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On-the-fly reduction of open loops

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in collaboration with F. Buccioni, J.-N. Lang, H. Zhang and S. Pozzorini

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Outline

I. Introduction: Numerical amplitude generation in OpenLoops

II. On-the-fly operations

   a) On-the-fly helicity summation

   b) On-the-fly merging

   c) On-the-fly reduction

III. Treatment of numerical instabilities due to small Gram determinants

IV. Performance and numerical stability benchmarks

V. Summary and Outlook

I. Numerical amplitude generation in OpenLoops

Fully automated numerical algorithm for tree and one-loop amplitudes (h = helicity configuration)

$$\mathcal{W}_0 = \sum_h \sum_{\mathrm{col}} |\mathcal{M}_0(h)|^2, \qquad \mathcal{W}_1 = \sum_h \sum_{\mathrm{col}} 2\,\mathrm{Re}\Big[\mathcal{M}_0^*(h)\mathcal{M}_1(h)\Big], \qquad \mathcal{W}_1^{\text{loop-ind}} = \sum_h \sum_{\mathrm{col}} |\mathcal{M}_1(h)|^2$$

- **OpenLoops 1** [Cascioli, Lindert, Maierhöfer, Pozzorini] available at openloops.hepforge.org
- **OpenLoops 2** [Buccioni, Lindert, Maierhöfer, Pozzorini, M.Z.] publication in preparation

Structure:

- **Process generator (Mathematica):** generate diagrams with `FeynArts` [Hahn], calculate and reduce colour factors, generate numerical recursion for amplitude calculation ($\rightarrow$ process library)
- **Numerical programme (Fortran):** Compute scattering probability densities from a process library and process-independent OpenLoops routines
- **Third party tools** for tensor reduction (OL 1) and MI evaluation (OL 1+2): `Collier` 1.2 [Denner, Dittmaier, Hofer '16], `Cuttools` 1.9.5 [Ossola, Papadopoulos, Pittau '08], `OneLoop` 3.6.1 [van Hameren '10]

▷ NLO QCD and NLO EW corrections fully automated

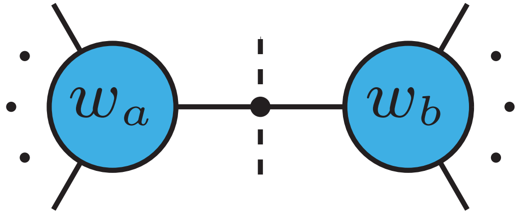
▷ OpenLoops is interfaced to Sherpa, Powheg, Herwig, Whizard, Geneva, Munich, Matrix

Amplitudes are sums of diagrams factorising into a colour factor and a colour-stripped amplitude

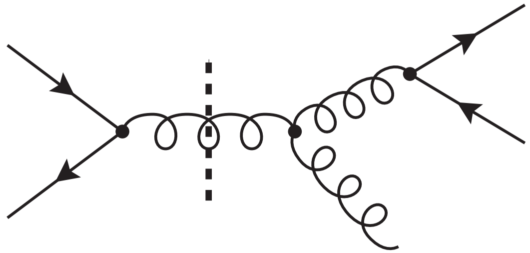
$$\mathcal{M}_l = \sum_d \mathcal{M}_l^{(d)} \qquad (l = 0, 1) \quad \text{with} \quad \mathcal{M}_l^{(d)} = \mathcal{C}_l^{(d)} \mathcal{A}_l^{(d)} .$$

Tree level amplitudes

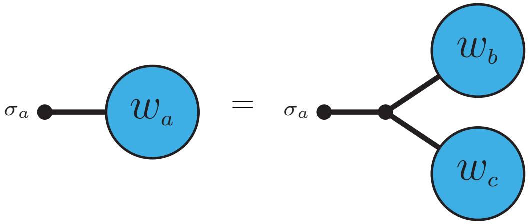
colour stripped $\mathcal{A}_0^{(d)}$ are split into subtrees by cutting an internal line:



for example



$\Rightarrow$ Numerical merging of subtrees performed recursively:



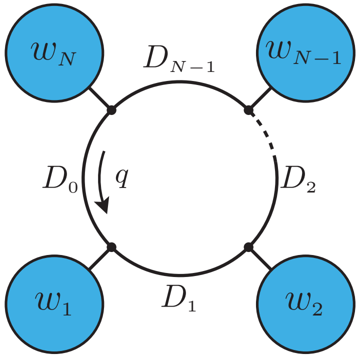
$$w_a^\alpha(k_a, h_a) = \frac{X^\alpha_{\beta\gamma}(k_b,k_c)}{k_a^2 - m_a^2}\, w_b^\beta(k_b, h_b)\, w_c^\gamma(k_c, h_c)$$

with momentum $k_a = k_b + k_c$ and for all possible helicity configurations $h_a = h_b + h_c$.

$\Rightarrow$ Once computed subtrees used in multiple Feynman diagrams at tree and loop level

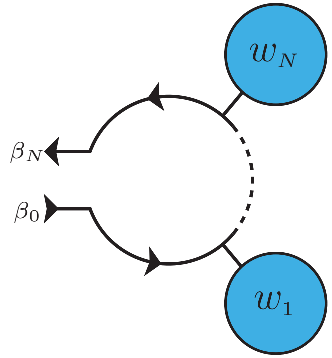
One-loop amplitudes

$$\mathcal{A}_1^{(d)} = \int \mathrm{d}^D q \, \frac{\mathrm{Tr}\Big[\mathcal{N}(q,h)\Big]}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{N-1}} =$$



cut open at $\bar{D}_0$ →

$$\Big[\mathcal{N}(q,h)\Big]^{\beta_N}_{\beta_0} =$$



propagators $D_i = (q+p_i)^2 - m_i^2$,

helicity configurations of subtree w_i: h_i

spinor/Lorentz indices $\beta_i \Rightarrow$ trace: contraction with $\delta^{\beta_0}_{\beta_N}$,

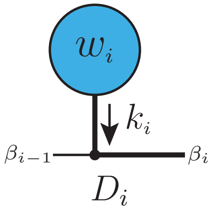
helicity configurations of $\mathcal{A}_1^{(d)}$: $h = h_1 + \ldots + h_N$

N-point numerator **factorises** into N **segments**:

$$\Big[\mathcal{N}(q,h)\Big]^{\beta_N}_{\beta_0} = \Big[\prod_{i=1}^{N} S_i(q,h_i)\Big]^{\beta_N}_{\beta_0} = \Big[S_1(q,h_1)\Big]^{\beta_1}_{\beta_0}\Big[S_2(q,h_2)\Big]^{\beta_2}_{\beta_1} \cdots \Big[S_N(q,h_N)\Big]^{\beta_N}_{\beta_{N-1}}$$

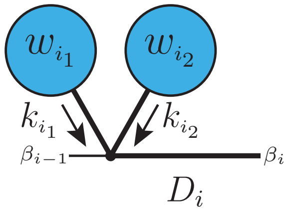
In the SM a segment (external subtree(s) + one loop vertex + propagator) is a q-polynomial of rank $r \leq 1$:

3-point segment: $\Big[S_i(q,h_i)\Big]^{\beta_i}_{\beta_{i-1}} =$



$$= \left\{\Big[Y^i_{\sigma_i}\Big]^{\beta_i}_{\beta_{i-1}} + \Big[Z^i_{\nu;\sigma_i}\Big]^{\beta_i}_{\beta_{i-1}} q^\nu\right\} w_i^{\sigma_i}(k_i,h_i)$$

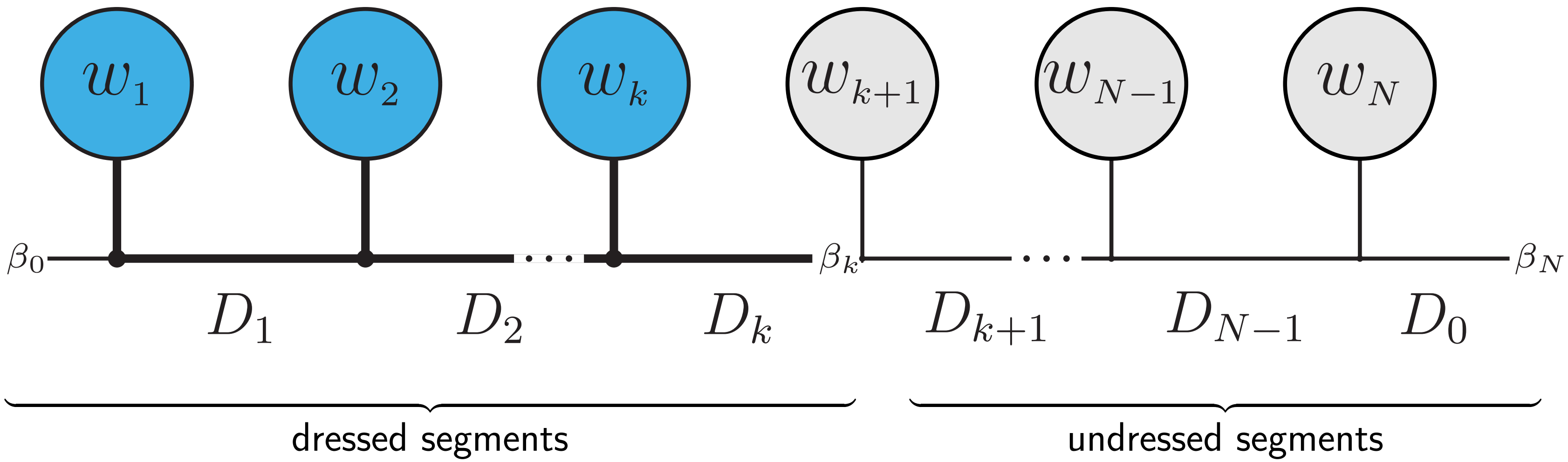
4-point segment: $\Big[S_i(q,h_i)\Big]^{\beta_i}_{\beta_{i-1}} =$



$$= \Big[Y^i_{\sigma_1\sigma_2}\Big]^{\beta_i}_{\beta_{i-1}} w_{i_1}^{\sigma_1}(k_{i_1},h_{i_1})\, w_{i_2}^{\sigma_2}(k_{i_2},h_{i_2}) \qquad (h_i = h_{i_1} + h_{i_2})$$

The OpenLoops dressing step

define partially dressed numerator $\mathcal{N}_n(q,\hat{h}_n) = S_1(q,h_1)\cdots S_n(q,h_n)$ $\quad (\hat{h}_n = \sum_{i=1}^{n} h_i)$



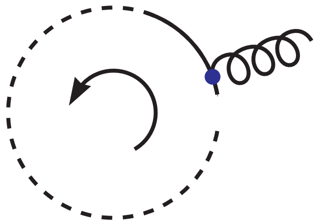
dressing step $\mathcal{N}_n(q,\hat{h}_n) = \mathcal{N}_{n-1}(q,\hat{h}_{n-1}) S_n(q,h_n)$ with initial condition $\mathcal{N}_0 = \mathbb{1}$

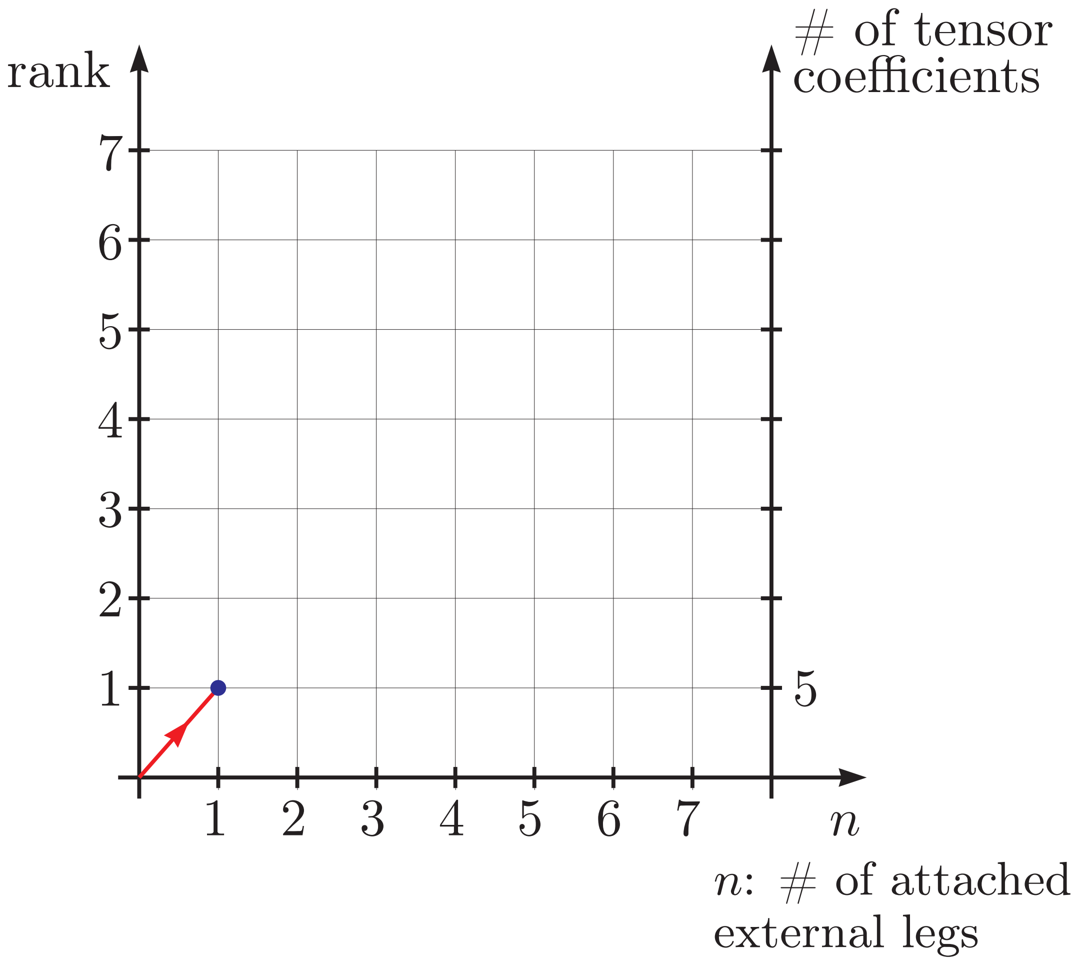
performed numerically for the tensor coefficients in

$$\mathcal{N}_n(q,\hat{h}_n) = \sum_{r=0}^{R} \mathcal{N}_{\mu_1\ldots\mu_r}(\hat{h}_n)\, q^{\mu_1}\cdots q^{\mu_r} \qquad (\text{rank } R \leq n)$$

Amplitude generation in OpenLoops 1

Example:





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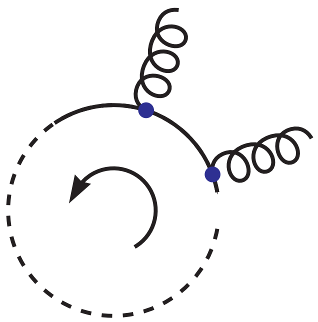
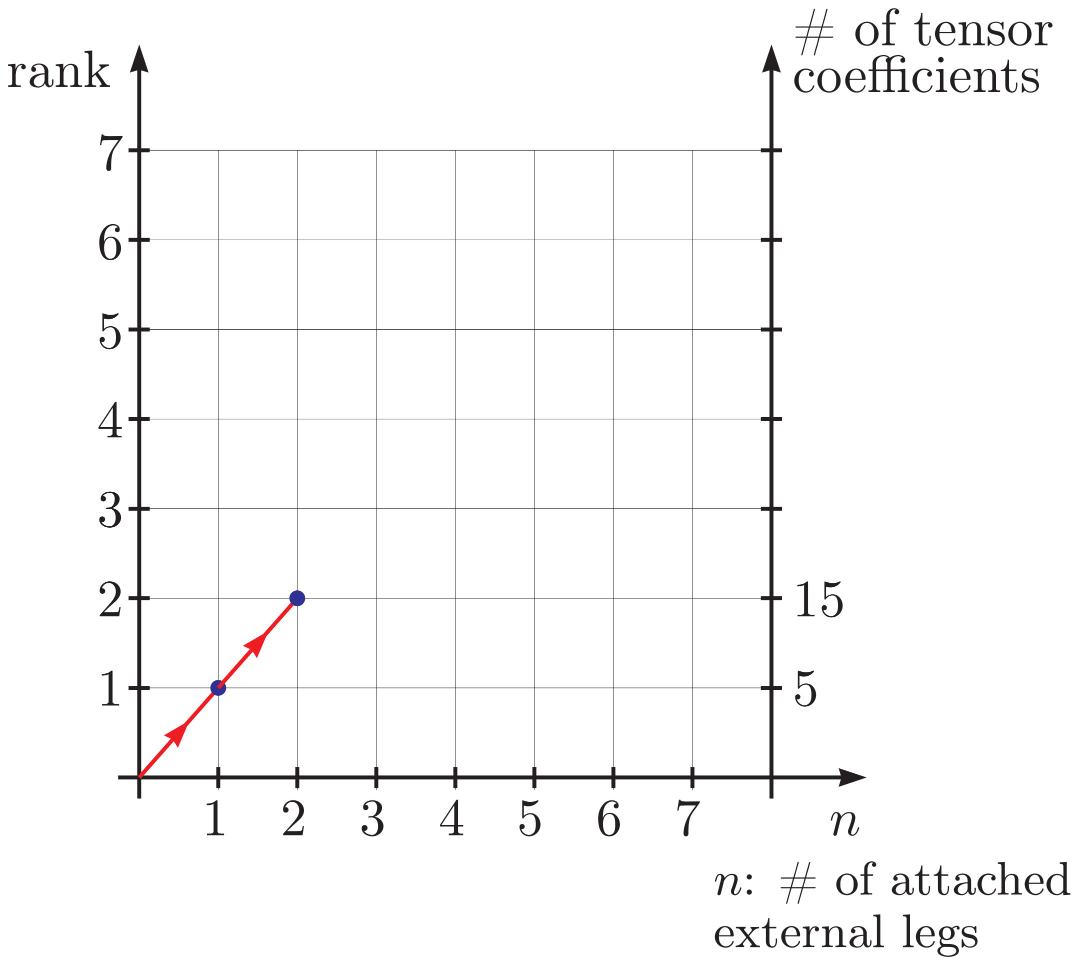
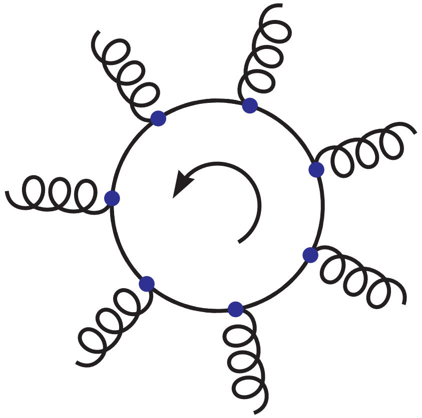


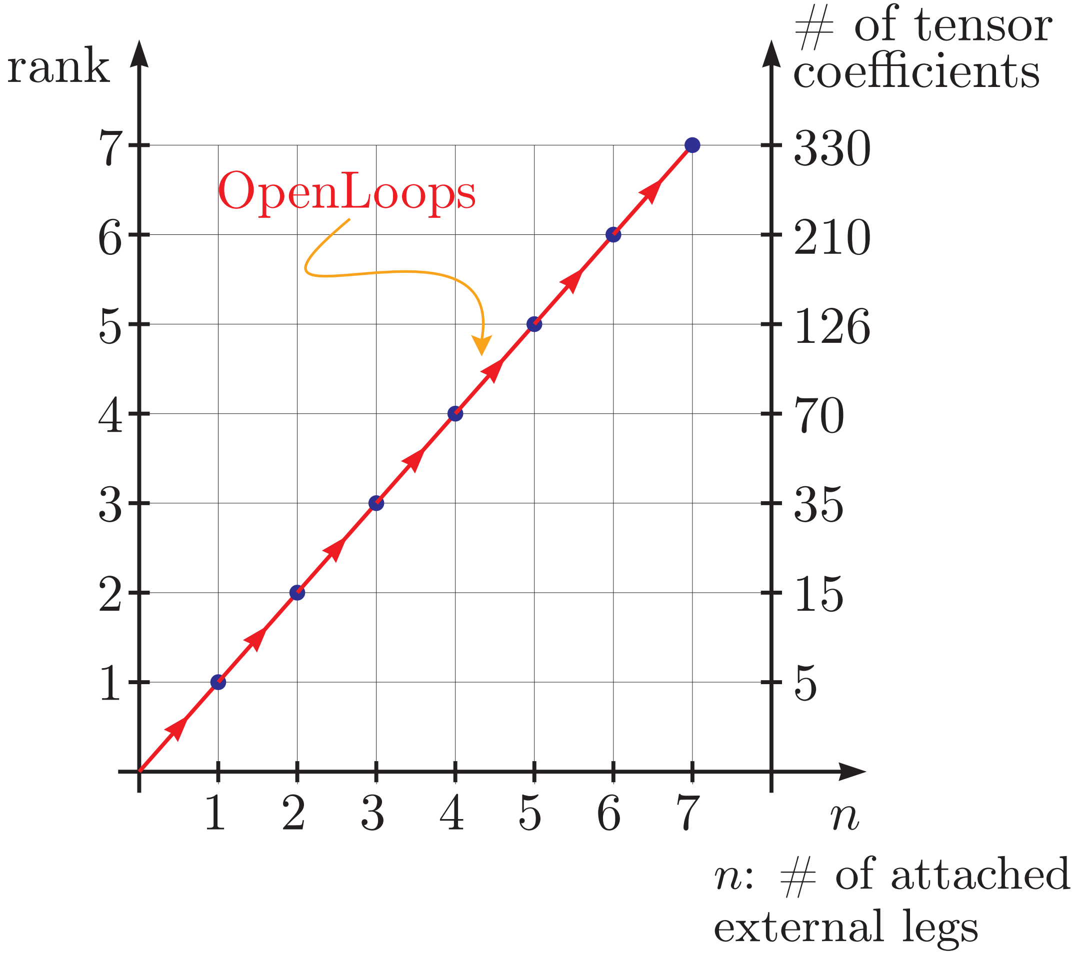


Amplitude generation in OpenLoops 1

Example:



complexity grows exponentially with tensor rank



Amplitude generation in OpenLoops 1

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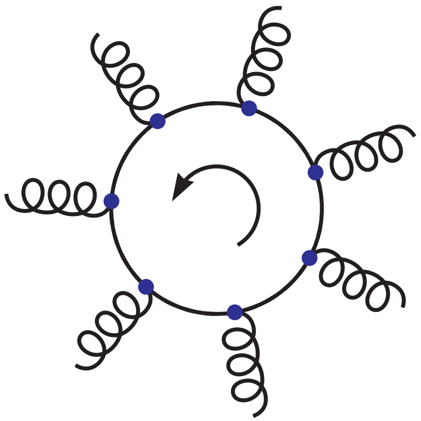

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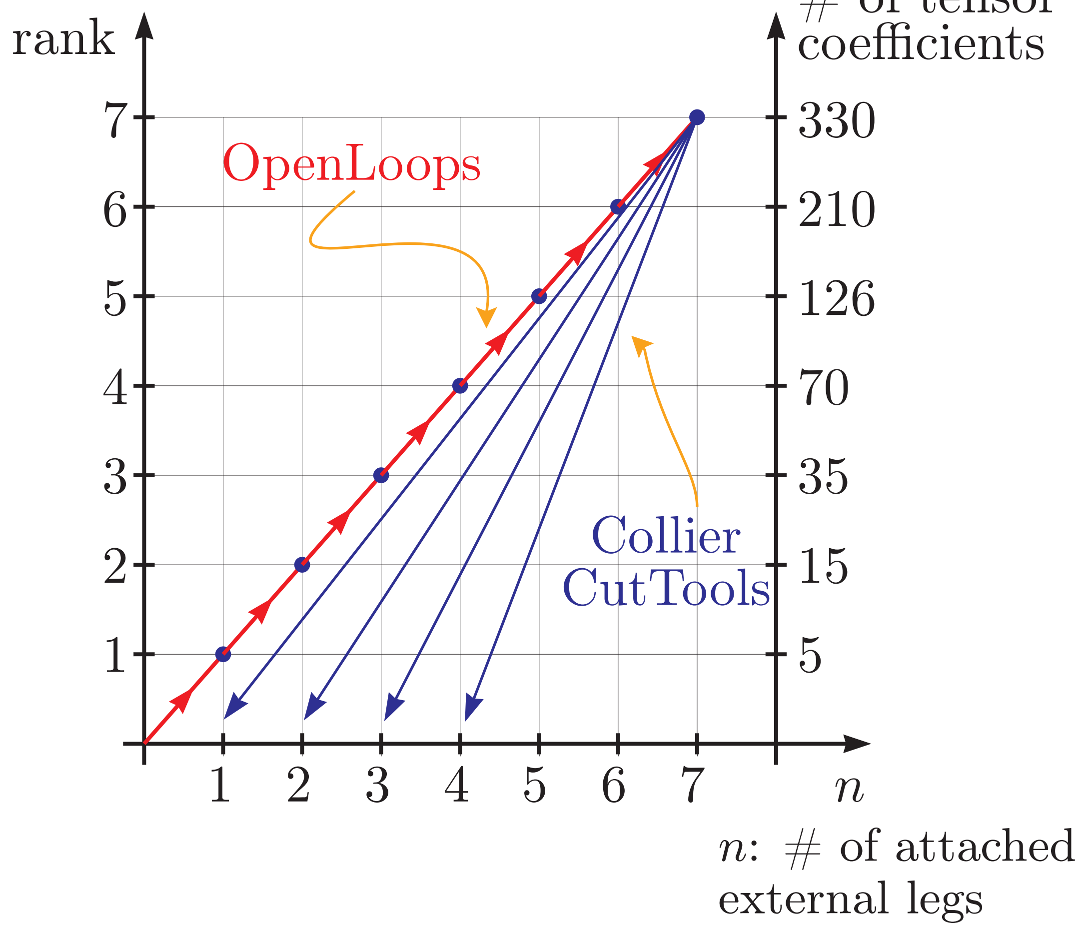
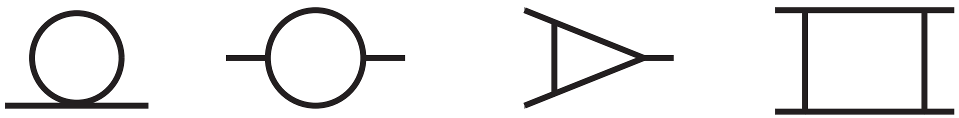


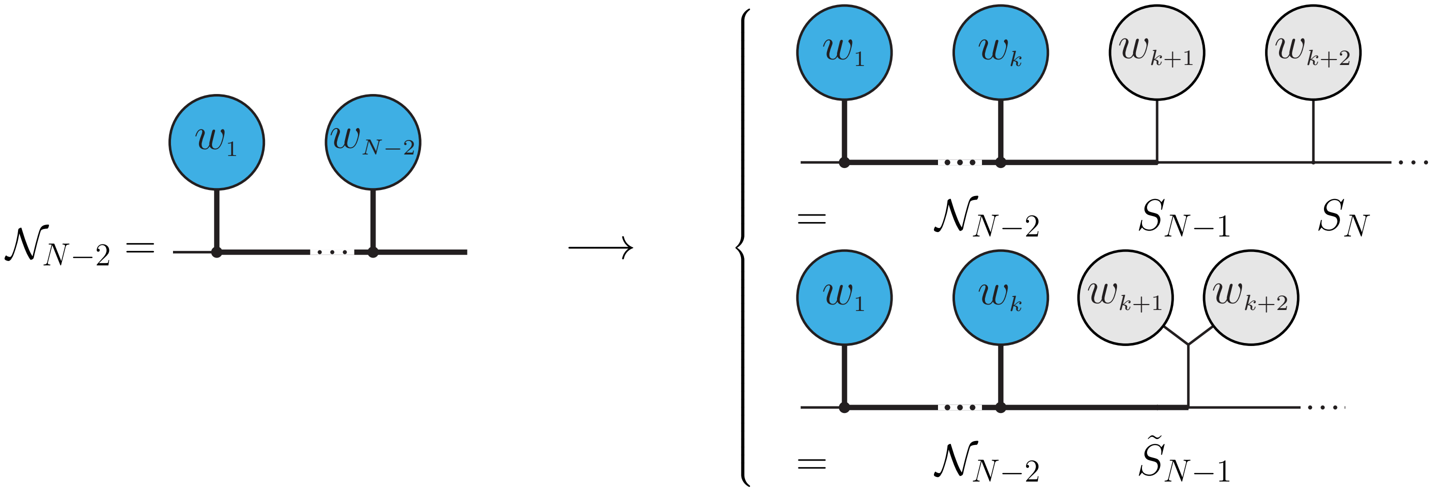
Numerical tensor integral reduction to scalar MI



The OpenLoops 1 algorithm

- For each diagram d and global helicity h configuration construct $\mathrm{Tr}\Big[\mathcal{N}_N^{(d)}(q,h)\Big]$
- Interference with Born: $\mathcal{V}_N^{(d)}(q,h) = 2\left(\sum_{\mathrm{col}} \mathcal{M}_0(h)^* \mathcal{C}^{(d)}\right) \mathrm{Tr}\Big[\mathcal{N}_N^{(d)}(q,h)\Big]$
- Helicity sum: $\mathcal{V}_N^{(d)}(q,0) = \sum_h \mathcal{V}_N^{(d)}(q,h)$
- Sum same topology diagrams, reduce and evaluate integrals: $\int \mathrm{d}^D q \sum_d \frac{\mathrm{Tr}\Big[\mathcal{V}_N^{(d)}(q,0)\Big]}{\bar{D}_0, \ldots, \bar{D}_{N-1}}$

$\Rightarrow$ parent-child recycling of colour-stripped partially dressed numerators



Issues:

- High tensor rank in loop momentum $q \Rightarrow$ high complexity
- Stability in the IR region is challenging for $2 \to 4$ processes

The on-the-fly method and OpenLoops 2

Long-term goal: NNLO automation for $2 \to 2$ and $2 \to 3$ processes

- 2 loop amplitude construction and reduction $\Rightarrow$ avoid high tensor rank complexity
- Numerical stability in IR regions at NLO for $2 \to 4$ is crucial

Introduce **On-the-fly operations** [Buccioni, Pozzorini, M.Z.] already during amplitude construction (see Eur. Phys. J. C **78** (2018) no.1, 70 [arXiv:1710.11452 [hep-ph]]):

Dressing steps are performed directly on $\mathcal{V}_n^{(d)}(q) = 2\left(\sum_{\text{col}} \mathcal{M}_0^* \mathcal{C}^{(d)}\right) \mathcal{N}_n(q)$, interleaved with

- On-the-fly helicity summation
- On-the-fly merging of open loops
- On-the-fly integrand reduction $\Rightarrow$ tensor rank ≤ 2 at all times,
  no external reduction libraries needed

Advantages:

- Significant gain in CPU efficiency.
- Stability issues addressed in a targeted way $\Rightarrow$ Significant gain in numerical stability.

will be available soon in **OpenLoops 2** [Buccioni, Lindert, Maierhöfer, Pozzorini, M.Z.]

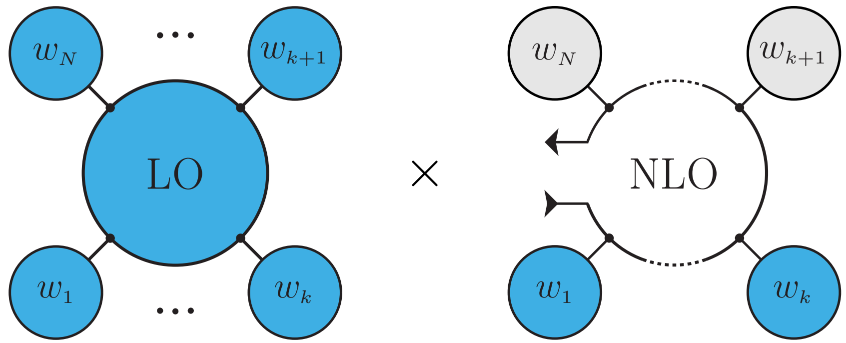
II.a) On-the-fly helicity summation

Consider color-helicity summed numerator ⇒ nested sums of helicities h_i of individual segments

$$\mathcal{V}_N(q,0) = \sum_h \underbrace{2\left(\sum_{\text{col}} \mathcal{M}_0(h)^*\mathcal{C}\right)}_{=\mathcal{V}_0(h)} \mathcal{N}_N(q,h) \quad = \quad \sum_{h_N}\left[\dots\sum_{h_2}\left[\sum_{h_1}\mathcal{V}_0(h)S_1(q,h_1)\right]S_2(q,h_2)\cdots\right]S_n(q,h_N).$$

- Interfere with colour factor and Born before dressing ⇒ initial open loop $\mathcal{V}_0(h)$
- Sum helicity dof of segment n during n-th dressing step $\mathcal{V}_n(q,\check{h}_n) = \sum_{h_n}\mathcal{V}_{n-1}(q,\check{h}_{n-1})S_n(q,h_n)$
- Partially dressed numerator depends on helicity of undressed segments $\check{h}_n = h_{n+1} + \cdots + h_N$:

$$\mathcal{V}_n(q,\check{h}_n) \quad = \quad \sum_{h_1\dots h_n}\sum_{\text{col}}$$



Note: Parent-child trick not possible (different colour factors) ⇒ On-the-fly merging instead

II.b) On-the-fly merging

Sum partially dressed open loops

$$\mathcal{V}_n(q,\check{h}_n) = \sum_{\alpha} \mathcal{V}_n^{(\alpha)}(q,\check{h}_n)$$

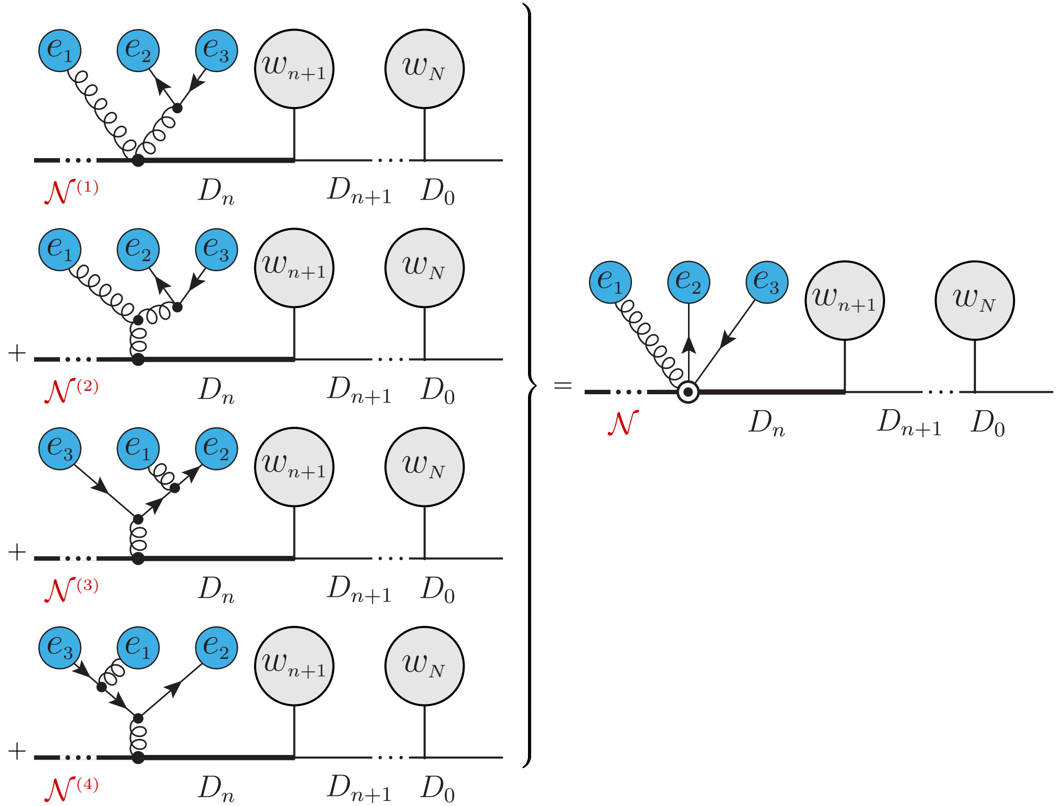
with

- the same topology $\bar{D}_0, \dots, \bar{D}_{N-1}$
- the same undressed segments $S_{n+1}, \dots, S_N$

since

$$\sum_{\alpha} \frac{\mathcal{V}_n^{(\alpha)} S_{n+1} \cdots S_{N-1}}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{N-1}} = \frac{\mathcal{V}_n S_{n+1} \cdots S_{N-1}}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{N-1}}$$

Example:



▷ dressing steps for $S_{n+1}, \dots, S_N$ performed only once for the merged object

▷ crucial for performance in combination with on-the-fly reduction

II.c) On-the-fly Reduction

Use reduction identities valid at integrand level [del Aguila, Pittau '05]

$$
\begin{aligned}
q^\mu q^\nu &= A^{\mu\nu} + B^{\mu\nu}_\lambda q^\lambda \\
&= \left[A^{\mu\nu}_{-1} + A^{\mu\nu}_0 D_0\right] + \left[B^{\mu\nu}_{-1,\lambda} + \sum_{i=0}^{3} B^{\mu\nu}_{i,\lambda} D_i\right] q^\lambda, \qquad D_i = (q+p_i)^2 - m_i^2
\end{aligned}
$$

in order to reduce the factorised open loop integrand:

$$
\frac{\mathcal{V}_N(q)}{D_0 \cdots D_N} = \frac{S_1(q) S_2(q) \cdots S_n(q) \cdots S_N(q)}{D_0 D_1 D_2 D_3 \cdots D_{N-1}}
$$

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$$q^\mu q^\nu = A^{\mu\nu} + B^{\mu\nu}_\lambda q^\lambda$$
$$= \left[A^{\mu\nu}_{-1} + A^{\mu\nu}_0 D_0\right] + \left[B^{\mu\nu}_{-1,\lambda} + \sum_{i=0}^{3} B^{\mu\nu}_{i,\lambda} D_i\right] q^\lambda, \qquad D_i = (q+p_i)^2 - m_i^2$$

in order to reduce the factorised open loop integrand:

$$\frac{\mathcal{V}_N(q)}{D_0 \cdots D_N} = \frac{S_1(q)S_2(q) \cdots S_n(q) \cdots S_N(q)}{D_0 D_1 D_2 D_3 \cdots D_{N-1}}$$

integrand reduction applicable after n steps $\forall n \geq 2$ (independently of future steps!)

$$\Rightarrow \quad \frac{\mathcal{V}^{\mu\nu} q_\mu q_\nu}{\bar{D}_0 \cdots \bar{D}_{N-1}} = \frac{\mathcal{V}^\mu_{-1} q_\mu + \mathcal{V}_{-1}}{\bar{D}_0 \cdots \bar{D}_{N-1}} + \sum_{i=0}^{3} \frac{\mathcal{V}^\mu_i q_\mu + \mathcal{V}_i}{\bar{D}_0 \cdots \bar{D}_{i-1} \bar{D}_{i+1} \cdots \bar{D}_{N-1}}$$

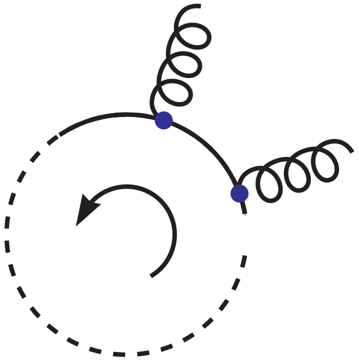
- q-dependence reconstructed in terms of 4 propagators $\Rightarrow$ new topologies with pinched propagators
- $A^{\mu\nu}, B^{\mu\nu}_\lambda$ depend on external momenta p_1, p_2, p_3

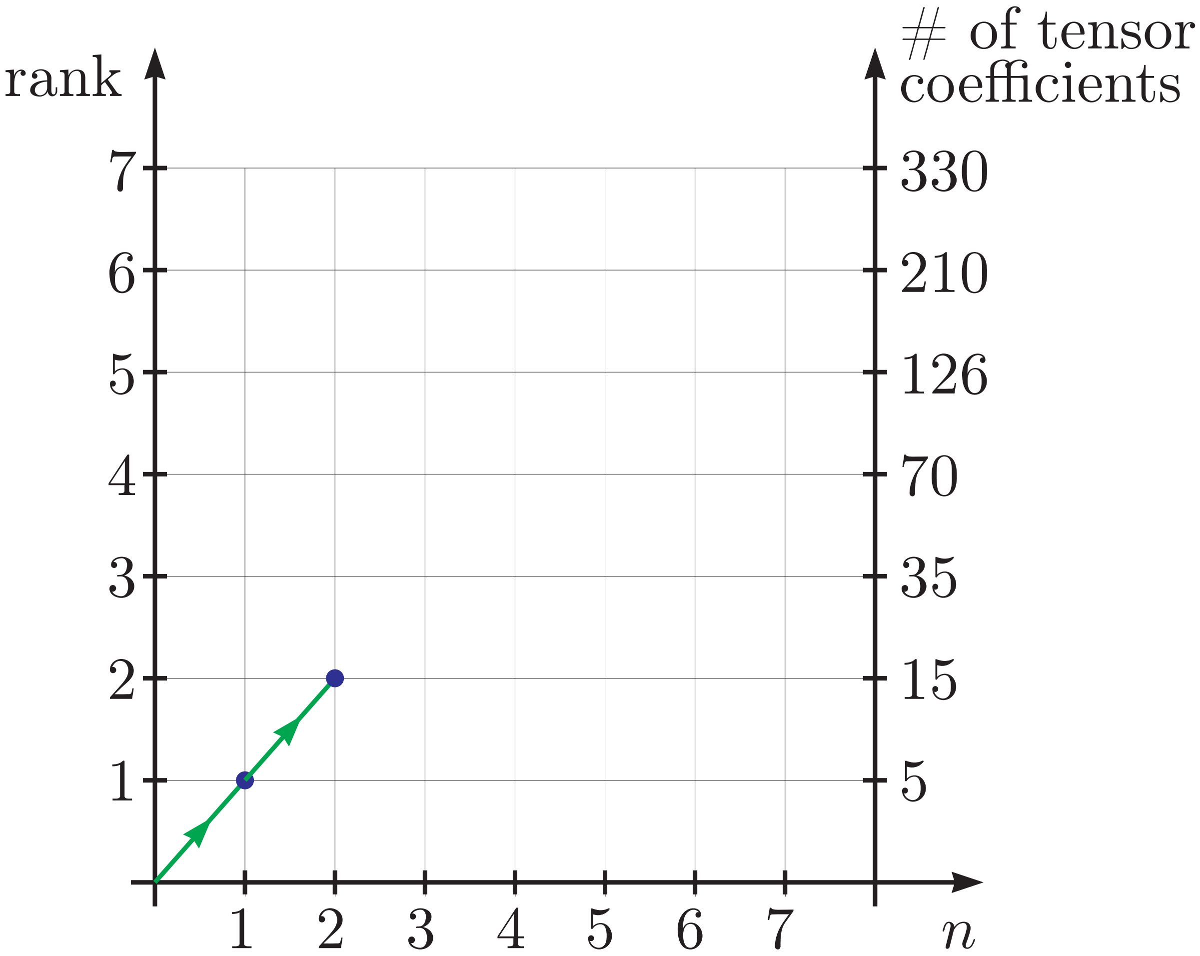
$\Rightarrow$ Compute with momentum space basis $l_1^\mu = p_1^\mu - \alpha_1 p_2^\mu, \quad l_2^\mu = p_2^\mu - \alpha_2 p_1^\mu, \quad l_3, l_4 \perp l_1, l_2, \quad l_i^2 = 0$

II.c) On-the-fly reduction

Amplitude generation and tensor reduction in OpenLoops 2

Example:





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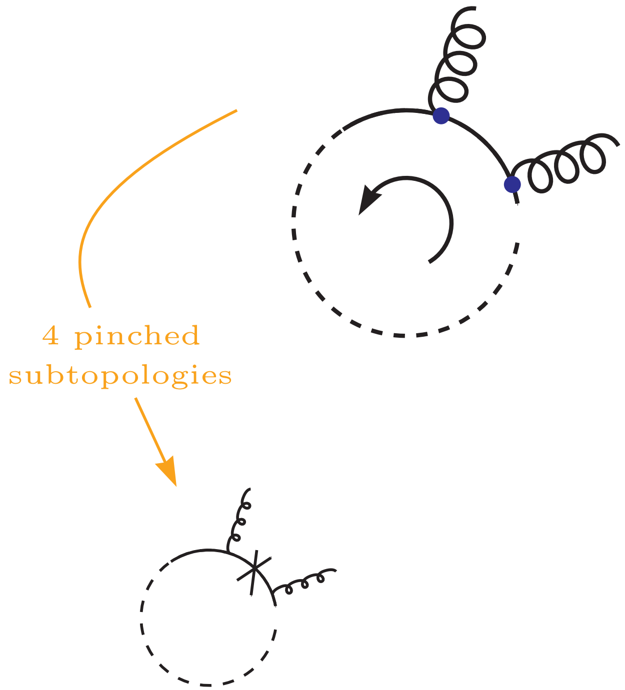

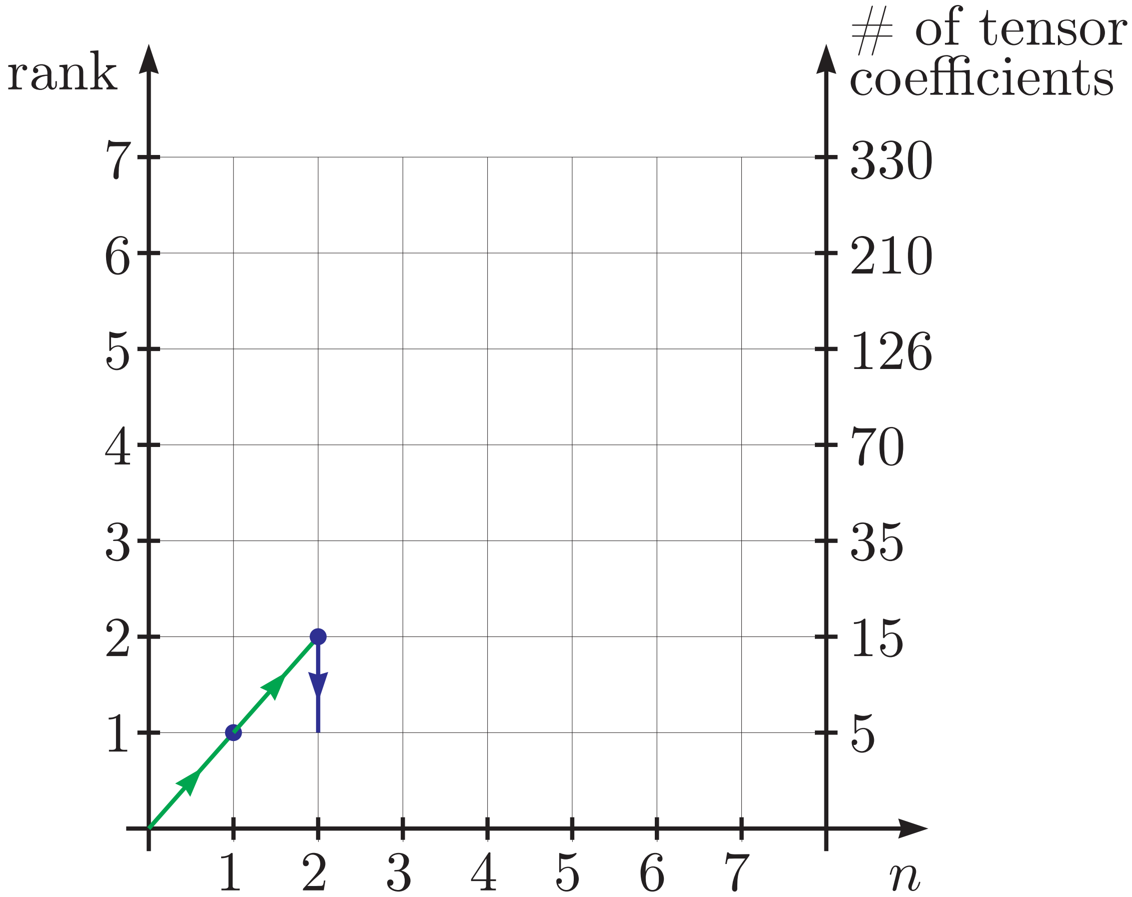

II.c) On-the-fly reduction

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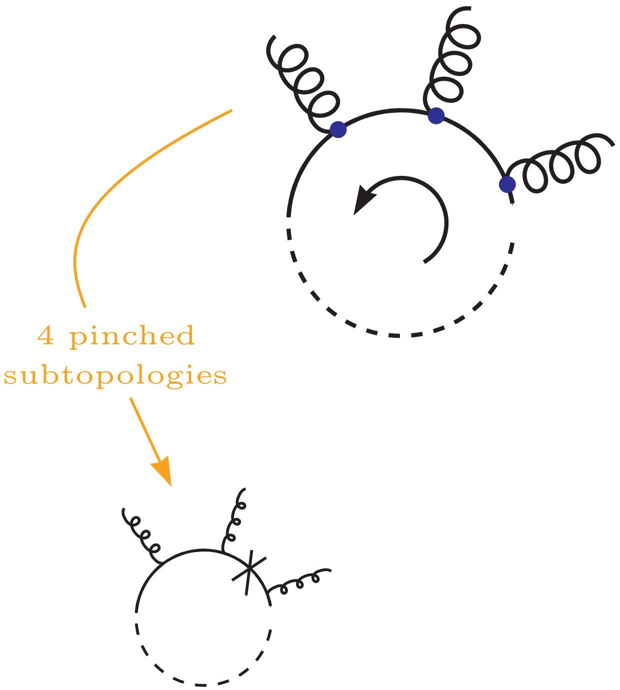

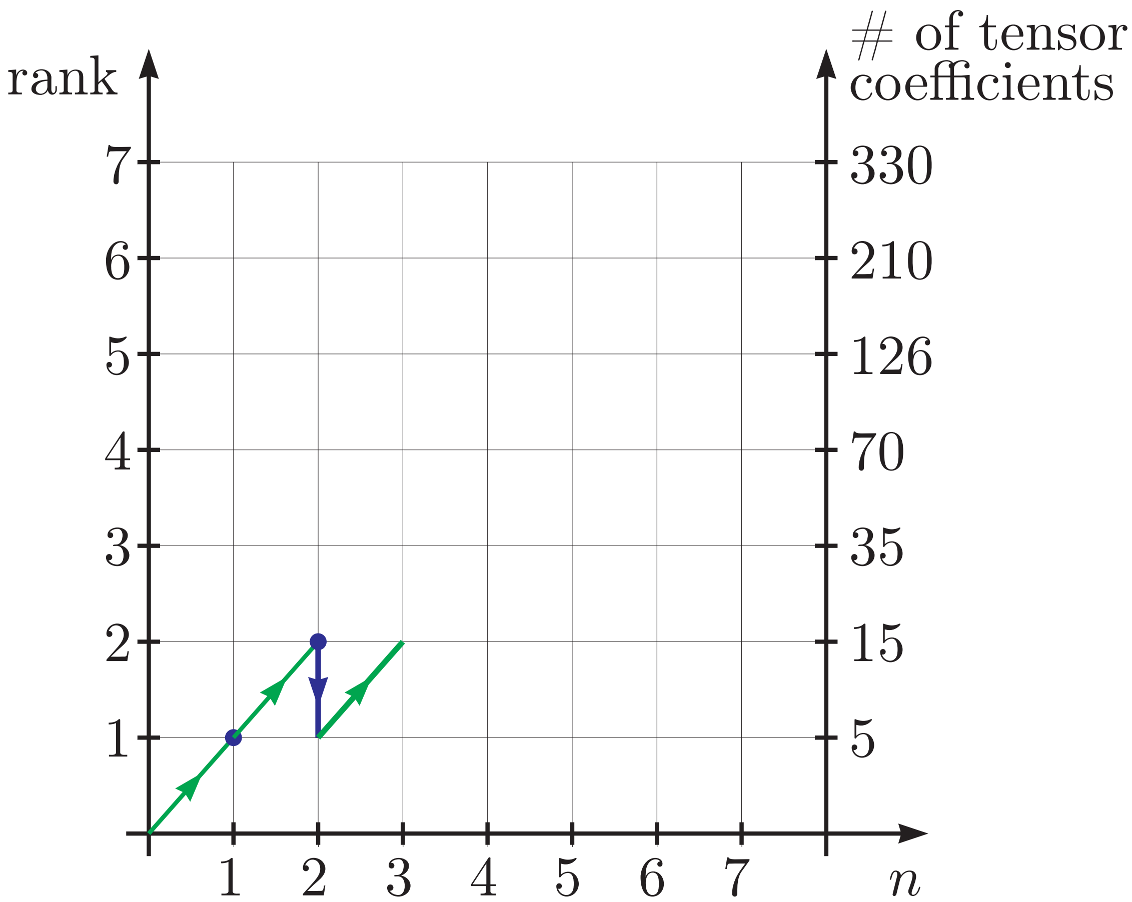

II.c) On-the-fly reduction

Amplitude generation and tensor reduction in OpenLoops 2

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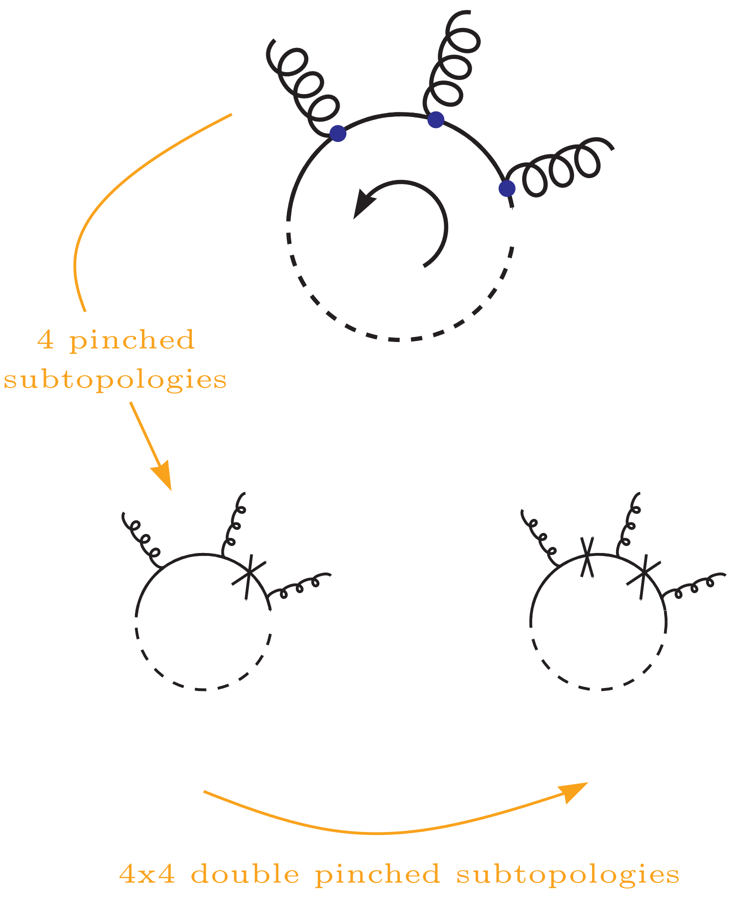
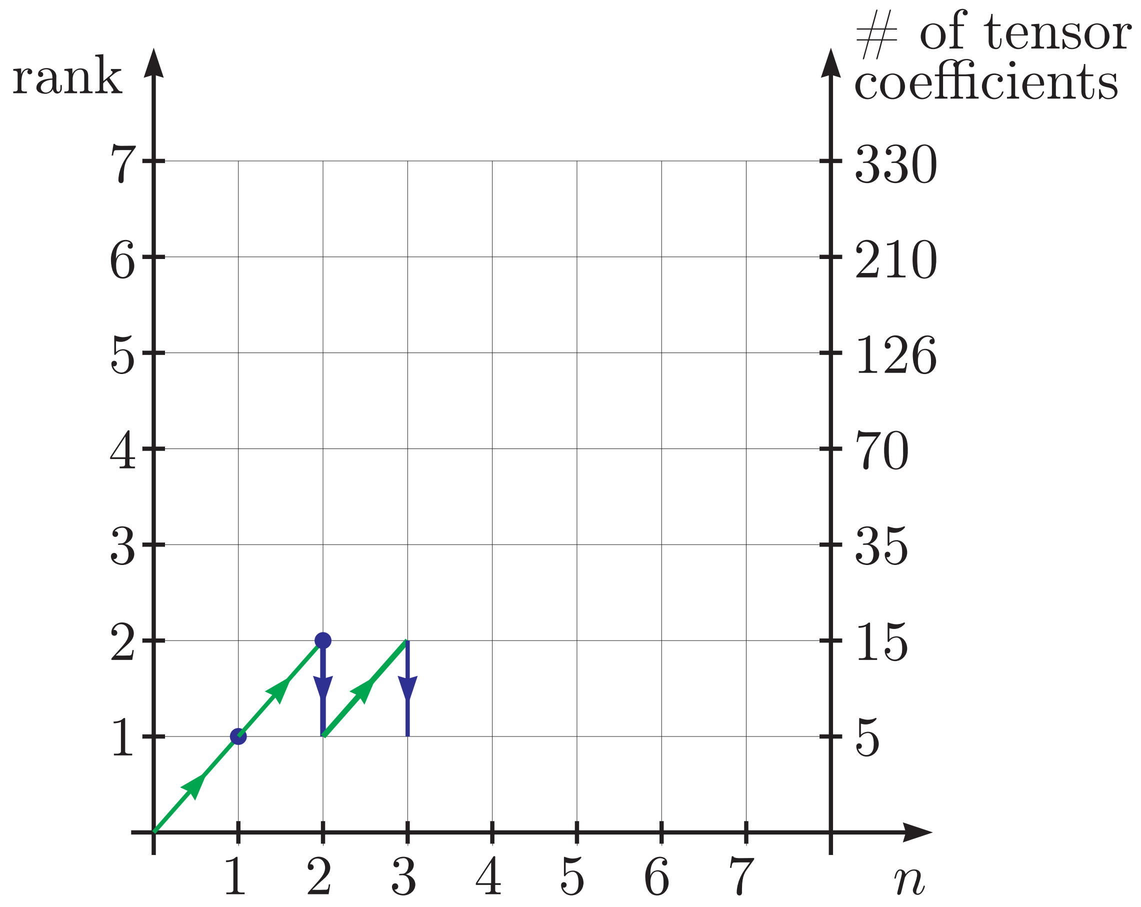
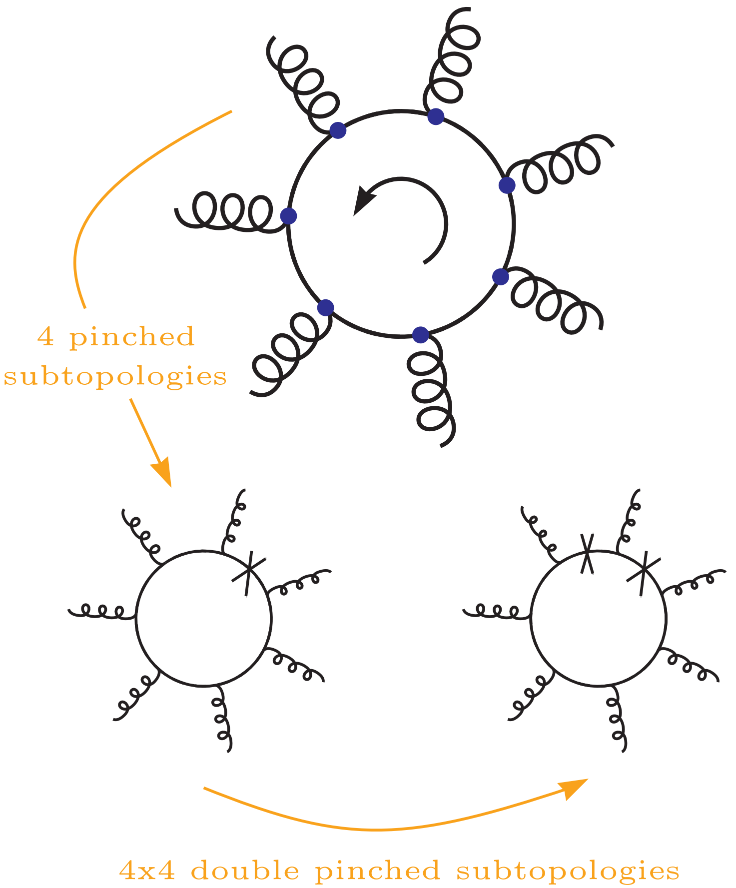


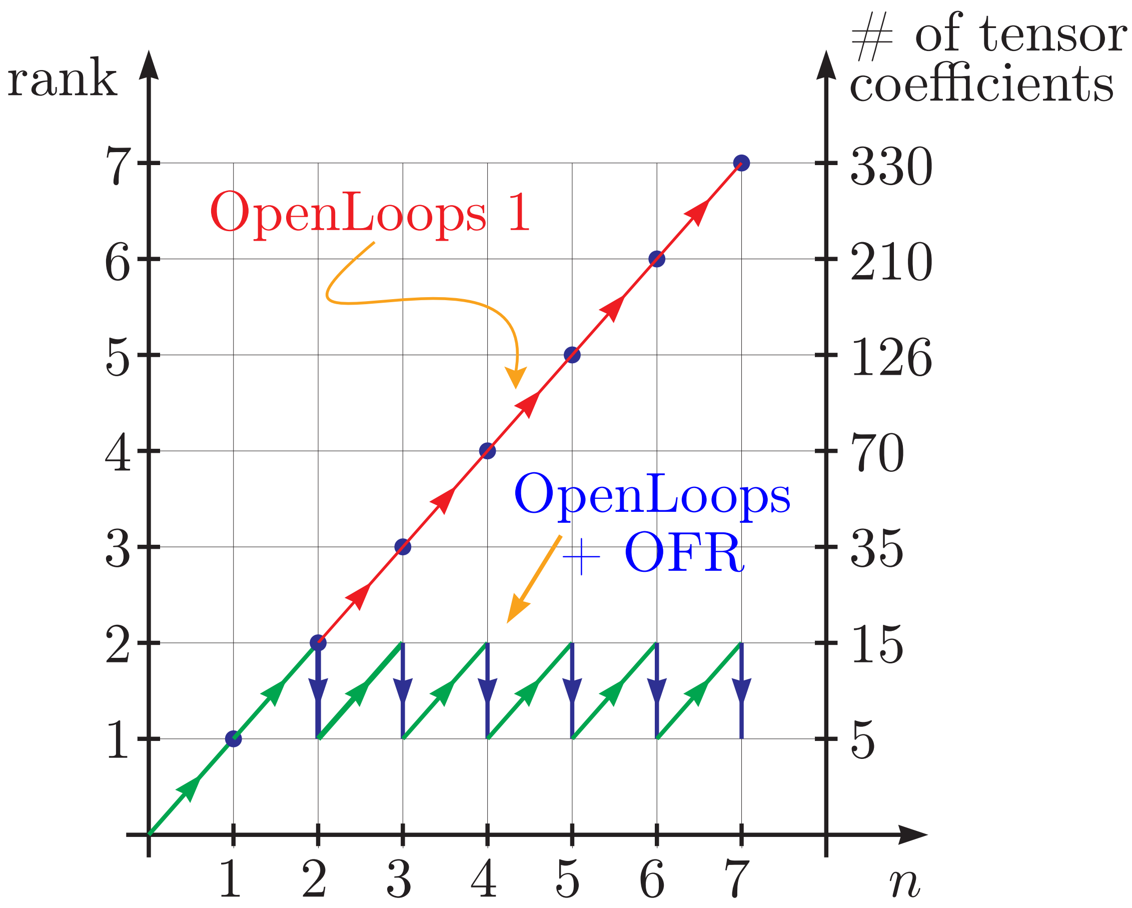


II.c) On-the-fly reduction

Amplitude generation and tensor reduction in OpenLoops 2

Example:

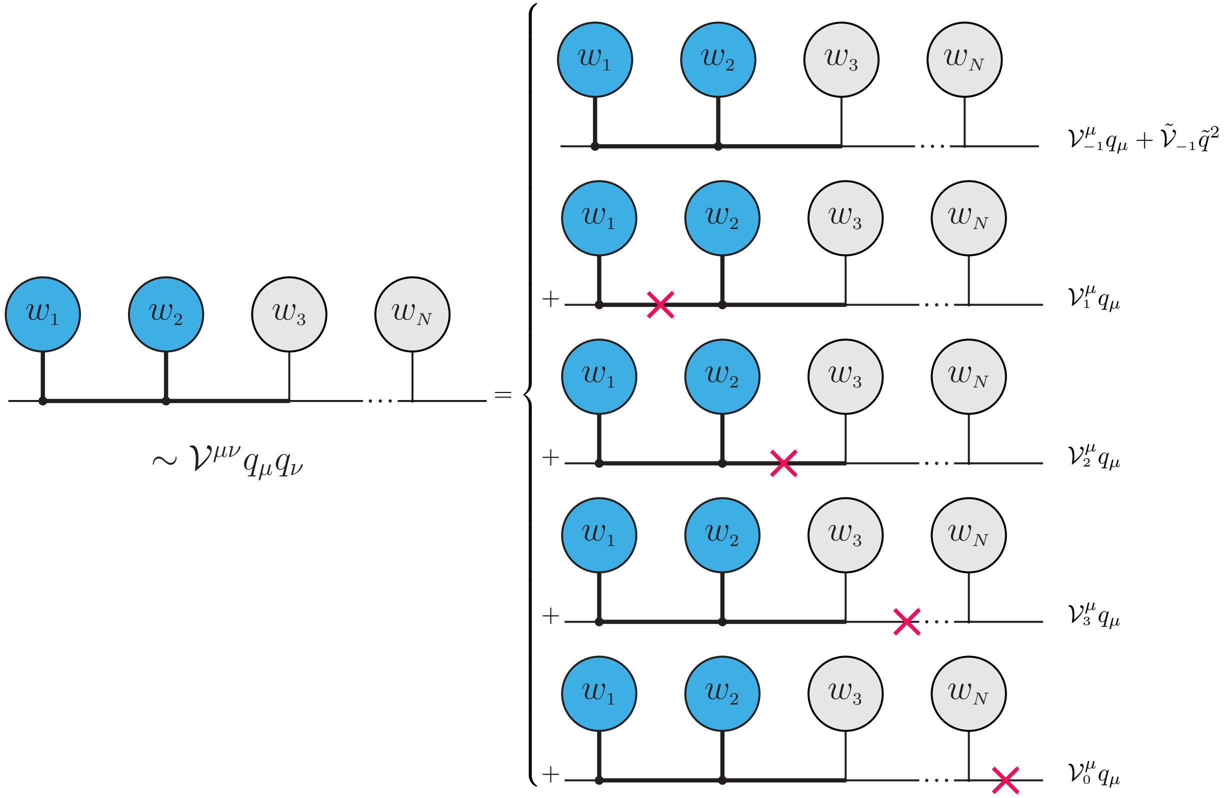




complexity associated with tensor rank remains small!

Problem: huge proliferation of topologies due to pinching of propagators

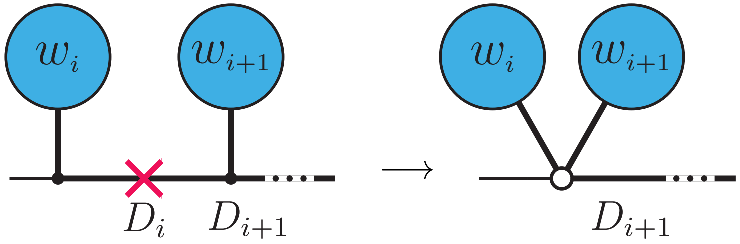
$$\Rightarrow \frac{\mathcal{V}_{\mu\nu}\, q^\mu q^\nu}{\bar{D}_0 \cdots \bar{D}_{N-1}} = \left[\underbrace{\left(\mathcal{V}^\mu_{-1} + \sum_{i=0}^{3} \mathcal{V}^\mu_i \bar{D}_i \right) q_\mu}_{\text{rank 1}} + \underbrace{\mathcal{V}_{-1} + \mathcal{V}_0 \bar{D}_0}_{\text{rank 0}} + \underbrace{\tilde{\mathcal{V}}_{-1} \tilde{q}^2}_{\text{rational term}} \right] \frac{1}{\bar{D}_0 \cdots \bar{D}_{N-1}}$$



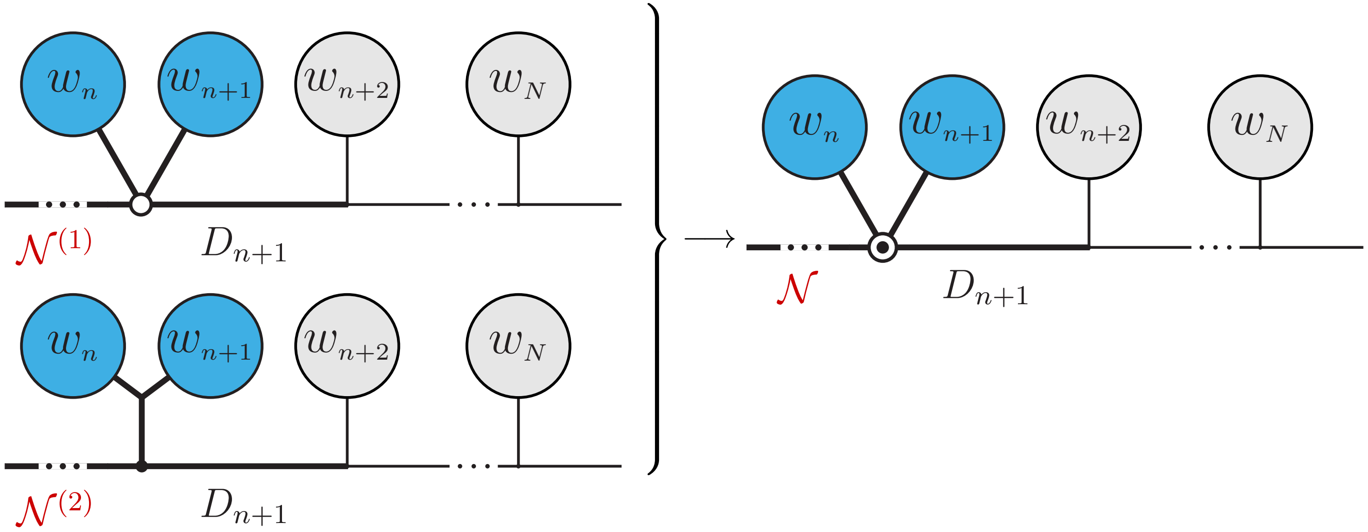
$\Rightarrow$ factor ~ 5 higher complexity after each reduction step!

Solution: On-the-fly merging

- Contract pinched propagator between dressed segments



- Merge with all (pinched and unpinched) diagrams with same topology and undressed segments



- No extra cost for pinched topologies after merging

OpenLoops 2 recursion step: dress one segment $\rightarrow$ reduce if necessary $\rightarrow$ merge

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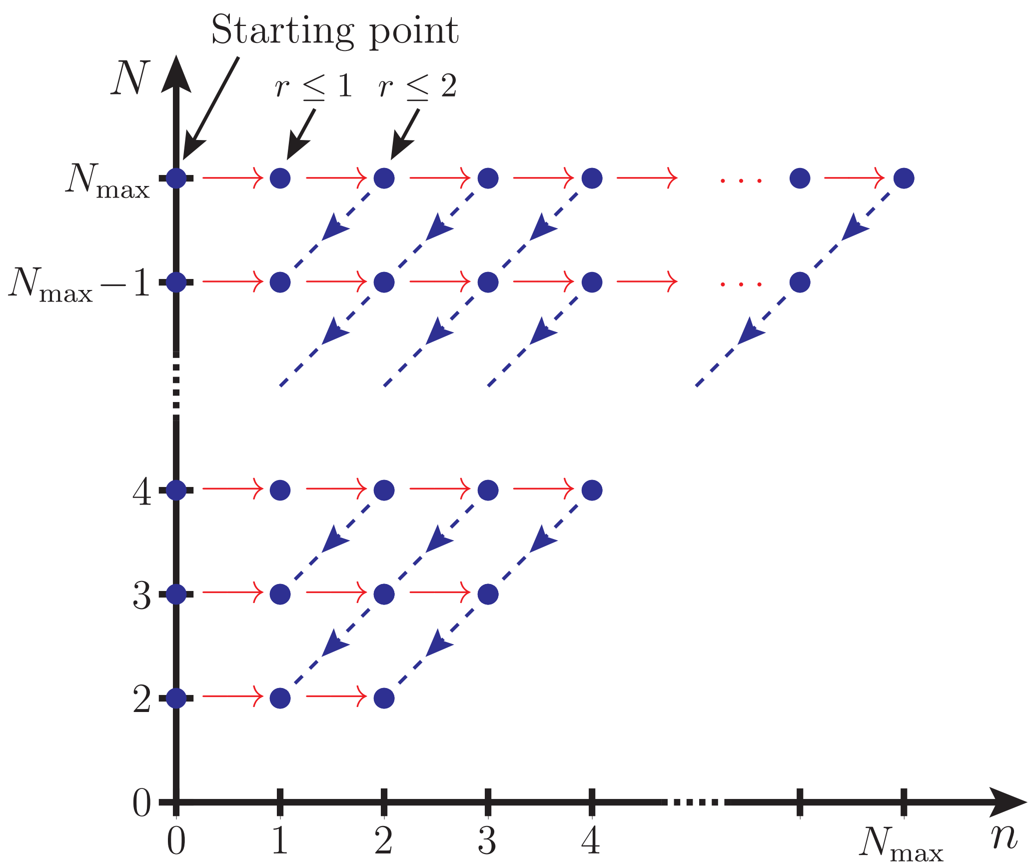
OpenLoops 2 algorithm

- Group open loops in (N, n)-sets with
  N: total number of segments,
  n: number of dressed segments.

- Start with $N = N_{max}$ and $n = 0$.

- Perform OpenLoops 2 recursion step on all open loops in an (N, n)-set until it is empty

  $\Rightarrow$ move on to $\begin{cases} (N, n+1) & \text{if } n < N \\ (N-1, 0) & \text{if } n = N \end{cases}$



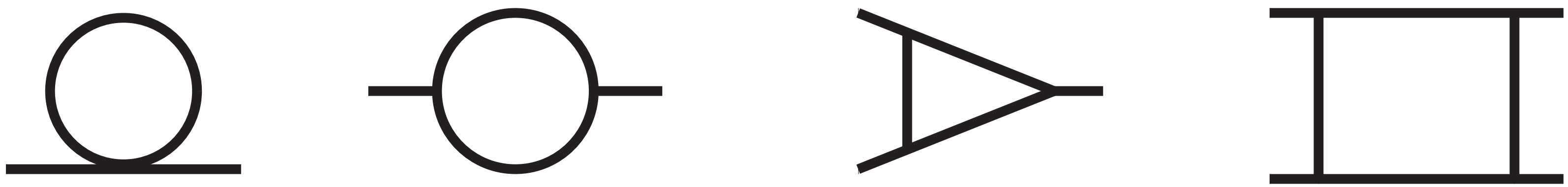
Final integral reduction

- reduce bubbles, rank-1 triangles and boxes with integral level identities [del Aguila, Pittau '05]

- reduce rank-1 and rank-0 integrals with $N \geq 5$ propagators to scalar boxes via simple OPP relations [Ossola, Papadopoulos, Pittau '07]

$$\frac{\mathcal{V} + \mathcal{V}_\mu q^\mu}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{N-1}} = \sum_{i_0<i_1<i_2<i_3}^{N-1} \frac{d(i_0 i_1 i_2 i_3)}{\bar{D}_{i_0} \bar{D}_{i_1} \bar{D}_{i_2} \bar{D}_{i_3}}$$

- use `Collier` 1.2 [Denner, Dittmaier, Hofer '16] or `OneLoop` 3.6.1 [van Hameren '10] for evaluation of scalar boxes, triangles, bubbles, tadpoles



III. Treatment of numerical instabilities due to small Gram determinants

$$q^\mu q^\nu = \left[A^{\mu\nu}_{-1} + A^{\mu\nu}_0 D_0\right] + \left[B^{\mu\nu}_{-1,\lambda} + \sum_{i=0}^{3} B^{\mu\nu}_{i,\lambda} D_i\right] q^\lambda$$

$A^{\mu\nu}_i, B^{\mu\nu}_{i,\lambda}$ computed from reduction basis $l_i(p_1, p_2)$ with $i = 1, 2, 3, 4$ and third momentum p_3

$$A^{\mu\nu}_i = \frac{1}{\gamma} a^{\mu\nu}_i,$$
$$B^{\mu\nu}_{i,\lambda} = \frac{1}{\gamma^2}\left[b^{(1)}_{i,\lambda}\right]^{\mu\nu} + \frac{1}{\gamma}\left[b^{(2)}_{i,\lambda}\right]^{\mu\nu}$$

Severe numerical instabilities for

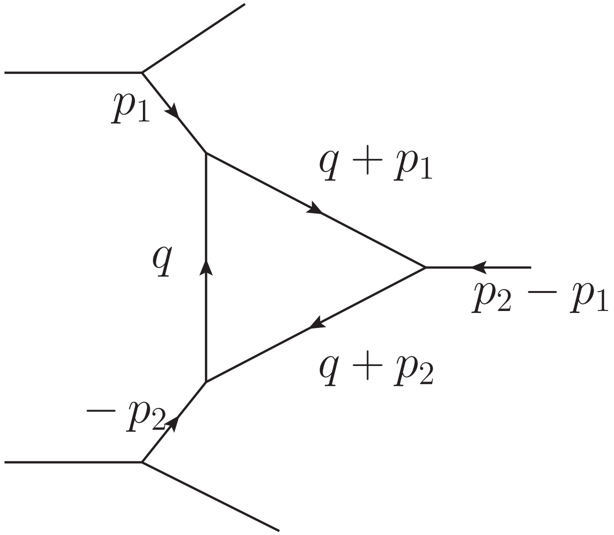
$\gamma = 4\frac{\Delta_{12}}{p_1 p_2 \pm \sqrt{\Delta_{12}}} \to 0$

with $\Delta_{12} = (p_1 p_2)^2 - p_1^2 p_2^2$

- For $N \geq 4$: Re-order $\{D_1, D_2, D_3\} \longrightarrow \{D_{i_1}, D_{i_2}, D_{i_3}\}$ such that $|\Delta_{i_1 i_2}|/Q^4_{i_1 i_2}$ is maximal ($Q^2_{ij} = \max\{|p_i \cdot p_j|, |p_i^2|, |p_j^2|\}$)

- For $N = 3$: Identify problematic kinematic configurations and use targeted expansions.

Triangles: For hard kinematics only one case with small Gram determinant: t-channel with



$$\begin{aligned} p_1^2 &= -p^2 < 0, \\ p_2^2 &= -p^2(1+\delta), \qquad 0 \le \delta \ll 1, \\ (p_2-p_1)^2 &= 0, \\ \Rightarrow \sqrt{\Delta} &= \frac{p^2}{2}\delta \\ \Rightarrow \gamma &= -p^2\delta^2 \end{aligned}$$

$\Rightarrow$ expand basis momenta l_i, reduction formula and scalar integrals in δ, e.g. massless rank 1:

$$\begin{aligned} C^\mu &= \frac{2}{\delta^2 p^2}\left\{B_0(-p^2)\left[-p_1^\mu(1+\delta)+p_2^\mu\right] + B_0\left(-p^2(1+\delta)\right)\left[(p_1^\mu - p_2^\mu)(1+\delta)\right]\right\} \\ &\quad + \frac{1}{\delta}C_0\left(-p^2, -p^2(1+\delta)\right)\left[-p_1^\mu(1+\delta)+p_2^\mu\right] \\ &= \frac{p_1^\mu + p_2^\mu}{2p^2}\left[-B_0(-p^2)+1\right] + \delta\,\frac{p_1^\mu + 2p_2^\mu}{6p^2}\left[B_0(-p^2)\right] + \mathcal{O}(\delta^2) \end{aligned}$$

with $C_0(p_1^2, p_2^2) \sim \int \mathrm{d}^D q \, \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2}$ and $B_0(p_1^2) \sim \int \mathrm{d}^D q \, \frac{1}{\bar{D}_0 \bar{D}_1}$

$\Rightarrow$ $\frac{1}{\delta}$-poles cancel (also for massive cases and higher rank)

All order expansions [in collaboration with J.-N. Lang, H. Zhang]

Expand B_0, C_0 in δ and cancel all poles, e.g.

$$\frac{1}{\delta^n} B_0(-p^2(1+\delta)) = \underbrace{\left(\frac{1}{\delta^n} B_0(-p^2) + \ldots + \frac{1}{\delta} B_0^{(n)}(-p^2)\right)}_{\text{poles} \to \text{cancel}} + \underbrace{B_{0,n}(-p^2, \delta)}_{\text{regular in } \delta}$$

with

$$\begin{aligned} B_{0,n}(-p^2,\delta) &= \sum_{m=n}^{\infty} \delta^{m-n} \left[\frac{1}{m!}\left(\frac{\partial}{\partial\delta}\right)^m B_0\left(-p^2(1+\delta)\right)\right]_{\delta=0} \\ C_{0,n}(-p^2,\delta) &= \sum_{m=n}^{\infty} \delta^{m-n} \left[\frac{1}{m!}\left(\frac{\partial}{\partial\delta}\right)^m C_0\left(-p^2, -p^2(1+\delta)\right)\right]_{\delta=0} \end{aligned}$$

Example:

$$C^\mu = (p_1 - p_2)^\mu \left[\frac{B_{0,1} + 2\,B_{0,2}}{p^2} - C_{0,1}\right] + p_1^\mu \left[\frac{B_{0,1}}{p^2} - C_0\right]$$

Closed formulas derived and implemented for $\left(\frac{\partial}{\partial\delta}\right)^m B_0$ and $\left(\frac{\partial}{\partial\delta}\right)^m C_0$ (all QCD mass configurations).

$\Rightarrow$ $B_{0,n}$ and $C_{0,n}$ computed to any n in order to reach given target precision!
Uncertainty due to truncation of series avoided entirely.

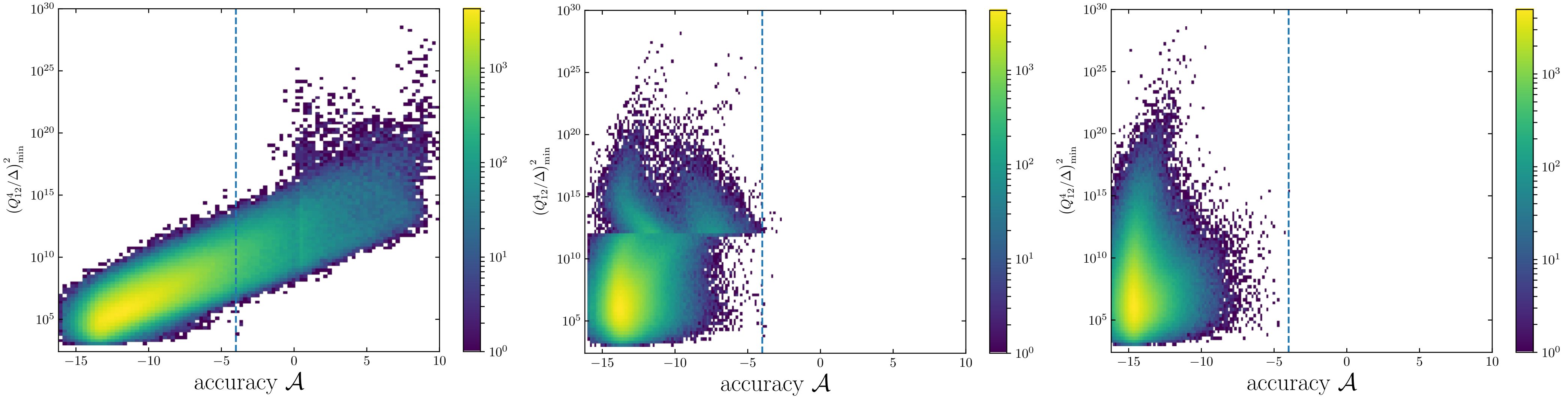
Highly efficient (complexity of series $B_{0,n}$ and $C_{0,n}$ scales like (number of terms)^2).

Accuracy improvements: Correlation between accuracy $\mathcal{A}$ and the largest $(Q^4/\Delta)^2$ in the event

from any rank-2 Gram determinant $\Delta = \Delta(p_i, p_j)$ with corresponding $Q^2 = \max\{|p_i \cdot p_j|, |p_i^2|, |p_j^2|\}$

$gg \to t\bar{t}gg$ with 10^6 events (OpenLoops 2 in double precision)

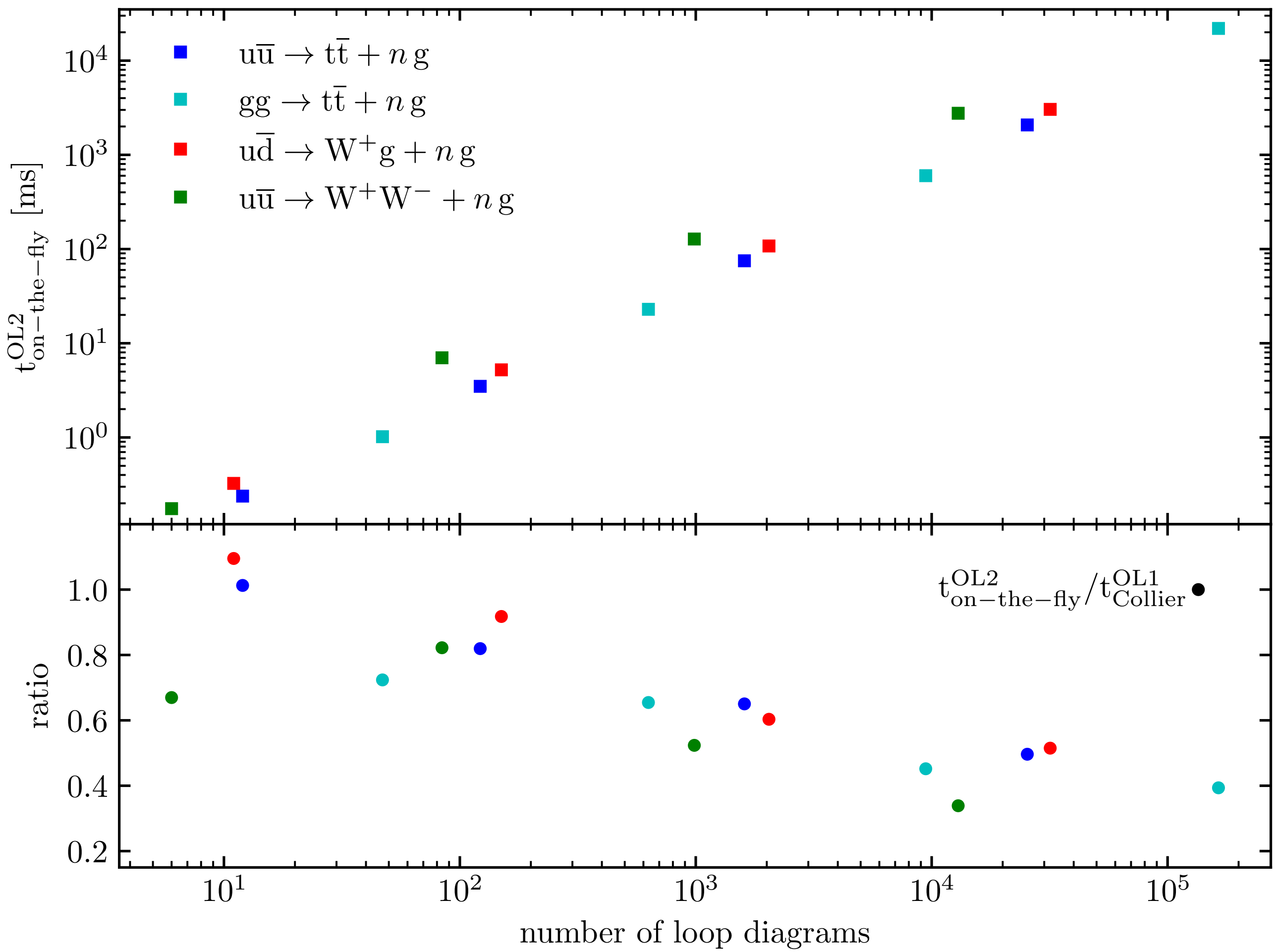
On-the-fly reduction with no stability improvements | (D_1, D_2, D_3)-permutation + expansions to $\mathcal{O}(\delta^2)$ | (D_1, D_2, D_3)-permutation + all order expansions



IV. Performance and numerical stability benchmarks

Runtime per phase space point – OpenLoops 1 with Collier vs OpenLoops 2:

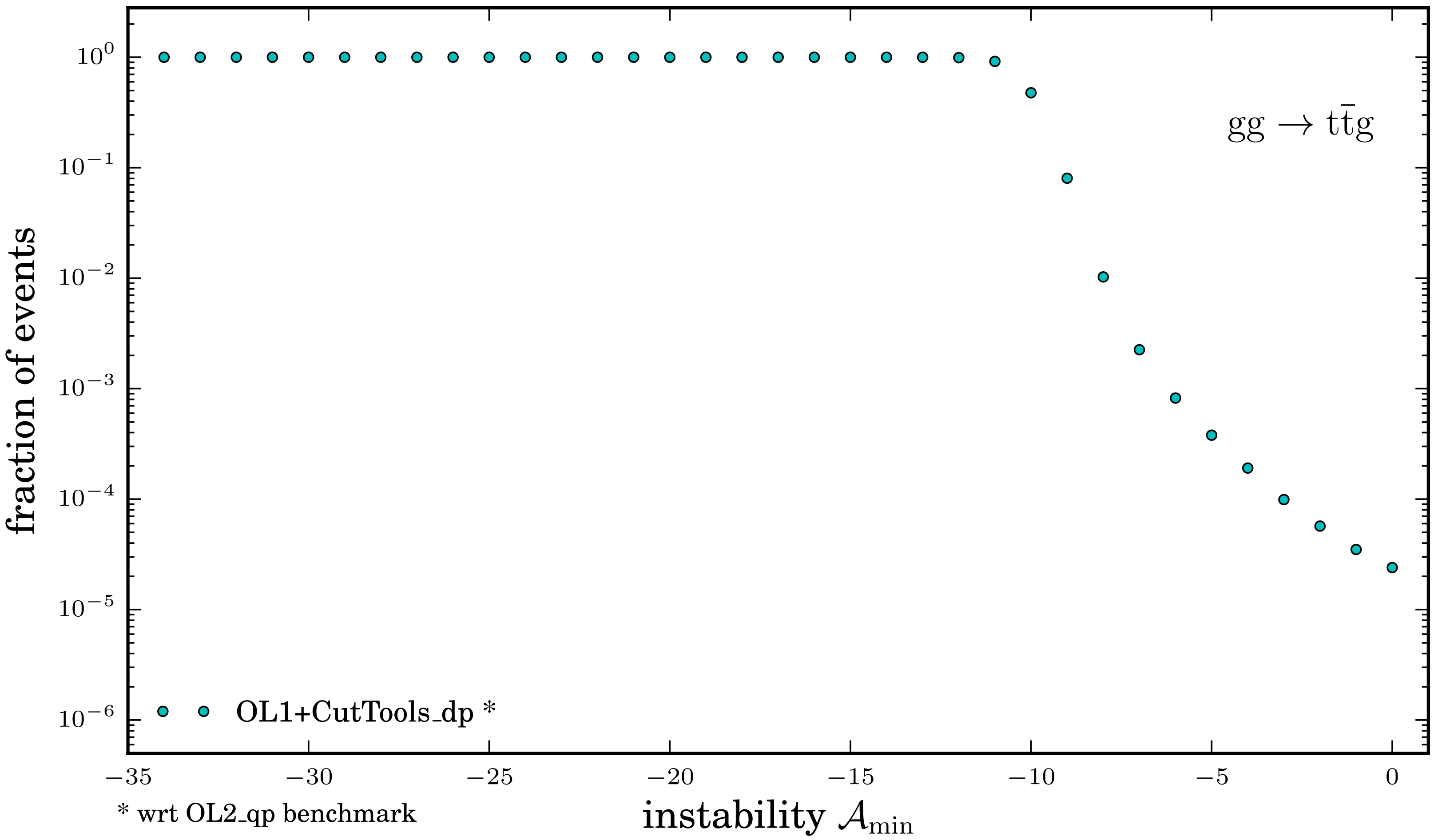
one-loop scattering probabilities for processes with $n = 0, 1, 2, 3$ gluons (up to $2 \to 5$ with $\sim 10^5$ diagrams)



Factor $\sim (2-3)$ speedup wrt OpenLoops 1 for nontrivial processes! (single Intel i7-4790K core, gfortran-4.8.5)

Stability of OpenLoops 1 and 2: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV (10^6 uniform random points)

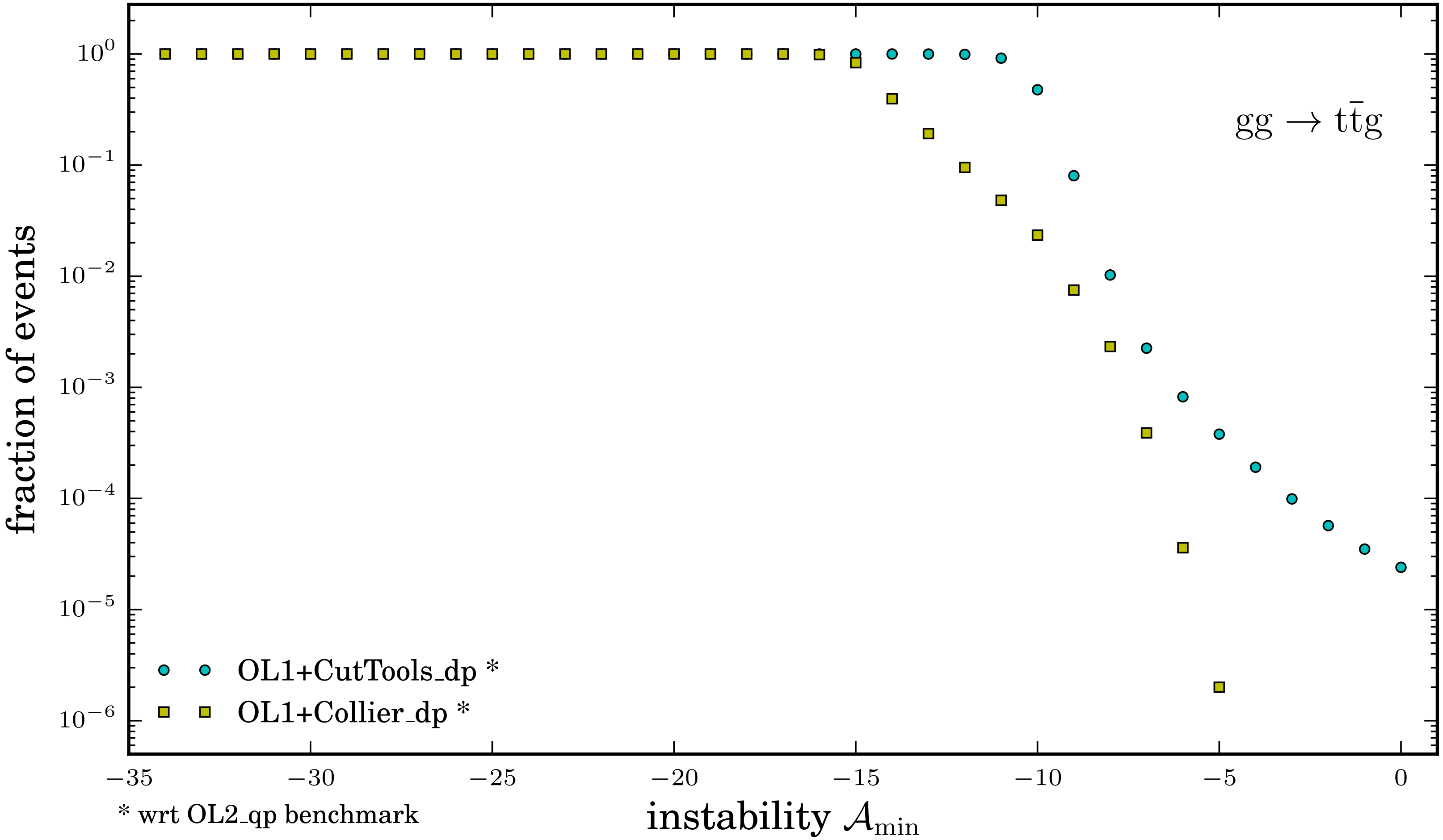
Probability of relative accuracy $\mathcal{A}$ in **OpenLoops 1+Cuttools in double precision (dp)** wrt quadruple precision benchmark



Hard cuts: $p_T > 50$ GeV and $\Delta R_{ij} => 0.5$ for final state QCD partons ($\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$)

Stability of OpenLoops 1 and 2: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV (10^6 uniform random points)

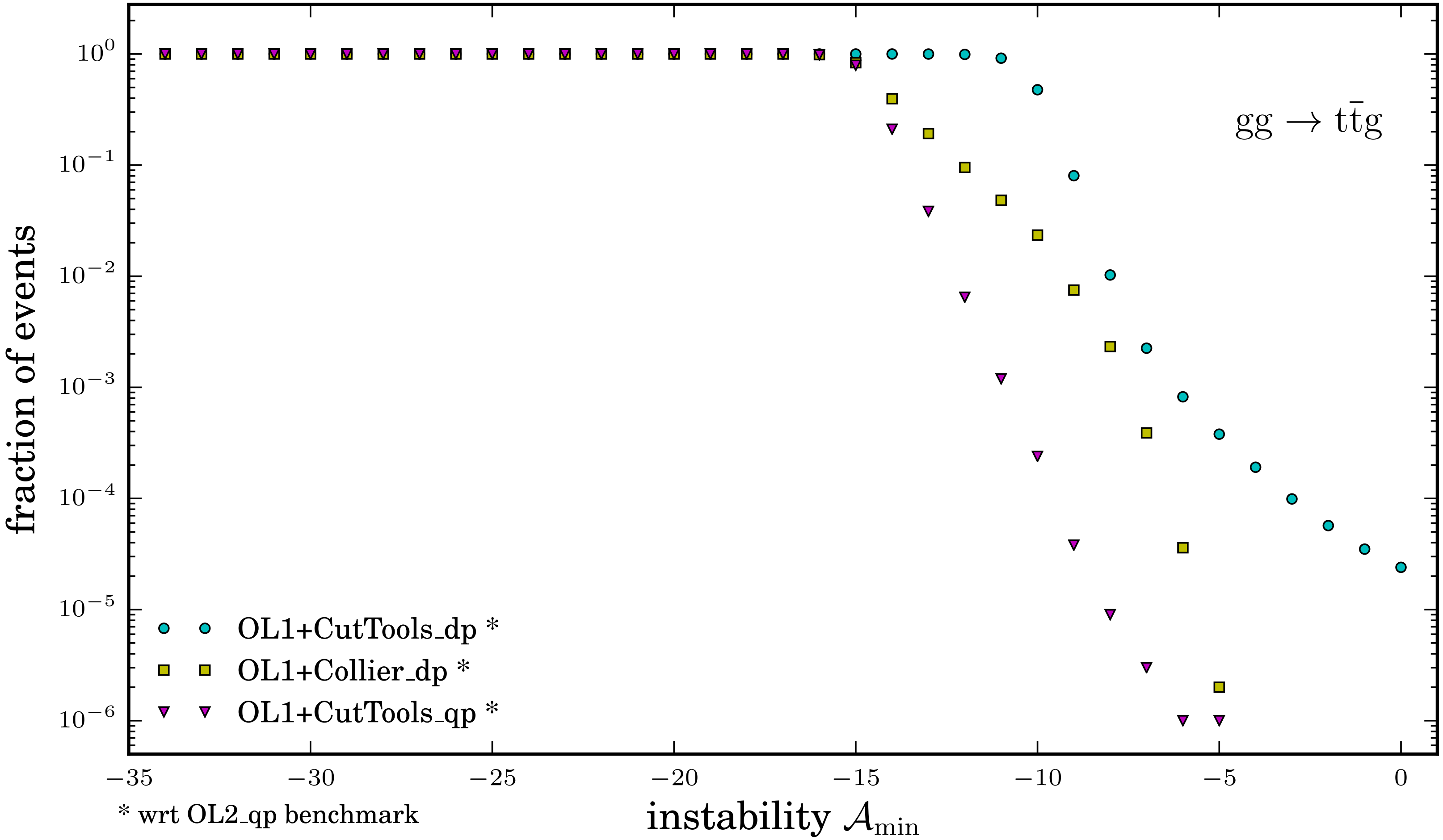
Probability of relative accuracy $\mathcal{A}$ in **OpenLoops 1+Collier in double precision (dp)** wrt quadruple precision benchmark



Hard cuts: $p_T > 50$ GeV and $\Delta R_{ij} => 0.5$ for final state QCD partons ($\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$)

Stability of OpenLoops 1 and 2: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV (10^6 uniform random points)

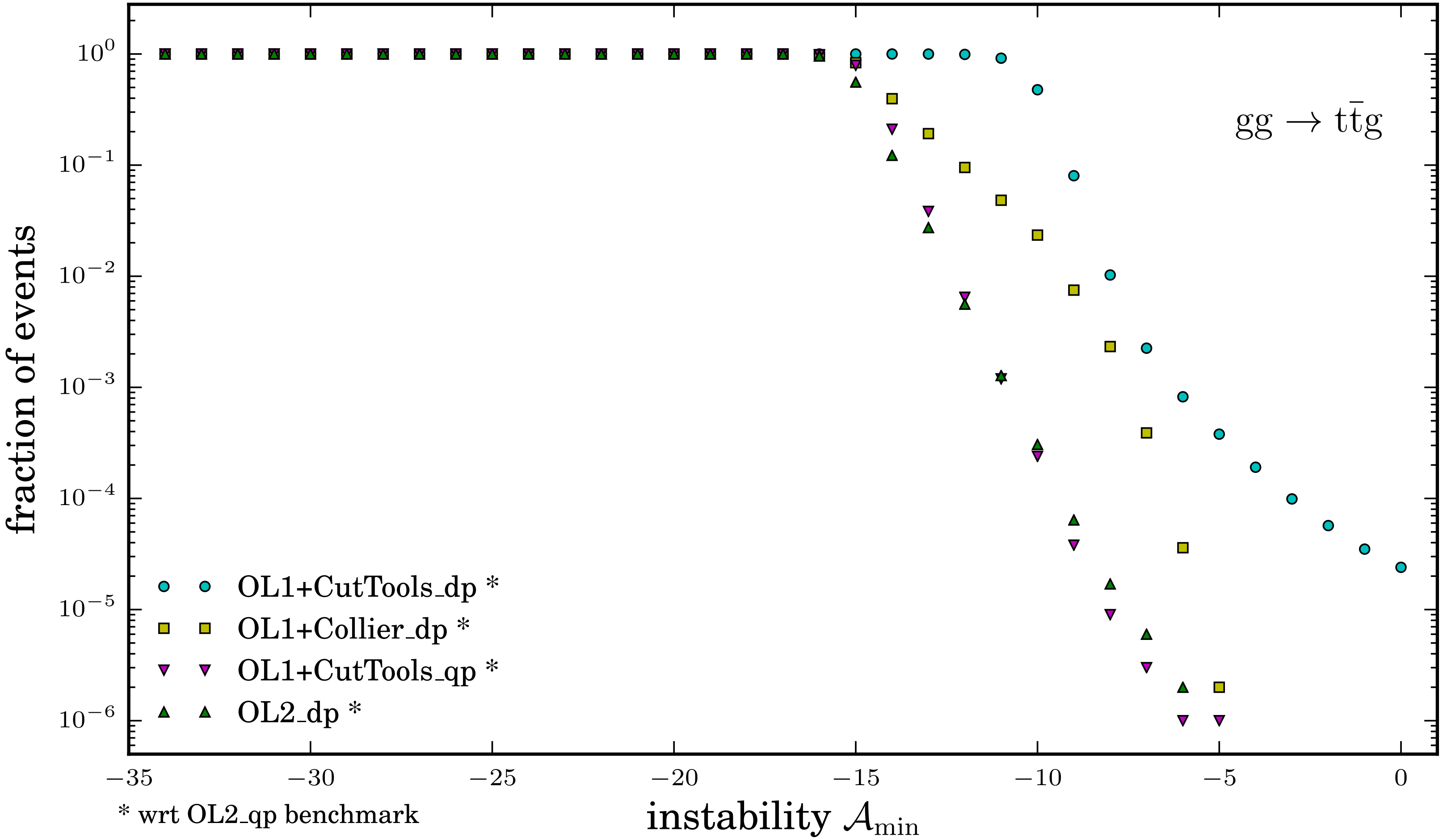
Probability of relative accuracy $\mathcal{A}$ in **OpenLoops 1+Cuttools in quadruple precision (qp)** wrt qp benchmark



Hard cuts: $p_T > 50$ GeV and $\Delta R_{ij} => 0.5$ for final state QCD partons ($\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$)

Stability of OpenLoops 1 and 2: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV (10^6 uniform random points)

Probability of relative accuracy $\mathcal{A}$ in **OpenLoops 2 in double precision (dp)** wrt quadruple precision benchmark

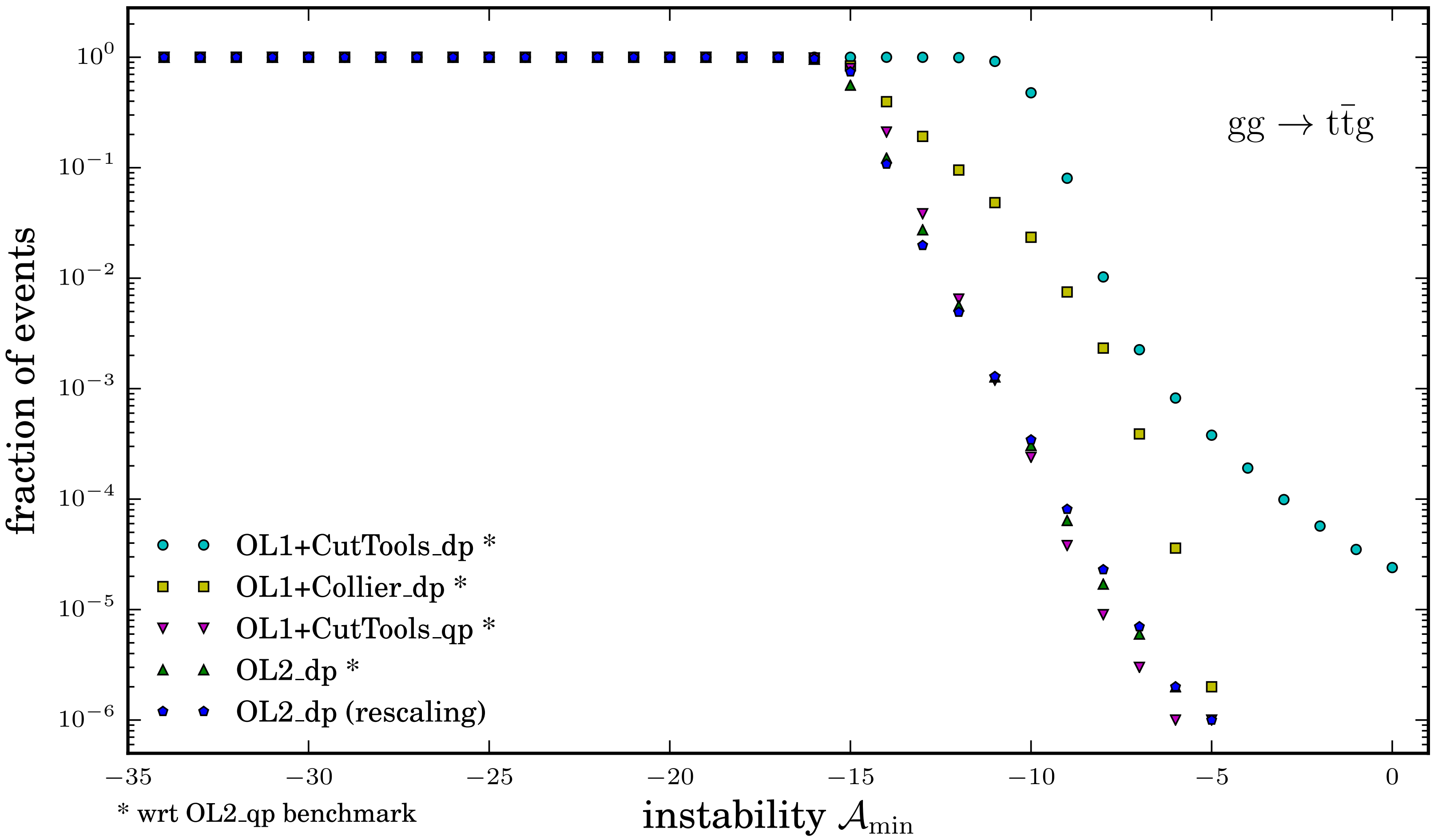


Hard cuts: $p_T > 50$ GeV and $\Delta R_{ij} => 0.5$ for final state QCD partons ($\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$)

Excellent stability thanks to on-the-fly reduction and dedicated all order expansions

Stability of OpenLoops 1 and 2: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV (10^6 uniform random points)

Probability of relative accuracy $\mathcal{A}$ in **OpenLoops 2 in double precision (dp)** from rescaling test

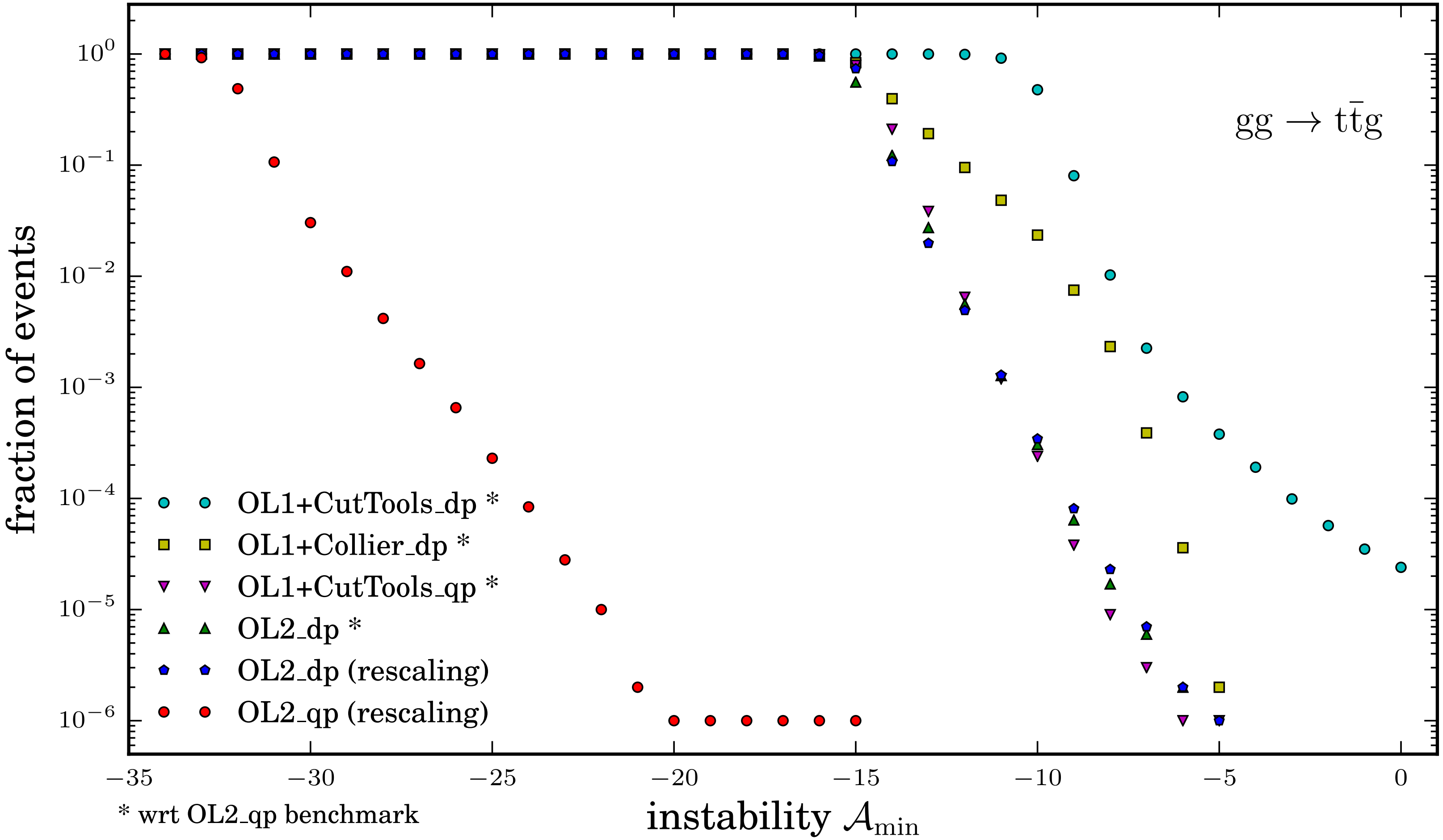


Hard cuts: $p_T > 50$ GeV and $\Delta R_{ij} => 0.5$ for final state QCD partons ($\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$)

No error from truncation of expansions $\Rightarrow$ Reliable rescaling test

Stability of OpenLoops 1 and 2: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV (10^6 uniform random points)

Probability of relative accuracy $\mathcal{A}$ in **OpenLoops 2 in quadruple precision (qp)** from rescaling test

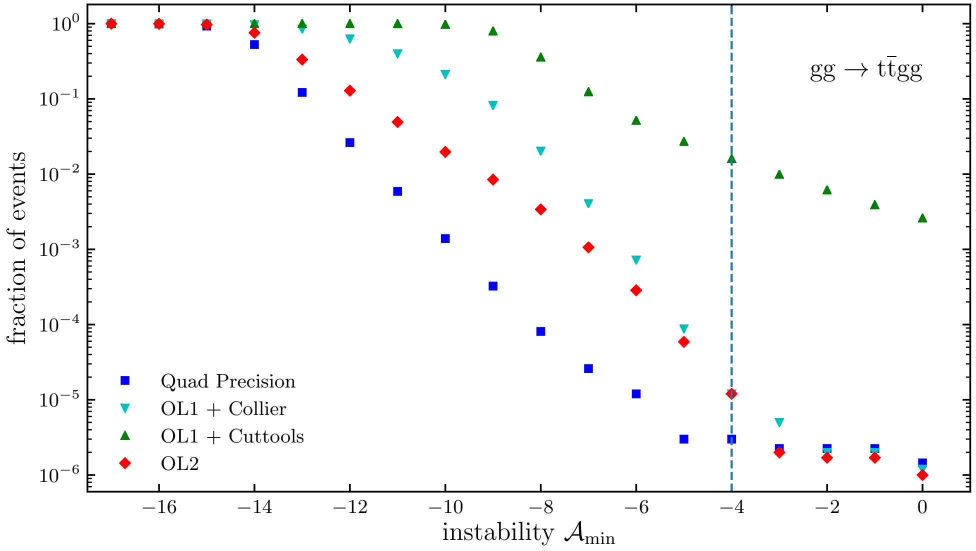


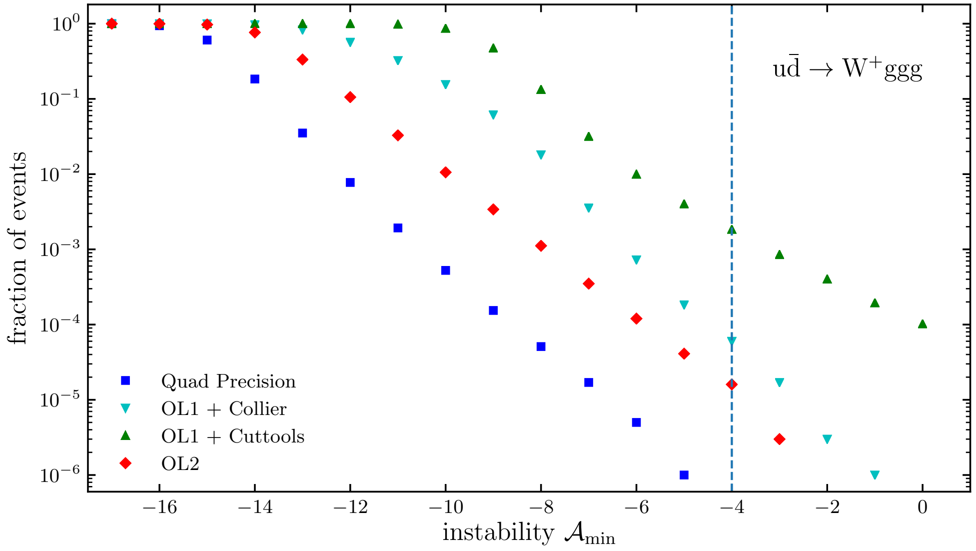
Hard cuts: $p_T > 50$ GeV and $\Delta R_{ij} => 0.5$ for final state QCD partons ($\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$)

Up to 32 digits thanks to on-the-fly reduction and all order expansions (no truncation error)

Stability of OpenLoops 1 and 2: $2 \to 4$ process at $\sqrt{\hat{s}} = 1$ TeV (10^6 uniform random points)

Probability of relative accuracy $\mathcal{A}$ (wrt OL1+Cuttools as a benchmark)

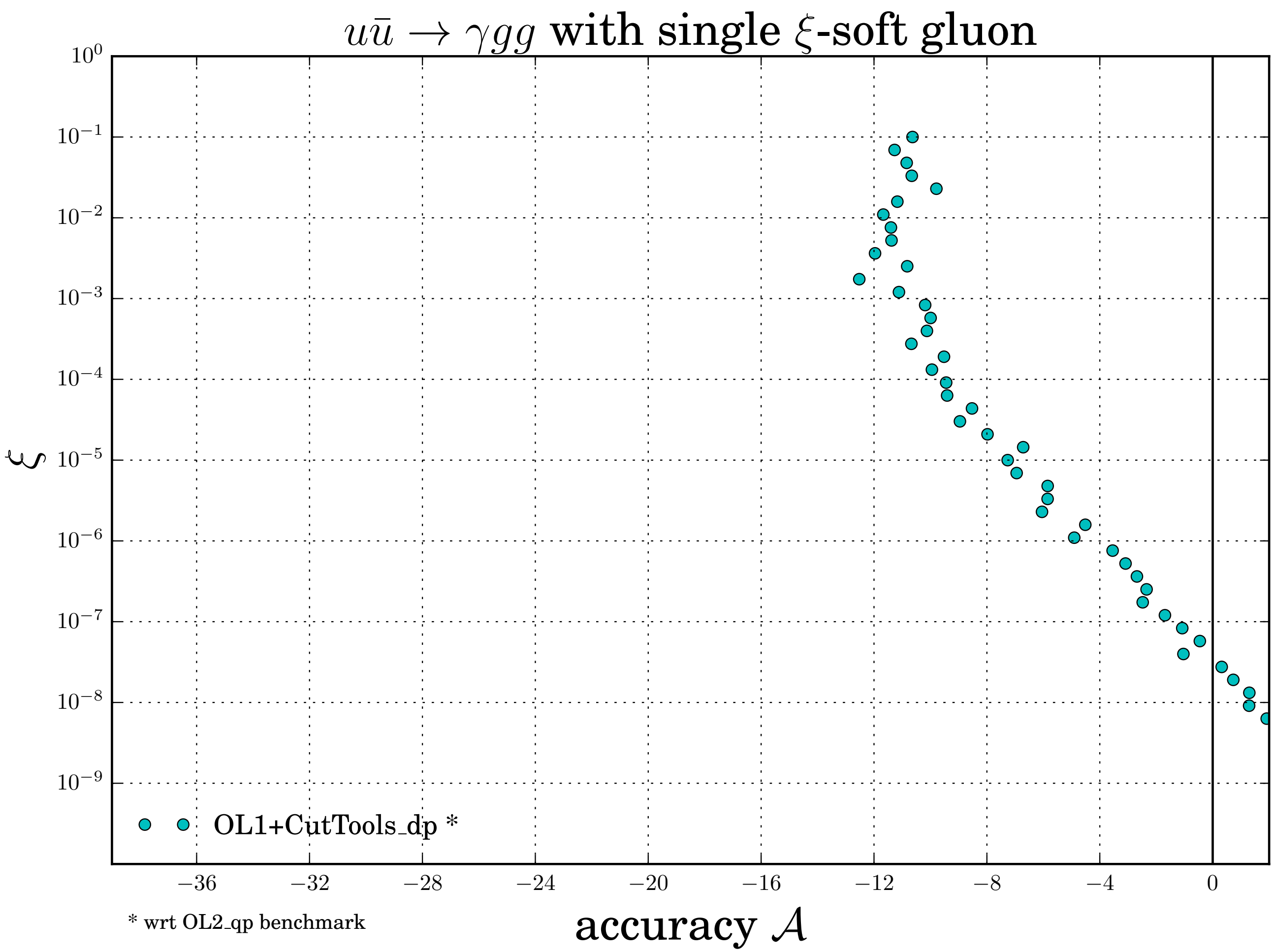




- **Hard cuts:** $p_T > 50\,$GeV and $\Delta R_{ij} => 0.5$ for final state QCD partons
- Orders of magnitude improvement wrt Cuttools and similar or better stability wrt Collier
- Further improvements in the tail under investigation

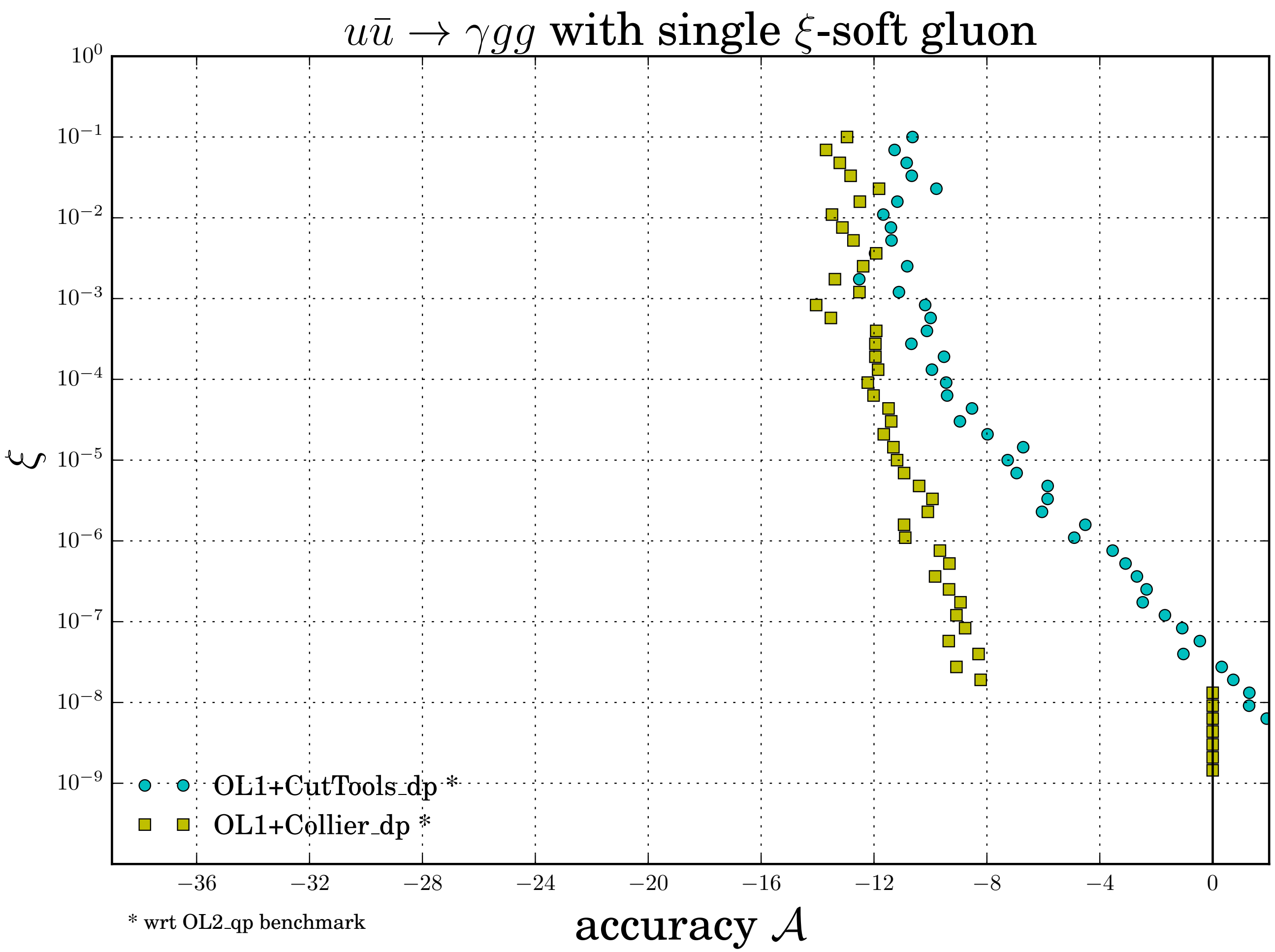
Very good stability thanks to on-the fly reduction and minimal Δ-expansions

Stability in the soft region: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV **OpenLoops1+Cuttools (dp)**



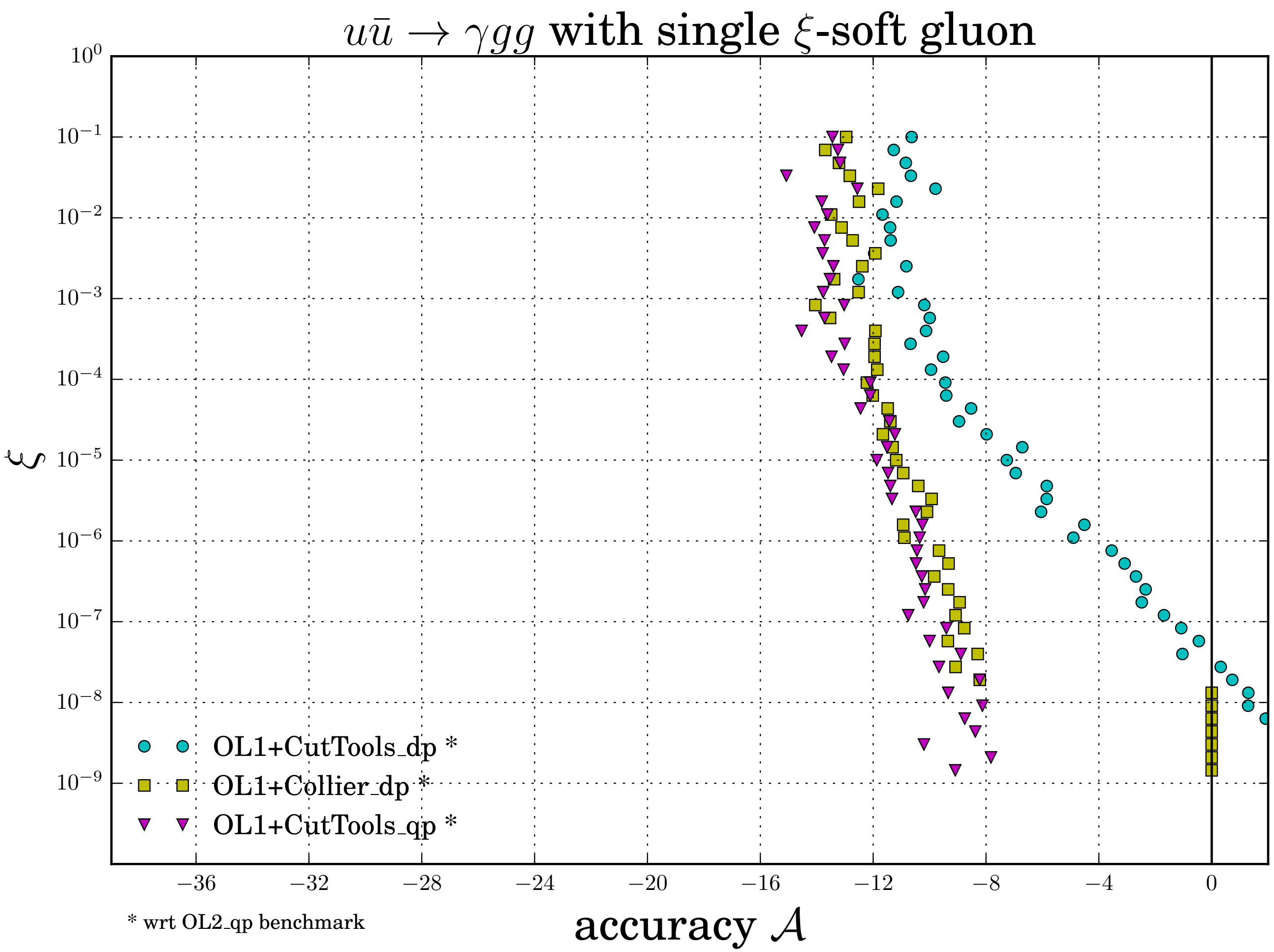
Single soft gluon with energy $E_{soft} = \xi\sqrt{\hat{s}}$. All other kinematic parameters fixed.

Stability in the soft region: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV **OpenLoops1+Collier (dp)**



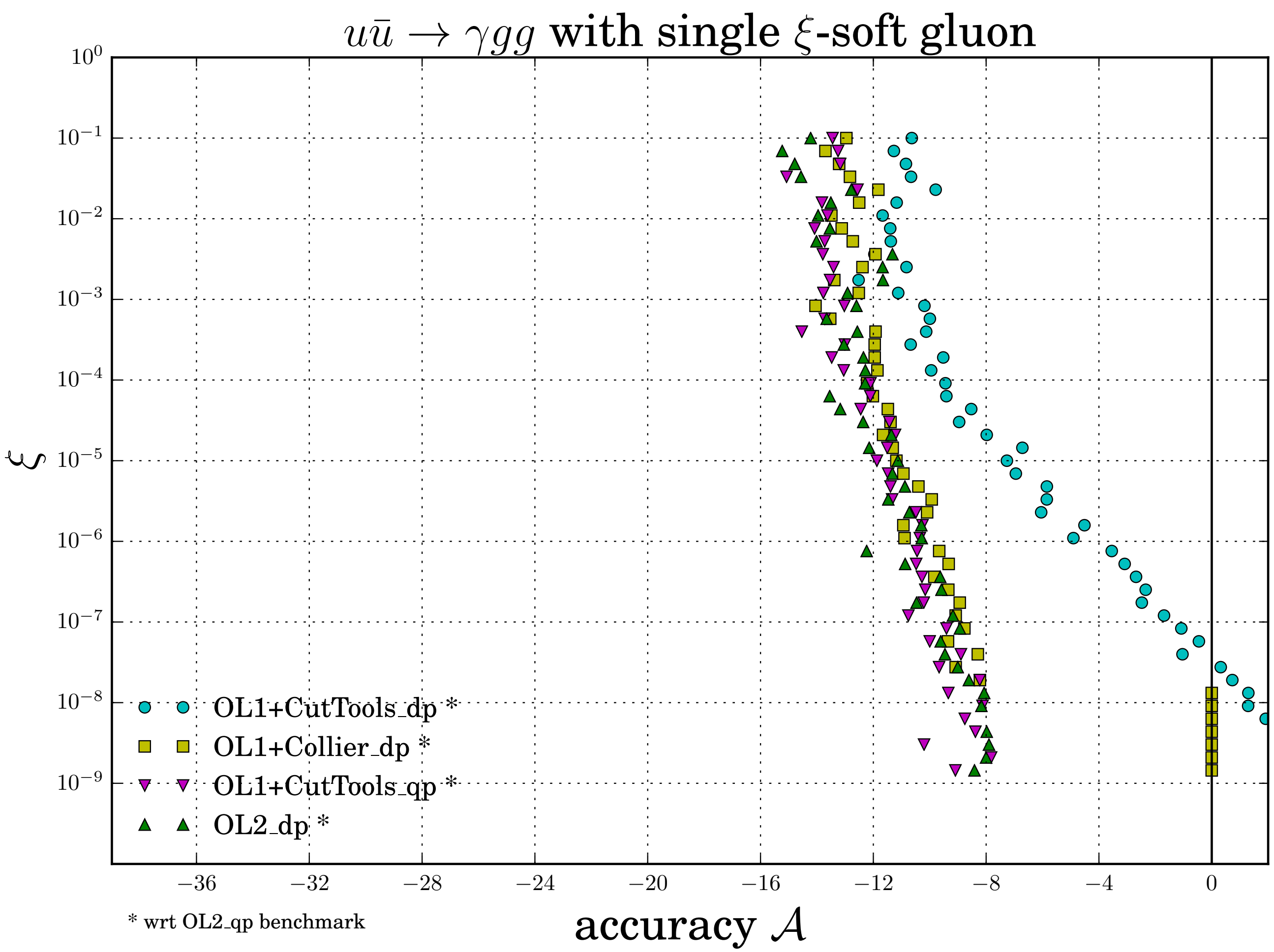
Single soft gluon with energy $E_{soft} = \xi\sqrt{\hat{s}}$. All other kinematic parameters fixed.

Stability in the soft region: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV **OpenLoops1+Cuttools (qp)**



Single soft gluon with energy $E_{soft} = \xi\sqrt{\hat{s}}$. All other kinematic parameters fixed.

Stability in the soft region: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV **OpenLoops2 (dp)**

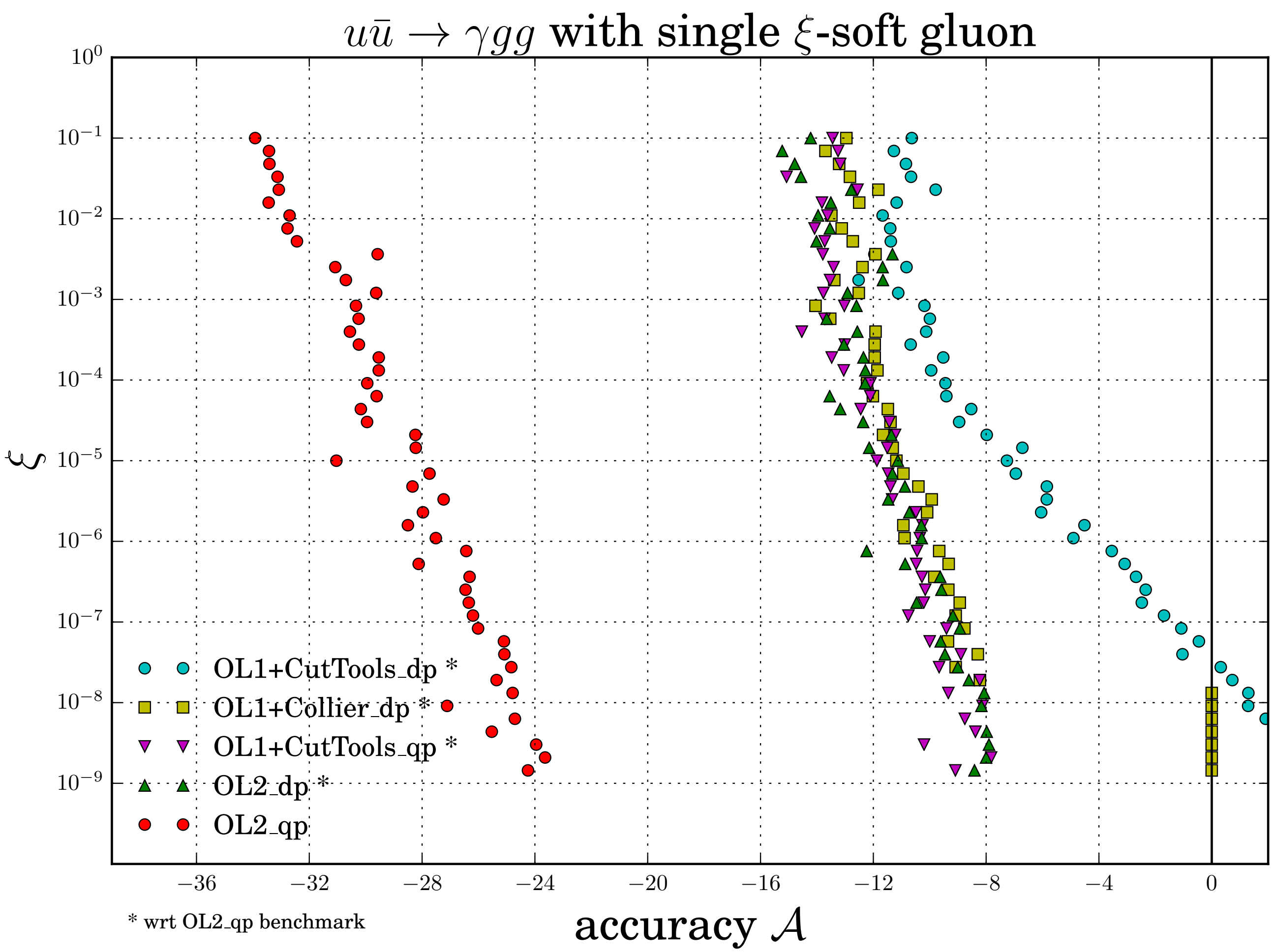


Single soft gluon with energy $E_{soft} = \xi\sqrt{\hat{s}}$. All other kinematic parameters fixed.

OpenLoops2 double precision similarly stable as OpenLoops1+Cuttools quad precision

Further systematic improvements for soft/collinear regions under investigation

Stability in the soft region: $2 \to 3$ process at $\sqrt{\hat{s}} = 1$ TeV **OpenLoops2 (qp)**



Single soft gluon with energy $E_{soft} = \xi\sqrt{\hat{s}}$. All other kinematic parameters fixed.

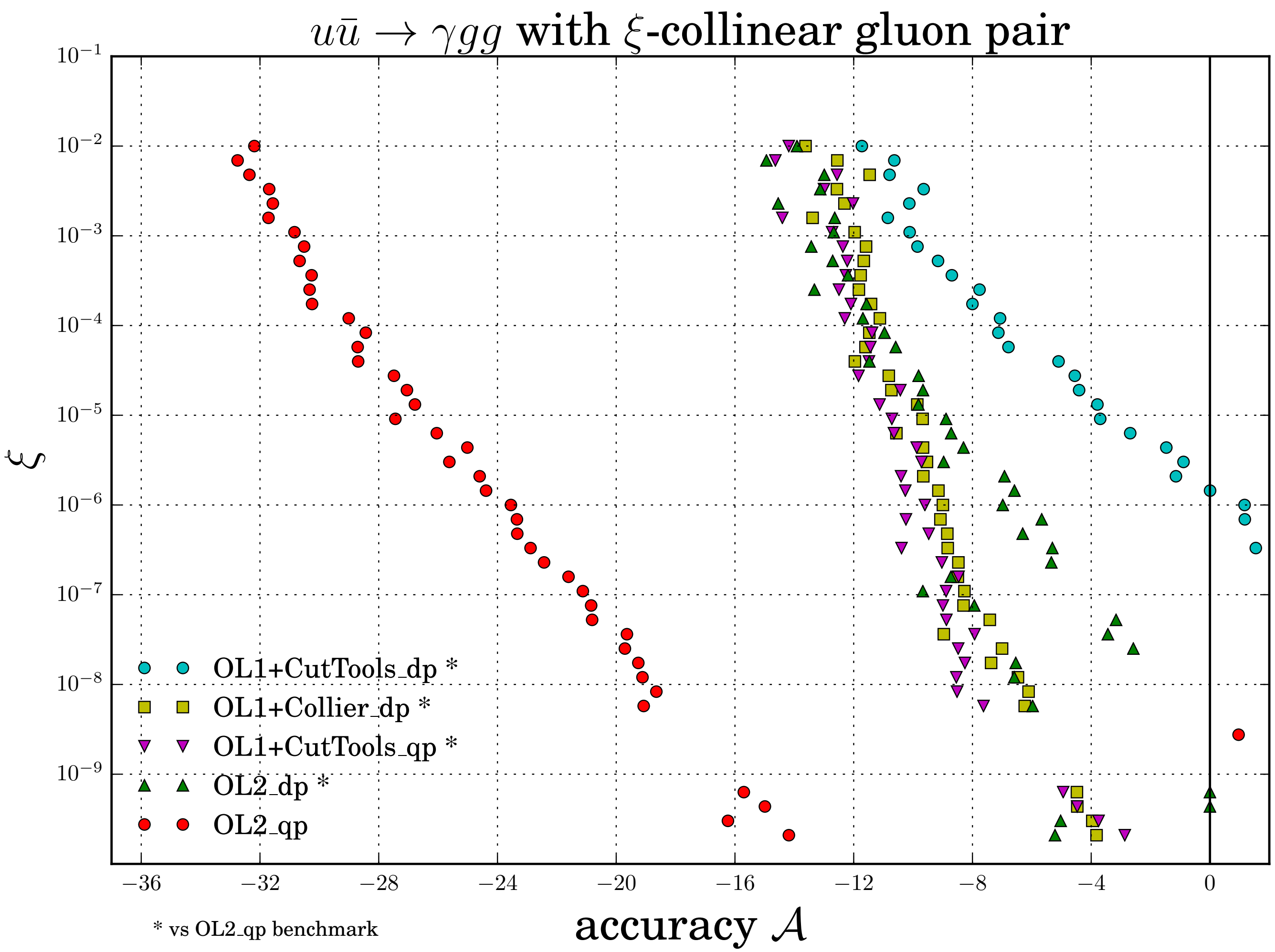
OpenLoops2 quadruple precision yields > 20 digits in deep IR region

Further systematic improvements for soft/collinear regions under investigation

V. Summary and Outlook

- New algorithm for construction and reduction of one-loop amplitudes in a single recursion
- Drastic reduction of complexity at all stages of the calculation (rank ≤ 2)
- On-the-fly helicity treatment and merging $\Rightarrow$ significant gain in CPU efficiency
- Full automation and same interface as OpenLoops 1
- Efficient solution of numerical instability issues possible in a single dressing and reduction tool
  $\Rightarrow$ Targeted all order expansions provide excellent numerical stability in the hard regions
- True quad precision benchmarks possible in this framework
- Ongoing/future projects:
  - improvement of stability in soft and collinear regions
  - extension to 2 loops

Backup: Stability in the collinear region: $2 \rightarrow 3$ process at $\sqrt{\hat{s}} = 1$ TeV



Collinear gluon pair with $\xi = \theta^2$ (angle between gluon pair). All other kinematic parameters fixed.