

# *Threshold and jet radius joint resummation for single-inclusive jet production*

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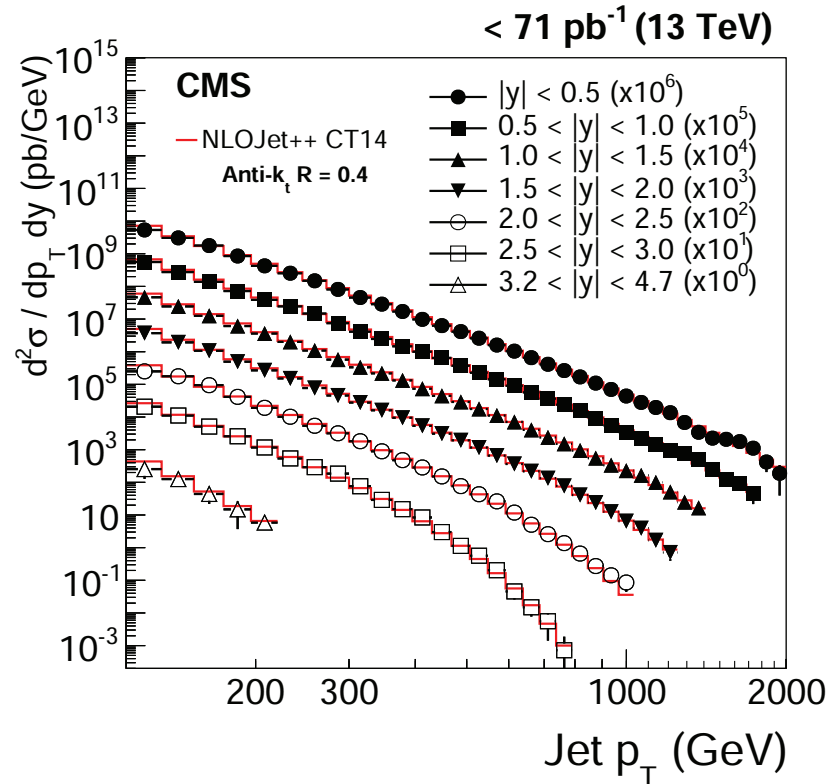
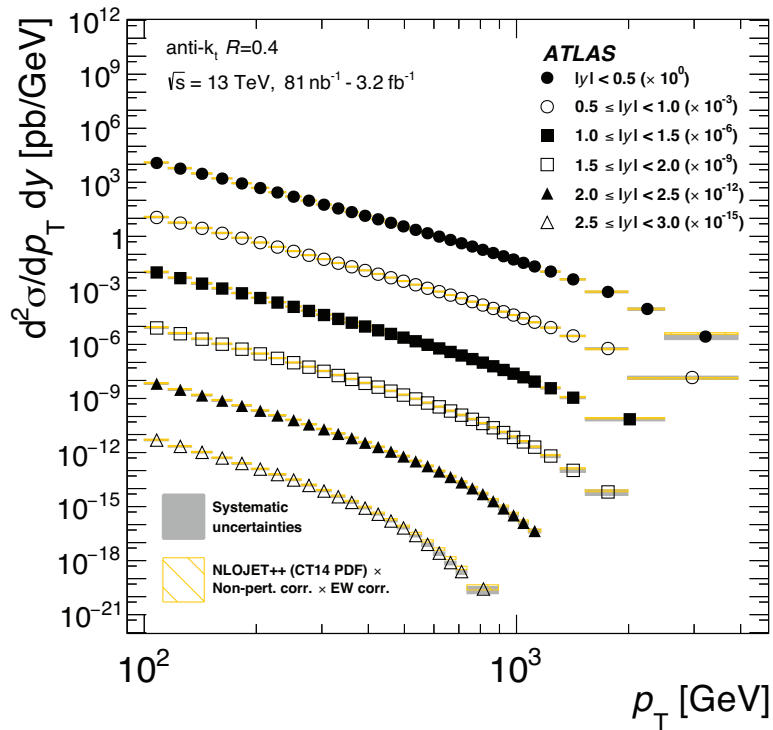
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DESY Workshop *Loops and Legs in Quantum Field Theory*, St. Goar, April 30, 2018

## *Based on work done in collaboration with:*

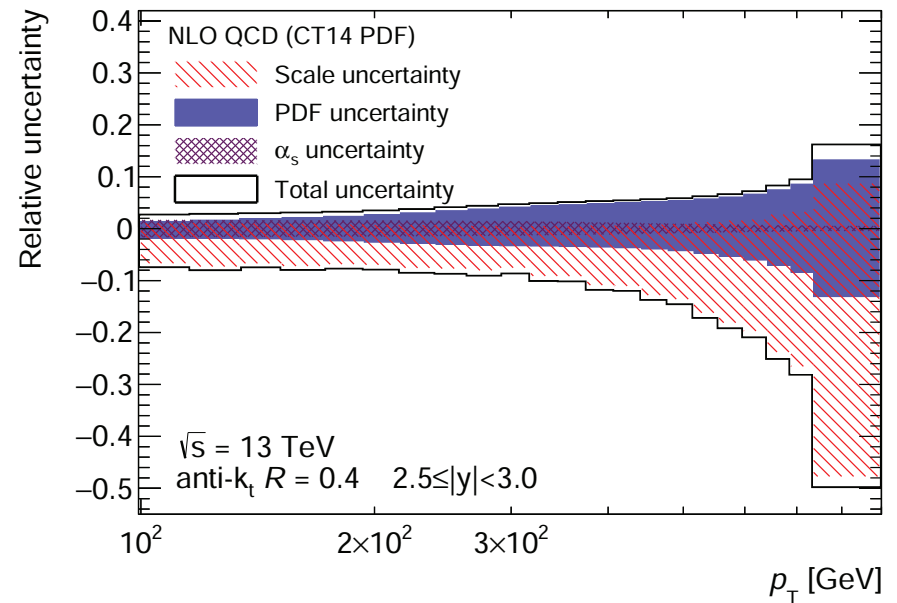
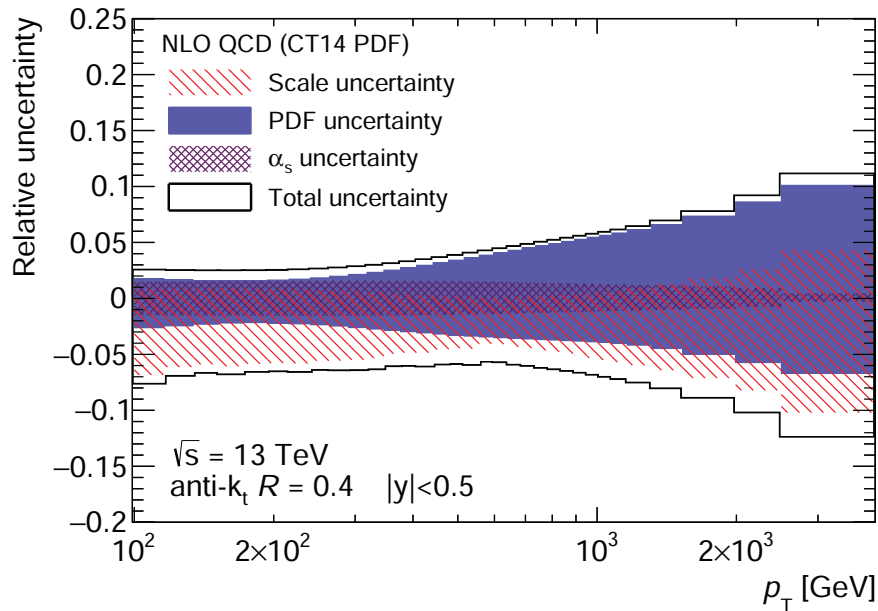
- *Threshold and jet radius joint resummation for single-inclusive jet production*  
Xiaohui Liu, S. M. and Felix Ringer *Phys.Rev.Lett.* 119 (2017) no.21, 212001  
[arXiv:1708.04641](#)
- *Phenomenology of single-inclusive jet production with jet radius and threshold resummation*  
Xiaohui Liu, S. M. and Felix Ringer [arXiv:1801.07284](#)

# Single-inclusive jet production



- Double differential cross section for  $pp \rightarrow \text{jet} + X$  at  $\sqrt{s} = 13 \text{ TeV}$ 
  - transverse momentum  $p_T$  and rapidity  $y$  of signal-jet
  - ATLAS arXiv:1711.02692 (left), CMS arXiv:1605.04436 (right)
- Comparison with NLO perturbative QCD predictions (NLOJET++ Nagy)
  - impressive agreement over several orders of magnitude

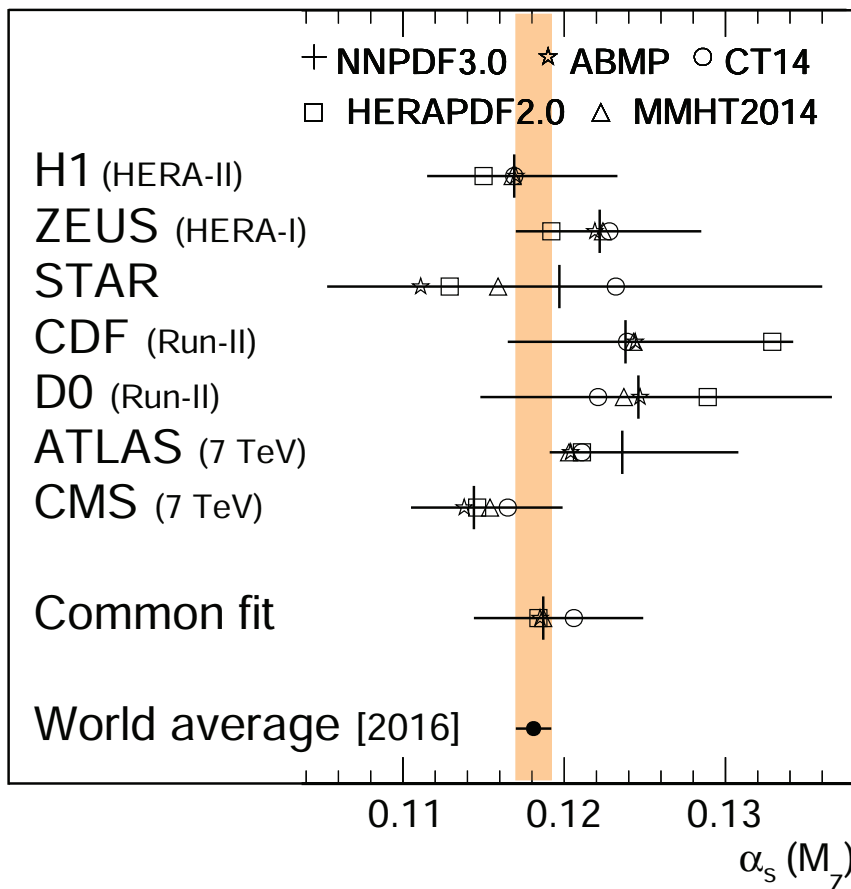
# Uncertainties



- Relative QCD uncertainties in single-inclusive jet cross-sections at NLO
  - first and last  $|y|$  bins for inclusive jet measurement
  - uncertainties due to renormalization and factorization scales,  $\mu_R$ ,  $\mu_F$ , the strong coupling  $\alpha_s$  with  $\Delta\alpha_s = 0.0015$  and PDFs CT14
- Sizable uncertainties
  - $\mathcal{O}(10\%)$  for central rapidities  $|y| < 0.5$
  - $\mathcal{O}(30 - 40\%)$  forward  $2.5 \leq |y| < 3.0$  at large  $p_T$

# Uses of inclusive jet data

- Determination of  $\alpha_s(M_Z)$  and PDFs (gluon at medium to large  $x$ )
  - partonic cross sections  $\hat{\sigma}_{ij \rightarrow \text{jet}} \propto \alpha_s^2(\mu)$
  - jet cross section  $d\sigma_{pp \rightarrow \text{jet}} = \alpha_s^2(\mu) \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes [\dots]$



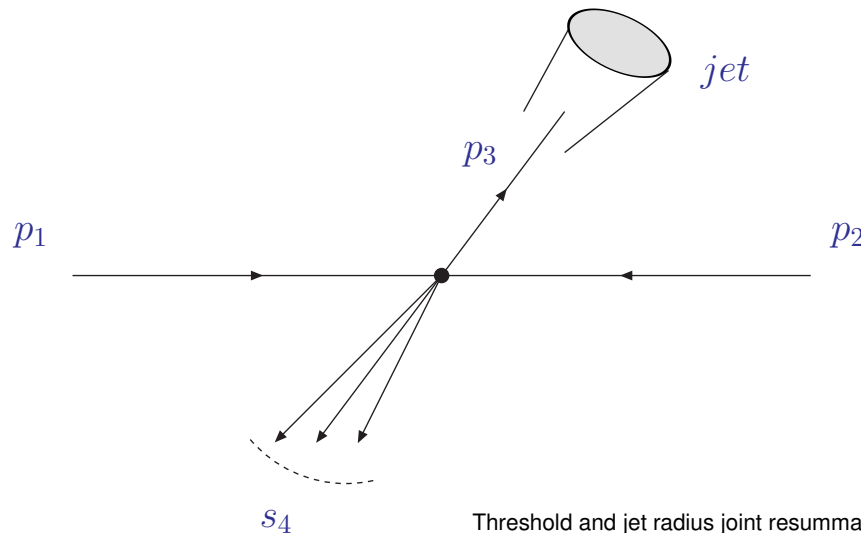
- $\alpha_s(M_Z)$  at NLO in QCD from inclusive jet cross section data  
Britzger, Rabbertz, Savoiu, Sieber '17
- correlations between PDFs and  $\alpha_s(M_Z)$  are important

# QCD factorization

- Double differential cross section for  $pp \rightarrow \text{jet} + X$ 
  - transverse momentum  $p_T$  and rapidity  $\eta$  of signal-jet

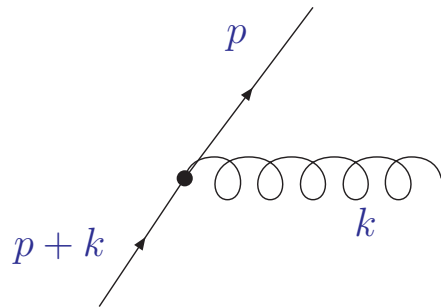
$$\frac{p_T^2 d^2\sigma}{dp_T^2 d\eta} = \sum_{i_1 i_2} \int_0^{V(1-W)} dz \int_{\frac{VW}{1-z}}^{1-\frac{1-V}{1-z}} dv x_1^2 f_{i_1}(x_1) x_2^2 f_{i_2}(x_2) \frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv dz}(v, z, p_T, R)$$

- PDFs  $f_i$  and variables  $V = 1 - p_T e^{-\eta} / \sqrt{S}$ ,  $VW = p_T e^{\eta} / \sqrt{S}$
- Partonic cross sections  $\hat{\sigma}_{i_1 i_2}$  dependent on partonic kinematic variables on  $s = x_1 x_2 S$ ,  $v = u/(u+t)$  and partonic threshold  $z = s_4/s \rightarrow 0$
- Mandelstam variables  $s = (p_1 + p_2)^2$ ,  $t = (p_1 - p_3)^2$  and  $u = (p_2 - p_3)^2$  with kinematics constraint  $s + t + u = s_4$



# Threshold logarithms

- Soft and collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for:  $E_g = 0$ , soft  $\theta_{qg} = 0$  collinear



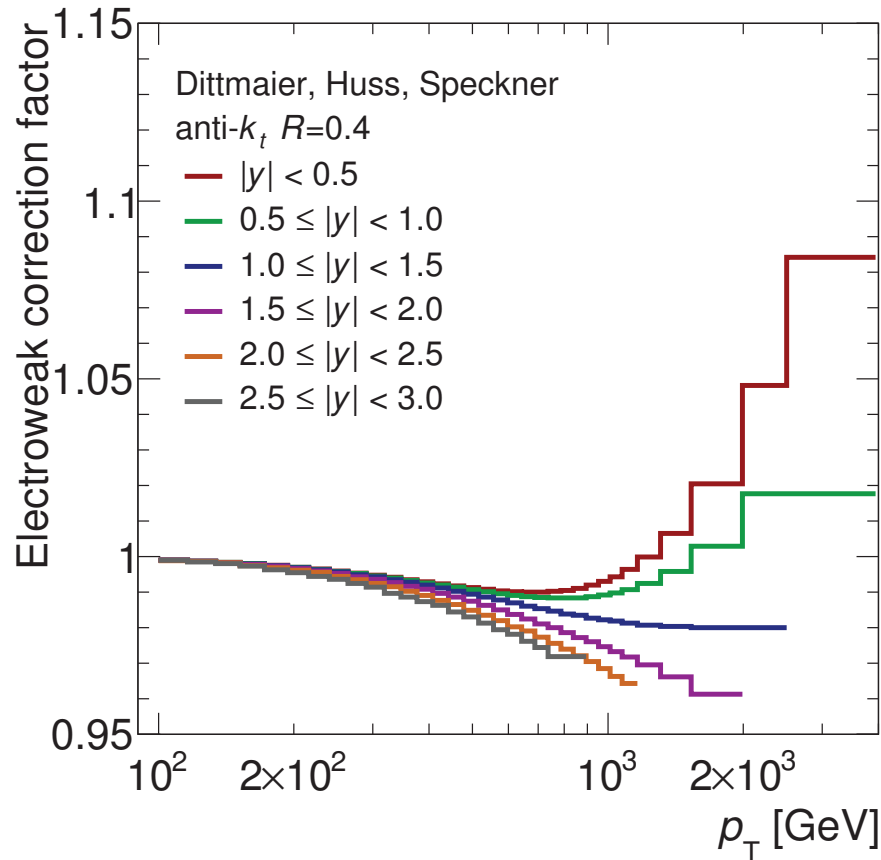
$$\alpha_s \int d^4 k \frac{1}{(p+k)^2} \quad \rightarrow \quad \alpha_s \int dE_g d\sin\theta_{qg} \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\rightarrow \alpha_s \ln^2(\dots)$$

- Large double-logarithmic corrections  $\ln(\dots) \gg 1$  near threshold
- Single-inclusive jet production with threshold logarithms
 
$$\alpha_s^n (\ln^{2n-1}(z)/z)_+$$
 for  $z = s_4/s \rightarrow 0$ 
  - positive corrections enhance partonic cross sections  $\hat{\sigma}_{i_1 i_2}$
  - long history of resummation [Sterman '87](#); [Catani, Trentadue '88](#); ...
- Same for weak radiative corrections: positive corrections for large  $p_T \gg M_W$  [Dittmaier, Huss, Speckner '12](#)

# Electroweak corrections at NLO

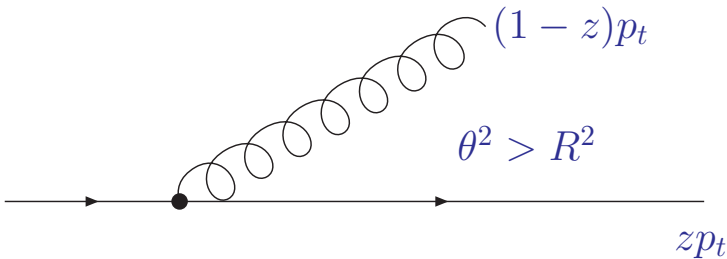


- Electroweak correction factors for inclusive jet cross-section as function of jet  $p_T$  for all  $|y|$  bins Dittmaier, Huss, Speckner '12
  - negative corrections  $\mathcal{O}(1 - 3\%)$  for  $p_T$  in range  $\mathcal{O}(\text{few } 100) \text{ GeV}$
  - sizeable (positive) effect  $\mathcal{O}(10\%)$  at large  $p_T \gg M_W$



# Jet radius logarithms (I)

- Collinear singularity when the jet becomes very narrow
  - partons radiated outside of jet (not recombined with jet by chosen jet algorithm) become more and more collinear to emitter
- Example
  - loss of transverse momentum for leading jet
  - quasi-collinear branching of quark with transverse momentum  $p_T$



$$\delta p_T = (1-z)p_T - p_T = -z p_T \text{ for } (1-z) > z$$

$$\delta p_T = (1-z)p_T - p_T = -z p_T \text{ for } z > (1-z)$$

- Perturbative radiation loss for average  $\langle \delta p_T \rangle_q$

$$\langle \delta p_T \rangle_q = p_T \frac{\alpha_s}{2\pi} \int_{R^2}^1 \frac{d\theta^2}{\theta^2} \int dz (\max[z, 1-z] - 1) P_{qq}(z)$$

# Jet radius logarithms (II)

- Leading order result for quark and gluon jets

Dasgupta, Magnea, Salam '07

$$\langle \delta p_T \rangle_q = C_F \frac{\alpha_s}{\pi} p_T \ln(R) \left( 2 \ln 2 - \frac{3}{8} \right) = 0.43 \alpha_s \ln(R)$$

$$\langle \delta p_T \rangle_g = \frac{\alpha_s}{\pi} p_T \ln(R) \left[ C_A \left( 2 \ln 2 - \frac{43}{96} \right) + T_f n_f \frac{7}{48} \right] = 1.02 \alpha_s \ln(R)$$

- Large single-logarithmic corrections  $\ln(1/R) \gg 1$  for small  $R$ 
  - negative corrections decrease partonic cross sections  $\hat{\sigma}_{i_1 i_2}$
  - resummation ...

# Joint resummation

## SCET factorization

- Factorization in small- $R$  and  $z \rightarrow 0$  threshold limit
  - assume anti- $k_t$  jet algorithm,  $z \sim R$ , and small finite mass of jet

$$\begin{aligned} \frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv dz} &= s \int ds_X ds_c ds_G \delta(zs - s_X - s_G - s_c) \\ &\quad \times \text{Tr} [\mathbf{H}_{i_1 i_2}(v, p_T, \mu_h, \mu) \mathbf{S}_G(s_G, \mu_{sG}, \mu)] J_X(s_X, \mu_X, \mu) \\ &\quad \times \sum_m \text{Tr} [J_m(p_T R, \mu_J, \mu) \otimes_{\Omega} S_{c,m}(s_c R, \mu_{sc}, \mu)] \end{aligned}$$

- Specific functions for individual kinematic regions
  - hard functions for  $2 \rightarrow 2$  scattering  $\mathbf{H}_{i_1 i_2}$  (known to 2-loops Broggio, Ferroglia, Pecjak, Zhang '14)
  - inclusive jet function  $J_X(s_X)$  dependent on invariant mass  $s_X$  of the recoiling collimated radiation (known to order  $\alpha_s^2$  Becher, Neubert '06, Becher, Bell '10)
  - global soft function  $\mathbf{S}_G$  accounts for wide-angle soft radiation which cannot resolve the small radius  $R$  (known to NLO Liu, S.M., Ringer '17)

# Joint resummation

## SCET factorization

- Factorization in small- $R$  and  $z \rightarrow 0$  threshold limit
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- Specific functions for individual kinematic regions
  - signal-jet function  $J(p_T R)$  accounts for energetic radiation inside jet  
Becher, Neubert, Rothen, Shao '15
  - soft-collinear (“coft”) function  $S_c(s_c R)$  captures soft radiation near jet boundary  
Becher, Neubert, Rothen, Shao '15, Chien, Hornig, Lee '15
- Sum runs over all collinear splittings and traces taken in color space
- ‘ $\otimes_{\Omega}$ ’ denotes associated angular integrals Becher, Neubert, Rothen, Shao '15

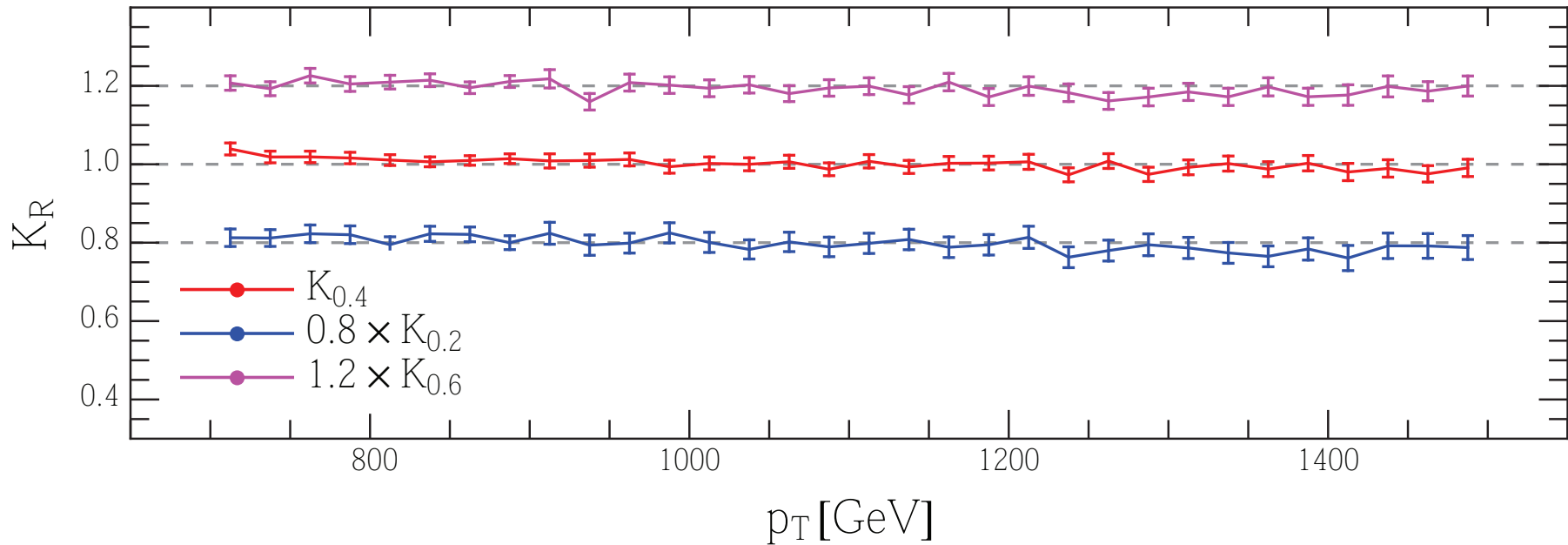
# Phenomenology

- Evaluation of cross section in SCET (resummation) with renormalization group equations
  - evolution of all functions from their natural scales  $\mu_i$  to common hard scale  $\mu = p_T^{\max}$
- Matching of NLL resummed results with full NLO calculation (need to avoid double counting)

$$d\sigma = d\sigma_{\text{NLL}} - d\sigma_{\text{NLO}_{\text{sin}}} + d\sigma_{\text{NLO}}$$

- resummed cross section  $d\sigma_{\text{NLL}}$
- fixed order NLO result in singular limit  $d\sigma_{\text{NLO}_{\text{sin}}}$
- complete fixed order NLO result  $d\sigma_{\text{NLO}}$

# Validation



- Comparison of fixed order NLO result in singular limit  $d\sigma_{\text{NLO}_{\text{sin}}}$  with complete fixed order NLO result  $d\sigma_{\text{NLO}}$
- Ratio  $K_R = \frac{d\sigma_{\text{NLO}_{\text{sin}}}}{d\sigma_{\text{NLO}}}$  as function of jet  $p_T$  for  $1.5 < |\eta| < 2$  at  $\sqrt{S} = 13 \text{ TeV}$  (error bars for numerical uncertainty)

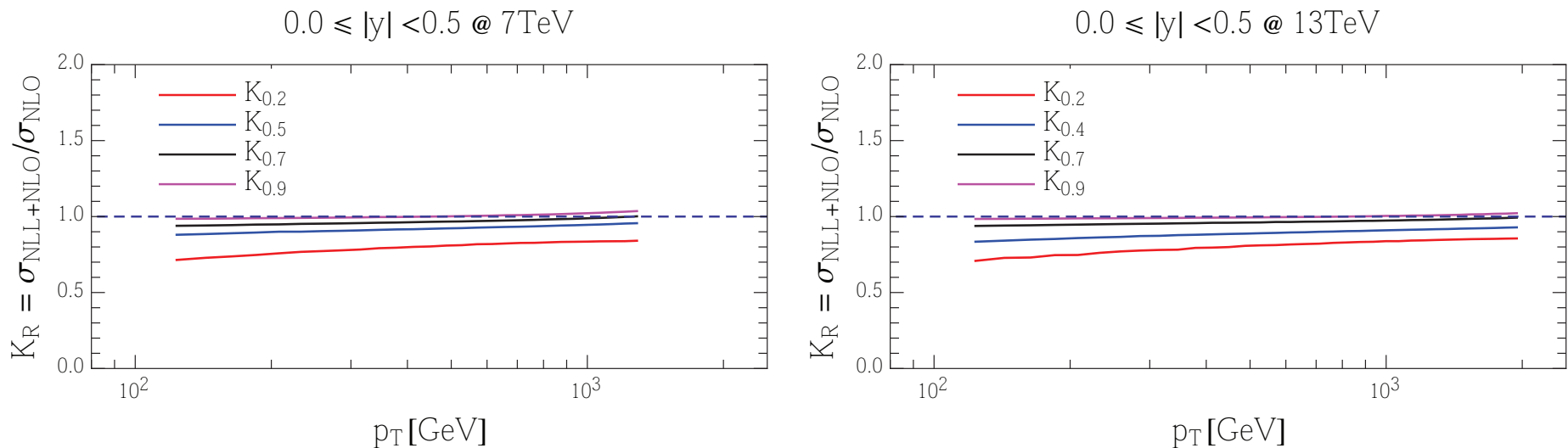
# Jet radius dependence (I)

## Resummation for various $R$

- Ratio  $K_R$  of NLO + NLL and NLO cross sections for different jet radii

$$K_R = \frac{\sigma_{\text{NLL+NLO}}(R)}{\sigma_{\text{NLO}}(R)}$$

- LHC at  $\sqrt{S} = 7 \text{ TeV}$  (left) and  $13 \text{ TeV}$  (right) with NLO PDF set of **MMHT**

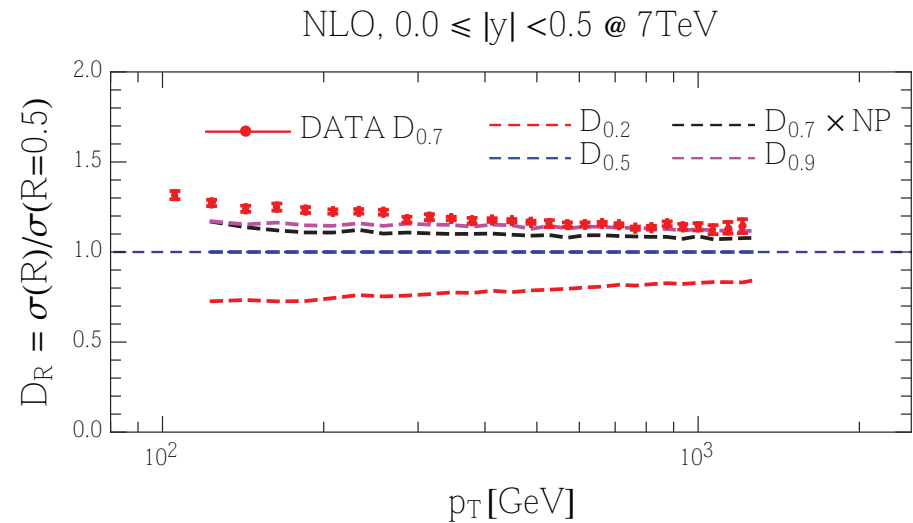
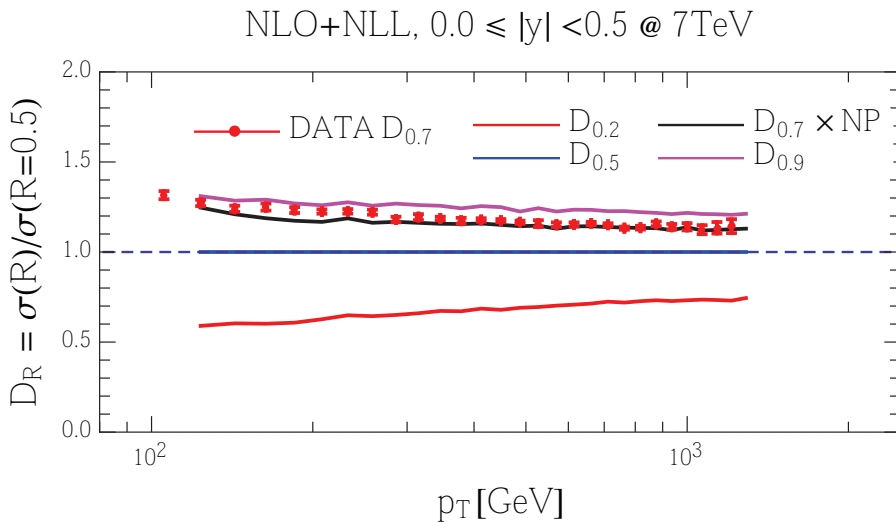


- Significant effect for small jet radii; reduction of  $\mathcal{O}(20\%)$  for  $R = 0.2$  in entire range of  $p_T$

# Jet radius dependence (II)

## Impact of joint resummation

- Cross section ratio  $D_R = \frac{\sigma(R)}{\sigma(R_{\text{fixed}})}$
- $D_R$  for NLO + NLL (left) and NLO (right) cross sections for  $R_{\text{fixed}} = 0.5$  at  $\sqrt{S} = 7 \text{ TeV}$  with NLO PDF set of **MMHT14**
- Single-inclusive jet data from collected at  $\sqrt{S} = 7 \text{ TeV}$  with  $R = 0.7$  by **CMS arXiv:1406.0324** (with NP correction factors)



- NLO result overshoots; joint resummation agrees with data for  $D_{R=0.7}$

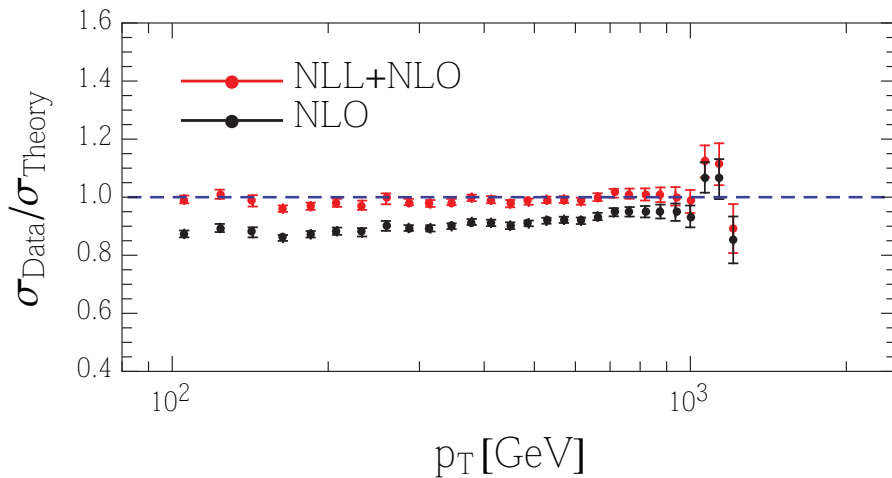


# Data vs. theory (I)

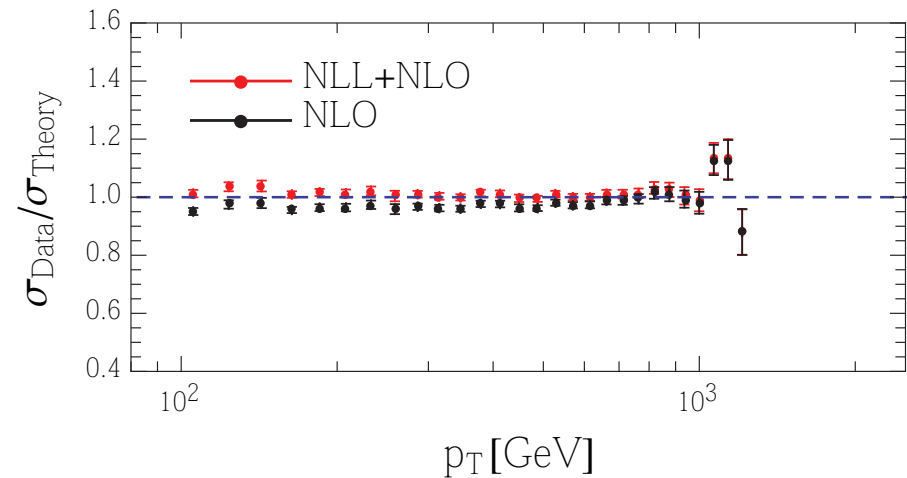
## LHC data at $\sqrt{S} = 7$ TeV

- Ratio  $\sigma_{\text{Data}}/\sigma_{\text{Theory}}$  with  $R = 0.5$  (left) and  $R = 0.7$  (right) to theoretical results at NLO (black dots) and at NLO + NLL (red dots) accuracy
  - NLO PDF set of [MMHT14](#)
  - data at  $\sqrt{S} = 7$  TeV by [CMS arXiv:1406.0324](#)

$0.0 \leq |\eta| < 0.5$ ,  $R = 0.5$  @ 7TeV



$0.0 \leq |y| < 0.5$ ,  $R = 0.7$  @ 7TeV



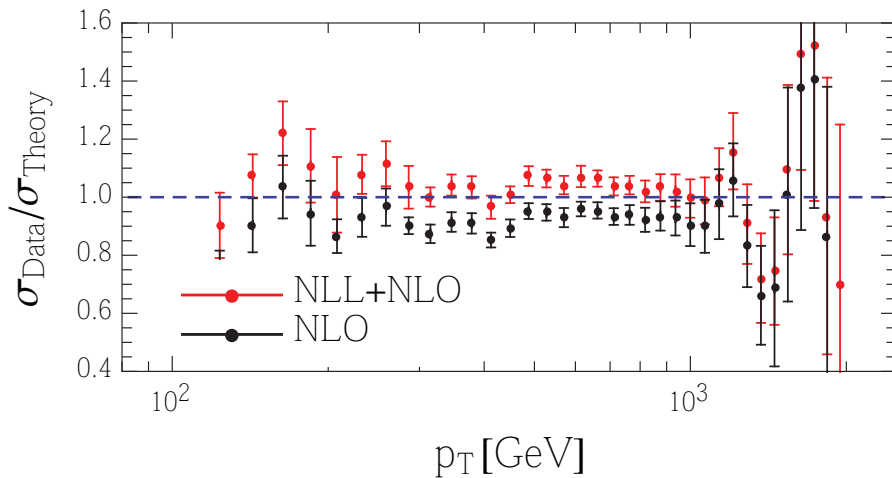
- Joint resummation agrees well with data; ratio with NLO predictions undershoots

# Data vs. theory (II)

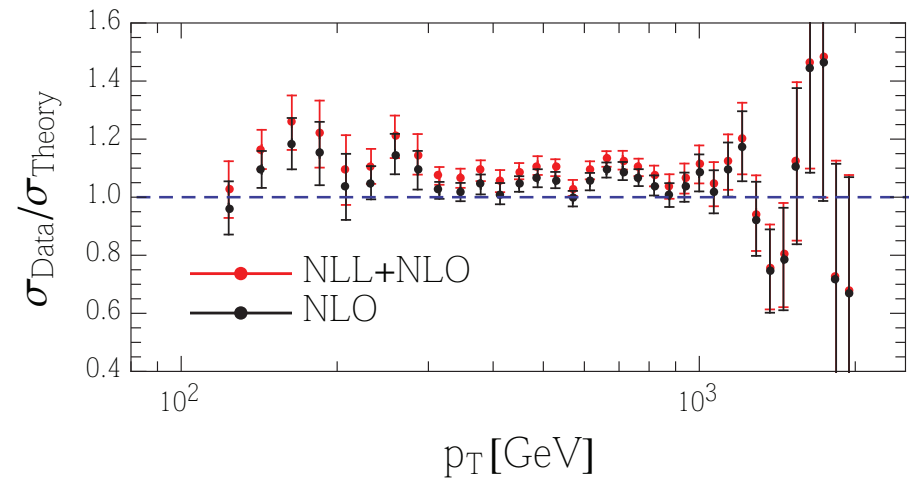
## LHC data at $\sqrt{S} = 13$ TeV

- Ratio  $\sigma_{\text{Data}}/\sigma_{\text{Theory}}$  with  $R = 0.4$  (left) and  $R = 0.7$  (right) to theoretical results at NLO (black dots) and at NLO + NLL (red dots) accuracy
  - NLO PDF set of **MMHT14**
  - data at  $\sqrt{S} = 13$  TeV by **CMS arXiv:1605.04436**

$0.0 \leq |y| < 0.5$ ,  $R = 0.4$  @ 13TeV



$0.0 \leq |y| < 0.5$ ,  $R = 0.7$  @ 13TeV

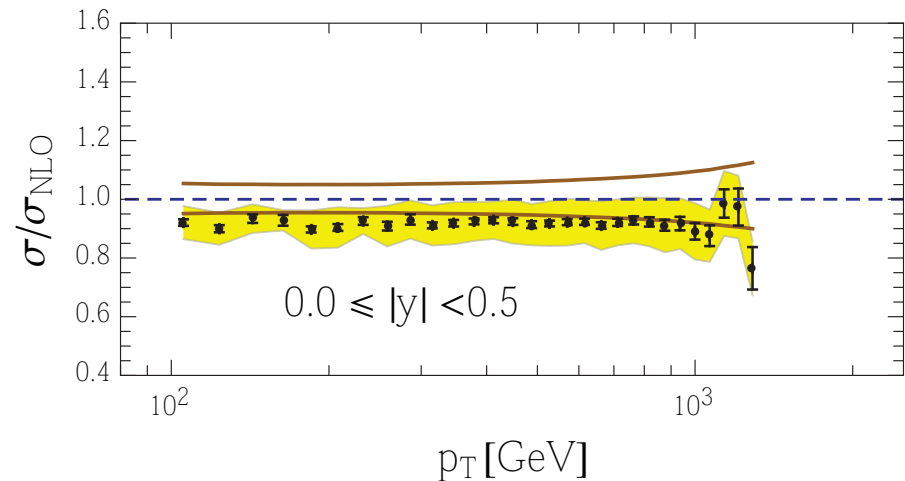
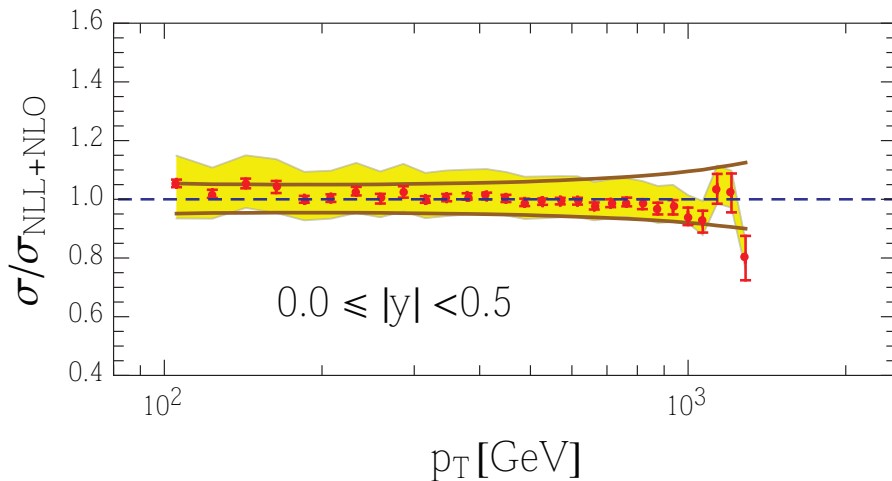


- Same trend as for data at  $\sqrt{S} = 7$  TeV, but still large experimental uncertainties

# Data vs. theory (III)

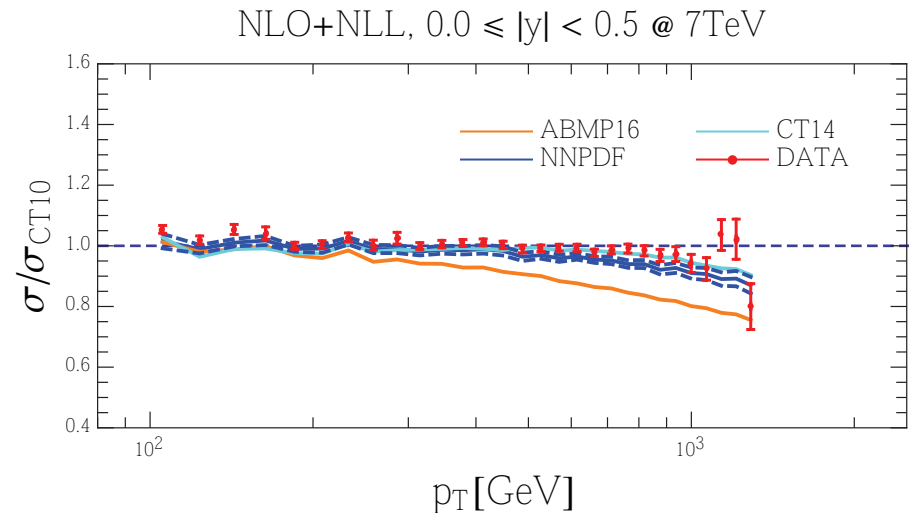
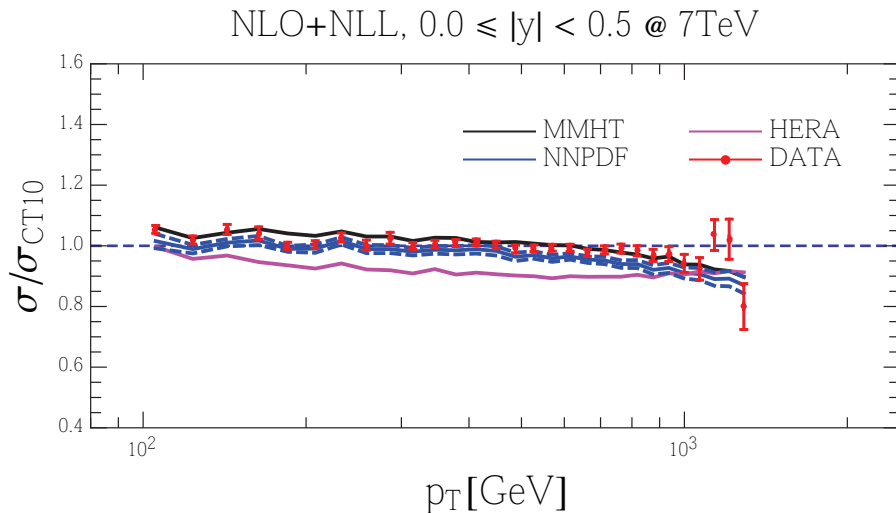
## Experimental and theoretical uncertainties

- Ratio  $\sigma_{\text{Data}}/\sigma_{\text{Theory}}$  with  $R = 0.5$  to theoretical results at NLO + NLL (left) and at NLO (right) accuracy at  $\sqrt{S} = 7 \text{ TeV}$  using the NLO PDF set by CT10 for data collected by CMS [arXiv:1406.0324](#) (with NP correction factors)
  - error bars represent experimental statistical errors
  - solid (brown) lines represent experimental systematic ones
  - band (yellow) indicates theoretical scale uncertainties



# PDF dependence

- Cross sections  $\sigma_{\text{NLO+NLL}}$  at NLL + NLO accuracy with  $R = 0.5$  at  $\sqrt{S} = 7 \text{ TeV}$  normalized to the one with the NLO PDF set by CT10
  - central NLO PDF sets HERAPDF2.0, MMHT14 and NNPDF3.1 (left)
  - central NLO PDF sets ABMP16, CT14 and NNPDF3.1 (right)
  - data collected by CMS [arXiv:1406.0324](https://arxiv.org/abs/1406.0324) (with NP correction factors) is superimposed



# Summary

## Theory framework

- Joint resummation of threshold logarithms  $\alpha_s^n (\ln^{2n-1}(z)/z)_+$  and small jet-radius logarithms  $\alpha_s^n \ln^n(R)$  to all orders at NLL accuracy
- Resummation at NLO + NLL accuracy in good agreement with CMS data
- Residual theoretical uncertainties from scale variation still of similar size as experimental systematics

## Uses of jet measurements

- Determinations of strong coupling constant  $\alpha_s(M_Z)$  and gluon PDF at medium to large- $x$  based on results fixed order perturbation theory likely to incur bias
- Precision of  $\lesssim 1\%$  makes resummed theoretical predictions at NNLO + NNLL accuracy in QCD mandatory