\ln\left(\frac{-S_{g\bar{q}}}{-M_H^2}\right)\\
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\end{aligned}$$

$$\begin{aligned}
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S_{g\bar{q}} &= (p_{A'}+k_g) \to -2\Lambda p\cdot k_g\\
S_{q\bar{q}} &= k_T^2
\end{aligned}$$

$$m^1(g^+,q^-,\bar{q}^+) \propto \left[\frac{1}{\epsilon}-\ln\left(\frac{-k_T^2}{\mu^2}\right)\right]\ln\Lambda+\cdots$$

$\emptyset \to gg\,g^*$ from $\emptyset \to gg\,q\bar{q}$

Ellis, Sexton 1986

$$c(s,t,u) = g^4(\mu^2)(\mu)^{4\epsilon}\Bigg\{c^{(4)}(s,t,u)\Bigg(1 + \frac{\alpha_S}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$
$$\left[\frac{V}{2N}\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-7\right)+N\left(-\frac{2}{\epsilon^2}-\frac{11}{3\epsilon}+\frac{11}{3}l(-\mu^2)\right)+T_R\left(\frac{4}{3\epsilon}-\frac{4}{3}l(-\mu^2)\right)\right]\Bigg)$$
$$+\frac{\alpha_S}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$
$$\Bigg[\frac{l(s)}{\epsilon}\left(\left((2N^2V+\frac{2V}{N^2}\right)\frac{t^2+u^2}{ut}-4V^2\frac{t^2+u^2}{s^2}\right)$$
$$+\frac{4N^2V}{\epsilon}\left(l(t)\left(\frac{u}{t}-\frac{2u^2}{s^2}\right)+l(u)\left(\frac{t}{u}-\frac{2t^2}{s^2}\right)\right)$$
$$-\frac{4V}{\epsilon}\left(\frac{u}{t}+\frac{t}{u}\right)(l(t)+l(u))\Bigg]$$
$$+\frac{\alpha_S}{2\pi}\left(f^c(s,t,u)+f^c(s,u,t)\right)\Bigg\}+O(\epsilon)$$

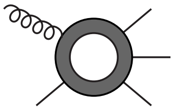
$$f^c(s,t,u) = 4NV\Bigg\{\frac{l(t)l(u)}{N}\left(\frac{t^2+u^2}{2tu}\right)$$
$$+l^2(s)\left(\frac{1}{4N^3}\frac{s^2}{tu}+\frac{1}{4N}\left(\frac{1}{2}+\frac{t^2+u^2}{tu}-\frac{t^2+u^2}{s^2}\right)-\frac{N}{4}\left(\frac{t^2+u^2}{s^2}\right)\right)$$
$$+l(s)\left(\left(\frac{5}{8}\frac{V}{N}-\frac{1}{2N}-\frac{1}{N^3}\right)-\left(N+\frac{1}{N^3}\right)\left(\frac{t^2+u^2}{2tu}\right)-\frac{V}{4N}\left(\frac{t^2+u^2}{s^2}\right)\right)$$
$$+\pi^2\left(\frac{1}{8N}+\frac{1}{N^3}\left(\frac{3(t^2+u^2)}{8tu}+\frac{1}{2}\right)+N\left(\frac{t^2+u^2}{8tu}-\frac{t^2+u^2}{2s^2}\right)\right)$$
$$+\left(N+\frac{1}{N}\right)\left(\frac{1}{8}-\frac{t^2+u^2}{4s^2}\right)$$
$$+l^2(t)\left(N\left(\frac{s}{4t}-\frac{u}{s}-\frac{1}{4}\right)+\frac{1}{N}\left(\frac{t}{2u}-\frac{u}{4s}\right)+\frac{1}{N^3}\left(\frac{u}{4t}-\frac{s}{2u}\right)\right)$$
$$+l(t)\left(N\left(\frac{t^2+u^2}{s^2}+\frac{3t}{4s}-\frac{5u}{4t}-\frac{1}{4}\right)-\frac{1}{N}\left(\frac{u}{4s}+\frac{2s}{u}+\frac{s}{2t}\right)-\frac{1}{N^3}\left(\frac{3s}{4t}+\frac{1}{4}\right)\right)$$
$$+l(s)l(t)\left(N\left(\frac{t^2+u^2}{s^2}-\frac{u}{2t}\right)+\frac{1}{N}\left(\frac{u}{2s}-\frac{t}{u}\right)+\frac{1}{N^3}\left(\frac{s}{u}-\frac{u}{2t}\right)\right)\Bigg\}$$

$$t = (p_A + k_2)^2 \to 2\Lambda p\cdot k_2$$
$$u = (p_A + k_3) \to -2\Lambda p\cdot k_2$$
$$s = k_T^2$$

$$\left[\frac{1}{\epsilon} - 1 - \ln\left(\frac{-k_T^2}{\mu^2}\right)\right]\ln\Lambda$$

Some four-point master integrals

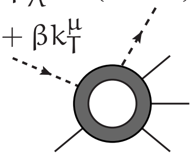
$k^\mu = xp^\mu + k_T^\mu$



$\Longrightarrow$

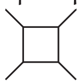
$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$

$p_{A'}^\mu = (x-\Lambda)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu$



$$\boxed{\begin{aligned} p_A^2 &= p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu &= k^\mu \end{aligned}}$$

$$[d\ell] = \frac{\Gamma(2-\varepsilon)\mu^{2\varepsilon}}{\Gamma^2(1-\varepsilon)\Gamma(1+\varepsilon)i\pi^{2-\varepsilon}}\, d^{4-2\varepsilon}\ell$$

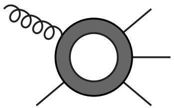


$$= \int [d\ell]\, \frac{\Lambda}{\ell^2\,(\ell + p_A + K_1)^2\,(\ell - K_3 - K_4)^2\,(\ell - K_4)^2}$$

Just use known expressions for regularized scalar integrals, put $(p_A + K_1)^2 \to 2\Lambda p\cdot K_1$, $(-p_A + K_2 + K_4)^2 \to -2\Lambda p\cdot(K_2 + K_4)$ etcetera, and take $\Lambda \to \infty$

Some four-point master integrals

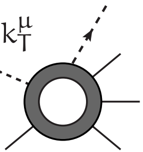
$$k^\mu = x p^\mu + k_T^\mu$$



$$\Longrightarrow$$

$$p_{A'}^\mu = (x-\Lambda)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu$$

$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$$



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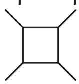
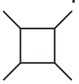


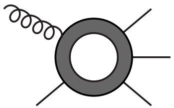
$$= \int [d\ell]\, \frac{\Lambda}{\ell^2\,(\ell + p_A + K_1)^2\,(\ell - K_3 - K_4)^2\,(\ell - K_4)^2}$$



$$= \frac{-1}{p\cdot K_4\, k_T^2}\left\{\left[\frac{1}{\varepsilon} - \ln\left(\frac{-k_T^2}{\mu^2}\right)\right]\ln\Lambda + \cdots\right\}$$

Some four-point master integrals

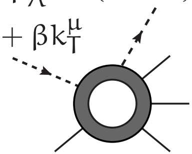
$$k^\mu = x p^\mu + k_T^\mu$$



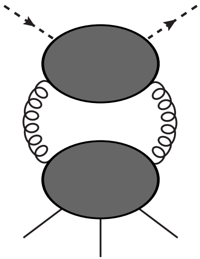
$$\Longrightarrow$$

$$p_{A'}^\mu = (x-\Lambda)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu$$

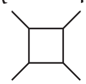
$$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$$



$$p_A^2 = p_{A'}^2 = 0$$
$$p_A^\mu + p_{A'}^\mu = k^\mu$$



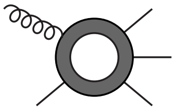
Boxes divergent in Λ only appear in graphs contributing to the one-shell limit $k_T \to 0$.



$$= \frac{-1}{p\cdot K_4\, k_T^2}\left\{\left[\frac{1}{\varepsilon} - \ln\left(\frac{-k_T^2}{\mu^2}\right)\right]\ln\Lambda + \cdots\right\}$$

Some triangles

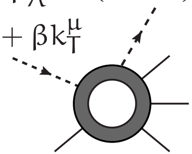
$k^\mu = x p^\mu + k_T^\mu$



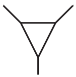
$\Longrightarrow$

$p_{A'}^\mu = (x - \Lambda) p^\mu - \alpha q^\mu + (1 - \beta) k_T^\mu$

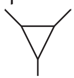
$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$



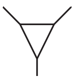
$$\boxed{\begin{array}{l} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = k^\mu \end{array}}$$



$$= \frac{1}{2p\cdot k_2}\left\{\frac{\ln^2\Lambda}{2} + \ln\left(\frac{-2p\cdot k_2}{\mu^2}\right)\ln\Lambda - \frac{\ln\Lambda}{\varepsilon} + \cdots\right\}$$

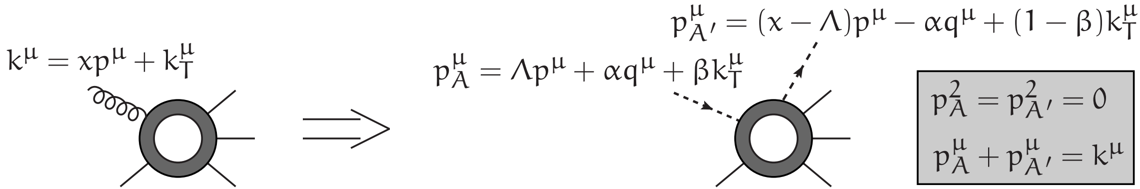


$$= \frac{1}{2p\cdot(K_1 - K_2)}\left\{\ln\left(\frac{-2p\cdot K_1}{-2p\cdot K_2}\right)\ln\Lambda + \cdots\right\}$$



$$= \frac{\Lambda}{k_T^2}\left\{\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\log\left(\frac{k_T^2}{-\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{k_T^2}{-\mu^2}\right)\right\}$$

Decomposition into master integrals



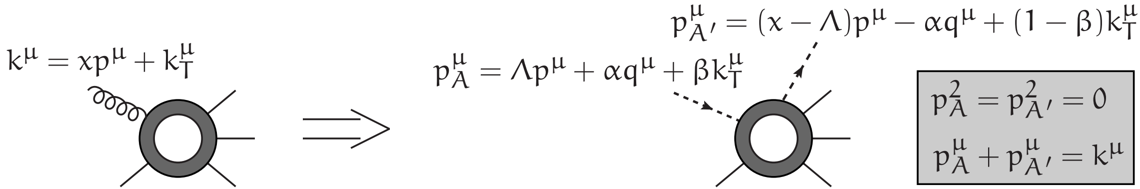
Well-known decomposition for on-shell one-loop amplitudes in terms of master integrals still holds for finite Λ.

$$\mathcal{A}^{(1)} = \int [d\ell] \, \frac{\mathcal{N}(\ell)}{\prod_i \mathcal{D}_i(\ell)} = \sum_{i,j,k,l} c_4(i,j,k,l)\, I_4(i,j,k,l) + \sum_{i,j,k} c_3(i,j,k)\, I_3(i,j,k)$$
$$+ \sum_{i,j} c_2(i,j)\, I_2(i,j) + \sum_{i} c_1(i)\, I_1(i) + \mathcal{R} + \mathcal{O}(\varepsilon)$$

$$I_4(i,j,k,l) = \int [d\ell] \, \frac{1}{\mathcal{D}_i(\ell)\mathcal{D}_j(\ell)\mathcal{D}_k(\ell)\mathcal{D}_l(\ell)} \quad , \quad \mathcal{D}_i(\ell) = (\ell + K_i)^2 - m_i^2 + i\eta$$

The coefficients $c_4, c_3, c_2.c_1$ are determined from the *integrand*.
(di)logarithms of external invariants and Λ appear in the master integrals I_4, I_3, I_2.

Decomposition into master integrals



Well-known decomposition for on-shell one-loop amplitudes in terms of master integrals still holds for finite Λ.

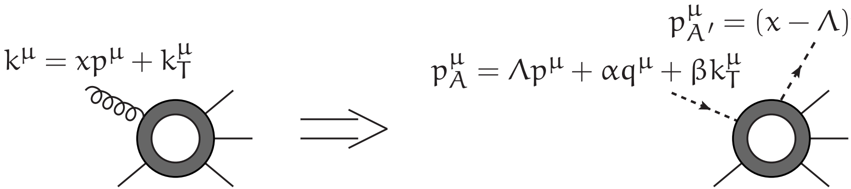
$$\mathcal{A}^{(1)} = \int [d\ell] \frac{\mathcal{N}(\ell)}{\prod_i \mathcal{D}_i(\ell)} = \sum_{i,j,k,l} c_4(i,j,k,l)\, I_4(i,j,k,l) + \sum_{i,j,k} c_3(i,j,k)\, I_3(i,j,k) + \sum_{i,j} c_2(i,j)\, I_2(i,j) + \sum_i c_1(i)\, I_1(i) + \mathcal{R} + \mathcal{O}(\varepsilon)$$

It is not completely correct to take $\Lambda \to \infty$ in the integrand before reduction, and just replace

$$\frac{1}{2p\cdot(\ell + K)} \to \frac{\Lambda}{(\ell + \Lambda p + K)^2}$$

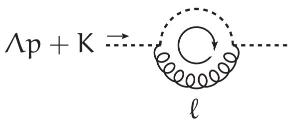
in the master integrals

Non-commuting limits



$$\boxed{\begin{array}{l} p_A^2 = p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu = k^\mu \end{array}}$$

Integrand-based reduction methods cannot be applied with naïve limit $\Lambda \to \infty$ on integrand. For example, the integrand of the following graph (Feynman gauge) vanishes in that limit, but the integral does not:



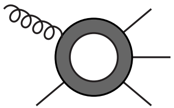
$$= \int [d\ell] \frac{\langle p|\gamma^\mu(\not{\ell} + \Lambda\not{p} + \not{K})\gamma_\mu|p]}{\ell^2(\ell + \Lambda p + K)^2}$$

$$= 2p\cdot K \left[\ln\Lambda - \frac{1}{\varepsilon} - 1 + \ln\left(-\frac{2p\cdot K}{\mu^2}\right) + \mathcal{O}(\varepsilon) \right]$$

But $\langle p|\gamma^\mu \not{p} \gamma_\mu|p] = 0$, so naïve power counting in Λ does not work.

Non-commuting limits

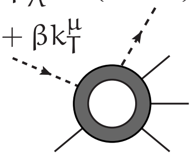
$k^\mu = xp^\mu + k_T^\mu$



$$\Longrightarrow$$

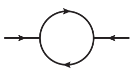
$p_{A'}^\mu = (x-\Lambda)p^\mu - \alpha q^\mu + (1-\beta)k_T^\mu$

$p_A^\mu = \Lambda p^\mu + \alpha q^\mu + \beta k_T^\mu$

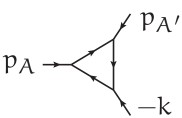


$$\boxed{\begin{aligned} p_A^2 &= p_{A'}^2 = 0 \\ p_A^\mu + p_{A'}^\mu &= k^\mu \end{aligned}}$$

For two-point master integrals and one three-point master integrals, integration does not commute with the limit $\Lambda \to \infty$: integration "eats" a power of Λ from the denominator.

$\Lambda p + K$  $-\Lambda p - K$

$$= \int \frac{[d\ell]}{\ell^2\,(\ell + \Lambda p + K)^2} \to \frac{1}{\varepsilon} + 2 - \log\left(\frac{2\Lambda p\cdot K}{-\mu^2}\right)$$



$$= \int \frac{[d\ell]}{\ell^2\,(\ell + p_A)^2\,(\ell + k)^2} \to \frac{1}{k_T^2}\left\{\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\log\left(\frac{k_T^2}{-\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{k_T^2}{-\mu^2}\right)\right\}$$

This complication manifests itself also in the fact that for these master integrals the solutions to the cut equations diverge with Λ.

Divergent solutions to cut equations

1) Boxes with 4 or 3 Λ-denominators, triangles with 3 or 2, bubbles with 2, eg:

$$\int \frac{[d\ell]}{(\ell+\Lambda p+K_0)^2(\ell+\Lambda p+K_1)^2(\ell+\Lambda p+K_2)^2(\ell+\Lambda p+K_3)^2}$$

has solutions $\ell^\mu = -\Lambda p^\mu + q^\mu$ with some finite q^μ. Looking at the corresponding residues, we see that the contribution of these masters to vanish.

2) Boxes with 2 Λ-denominators

$$\int \frac{[d\ell]\,\Lambda^2}{(\ell+K_0)^2(\ell+K_1)^2(\ell+\Lambda p+K_2)^2(\ell+\Lambda p+K_3)^2}$$

give but non-vanishing contribution. Divergence reflects the fact that there is no solution to $p\cdot(\ell+K_2) = p\cdot(\ell+K_3) = 0$, because this implies $p\cdot K_2 = p\cdot K_3$. This then implies that these are not master intergrals, and decompose into triangles.

3) Bubbles and the special triangle

$$\int \frac{[d\ell]}{\ell^2\,(\ell+\Lambda p+K)^2} \quad , \quad \int \frac{[d\ell]}{\ell^2\,(\ell+p_A)^2\,(\ell+p_A+p_{A'})^2}$$

give a finite contribution.

Summary

- k_T-dependent factorization gives the opportunity to have complete kinematics at lowest order in perturbative calculations
- it allows for the application of initial-state parton showers without changing the hard kinematics
- hard scattering amplitudes are well defined and computable at tree-level
- there is a natural regularization for the singularities at one loop related to linear denominators which, however, does not trivialize the calculation