Karlsruhe Institute of Technology

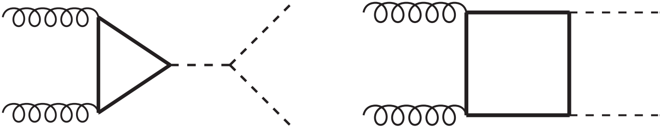
Double Higgs Production in the High Energy Limit

Loops and Legs in Quantum Field Theory 2018 | JHEP 03 (2018) 048

Joshua Davies, Go Mishima, Matthias Steinhauser, David Wellmann | April 30, 2018

INSTITUTE FOR THEORETICAL PARTICLE PHYSICS (TTP)

Introduction



Dominant channel at a hadron collider: gluon fusion

Gives access to the Higgs self-coupling λ_{HHH}

- experimentally challenging measurement (small cross-section)
- perhaps feasible with HL-LHC?

Theory Results

LO

- full result [Glover,van der Bij '88][Plehn,Spira,Zerwas '98]

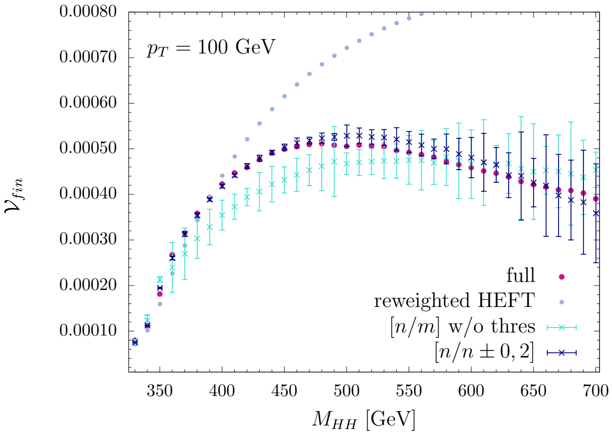
NLO

- large-m_t limit [Dawson,Dittmaier,Spira '98] [Grigo,Hoff,Melnikov,Steinhauser '13] [Degrassi,Giardine,Gröber '16]
- numerical result [Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Zicke '16]
- Padé approximation [Gröber,Maier,Rauh '17]

NNLO

- large-m_t limit [de Florian,Mazzitelli '13] [Grigo,Melnikov,Steinhauser '14] [Grigo,Hoff,Steinhauser '15]
- finite-m_t estimate [Grazzini,Heinrich,Jones,Kallweit,Kerner,Lindert,Mazzitelli '18]

NLO Virtual Cross Section

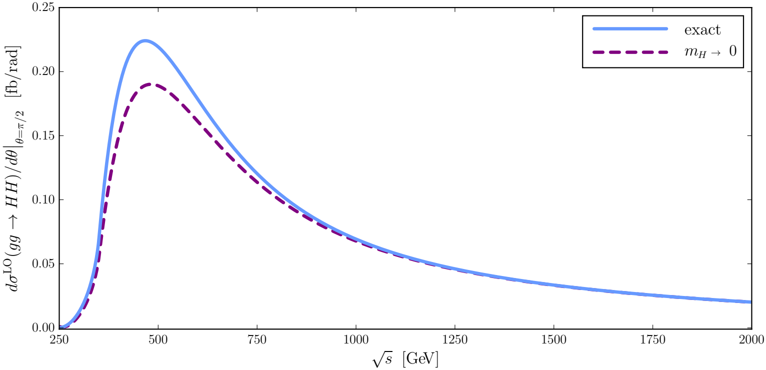


[Gröber,Maier,Rauh '17]

High Energy Limit

Assume $s, t \gg m_t^2 > m_H^2$, expand in this region.

- Easier integral reduction
- Analytic expressions, in terms of Harmonic Polylogarithms

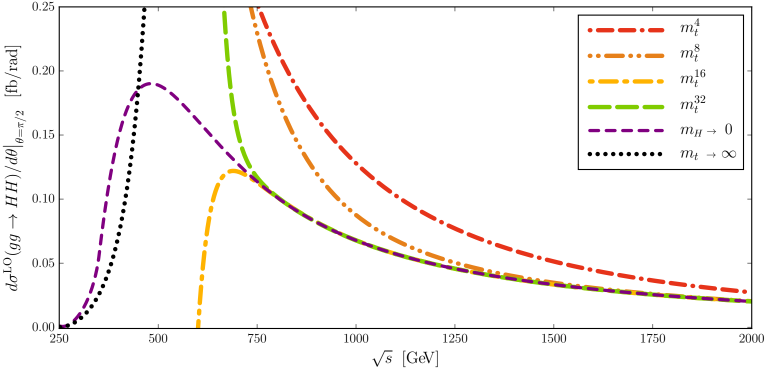


$t = -\frac{s}{2}(1 + \cos\theta)$

High Energy Limit

Assume $s, t \gg m_t^2 > m_H^2$, expand in this region.

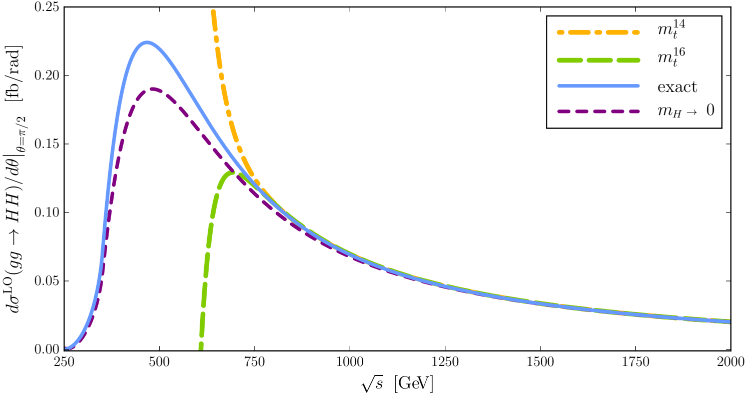
- $\mathbf{I(m_t^2) = \sum_{m,n} C_{m,n} \, (m_t^2)^m \, \log{(m_t^2)}^n}$
- $I(m_H^2) = I(0) + m_H^2 I'(0) + \cdots$



High Energy Limit

Assume $s, t \gg m_t^2 > m_H^2$, expand in this region.

- $I(m_t^2) = \sum_{m,n} C_{m,n} \, (m_t^2)^m \, \log{(m_t^2)}^n$
- $\mathbf{I(m_H^2) = I(0) + m_H^2 I'(0)} + \cdots$



Amplitude and Reduction

Diagram generation	`qgraf`	[Nogueira '93]
Topology mapping	`q2e/exp`	[Harlander,Seidelsticker,Steinhauser '97]
Physics, projection	`TFORM 4.2`	[Ruijl,Ueda,Vermaseren '17]
m_H^2 expansion	`LiteRed`	[Lee '13]
IBP Reduction	`FIRE 5.2`	[Smirnov '14]
	(`LiteRed`)	[Lee '13]

Reduction is performed for exact m_t^2, but $m_H^2 = 0$.

Feynman Diagrams: $\mathbf{8}^{LO} + \mathbf{118}^{NLO}$

$\downarrow$

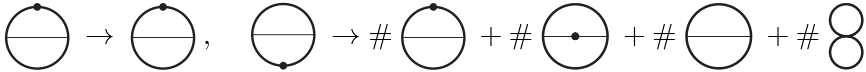
Scalar Integrals: **26K** (+120K (m_H^2 exp, one term))

$\downarrow$

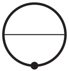
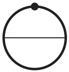
Masters Integrals: $\mathbf{10}^{LO} + \mathbf{221}^{NLO}$

A Minimal Set of Master Integrals?

FIRE yields **221**^{NLO} two-loop masters: this is not a minimal set! E.g.,



Make use of FIRE command FindRules. Finds equivalent integrals:

FindRules[] =  .

For any integral I we should have that, after applying reduction rules,

$$\texttt{FindRules}[I] = I$$

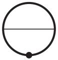
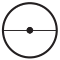
Method: build such equations for many integrals

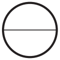
- use to derive extra reduction relations
- final result: **161**^{NLO} master integrals

A Simple Example

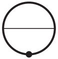
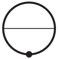
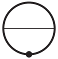
Reduction rules:

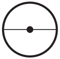
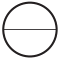
 $\rightarrow$ ,

 $\rightarrow -$  $+ \dfrac{\epsilon(4m_t^2 - s)s}{2(2\epsilon - 1)(m_t^2 - s)m_t^2}$ 

$+ \dfrac{(3\epsilon - 2)(2m_t^2 - s)}{2(m_t^2 - s)m_t^2}$  $+ \dfrac{(\epsilon - 1)^2}{(2\epsilon - 1)(m_t^2 - s)m_t^2}$  .

Then,

FindRules[] $-$  $=$  $-$  $= \; 0 \Longrightarrow$

 $\rightarrow \dfrac{\epsilon(4m_t^2 - s)s}{4(2\epsilon - 1)(m_t^2 - s)m_t^2}$  $+ \dfrac{(3\epsilon - 2)(2m_t^2 - s)}{4(m_t^2 - s)m_t^2}$ 

$+ \dfrac{(\epsilon - 1)^2}{2(2\epsilon - 1)(m_t^2 - s)m_t^2}$  .

Compute Masters: Differential Equations

Differentiate master integrals wrt $X \in \{s,t,m_t^2\}$. IBP reduce result:

$$\frac{\mathrm{d}}{\mathrm{d}X}\vec{J} = M(s,t,m_t^2,\epsilon)\cdot\vec{J}.$$

m_t^2 **equation:** substitute high-energy ansatz for each master integral,

$$J = \sum_i \sum_j \sum_k C_{ijk}(s,t)\,\epsilon^i\,(m_t^2)^j\,\log\left(m_t^2\right)^k.$$

Obtain a system of linear equations for coefficients $C_{ijk}(s,t)$. **Solve!**

... we require **Boundary Conditions**

- determine leading powers in $m_t^2 \to$ fixes some $C_{ijk}(s,t)$

Boundary Conditions 1

Expansion by Regions method yields Mellin-Barnes integrals for the leading, next-to-leading, behaviour in m_t^2. [`asy.m` Pak,Smirnov '11]

Integrands can depend on (s, t). Two ways to proceed:

1. Set $s = t = -1$
   - Evaluate MB integrals numerically, to high precision (300+200 digits)
   - Fit to basis using PSLQ: $\{1, \ln 2, \pi^2, \zeta_3, \pi^4, \ldots\}$, obtain $C_{ijk}(-1, -1)$

Example:

$\epsilon^{\mathbf{0}} (m_t^2)^{\mathbf{0}}$

$$C_{\mathbf{0}\,\mathbf{0}\,0} = -\,237.5961\cdots = -24 - 20\pi^2/3 - 16\pi^2 \ln 2 - 40\zeta_3 + \pi^4/10$$
$$C_{\mathbf{0}\,\mathbf{0}\,1} = -\,110.0840\cdots = -32\pi^2/3 - 4\zeta_3$$
$$C_{\mathbf{0}\,\mathbf{0}\,2} = +\,13.42026\cdots = 20 - 2\pi^2/3$$
$$C_{\mathbf{0}\,\mathbf{0}\,3} = +\,10.66666\cdots = 32/3$$
$$C_{\mathbf{0}\,\mathbf{0}\,4} = +\,1.000000\cdots = 1$$

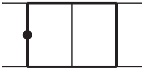
   - Integrate t differential equation to obtain leading $C_{ijk}(-1, t)$

Boundary Conditions 2

2. Set $s = -1$, keep t dependence
   - Expand MB integrals about $t = 0$ (50+200 terms)
   - Fit to basis of HPLs to obtain leading $C_{ijk}(-1, t)$

Example:

$\epsilon^{\mathbf{0}}(m_t^2)^{\mathbf{0}} \log(m_t^2)^{\mathbf{0}}$



$$
\begin{aligned}
C_{\mathbf{0\,0\,0}} =& -8\zeta_3 - 24 - 4\pi^2 - 7\pi^4/15 + (8\zeta_3 - 8 + 20\pi^2/3)\mathbf{t} \\
& - (5\pi^2 + 18)\mathbf{t^2} - (44/9 + 16\pi^2/9)\mathbf{t^3} - (41/18 + 11\pi^2/12)\mathbf{t^4} \\
& - (33/25 + 14\pi^2/25)\mathbf{t^5} - (194/225 + 17\pi^2/45)\mathbf{t^6} \\
& - (4/9 + 40\pi^2/147)\mathbf{t^7} + \cdots + \mathcal{O}(\mathbf{t^8}) \\
=& -8(1-t)\zeta_3 - 24 - 4\pi^2 - 7\pi^4/15 + 8\pi^2 t/3 \\
& + 8\pi^2(1-t)H_1(t) - 4\pi^2 H_2(t) + 16(1-t)H_3(t) - 24H_4(t) \\
& + \cdots .
\end{aligned}
$$

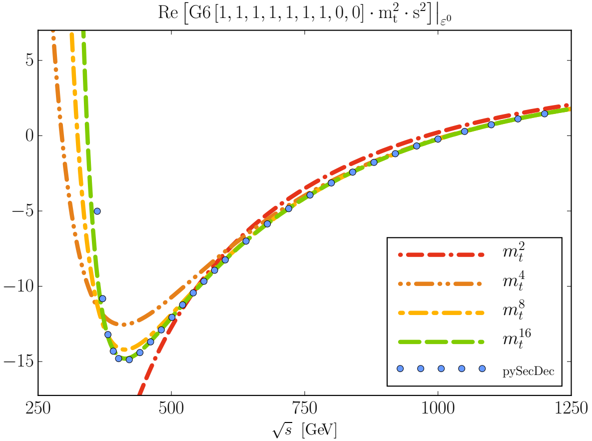
Method 2 is easier here (we don't need to integrate t differential equation). Both methods agree for easier (lower-line) integrals.

Example Master Integral

Comparison: m_t^2 expansion vs. pySecDec numerical values.

[Borowka,Heinrich,Jahn,Jones,Kerner,Schlenk,Zirke '18]

double box:
6 massive (m_t),
1 massless prop.
Real part, ϵ^0



Example Master Integral

Comparison: m_t^2 expansion vs. `pySecDec` numerical values.

[Borowka,Heinrich,Jahn,Jones,Kerner,Schlenk,Zirke '18]

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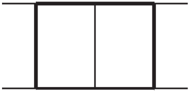
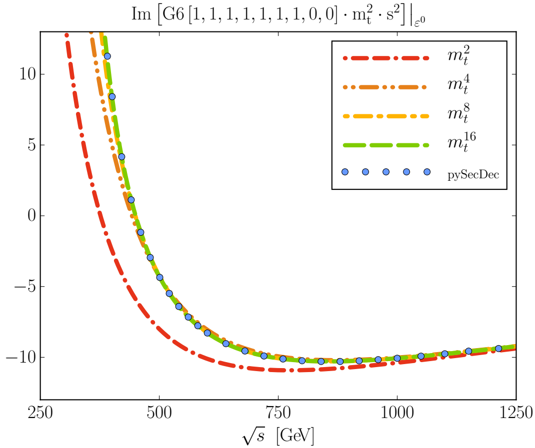
1 massless prop.
Imaginary part, ϵ^0





Amplitude

$$\mathcal{M}^{\mu\nu} \sim \mathcal{A}_1^{\mu\nu}(\mathcal{F}_{tri} + \mathcal{F}_{box1}) + \mathcal{A}_2^{\mu\nu}(\mathcal{F}_{box2})$$

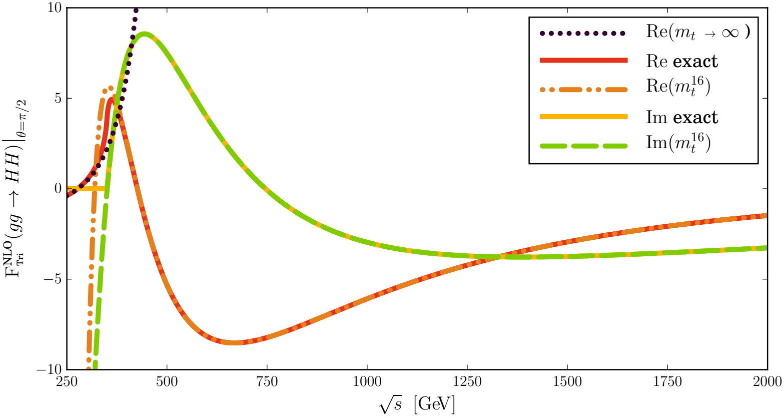
Projectors: $\mathcal{F}_{tri} + \mathcal{F}_{box1} = P_{1\,\mu\nu}\mathcal{M}^{\mu\nu}$, $\mathcal{F}_{box2} = P_{2\,\mu\nu}\mathcal{M}^{\mu\nu}$

At two loops, form factors ($\mathcal{F}$) have C_F and C_A contributions.

- C_F only from planar master integrals... complete ✓
- C_A also from non-planar master integrals... to come

C_F contribution to $\mathcal{F}_{tri}^{NLO}$

Exact curves from NLO $gg \to H$ [Harlander,Kant '13]



C_F contribution to $\mathcal{F}^{NLO}_{box1}$

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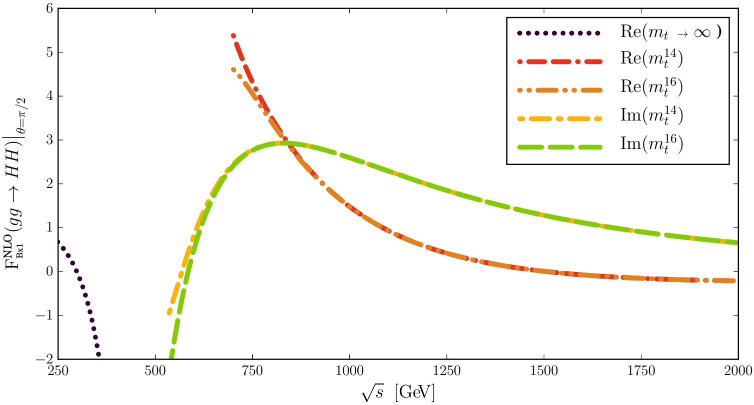
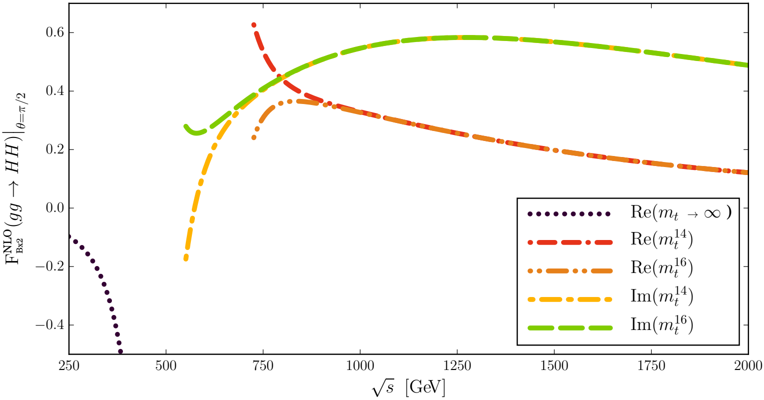


C_F contribution to $\mathcal{F}^{NLO}_{box2}$



Conclusions and Outlook

Progress towards $gg \to HH$ amplitude in the limit $s, t \gg m_t^2 > m_H^2$

- contributions from planar master integrals complete
  - C_F terms of $\mathcal{F}_{tri}$ reproduce known $gg \to H$ result
- contributions from non-planar master integrals in progress
  - amplitude ϵ poles known and consistent

Final result will give the amplitude in a region which is not well described by current results.

- combine with existing Padé approximation?