4-Gluon Amplitude at 3-loops in QCD

Taushif Ahmed
THEP
Johannes Gutenberg Universität Mainz

St. Goar
Loops and Legs 2018

In collaboration with
Johannes Henn & Bernhard Mistlberger

Ref: xxxx.xxxxx

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

PROLOGUE

- Scattering amplitude is the basic building block in QFT
- Topic of interest for decades.
- More precise theoretical predictions are crying need at the LHC
- Necessary to reveal the underlying structures in QFT

PROLOGUE

- Scattering amplitude is the basic building block in QFT
- Topic of interest for decades.
- More precise theoretical predictions are crying need at the LHC
- Necessary to reveal the underlying structures in QFT
- We focus on a 4-gluon process involving on-shell partons in QCD

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0$$

- Ingredient for the di-jet production
- State-of-the-art
  - 1-loop: Ellis, Sexton '86
  - 2-loop Helicity amplitude: Bern, Dixon, Kosower '00
  - 2-loop full amplitude: Glover, Oleari, Yeomans '01

Prologue

- Scattering amplitude is the basic building block in QFT
- Topic of interest for decades.
- More precise theoretical predictions are crying need at the LHC
- Necessary to reveal the underlying structures in QFT
- We focus on a 4-gluon process involving on-shell partons in QCD

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0$$

- Ingredient for the di-jet production
- State-of-the-art
  - 1-loop: Ellis, Sexton '86
  - 2-loop Helicity amplitude: Bern, Dixon, Kosower '00
  - 2-loop full amplitude: Glover, Oleari, Yeomans '01
- Next challenging goal: go for 3-loop!
- First attempt in N=4 by Henn, Mistlberger in '16
- We address this at 3-loop for the first time in QCD

Our goal

Plan of the Talk

- Tensor Structure & Projectors
- Helicity Amplitude
- Planar Limit
- Details of Computation
- Master Integrals in UT Basis
- UV Renormalisation
- IR Factorisation
- A Glimpse at the Preliminary Result (Not completed yet)

Process of Interest

- We consider

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0 \qquad \text{on-shell in QCD}$$

with arbitrary number of light fermions in fund represent of SU(N) gauge group

Process of Interest

- We consider

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0 \qquad \text{on-shell in QCD}$$

with arbitrary number of light fermions in fund represent of SU(N) gauge group

- Goal: Compute the amplitude and helicity amplitudes in planar limit
- One approach: decompose the amplitude into linearly independent tensor structures

Process of Interest

- We consider

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0 \qquad \text{on-shell in QCD}$$

with arbitrary number of light fermions in fund represent of SU(N) gauge group

- Goal: Compute the amplitude and helicity amplitudes in planar limit

- One approach: decompose the amplitude into linearly independent tensor structures

- General Tensor Structure

$$|\mathcal{M}(p_1, p_2, p_3)\rangle = S_{\mu\nu\rho\sigma}(\{p_i\})\epsilon_1^\mu(p_1)\epsilon_2^\nu(p_2)\epsilon_3^\rho(p_3)\epsilon_4^\sigma(p_4)$$

$$S^{\mu\nu\rho\sigma} \propto \{g^{\mu\nu}g^{\rho\sigma}, \;\; g^{\mu\nu}p_{i_1}^\rho p_{i_2}^\sigma, \;\; p_{i_1}^\mu p_{i_2}^\nu p_{i_3}^\rho p_{i_4}^\sigma\}$$

138 Tensorial Structures ⟶ Not all of them contribute!

- Transversality Condition

$$\epsilon_i . p_i = 0$$

Number of tensorial structures reduce to 43

PROJECTORS

- Ward Identities

$$S_{\mu\nu\rho\sigma}(\{p_i\})p_1^\mu \epsilon_2^\nu(p_2)\epsilon_3^\rho(p_3)\epsilon_4^\sigma(p_4) = 0$$
$$S_{\mu\nu\rho\sigma}(\{p_i\})\epsilon_1^\mu(p_1)p_2^\nu \epsilon_3^\rho(p_3)\epsilon_4^\sigma(p_4) = 0$$
$$S_{\mu\nu\rho\sigma}(\{p_i\})\epsilon_1^\mu(p_1)\epsilon_2^\nu(p_2)p_3^\rho \epsilon_4^\sigma(p_4) = 0$$
$$S_{\mu\nu\rho\sigma}(\{p_i\})\epsilon_1^\mu(p_1)\epsilon_2^\nu(p_2)\epsilon_3^\rho(p_3)p_4^\sigma = 0$$

PROJECTORS

- Ward Identities

$$S_{\mu\nu\rho\sigma}(\{p_i\})p_1^\mu\epsilon_2^\nu(p_2)\epsilon_3^\rho(p_3)\epsilon_4^\sigma(p_4) = 0$$
$$S_{\mu\nu\rho\sigma}(\{p_i\})\epsilon_1^\mu(p_1)p_2^\nu\epsilon_3^\rho(p_3)\epsilon_4^\sigma(p_4) = 0$$
$$S_{\mu\nu\rho\sigma}(\{p_i\})\epsilon_1^\mu(p_1)\epsilon_2^\nu(p_2)p_3^\rho\epsilon_4^\sigma(p_4) = 0$$
$$S_{\mu\nu\rho\sigma}(\{p_i\})\epsilon_1^\mu(p_1)\epsilon_2^\nu(p_2)\epsilon_3^\rho(p_3)p_4^\sigma = 0$$

- Number of tensorial structures reduce to 10
- Issue: Huge number of terms at intermediate steps
- Make specific gauge choice of external gauge bosons
- Unaffected final result due to gauge invariance

Not imposing gauge invariance on tensor structures, fix the gauge of external bosons

PROJECTORS

Gauge Fixed Tensor

$$\epsilon_1.p_2 = \epsilon_2.p3 = \epsilon_3.p_4 = \epsilon_4.p_1 = 0$$

- Choice is arbitrary
- Number of tensorial structures reduce to 10
- Total number is same as earlier: consistent

$$T_1 = \epsilon_1 \cdot \epsilon_2 \, \epsilon_3 \cdot \epsilon_4 \,, \qquad T_2 = \epsilon_1 \cdot \epsilon_3 \, \epsilon_2 \cdot \epsilon_4 \,,$$

$$T_3 = \epsilon_1 \cdot \epsilon_4 \, \epsilon_2 \cdot \epsilon_3 \,, \qquad T_4 = \epsilon_1 \cdot \epsilon_2 \, p_1 \cdot \epsilon_3 \, p_3 \cdot \epsilon_4 \,,$$

$$T_5 = \epsilon_1 \cdot \epsilon_3 \, p_1 \cdot \epsilon_2 \, p_3 \cdot \epsilon_4 \,, \qquad T_6 = \epsilon_1 \cdot \epsilon_4 \, p_1 \cdot \epsilon_2 \, p_1 \cdot \epsilon_3 \,,$$

$$T_7 = \epsilon_2 \cdot \epsilon_3 \, p_3 \cdot \epsilon_1 \, p_3 \cdot \epsilon_4 \,, \qquad T_8 = \epsilon_2 \cdot \epsilon_4 \, p_3 \cdot \epsilon_1 \, p_1 \cdot \epsilon_3 \,,$$

$$T_9 = \epsilon_3 \cdot \epsilon_4 \, p_1 \cdot \epsilon_2 \, p_3 \cdot \epsilon_1 \,, \qquad T_{10} = p_3 \cdot \epsilon_1 \, p_1 \cdot \epsilon_2 \, p_1 \cdot \epsilon_3 \, p_3 \cdot \epsilon_4$$

Matrix element $|\mathcal{M}\rangle = \sum_{i=1}^{10} A_i(\{s_{ij}\}) \, T_i \quad \text{with} \quad s_{ij} = (p_i + p_j)^2$

Goal: calculate $\{A_i\}$ for $i = 1, 2, \cdots, 10$

Projectors

$$\sum_{pol} P(A_i)|\mathcal{M}\rangle = A_i$$

Projection must be done in d-dimensions

- Polarisation sum must be consistent with gauge choice

$$\sum_{pol} \epsilon_1^{*\mu}(p_1)\, \epsilon_1^{\nu}(p_1) = -g^{\mu\nu} + \frac{p_1^{\mu}\, p_2^{\nu} + p_1^{\nu}\, p_2^{\mu}}{p_1 \cdot p_2}$$

Projectors themselves can be decomposed in tensor basis: $P(A_i) = \sum_{j=1}^{10} X_{ij} T_j^*$

$$\sum_{pol} \sum_{j=1}^{10} X_{ij} T_j^* T_k = \delta_{ik}$$

Solve for $X_{ij}(d, s, t)$

- Specify the helicities of external gluons
- Choice of reference momentum must be consistent with gauge choice

$$\epsilon_1^{+,\mu}(p_1, q_1) \quad \text{: Reference momentum} \quad q_1 = p_2$$

- All plus amplitude

$$|\mathcal{M}_{++++}\rangle = \sum_{i=1}^{10} A_i T_{i,++++} = \frac{1}{4\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\Big\{ -4(A_1 + A_3)st - 4A_2u^2 - 2A_4stu + 2A_5su^2 - 2A_6stu$$
$$-2A_7stu + 2A_8tu^2 - 2A_9stu - A_{10}stu^2\Big\}$$

Goal: calculate all plus helicity amplitudes

Calculate $\{A_i\}$

Perturbative Expansion

A_i Can be expanded in powers of strong coupling constant

$$A_i(s,t,\epsilon) = a_s \sum_{L=0}^{\infty} a_s^L A_i^{(L)}(s,t,\epsilon)\,, \qquad a_s = g_s^2/(16\pi)^2$$

$$d = 4 - 2\epsilon$$

Goal: Calculate the three loop contribution $A_i^{(3)}$ in the planar limit

First time in QCD!

LARGE N LIMIT

- Amplitude is a tensor in Color space
- Can be decomposed in terms of traces of fundamental color generators of SU(N)
- Pure gauge amplitude can be expressed in terms of six color structures

$$C_1 = tr(1234) + tr(1432) \qquad C_4 = tr(12)tr(34)$$
$$C_2 = tr(1243) + tr(1342) \qquad C_5 = tr(13)tr(24)$$
$$C_3 = tr(1423) + tr(1324) \qquad C_6 = tr(14)tr(23)$$

$$tr(T^{a_1}T^{a_2}T^{a_3}T^{a_4}) = tr(1234)$$

It can be written as

$$A_i^{(L)} = \sum_{\lambda=1}^{3}\left\{\sum_{k=0}^{\lfloor \frac{L}{2} \rfloor} N^{L-2k} A_{i,\lambda}^{(L),2k}\right\} C_\lambda + \sum_{\lambda=4}^{6}\left\{\sum_{k=0}^{\lfloor \frac{L-1}{2} \rfloor} N^{L-2k-1} A_{i,\lambda}^{(L),2k+1}\right\} C_\lambda$$

$A_{i,\lambda}^{(L),0}$ are leading order in N. Others are sub-leading

Only $A_{i,\lambda}^{(L),0}$ contributes in the large N-limit

Our goal

Only planar diagrams contribute: Planar limit

Large N Limit

- In presence of light fermions: planar diagrams involving fermions should be included
- n_f should be counted same as N
- Include all the terms satisfying $n_f^{a_1} N^{a_2} a_s^{a_3}$ with $a_1 + a_2 = a_3$
- Symmetry of the process: one particular color ordered amplitude is sufficient

Computation

- Age-old Feynman diagrammatic approach, however, lots of challenges!

- Huge expressions, complicated reductions!

- Feynman diagrams using `Qgraf`: 48723 three-loop, 39k planar topologies — Noguira '06

- Discard non-planar diagrams by removing sub-leading colors.

- Cross-checked using `REDUZE2` using the liberty of shifting loop momenta — Monteuffel, Studerus '12

- In-house `FORM` code: convert `Qgraf` raw output to `FORM` — Vermaseren

  SU(N) color simplification

  Lorentz algebra
  Dirac algebra
  } in d-spacetime dimensions

COMPUTATION

- Removing unphysical DOF of gluons: Internal: ghost loops

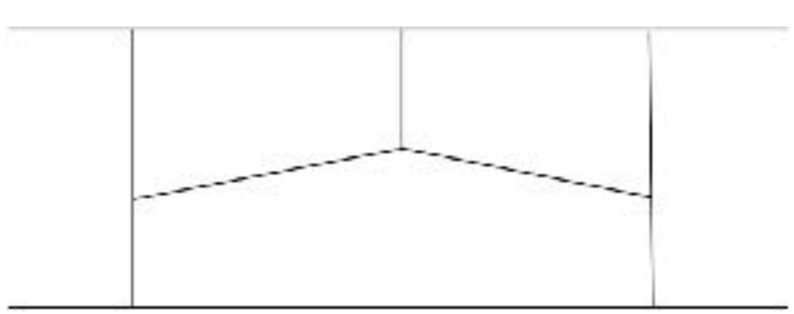
  External: pol sum in axial gauge

- Millions of 3-loop integrals with very high power of numerators: highest 6!

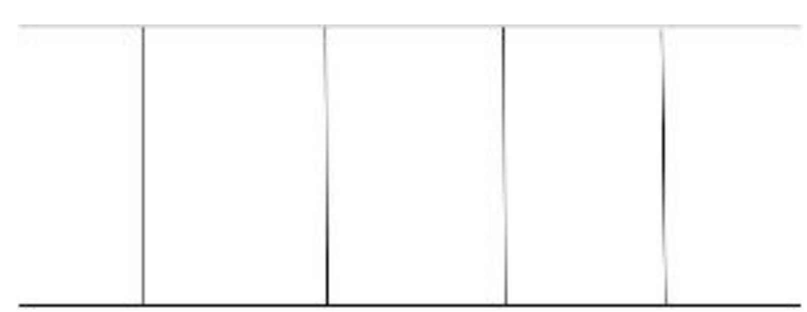
$$J[a_1, a_2, \cdots, a_{15}] = \int \prod_{L=1,2,3} \left[\frac{d^d k_L}{(2\pi)^d} \right] \frac{1}{D_1^{a_1} D_2^{a_2} \cdots D_{15}^{a_{15}}}$$

D are the inverse of propagators

Most complicated $\sum |a_i| = 16$



Tennis court



Tripple ladder

COMPUTATION

- IBP Reduction

  `FIRE5.2 C++` along with `LiteRed1.82`

  2-step reductions:

  1. `LiteRed` along with `Mint`: symbolic rules & 89 MIs
  2. Non-minimal set : Huge reduction file!
  3. Reduce again using `FIRE` c++: 81 MIs — Boels, Jin, Luo ‘18
  4. Minimal set: reduction file size reduced 1/10

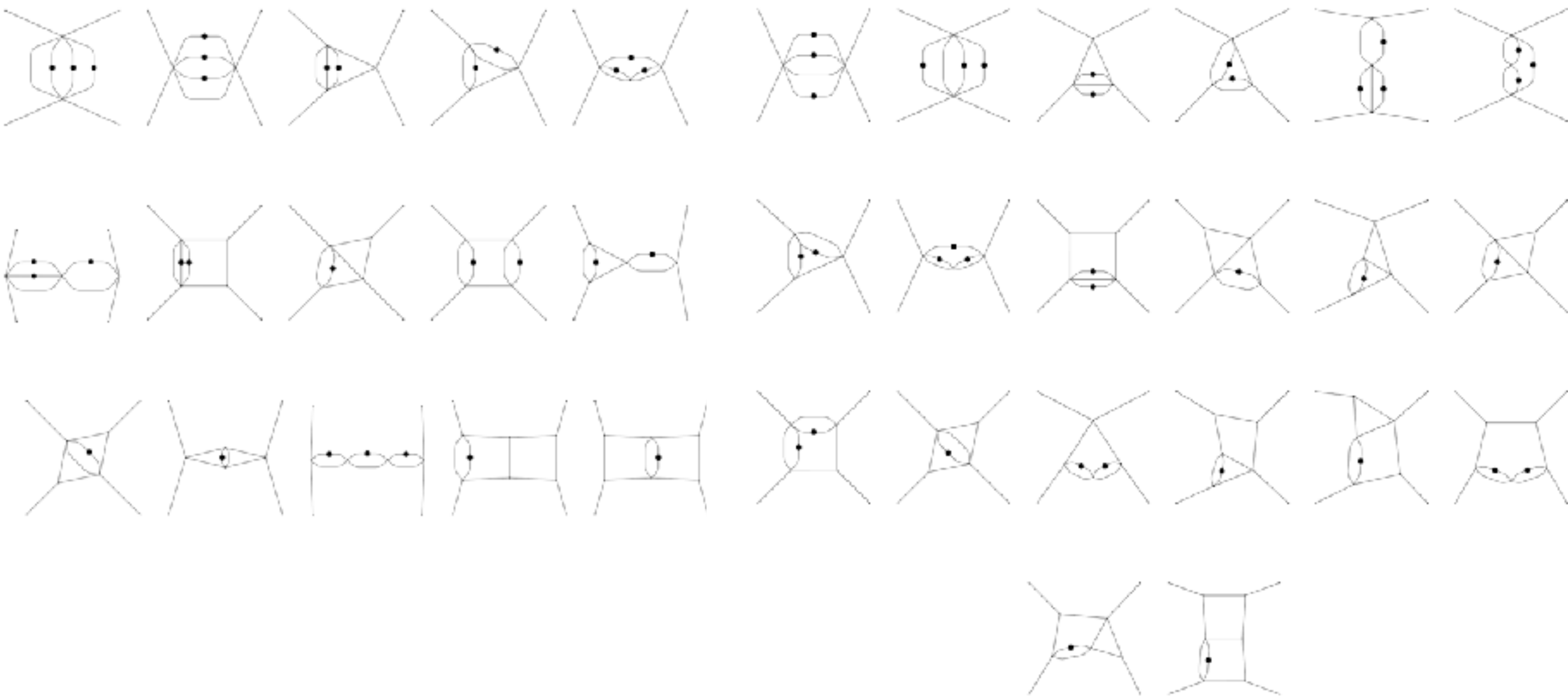
  Advantage: substantial improvement of total reduction time

  Comments: Non-minimal set of MIs is inefficient

- 21 MIs with one double prop
- 2 MIs with one numerator

MIs in UT-Basis

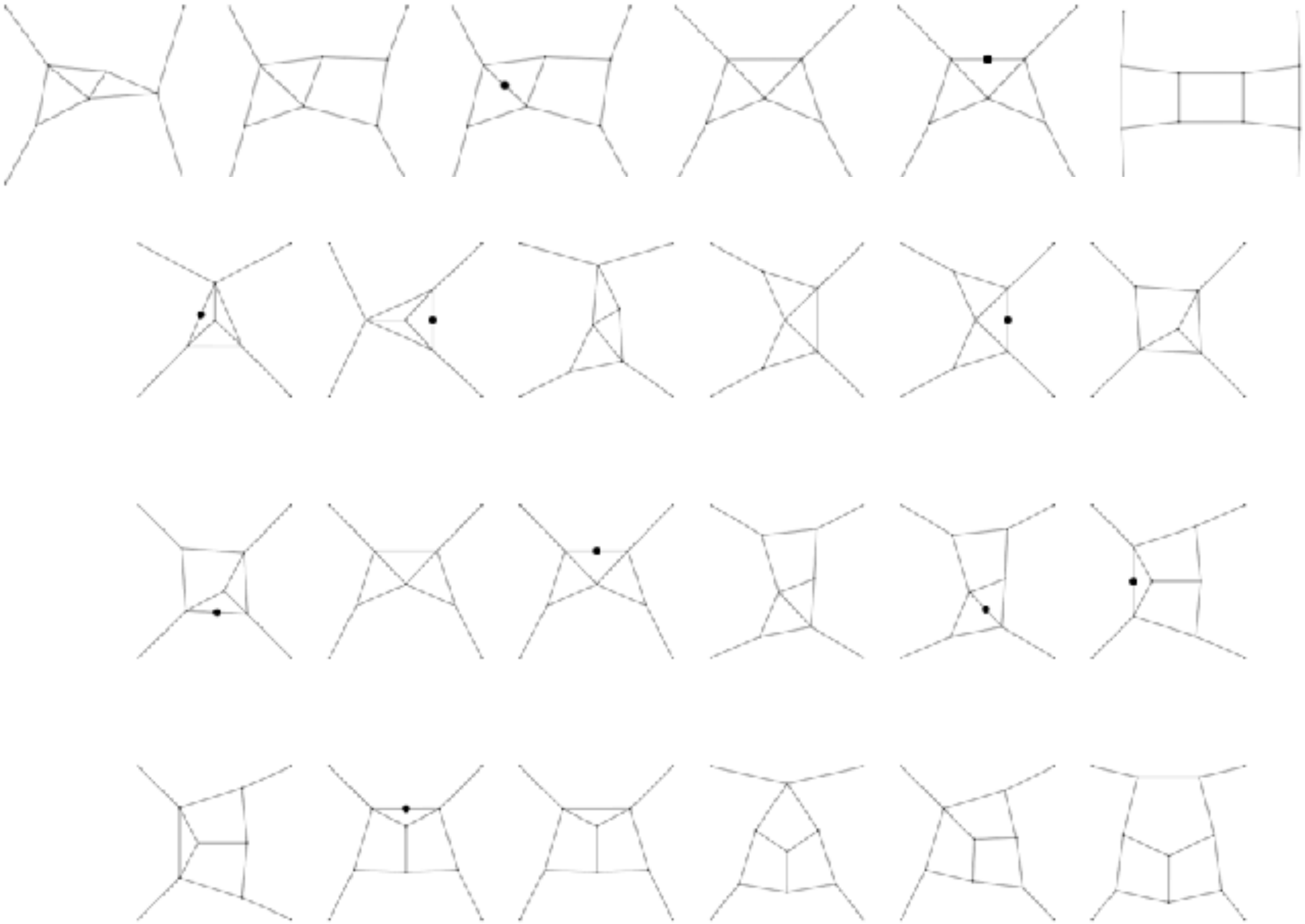
Henn, Smirnov, Smirnov '13



With Bubble Sub-diagrams

MIs in UT-Basis

Henn, Smirnov, Smirnov '13



MIs without Bubble sub-integrals

- Remaining integrals are obtained by interchanging s & t

UV Renormalisation

- Dimensional regularisation: $d = 4 - 2\epsilon$
- UV structures of amplitude and all plus amplitude are different

$\mathcal{M}^{(0)}(s,t,\epsilon) \neq 0$ UV divergent 1-loop

$\mathcal{M}^{(0)}_{++++}(s,t,\epsilon) = 0$ All plus amplitude vanishes at tree level → UV Finite 1-loop

UV Renormalisation

- Dimensional regularisation: $d = 4 - 2\epsilon$
- UV structures of amplitude and all plus amplitude are different

$\mathcal{M}^{(0)}(s,t,\epsilon) \neq 0$ UV divergent 1-loop

$\mathcal{M}^{(0)}_{++++}(s,t,\epsilon) = 0$ All plus amplitude vanishes at tree level → UV Finite 1-loop

- UV Renormalisation is done in $\overline{\mathrm{MS}}$ Scheme

$$\hat{a}_s S_\epsilon = a_s(\mu_R^2) Z_{a_s}(\mu_R^2) \left(\frac{\mu^2}{\mu_R^2}\right)^{-\epsilon}$$

- Bare strong coupling constant: $\hat{a}_s = \hat{\alpha}_s/(4\pi) = \hat{g}_s^2/(16\pi)^2$
- Renormalised: a_s
- μ : introduced to maintain the dimensionless of coupling in d-dimensions
- μ_R : renormalisation scale

$$Z_{a_s}(\mu_R^2) = 1 + a_s(\mu_R^2)\left\{-\frac{1}{\epsilon}\beta_0\right\} + a_s^2(\mu_R^2)\left\{\frac{1}{\epsilon^2}\beta_0^2 - \frac{1}{2\epsilon}\beta_1\right\} + a_s^3(\mu_R^2)\left\{-\frac{1}{\epsilon^3}\beta_0^3 + \frac{7}{6\epsilon^2}\beta_0\beta_1 - \frac{1}{3\epsilon}\beta_2\right\}$$

Tarasov, Vladimirov, Zharkov '80

- β_i : coefficients of QCD-beta function

IR Factorisation

- UV Renormalised amplitude contains soft & collinear divergences

- IR structures for the amplitude and all plus amplitude for the same reason

$$\mathcal{M}^{(L)}(s,t,\epsilon)|_{poles} = \frac{\mathcal{M}^{(L)}_{-2L}}{\epsilon^{2L}} + \cdots + \frac{\mathcal{M}^{(L)}_{-1}}{\epsilon}\,, \quad L \geq 1$$

$$\mathcal{M}^{(L)}_{++++}(s,t,\epsilon)|_{poles} = \frac{\mathcal{M}^{(L)}_{-L}}{\epsilon^{2(L-1)}} + \cdots + \frac{\mathcal{M}^{(L)}_{-1}}{\epsilon}\,, \quad L \geq 2$$

IR FACTORISATION

- UV Renormalised amplitude contains soft & collinear divergences

- IR structures for the amplitude and all plus amplitude for the same reason

$$\mathcal{M}^{(L)}(s,t,\epsilon)|_{poles} = \frac{\mathcal{M}^{(L)}_{-2L}}{\epsilon^{2L}} + \cdots + \frac{\mathcal{M}^{(L)}_{-1}}{\epsilon} \,, \quad L \geq 1$$

$$\mathcal{M}^{(L)}_{++++}(s,t,\epsilon)|_{poles} = \frac{\mathcal{M}^{(L)}_{-L}}{\epsilon^{2(L-1)}} + \cdots + \frac{\mathcal{M}^{(L)}_{-1}}{\epsilon} \,, \quad L \geq 2$$

- These poles are universal and were first predicted by Catani up to 2-loop (except single pole)

Catani '98

Sterman, Yeomans '03

IR Factorisation

- UV Renormalised amplitude contains soft & collinear divergences

- IR structures for the amplitude and all plus amplitude for the same reason

$$\mathcal{M}^{(L)}(s,t,\epsilon)|_{poles} = \frac{\mathcal{M}^{(L)}_{-2L}}{\epsilon^{2L}} + \cdots + \frac{\mathcal{M}^{(L)}_{-1}}{\epsilon}\,, \quad L \geq 1$$

$$\mathcal{M}^{(L)}_{++++}(s,t,\epsilon)|_{poles} = \frac{\mathcal{M}^{(L)}_{-L}}{\epsilon^{2(L-1)}} + \cdots + \frac{\mathcal{M}^{(L)}_{-1}}{\epsilon}\,, \quad L \geq 2$$

- These poles are universal and were first predicted by Catani up to 2-loop (except single pole)

Catani '98

Sterman, Yeomans '03

- Using SCET, Becher & Neubert derived an exact formula for IR divergences for any loops & legs in massless QCD involving the single pole. '09

- Using Wilson lines for hard partons and soft-eikonal jet functions, Gardi & Magnea Also arrived at similar result '09

IR Factorisation

- From SCET

$$\mathcal{M}^{fin}(s,t) = \lim_{\epsilon\to 0} \mathbf{Z}^{-1}(\epsilon,s,t)\mathcal{M}(\epsilon,s,t)$$

UV finite

All the IR divergences are governed by the matrix $\mathbf{Z}$

IR FACTORISATION

- From SCET

$$\mathcal{M}^{fin}(s,t) = \lim_{\epsilon \to 0} \mathbf{Z}^{-1}(\epsilon, s, t)\mathcal{M}(\epsilon, s, t)$$

UV finite

All the IR divergences are governed by the matrix $\mathbf{Z}$

The all order solution:

$$\mathbf{Z}(\epsilon, s, t, \mu) = \mathcal{P} \exp\left[\int_{\mu}^{\infty} \frac{d\mu'}{\mu'} \mathbf{\Gamma}(s, t, \mu')\right]$$

Becher & Neubert '09

Anomalous dimension matrix

$$\mathbf{\Gamma} = \sum_{i<j} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{cusp}(a_s) \log\left(\frac{\mu^2}{-s_{ij}}\right) + \sum_i \gamma^i(a_s)$$

$\mathbf{T}_i$ Color generator matrix associated with i-th Parton

γ_{cusp} Cusp anomalous dimensions

γ^i Anomalous dimensions for partons

IR FACTORISATION

Explicit solution up to 3-loop

$$\boldsymbol{Z} = 1 + a_s\left(\frac{\Gamma'_0}{4\epsilon^2} + \frac{\boldsymbol{\Gamma}_0}{2\epsilon}\right) + a_s^2\left[\frac{(\Gamma'_0)^2}{32\epsilon^4} + \frac{\Gamma'_0}{8\epsilon^3}\left(\boldsymbol{\Gamma}_0 - \frac{3}{2}\beta_0\right) + \frac{\boldsymbol{\Gamma}_0}{8\epsilon^2}(\boldsymbol{\Gamma}_0 - 2\beta_0) + \frac{\Gamma'_1}{16\epsilon^2} + \frac{\boldsymbol{\Gamma}_1}{4\epsilon}\right]$$

$$+ a_s^3\left[\frac{(\Gamma'_0)^3}{384\epsilon^6} + \frac{(\Gamma'_0)^2}{64\epsilon^5}(\boldsymbol{\Gamma}_0 - 3\beta_0) + \frac{\Gamma'_0}{32\epsilon^4}\left(\boldsymbol{\Gamma}_0 - \frac{4}{3}\beta_0\right)\left(\boldsymbol{\Gamma}_0 - \frac{11}{3}\beta_0\right) + \frac{\Gamma'_0\Gamma'_1}{64\epsilon^4}\right.$$

$$+ \frac{\boldsymbol{\Gamma}_0}{48\epsilon^3}(\boldsymbol{\Gamma}_0 - 2\beta_0)(\boldsymbol{\Gamma}_0 - 4\beta_0) + \frac{\Gamma'_0}{16\epsilon^3}\left(\boldsymbol{\Gamma}_1 - \frac{16}{9}\beta_1\right) + \frac{\Gamma'_1}{32\epsilon^3}\left(\boldsymbol{\Gamma}_0 - \frac{20}{9}\beta_0\right)$$

$$\left. + \frac{\boldsymbol{\Gamma}_0\boldsymbol{\Gamma}_1}{8\epsilon^2} - \frac{\beta_0\boldsymbol{\Gamma}_1 + \beta_1\boldsymbol{\Gamma}_0}{6\epsilon^2} + \frac{\Gamma'_2}{36\epsilon^2} + \frac{\boldsymbol{\Gamma}_2}{6\epsilon}\right].$$

Where $\Gamma' = -\gamma_{cusp}(a_s)\sum_i \mathbf{T}_i^2$

Explicit solution up to 3-loop

$$\boldsymbol{Z} = 1 + a_s\left(\frac{\Gamma'_0}{4\epsilon^2} + \frac{\boldsymbol{\Gamma}_0}{2\epsilon}\right) + a_s^2\left[\frac{(\Gamma'_0)^2}{32\epsilon^4} + \frac{\Gamma'_0}{8\epsilon^3}\left(\boldsymbol{\Gamma}_0 - \frac{3}{2}\beta_0\right) + \frac{\boldsymbol{\Gamma}_0}{8\epsilon^2}(\boldsymbol{\Gamma}_0 - 2\beta_0) + \frac{\Gamma'_1}{16\epsilon^2} + \frac{\boldsymbol{\Gamma}_1}{4\epsilon}\right]$$
$$+ a_s^3\left[\frac{(\Gamma'_0)^3}{384\epsilon^6} + \frac{(\Gamma'_0)^2}{64\epsilon^5}(\boldsymbol{\Gamma}_0 - 3\beta_0) + \frac{\Gamma'_0}{32\epsilon^4}\left(\boldsymbol{\Gamma}_0 - \frac{4}{3}\beta_0\right)\left(\boldsymbol{\Gamma}_0 - \frac{11}{3}\beta_0\right) + \frac{\Gamma'_0\Gamma'_1}{64\epsilon^4}\right.$$
$$+ \frac{\boldsymbol{\Gamma}_0}{48\epsilon^3}(\boldsymbol{\Gamma}_0 - 2\beta_0)(\boldsymbol{\Gamma}_0 - 4\beta_0) + \frac{\Gamma'_0}{16\epsilon^3}\left(\boldsymbol{\Gamma}_1 - \frac{16}{9}\beta_1\right) + \frac{\Gamma'_1}{32\epsilon^3}\left(\boldsymbol{\Gamma}_0 - \frac{20}{9}\beta_0\right)$$
$$\left. + \frac{\boldsymbol{\Gamma}_0\boldsymbol{\Gamma}_1}{8\epsilon^2} - \frac{\beta_0\boldsymbol{\Gamma}_1 + \beta_1\boldsymbol{\Gamma}_0}{6\epsilon^2} + \frac{\Gamma'_2}{36\epsilon^2} + \frac{\boldsymbol{\Gamma}_2}{6\epsilon}\right].$$

Where $\Gamma' = -\gamma_{cusp}(a_s)\sum_i \mathbf{T}_i^2$

For 3-loop all ++++ amplitude, we need up to $\mathcal{O}(a_s^2)$

However, for the full amplitude we need up to $\mathcal{O}(a_s^3)$

In the planar limit, the IR factorisation simplifies

Almelid, Duhr & Gardi '15

$$\mathbf{Z}^{-1}(\epsilon, s, t) = z^{-1}(\epsilon, s) z^{-1}(\epsilon, t)$$

z are simple scalar functions

$$z(s) = \exp\left[\sum_{L\geq 1} a_s^L Y^{(L)} (-s)^{L\epsilon}\right] \qquad \text{where} \qquad Y^{(L)} = \log(F_g)|_{poles}$$

Gluon form factor

These are

$$Y^{(1)} = \frac{1}{\epsilon^2}\left[-\frac{1}{2} C_A \gamma^0_{cusp}\right] + \frac{1}{\epsilon}\left[\gamma^0_g\right]$$

$$Y^{(2)} = \frac{1}{\epsilon^3}\left[\frac{3}{8}\beta_0 C_A \gamma^0_{cusp}\right] + \frac{1}{\epsilon^2}\left[-\frac{1}{2}\beta_0 \gamma^0_g - \frac{1}{8} C_A \gamma^1_{cusp}\right] + \frac{1}{\epsilon}\left[\frac{\gamma^1_g}{2}\right]$$

and so on

Results

- Our 1 and 2-loop results agree with the universal IR structure.
- We have checked the amplitude as well as ++++ amplitude
- They exhibit the necessary symmetry between s & t
- We have extended the 2-loop results to ϵ^5 order
- 3-loop reduction is done!
- Result in terms of Master integrals is at hand.
- Just waiting for the final result! Will be available soon!

Concluding Remarks

- We have computed the 4-gluon 3-loop amplitude in QCD in the planar limit
- Results will be available soon
- First step towards the full computation
- This result will be used as a first check-up for the full computation
- Will be used to compute the di-jet production
- First ever attempt in QCD!

Concluding Remarks

- We have computed the 4-gluon 3-loop amplitude in QCD in the planar limit
- Results will be available soon
- First step towards the full computation
- This result will be used as a first check-up for the full computation
- Will be used to compute the di-jet production
- First ever attempt in QCD!

Thank you!

Preliminary Result (Partial of course)

Coefficient of. $n_f^3\ Tr(T^{a_1}T^{a_3}T^{a_4}T^{a_2})$ of the Amplitude (Unrenormalised)

$$\frac{1}{\epsilon}\frac{1}{(1+x)^3}\Bigg[T_1\left(\frac{8x^4}{27}+\frac{28x^3}{27}+\frac{4x^2}{3}+\frac{20x}{27}+\frac{4}{27}\right)+T_2\left(-\frac{4x^3}{27}-\frac{4x^2}{27}+\frac{4x}{27}+\frac{4}{27}\right)+T_5\left(\frac{8}{27t}-\frac{8x^2}{27t}\right)$$
$$+T_8\left(\frac{8}{27t}-\frac{8x^2}{27t}\right)+T_{10}\left(\frac{16}{27t^2}-\frac{16x}{27t^2}\right)\Bigg]$$

$$+\frac{1}{(1+x)^3}\Bigg[T_1\left\{\left(-\frac{8x^4}{9}-\frac{28x^3}{9}-4x^2-\frac{20x}{9}-\frac{4}{9}\right)\left(H(\{0\},x)+\log(-s)-3\right)\right\}$$
$$+T_2\left\{\left(\frac{4x^3}{9}+\frac{4x^2}{9}-\frac{4x}{9}-\frac{4}{9}\right)\left(\log(-s)-3\right)\right\}$$
$$+T_5\left\{\left(\frac{8x^2}{9t}-\frac{8}{9t}\right)\left(\log(-s)-3\right)\right\}+T_8\left\{\left(\frac{8x^2}{9t}-\frac{8}{9t}\right)\left(\log(-s)-3\right)\right\}$$
$$+T_{10}\left\{\left(\frac{16x}{9t^2}-\frac{16}{9t^2}\right)\left(\log(-s)-3\right)\right\}\Bigg]$$

Up to an overall normalisation factor $x=\frac{u}{t}\,,t=2p_1.p_3,u=2p_2.p_3$

Absolutely no check has been performed! Will be done soon!