On the gauge dependence of Quantum Electrodynamics

How to go beyond the Feynman gauge

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The Landau-Khalatnikov-Fradkin transform

$$G'(x-y) = G(x-y)\exp\left[ie(M(x-y) - M(0))\right]$$

- $M(x-y)$: describes the propagator of a scalar field & characterizes the gauge dependence

- gauge dependent terms factorize
- gauge dependent terms exponentiate

Renormalized self-energy of the electron

$$-\Sigma\left(\alpha, \xi, L = \ln\left[{}^{-p^2}/{}_{\mu^2}\right]\right)_{\widetilde{\mathrm{MOM}}}$$

$$=$$

$$-\frac{1}{3!}\xi^3 L^3 \quad -\frac{3}{2}\xi L^2 \quad -\frac{3}{2}L \quad \left(\frac{\alpha}{4\pi}\right)^3$$

$$+\frac{1}{2!}\xi^2 L^2 \quad +\frac{3}{2}L \quad \left(\frac{\alpha}{4\pi}\right)^2$$

$$-\frac{1}{1!}\xi L \quad \left(\frac{\alpha}{4\pi}\right)^1$$

Renormalized self-energy of the electron

$$-\Sigma\left(\alpha, \xi, L = \ln\left[{}^{-p^2}/_{\mu^2}\right]\right)_{\widetilde{\text{MOM}}} \overset{?}{=} -\Sigma(\alpha, 0, L)_{\widetilde{\text{MOM}}} \exp\left[-\xi L \left(\frac{\alpha}{4\pi}\right)\right]$$

$$=$$

$$+\frac{1}{4!}\xi^4 L^4 + \frac{3}{2}\frac{\xi^2}{2!}L^3 + \left(\frac{3}{2}\xi \qquad\right)L^2 \qquad \left(\frac{\alpha}{4\pi}\right)^4$$

$$-\frac{1}{3!}\xi^3 L^3 \qquad -\frac{3}{2}\xi L^2 \qquad -\frac{3}{2}L \qquad \left(\frac{\alpha}{4\pi}\right)^3$$

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$$=$$

$$+\frac{1}{4!}\xi^4 L^4 + \frac{3}{2}\frac{\xi^2}{2!} L^3 + \left(\frac{3}{2}\xi + \frac{9}{8}\right) L^2 + \left(\frac{1027}{8} + 400\zeta(3) - 640\zeta(5)\right) L \qquad \left(\frac{\alpha}{4\pi}\right)^4$$

$$-\frac{1}{3!}\xi^3 L^3 \qquad -\frac{3}{2}\xi L^2 \qquad -\frac{3}{2} L \qquad \left(\frac{\alpha}{4\pi}\right)^3$$

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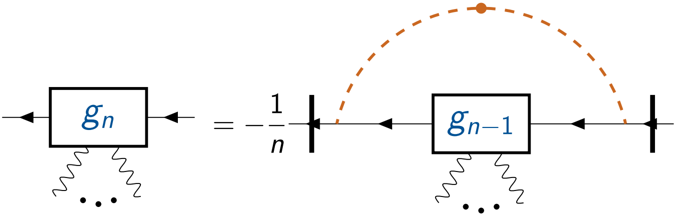
Characterization of the ξ dependence

$$G_c(\varepsilon, \alpha, \xi) = g_0(\varepsilon, \alpha) + g_1(\varepsilon, \alpha)\xi + g_2(\varepsilon, \alpha)\xi^2 + \dots$$

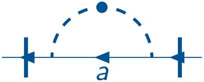


Beyond the Feynman gauge: an algorithm

1. derive ε-expansion of the connected GF in the Feynman gauge

$$g_0(\varepsilon, \alpha) = c_1(\varepsilon)\alpha + c_2(\varepsilon)\alpha^2 + c_3(\varepsilon)\alpha^3 + \dots$$

2. derive $F(\varepsilon, a)$ of the auxiliary graph



3. $\xi^0 \Rightarrow \xi^1$ terms:

$$g_1(\varepsilon, \alpha) = F(\varepsilon, 1)\alpha + F(\varepsilon, 1+\varepsilon)c_1(\varepsilon)\,\alpha^2 + F(\varepsilon, 1+2\varepsilon)c_2(\varepsilon)\,\alpha^3 + \dots$$

4. for $\xi^2, \xi^3, \dots$ terms: iterate step 3)

Higher dimensional QED

$$\mathscr{L}_{d=6} = -\frac{1}{4}(\partial_\mu F_{\nu\rho})(\partial^\mu F^{\nu\rho}) - \frac{1}{2\xi}(\partial_\mu \partial^\nu A_\nu)(\partial^\mu \partial^\rho A_\rho) + i\bar{\psi}\not{D}\psi$$

$$\mathscr{L}_{d=8} = -\frac{1}{4}(\partial_\mu \partial_\nu F_{\rho\sigma})(\partial^\mu \partial^\nu F^{\rho\sigma}) - \frac{1}{2\xi}(\partial_\mu \partial_\nu \partial^\rho A_\rho)(\partial^\mu \partial^\nu \partial^\sigma A_\sigma) + i\bar{\psi}\not{D}\psi + \frac{g_2^2}{32}(F_{\mu\nu}F^{\mu\nu})(F_{\rho\sigma}F^{\rho\sigma}) + \frac{g_3^2}{8}F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}$$

(Gracey '16)

- photon propagator decomposes as usual
- photon self interactions are transversal
- ξ characterization applies across dimensions

The renormalized electron propagator

$$\frac{\partial}{\partial \xi} S_0(\alpha, \xi, L) = ie^2 \int \frac{d^D p}{(2\pi)^D} \frac{1}{\left[(q+p)^2\right]^2} \frac{1}{\not{q}} S_0(\alpha, \xi, \ln {}^{-q^2}/_{\mu^2}) \frac{1}{\not{q}}$$

- IR divergence: subtract counterterm $\underbrace{\text{which}}$ has same
  IR behaviour and respects $\widetilde{\text{MOM}}$ condition

- $\frac{\partial}{\partial \xi} S(\alpha, \xi, L) = L \frac{\alpha}{4\pi} S(\alpha, \xi, L)$

Proof of folklore statement

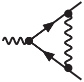
- $S(\alpha, \xi, L) = S(\alpha, 0, L) \exp\left[\xi L \frac{\alpha}{4\pi}\right]$
- $\gamma(\alpha, \xi) - \gamma(\alpha, 0) = \xi \frac{\alpha}{4\pi}$

The gauge dependence-infrared puzzle

gauge parameter

QED $\quad \exp\left(\xi L \frac{\alpha}{4\pi}\right)$

The gauge dependence-infrared puzzle

	gauge parameter	IR divergences	
QED	$\exp\left(\xi L \frac{\alpha}{4\pi}\right)$	exp ()	(Korthals Altes, de Rafael '76)

The gauge dependence-infrared puzzle

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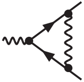
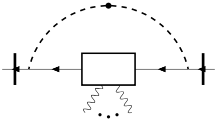
QED	$\exp\left(\xi L \frac{\alpha}{4\pi}\right)$	exp ()	(Korthals Altes, de Rafael '76)
QCD	?	$\exp\left(\sum \text{webs}\right)$	

Conclusion



1. simple characterization of ξ dependence
2. method applies to higher dimensional field theories
3. anomalous dimension of the electron depends on ξ at α^1 only
$$\gamma(\alpha,\xi) - \gamma(\alpha,0) = \xi \frac{\alpha}{4\pi}$$
4. connects to IR divergences