Loops and Legs in Quantum Field Theory
Sankt Goar, Apr 29 - May 4, 2018

Loop-tree duality at two loops

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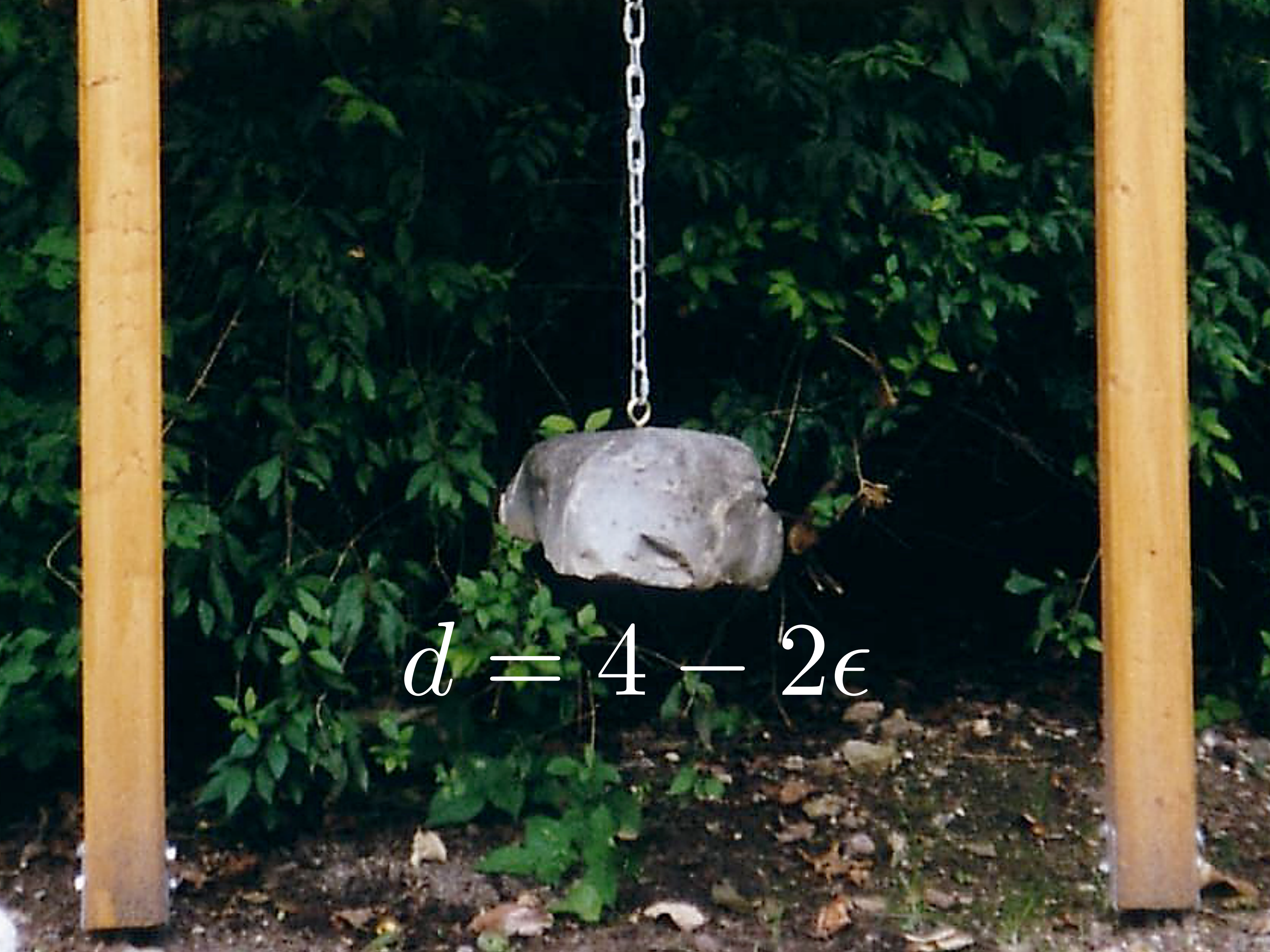
Quantum - Vorhersage - Station

	bedindungen	vorhersage
SM extrapolated to **infinite energy** in loop corrections >> **MPlank**		**UV** Erdbeben
Quantum state with N partons not = quantum state with **zero energy emission** of extra partons		**soft** Nebel
partons emitted in **exactly the same direction**		**not enough space** for two, three …


$d = 4 - 2\epsilon$

unsubtraction
in the IR (degenerate states together)
subtract the UV

- **LTD:** open loops to trees
- **FDU:** mapping of V ▶R kinematics

⇓

- local cancellation of singularities in d=4 space-time
- R+V simultaneous:
  - more efficient event generators
  - though can deal with amplitudes, focused on $d\sigma$

The loop-tree duality theorem

[Catani et al. 2008]

One-loop integrals (or scattering amplitudes in any relativistic, local and unitary QFT) represented by a linear combination of *N* **single-cut phase-space** integrals

$$\int_\ell \prod G_F(q_i) = -\sum \int_\ell \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j)$$

- $\tilde{\delta}(q_i) = i\, 2\pi\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$ sets internal line on-shell, positive energy mode
- $G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\, \eta\, k_{ji}}$ **dual propagator,** $k_{ji} = q_j - q_i$

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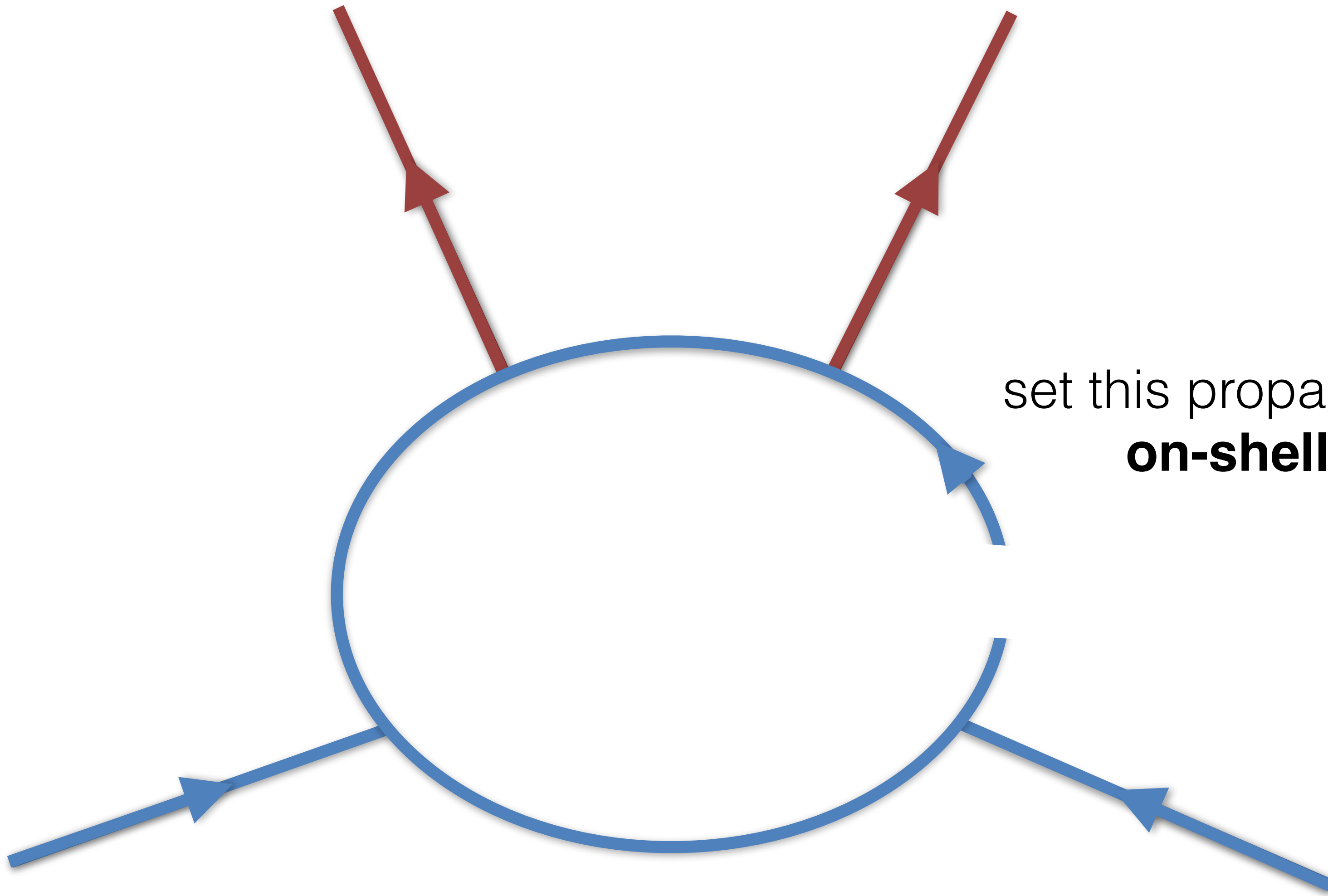
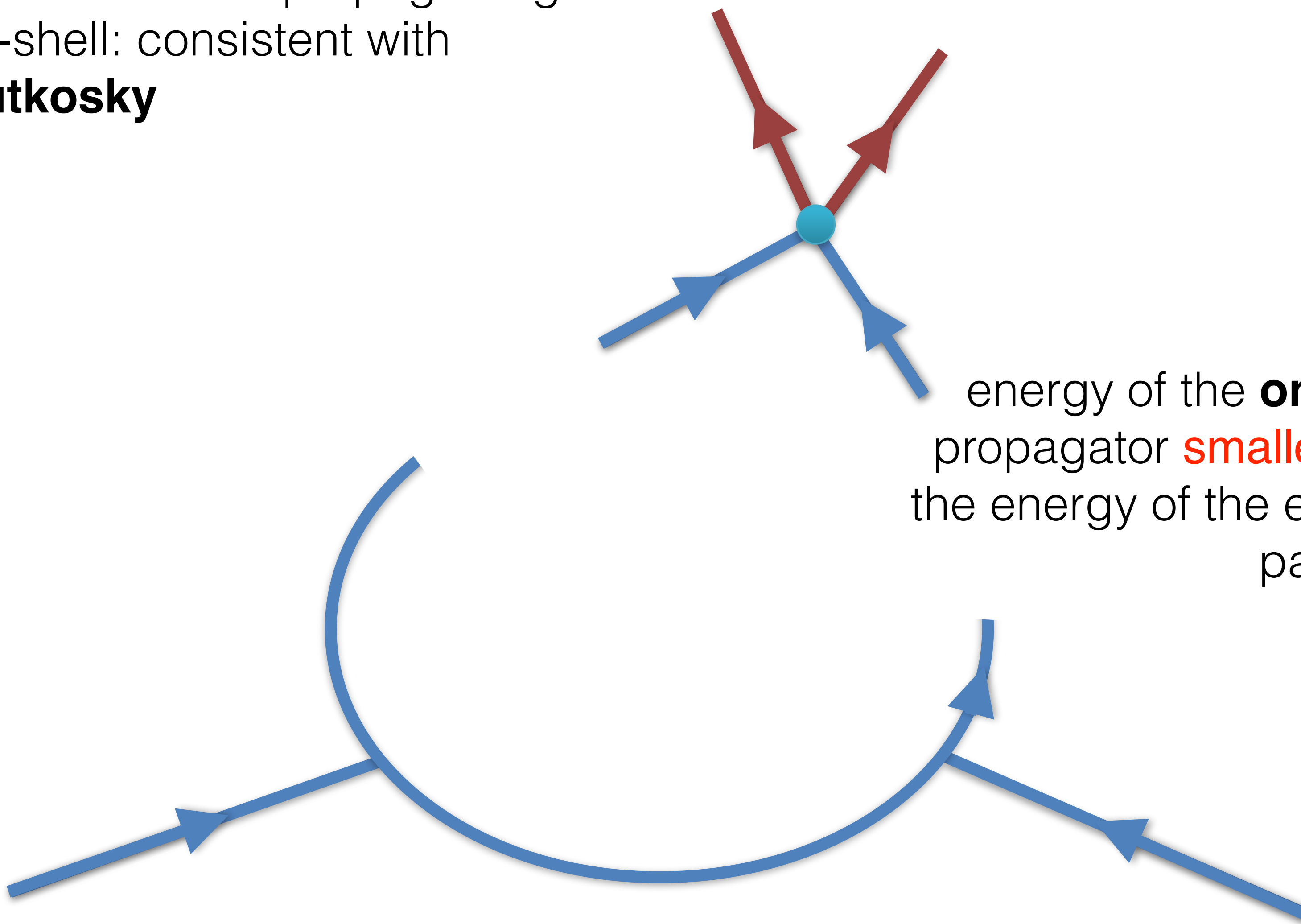
- Lorentz-covariant dual prescription with η a **future-like** vector; only the **sign** matters
- best choice $\eta^\mu = (1,\mathbf{0})$: energy component integrated out, remaining integration in **Euclidean space**

- The dual loop **integrand becomes singular** when subsets (>=2) of internal propagators go on-shell



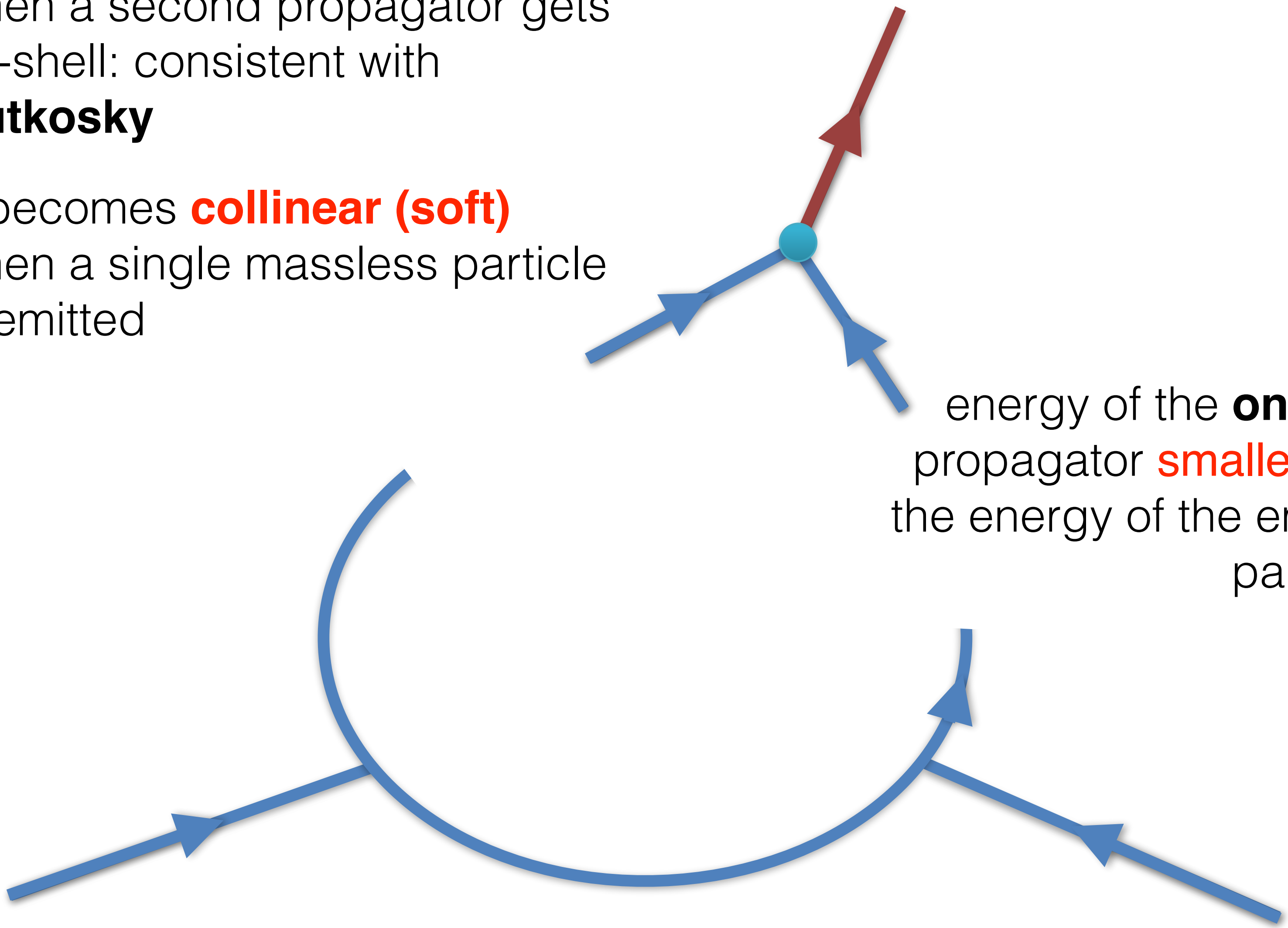
- **Threshold** singularities occur when a second propagator gets on-shell: consistent with **Cutkosky**



energy of the **on-shell** propagator smaller than the energy of the emitted particles

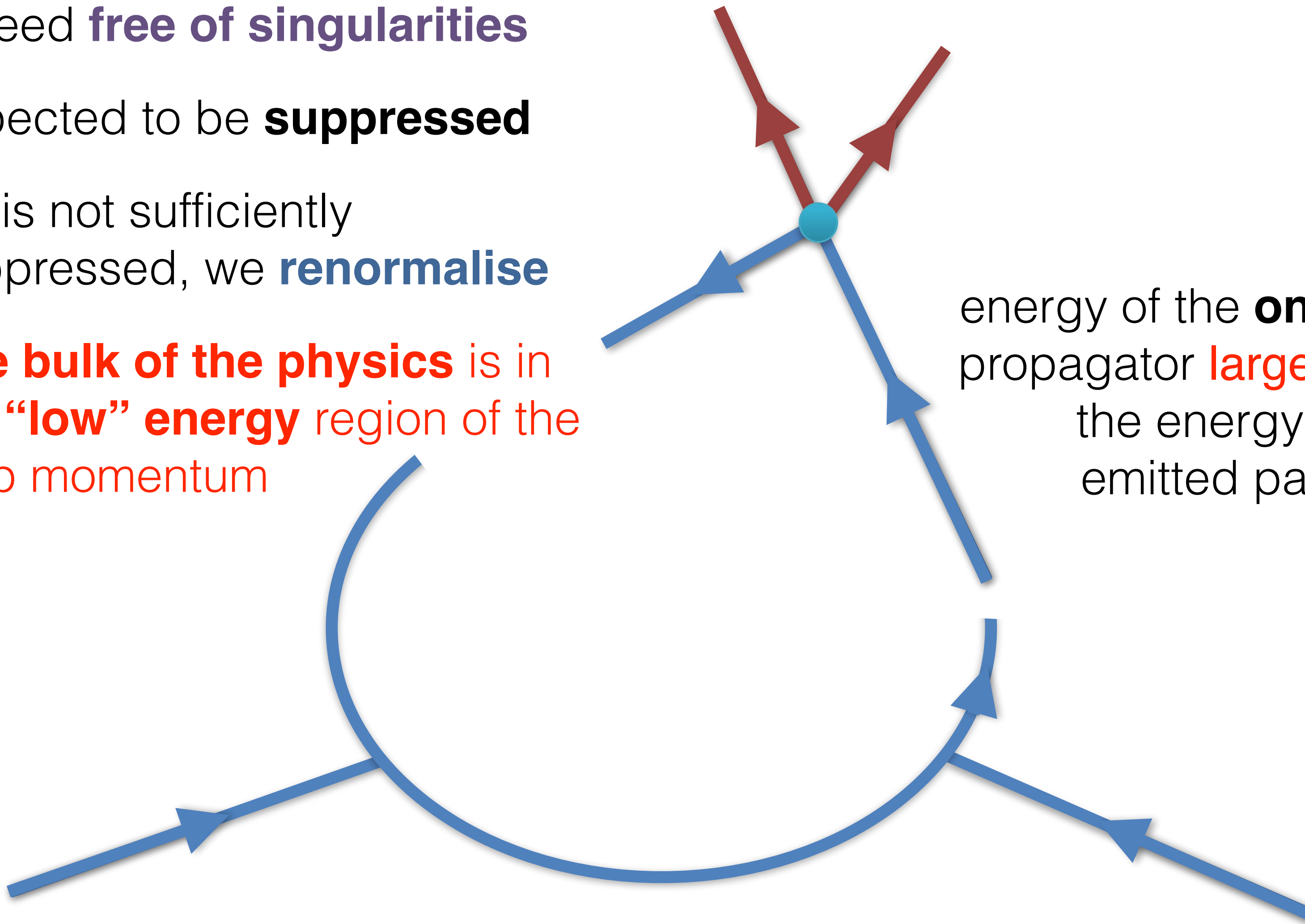
- **Threshold** singularities occur when a second propagator gets on-shell: consistent with **Cutkosky**

- It becomes **collinear (soft)** when a single massless particle is emitted



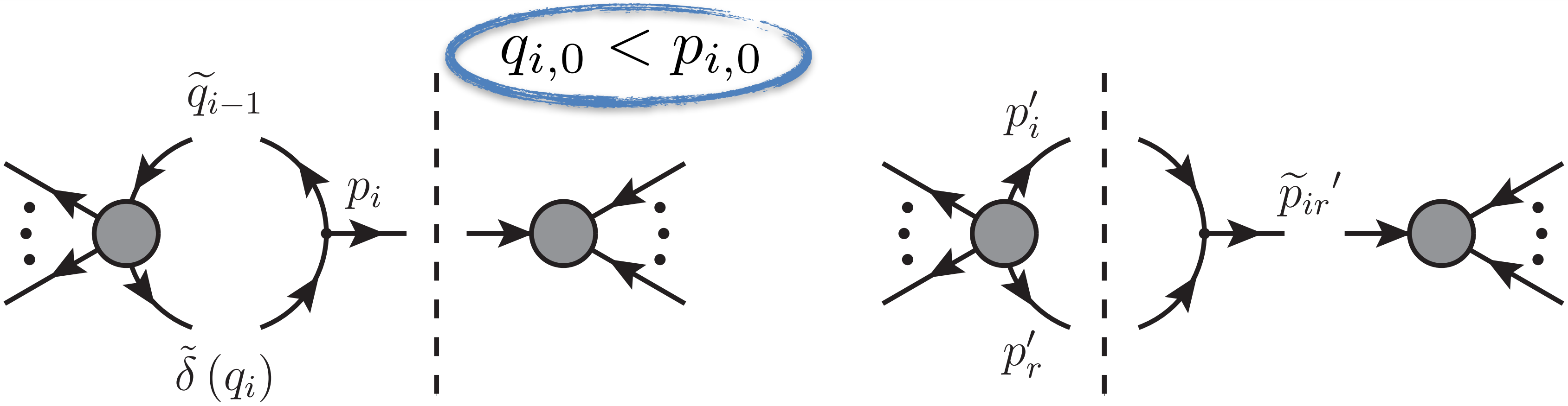
energy of the **on-shell** propagator smaller than the energy of the emitted particles

- Virtual particle emitted and absorbed
- Indeed **free of singularities**
- Expected to be **suppressed**
- If it is not sufficiently suppressed, we **renormalise**
- **The bulk of the physics** is in the **“low” energy** region of the loop momentum



energy of the **on-shell** propagator larger than the energy of the emitted particles

Momentum mapping: multi-leg



- Motivated by the **factorisation properties of QCD**: assuming q_i^μ on-shell, and close to collinear with p_i^μ ,we define the momentum mapping

$$p_r'^\mu = q_i^\mu \; ,$$

$$p_i'^\mu = p_i^\mu - q_i^\mu + \alpha_i \, p_j^\mu \; , \qquad \alpha_i = \frac{(q_i - p_i)^2}{2 p_j \cdot (q_i - p_i)} \; ,$$

$$p_j'^\mu = (1 - \alpha_i) \, p_j^\mu \; , \qquad p_k'^\mu = p_k^\mu \; , \qquad k \neq i, j$$

- All the primed momenta (real process) on-shell and momentum conservation
- p_i^μ is the **emitter**, p_j^μ the **spectator** needed to absorb momentum recoil
- **Quasi-collinear configurations** can also be conveniently mapped such that the massless limit is smooth

UV renormalisation: local subtraction

- Expand propagators and numerators around a UV propagator [Weinzierl et al.]

$$G_F(q_i) = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0} \left[1 - \frac{2 q_{\rm UV} \cdot k_i + k_i^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0} + \frac{(2 q_{\rm UV} \cdot k_i)^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2} \right] + \ldots$$

$$q_{\rm UV} = \ell + k_{\rm UV} \qquad k_i = q_i - q_{\rm UV}$$

- and adjust **subleading** terms to subtract only the pole ($\overline{\rm MS}$ **scheme**), or to define any other renormalisation scheme. For the scalar two point function

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2}$$

- Dual representation needs to deal with **multiple poles** [Bierenbaum et al.]

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2 \left(q_{\rm UV,0}^{(+)} \right)^2}$$

$$q_{\rm UV,0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$

Hernández-Pinto, Sborlini, GR, JHEP **1602,** 044

- Integration on the UV on-shell hyperboloid: loop three-momentum unconstrained, but **loop contributions suppressed** for loop energies larger than $\mu_{\rm UV}$

LTD unsubtraction: multi-leg

Sborlini, Driencourt-Mangin, Hernández-Pinto, GR, JHEP **1608,** 160

- The **dual representation** of the renormalised loop cross-section: one single integral in the loop three-momentum

$$\int_N d\sigma_{\rm V}^{(1,{\rm R})} = \int_N \int_{\vec{\ell}_1} 2\,{\rm Re}\,\langle \mathcal{M}_N^{(0)} | \left(\sum_i \mathcal{M}_N^{(1)}(\tilde{\delta}(q_i)) \right) - \mathcal{M}_{\rm UV}^{(1)}(\tilde{\delta}(q_{\rm UV})) \rangle$$

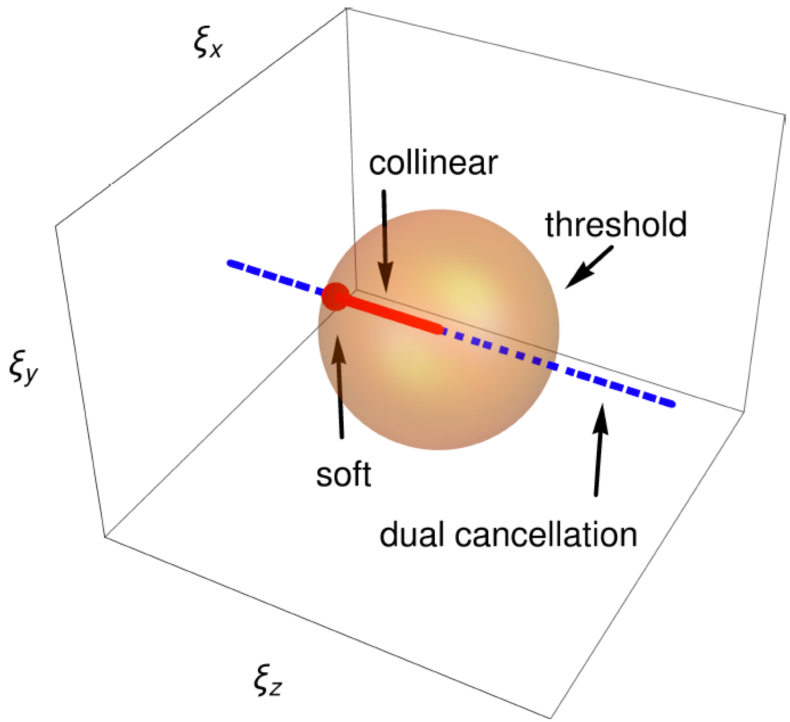
- A **partition** of the real phase-space

$$\sum_i \mathcal{R}_i(\{p'_j\}_{N+1}) = 1$$

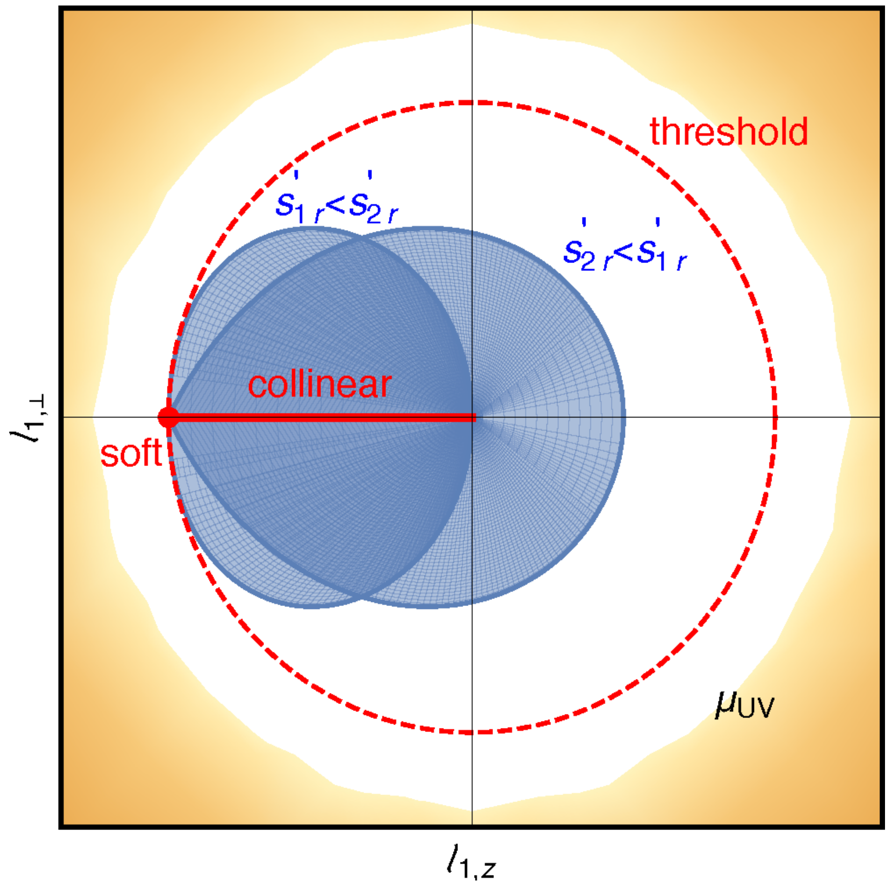
- The real contribution **mapped** to the Born kinematics + loop three-momentum

$$\int_{N+1} d\sigma_{\rm R}^{(1)} = \int_N \int_{\vec{\ell}_1} \sum_i \mathcal{J}_i(q_i)\, \mathcal{R}_i(\{p'_j\})\, |\mathcal{M}_{N+1}^{(0)}(\{p'_j\})|^2 \Big|_{\{p'_j\}_{N+1} \to (q_i, \{p_k\}_N)}$$

IR singularities and mapping regions: $1 \to 2$



$$\ell^\mu = \frac{\sqrt{s_{12}}}{2}\,(\xi_0, \xi_x, \xi_y, \xi_z)$$



- physics is in a region of the loop momentum which is of the size of the **hard scale**

LTD unsubtraction: NNLO

- At **NNLO**

$$\sigma^{\rm NNLO} = \int_N d\sigma_{\rm VV}^{(2,{\rm R})} + \int_{N+1} d\sigma_{\rm VR}^{(2,{\rm R})} + \int_{N+2} d\sigma_{\rm RR}^{(2)}$$

where the **VV** contribution reads

$$d\sigma_{\rm VV}^{(2)} = \int_{\vec{\ell}_1} \int_{\vec{\ell}_2} \sum_{i,j} \Big[2\,{\rm Re}\,\langle \mathcal{M}_N^{(0)} | \mathcal{M}_N^{(2)}(\tilde{\delta}(q_i, q_j)) \rangle + \langle \mathcal{M}_N^{(1)}(\tilde{\delta}(q_i)) | \mathcal{M}_N^{(1)}(\tilde{\delta}(q_j)) \rangle \Big] \mathcal{O}(\{p_k\})$$

- Need the **RV** and **RR** contributions **mapped** to the Born kinematics + the two independent loop three-momenta
- **Known two-loop amplitudes** not suitable: requires LTD unintegrated representation

LTD at two-loops and beyond

- **Iterative** application of LTD at higher orders

$$G_F(\alpha_k) = \sum_{i\in\alpha_k} G_F(q_i) \ , \qquad G_D(\alpha_k) = \sum_{i\in\alpha_k} \tilde{\delta}(q_i) \prod_{j\neq i} G_D(q_i;q_j) \ ,$$

- At one loop:

$$\int_{\ell_1} G_F(\alpha_1) = -\int_{\ell_1} G_D(\alpha_1)$$

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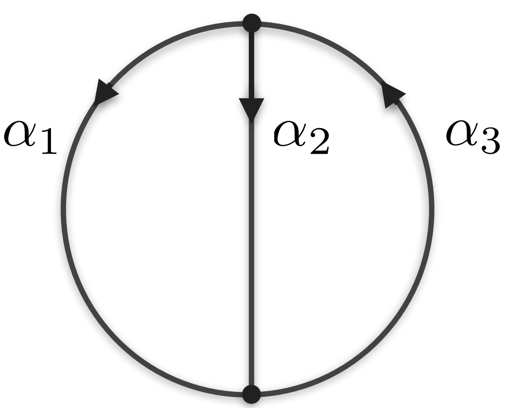
- At two-loops:



$$\int_{\ell_1}\int_{\ell_2} G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3)$$
$$= -\int_{\ell_1}\int_{\ell_2} G_F(\alpha_1)\, G_D(\alpha_2 \cup \alpha_3)$$

$$G_D(\alpha_2)\, G_D(\alpha_3) + G_D(\alpha_2)\, G_F(\alpha_3) + G_F(\alpha_2)\, G_D(\alpha_3)$$

two cuts ✔

$$-G_D(\alpha_1 \cup \alpha_3) \qquad -G_D(-\alpha_2 \cup \alpha_1)$$

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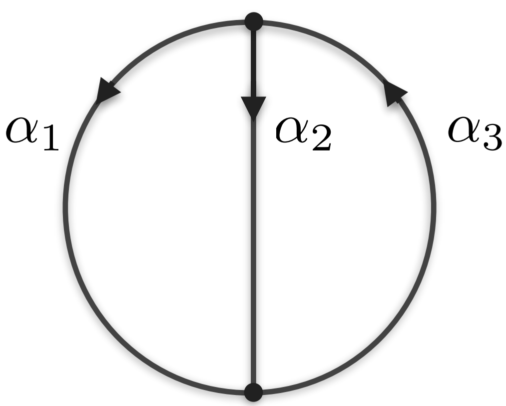
- At two-loops:

$$\int_{\ell_1}\int_{\ell_2} G_F(\alpha_1\cup\alpha_2\cup\alpha_3) = \int_{\ell_1}\int_{\ell_2} \Big\{ G_D(\alpha_2)\, G_D(\alpha_1\cup\alpha_3) \\ G_F(-\alpha_2\cup\alpha_1)\, G_D(\alpha_3) - G_F(\alpha_1)\, G_D(\alpha_2)\, G_D(\alpha_3) \Big\}$$



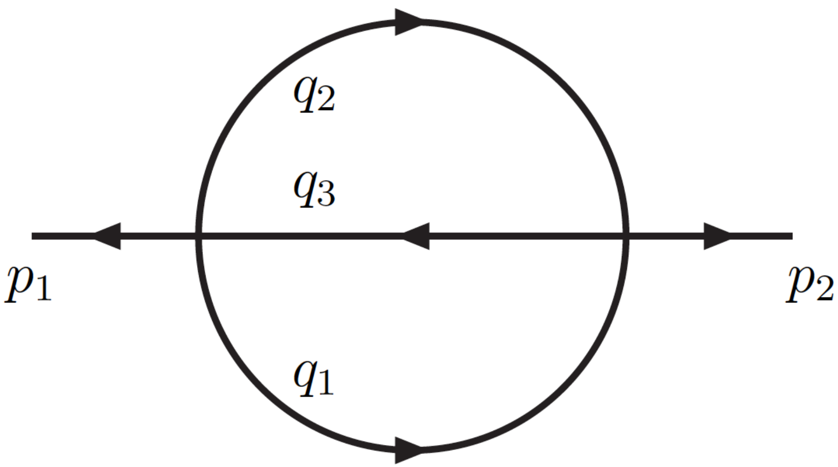
With a **number of cuts equal to the number of loops** the loop amplitude opens to a tree-level like object

Massless sunrise two-loop

Bierenbaum et al., 2010

$$L^{(2)}(p_1,p_2) = \int_{\ell_1}\int_{\ell_2} \tilde{\delta}(\ell_1)\,\tilde{\delta}(\ell_2)\left\{G_F(\ell_1+\ell_2+p_1)+G_F(\ell_1+\ell_2-p_1)+G_F(\ell_1-\ell_2-p_1)\right.$$
$$\left.+\tilde{\delta}(\ell_1+\ell_2+p_1)+\tilde{\delta}(\ell_1+\ell_2-p_1)\right\}$$

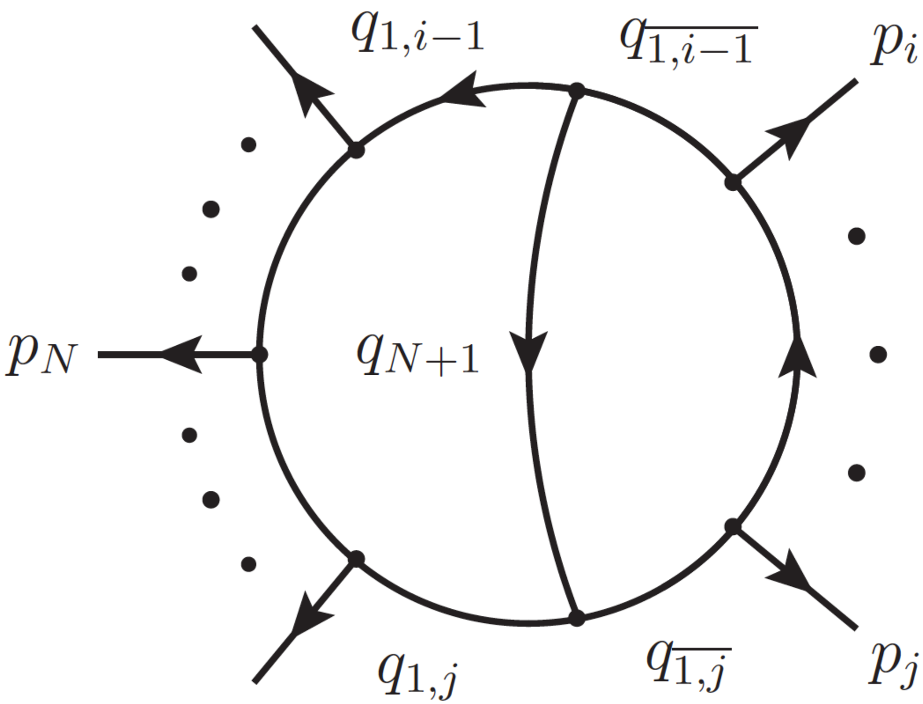
$$L^{(2)}(p_1,p_2) = d_\Gamma \int_{\ell_2} \tilde{\delta}(\ell_2)\left\{[(\ell_2+p_1)^2+i0]^{-\epsilon}\left(e^{i2\pi\epsilon}+1\right)\right.$$
$$\left.+[(\ell_2-p_1)^2+i0]^{-\epsilon}\left[e^{i2\pi\epsilon}-\theta((\ell_2-p_1)^2)\,\theta((\ell_2-p_1)_0)\left(e^{i2\pi\epsilon}-1\right)\right]\right\}$$



$$L^{(2)}(p_1,p_2) = -G_2(-p_1^2-i0)^{1-2\epsilon}$$

$$G_2 = \frac{\Gamma(-1+2\epsilon)\,\Gamma(1-\epsilon)^3}{(4\pi)^{4-2\epsilon}\,\Gamma(3-3\epsilon)}$$

Planar diagrams at two loops



$$\alpha_1 : \qquad q_{1,i} = \ell_1 + p_{1,i}$$

$$\alpha_2 : \qquad q_{N+1} = \ell_2$$

$$\alpha_3 : \qquad q_{\overline{1,j}} = \ell_1 + \ell_2 + p_{1,j}$$

Dual amplitude for $H \to \gamma\gamma$ at two loops

Driencourt-Mangin, Sborlini, Torres-Bobadilla, GR

- Simplest two loop amplitude: **proof of concept** for other amplitudes with higher multiplicities
- Well known numerically/analytically, however, **known amplitude not suitable** within LTD/FDU, requires unintegrated amplitude
- **IBP** would modify the **local behaviour** of the integrand: not suitable
- Dual propagators are **linear in the loop momenta:** tensor reduction simpler (reduction to master integrals not necessary)
- For Higgs on-shell **below threshold:** dual prescriptions can be ignored, numerical integration stable
- **Universality** also holds at two-loops ?

Dual amplitude for $H \to \gamma\gamma$

Driencourt-Mangin,GR, Sborlini, EPJC 2018

- **Universality** and **compactness** of the dual representation. In four space-time dimensions after **local renormalization**

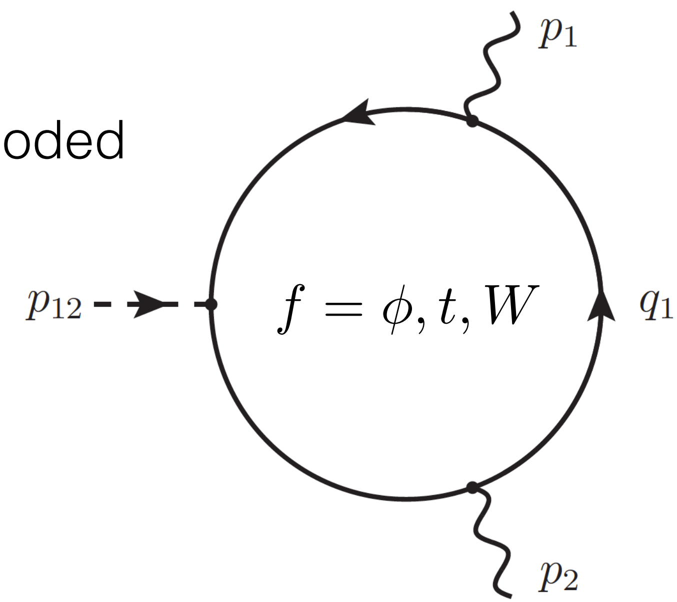
$$
\mathcal{A}^{(1,f)}_{1,\mathrm{R}}\Big|_{d=4} = g_f\, s_{12} \int_\ell \left[\frac{1}{2\ell_0^{(+)}} \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{2,0}^{(+)}} + \frac{2(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \right.
$$

$$
\left. \times \frac{M_f^2}{(2\ell\cdot p_1)(2\ell\cdot p_2)}\, c_1^{(f)} + \frac{3\mu_{\mathrm{UV}}^2}{4(q_{\mathrm{UV},0}^{(+)})^5}\, \hat{c}_{23}^{(f)} \right] \qquad q_{i,0}^{(+)} = \sqrt{\mathbf{q}_i^2 + m_i^2 - \imath 0}
$$

- The flavour of the internal particles is encoded by **two scalar coefficients**

$$
c_1^{(f)} = \left(2, -4 + \frac{s_{12}}{M_t^2}, 6 - \frac{3 s_{12}}{M_W^2}\right)
$$

$$
\hat{c}_{23}^{f} = \frac{c_{23}^{(f)}}{d-4}\Big|_{d=4} = \left(1, -2, 3 + \frac{s_{12}}{2M_W^2}\right)
$$



Dual amplitude for $H \to \gamma\gamma$ at two loops

Driencourt-Mangin, Sborlini, Torres-Bobadilla, GR

- Below threshold

$$\mathcal{A}^{(2)}(\{p_i\}) = \int_{\ell_1}\int_{\ell_2} \mathcal{N}(\ell_1, \ell_2, \{p_i\}) \Big\{ G_D(\alpha_1)\, G_D(\alpha_2)\, G_F(\alpha_3)$$

$$G_F(\alpha_1)\, G_D(-\alpha_2)\, G_D(\alpha_3) + G_D(\alpha_1)\, G_F(\alpha_2)\, G_D(\alpha_3) \Big\}$$

- There are **22 double cuts** (although 7 cuts obtained from 1<->2)
- Self-energy insertions in internal lines introduce double poles
- Massless snail diagrams: integrate to zero but relevant to achieve universality
- Integrand expressions in terms of **O(20) scalar coefficients**
- soon to be published

Conclusions

- First attempts towards **LTD/FDU at NNLO**
- **IBP not suitable** because it modifies the local behaviour of the integrand
- First results for the LTD representation of the $H \to \gamma\gamma$ **two-loop amplitude** with different internal flavours: scalar, top quark, and W boson.
- Compact integrand expressions with similar functional form in terms of flavour dependent **O(20) scalar coefficients**
- **proof-of-concept** for other processes with higher multiplicities.

COST Action CA16201

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https://particleface.eu

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