

Electroweak radiative corrections to polarized Bhabha scattering

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OUTLINE

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MOTIVATION

Motivation:

- Development of the physical program for future high-energy e^+e^- colliders
- Having high-precision theoretical description of Bhabha scattering is of crucial importance
- Many of the future e^+e^- colliders foresee running with polarized beam(s)

QUESTIONS:

- What we have?
- What we need?
- What to do?

FUTURE e^+e^- COLLIDER PROJECTS

Linear Colliders

- ILC, CLIC
- ILC: technology is ready, to be built in Japan (?)

E_{tot}

- ILC: 91; 250 GeV — 1 TeV
- CLIC: 500 GeV — 3 TeV

$$\mathcal{L} \approx 2 \cdot 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

Stat. uncertainty $\sim 10^{-3}$

Beam polarization:

e^- beam: $P = 80 - 90\%$

e^+ beam: $P = 30 - 60\%$

Circular Colliders

- FCC-ee, TLEP
- CEPC
- muon collider (?)

E_{tot}

- 91; 160; 240; 350 GeV

$$\mathcal{L} \approx 2 \cdot 10^{36} \text{ cm}^{-2}\text{s}^{-1} \text{ (4 exp.)}$$

Stat. uncertainty $< 10^{-3}$

Beam polarization: desirable

SUPER CHARM-TAU FACTORY PROJECT

Budker Institute of Nuclear Physics in Novosibirsk

Colliding electron-positron beams with c.m.s. energies from 2 to 5 GeV with unprecedented high **luminosity** $10^{35} \text{cm}^{-2} \text{s}^{-1}$

The electron beam will be **longitudinally polarized**

The main goal of experiments at the Super Charm-Tau factory is to study the processes charmed mesons and tau leptons, using a data set that is 2 orders of magnitude more than the one collected by BES III

ESTIMATED EXPERIMENTAL PRECISION

Now:

Quantity	Theory error	Exp. error
M_W [MeV]	4	15
$\sin^2 \theta_{eff}^l$ [10^{-5}]	4.5	16
Γ_Z [MeV]	0.5	2.3
R_b [10^{-5}]	15	66

Quantity	ILC	FCC-ee	CEPC	Projected theory error
M_W [MeV]	3–4	1	3	1
$\sin^2 \theta_{eff}^l$ [10^{-5}]	1	0.6	2.3	1.5
Γ_Z [MeV]	0.8	0.1	0.5	0.2
R_b [10^{-5}]	14	6	17	5–10

The estimated error for the theoretical predictions of these quantities is given, under the assumption that $O(\alpha\alpha_s^2)$, fermionic $O(\alpha^2\alpha_s)$, fermionic $O(\alpha^3)$, and leading four-loop corrections entering through the ρ -parameter will become available.

SANC concentrates now on *polarization effects*.

INTRODUCTION TO SANC

- The SANC system implements calculations of complete (real and virtual) NLO QCD and EW corrections for various processes at the partonic level
- All calculations are performed within the OMS (on-mass-shell) renormalization scheme in the R_ξ gauge which allows an explicit control of the gauge invariance by examining cancellation of the gauge parameters in the analytic expression of the matrix element
- Cross-sections of the processes at hadron level obtained by convoluting the partonic level cross-sections with PDFs
- The list of processes implemented in the MCSANC integrator includes Drell-Yan processes (inclusive), associated Higgs and gauge boson production and single-top quark production in s- and t-channel (v1.01 – CPC 184 (2013) 2343), photon-induced contribution, EW corrections beyond NLO approximation to DY (v1.20 – JETP Lett. 103 (2016) 131)

SANC FOR PROCESSES WITH POLARIZED BEAMS

- NLO EW corrections for polarized e^+e^- scattering:
 - Bhabha scattering ([arXiv:1801.00125](#))
 - $e^+e^- \rightarrow \mu^+\mu^-$ (or $\tau^+\tau^-$) (**preliminary**)
 - $e^+e^- \rightarrow Z\gamma$ (**preliminary**)
 - $e^+e^- \rightarrow t\bar{t}$ (**in progress**)
 - $e^+e^- \rightarrow ZH$ (**in progress**)
 - $e^+e^- \rightarrow \gamma\gamma$ (**in progress**)
 - $e^+e^- \rightarrow ZZ$ (**in progress**)
 - $e^+e^- \rightarrow f\bar{f}\gamma$ (future plans)
 - $e^+e^- \rightarrow f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
 - $\gamma\gamma \rightarrow \gamma\gamma$ (future plans)
 - $\gamma\gamma \rightarrow Z\gamma$ (future plans)
 - $\gamma\gamma \rightarrow ZZ$ (future plans)

BHABHA MATRIX ELEMENT SQUARED

$$\begin{aligned}
 |\mathcal{M}|^2 = \frac{1}{4} & \left\{ (1 - P_{e^-}^{\parallel})(1 + P_{e^+}^{\parallel})|\mathcal{H}_{-+}|^2 + (1 + P_{e^-}^{\parallel})(1 - P_{e^+}^{\parallel})|\mathcal{H}_{-+}|^2 \right. \\
 & + (1 - P_{e^-}^{\parallel})(1 - P_{e^+}^{\parallel})|\mathcal{H}_{--}|^2 + (1 + P_{e^-}^{\parallel})(1 - P_{e^+}^{\parallel})|\mathcal{H}_{++}|^2 \\
 & - 2P_{e^-}^T P_{e^+}^T \left[\cos(\phi_- - \phi_+) \operatorname{Re}(\mathcal{H}_{++}\mathcal{H}_{--}^*) + \cos(\phi_- + \phi_+ - 2\phi) \operatorname{Re}(\mathcal{H}_{-+}\mathcal{H}_{+-}^*) \right. \\
 & \left. + \sin(\phi_- + \phi_+ - 2\phi) \operatorname{Im}(\mathcal{H}_{-+}\mathcal{H}_{+-}^*) + \sin(\phi_- - \phi_+) \operatorname{Im}(\mathcal{H}_{++}\mathcal{H}_{--}^*) \right] \\
 & + 2P_{e^-}^T \left[\cos(\phi_- - \phi) \left((1 - P_{e^+}^{\parallel}) \operatorname{Re}(\mathcal{H}_{+-}\mathcal{H}_{--}^*) + (1 + P_{e^+}^{\parallel}) \operatorname{Re}(\mathcal{H}_{++}\mathcal{H}_{-+}^*) \right) \right. \\
 & \left. + \sin(\phi_- - \phi) \left((1 - P_{e^+}^{\parallel}) \operatorname{Im}(\mathcal{H}_{+-}\mathcal{H}_{--}^*) + (1 + P_{e^+}^{\parallel}) \operatorname{Im}(\mathcal{H}_{++}\mathcal{H}_{-+}^*) \right) \right] \\
 & - 2P_{e^+}^T \left[\cos(\phi_+ - \phi) \left((1 - P_{e^-}^{\parallel}) \operatorname{Re}(\mathcal{H}_{-+}\mathcal{H}_{--}^*) + (1 + P_{e^-}^{\parallel}) \operatorname{Re}(\mathcal{H}_{++}\mathcal{H}_{+-}^*) \right) \right. \\
 & \left. - \sin(\phi_+ - \phi) \left((1 - P_{e^-}^{\parallel}) \operatorname{Im}(\mathcal{H}_{-+}\mathcal{H}_{--}^*) + (1 + P_{e^-}^{\parallel}) \operatorname{Im}(\mathcal{H}_{++}\mathcal{H}_{+-}^*) \right) \right] \left. \right\},
 \end{aligned}$$

where \mathcal{H}_{++} , \mathcal{H}_{--} , \mathcal{H}_{+-} , and \mathcal{H}_{-+} are helicity amplitudes

G. Moortgat-Pick et al. Phys. Rept. 2008

POLARIZED BHABHA SCATTERING: NOTATION

We consider a scattering of two polarized e^+ and e^- beams with four momentum of incoming particles p_1 and p_2 , outgoing particles p_3 and p_4 at the one-loop EW level

$$e^+(p_1) + e^-(p_2) \longrightarrow e^+(p_3) + e^-(p_4) + (\gamma(p_5))$$

The cross-section of this process at one-loop can be divided into four parts:

$$\sigma^{1\text{-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

where σ^{Born} — Born level cross-section, σ^{virt} — contribution of virtual(loop) corrections, σ^{soft} — contribution due to soft photon emission, σ^{hard} — contribution due to hard photon emission (with energy $E_\gamma > \omega$).

Auxiliary parameters λ ("photon mass") and ω (soft-hard separator) are cancelled out in the sum

BHABHA: HA FOR BORN AND VIRTUAL CONTRIBUTIONS

At one-loop level we have six non-zero HAs (four independent):

$$\begin{aligned} \mathcal{H}_{++++} &= \mathcal{H}_{----} = -2e^2 \frac{s}{t} \left[\mathcal{F}_{QQ}^{(\gamma,Z)}(t, s, u) - \chi_Z^t \delta_e \mathcal{F}_{QL}^Z(t, s, u) \right], \\ \mathcal{H}_{+--+} &= \mathcal{H}_{-+-+} = -e^2 c_- \left[\mathcal{F}_{QQ}^{(\gamma,Z)}(s, t, u) - \chi_Z^s \delta_e \mathcal{F}_{QL}^Z(s, t, u) \right], \\ \mathcal{H}_{+---} &= -e^2 c_+ \left(\left[\mathcal{F}_{QQ}^{(\gamma,Z)}(s, t, u) + \chi_Z^s (\mathcal{F}_{LL}^Z(s, t, u) - 2\delta_e \mathcal{F}_{QL}^Z(s, t, u)) \right] \right. \\ &\quad \left. + \frac{s}{t} \left[\mathcal{F}_{QQ}^{(\gamma,Z)}(t, s, u) + \chi_Z^t (\mathcal{F}_{LL}^Z(t, s, u) - 2\delta_e \mathcal{F}_{QL}^Z(t, s, u)) \right] \right), \\ \mathcal{H}_{-++-} &= -e^2 c_+ \left(\left[\mathcal{F}_{QQ}^{(\gamma,Z)}(s, t, u) \right] + \frac{s}{t} \left[\mathcal{F}_{QQ}^{(\gamma,Z)}(t, s, u) \right] \right), \end{aligned}$$

where $c_+ = 1 + \cos \theta$, $c_- = 1 - \cos \theta$,

$$\chi_Z^s = \frac{1}{4s_W^2 c_W^2} \frac{s}{s - M_Z^2 + iM_Z \Gamma_Z}, \quad \chi_Z^t = \frac{1}{4s_W^2 c_W^2} \frac{t}{t - M_Z^2}, \quad \delta_e = v_e - a_e = 2s_W^2,$$

$$\mathcal{F}_{QQ}^{(\gamma,Z)}(a, b, c) = \mathcal{F}_{QQ}^\gamma(a, b, c) + \chi_Z^a \delta_e^2 \mathcal{F}_{QQ}^Z(a, b, c).$$

We get the Born level HAs by replacing $\mathcal{F}_{LL}^Z \rightarrow 1$, $\mathcal{F}_{QL}^Z \rightarrow 1$, $\mathcal{F}_{QQ}^Z \rightarrow 1$ and $\mathcal{F}_{QQ}^\gamma \rightarrow 1$.

BREMSSTRAHLUNG HELICITY AMPLITUDES

Bremsstrahlung HA: contributions

$$\mathcal{H}^{\text{hard}} = \mathcal{H}^{\text{isr}} + \mathcal{H}^{\text{fsr}} + \mathcal{H}^{\text{esr}} + \mathcal{H}^{\text{psr}}$$

Crossing symmetry

$$\mathcal{H}_{\chi_1\chi_2\chi_3\chi_4\chi_5}^{\text{fsr}}(p_1, p_2, p_3, p_4) = +\mathcal{H}_{-\chi_4-\chi_3-\chi_2-\chi_1\chi_5}^{\text{isr}}(-p_4, -p_3, -p_2, -p_1)$$

$$\mathcal{H}_{\chi_1\chi_2\chi_3\chi_4\chi_5}^{\text{esr}}(p_1, p_2, p_3, p_4) = -\mathcal{H}_{+\chi_1-\chi_3-\chi_2+\chi_4\chi_5}^{\text{isr}}(+p_1, -p_3, -p_2, +p_4)$$

$$\mathcal{H}_{\chi_1\chi_2\chi_3\chi_4\chi_5}^{\text{psr}}(p_1, p_2, p_3, p_4) = -\mathcal{H}_{-\chi_4+\chi_2+\chi_3-\chi_1\chi_5}^{\text{isr}}(-p_4, +p_2, +p_3, -p_1)$$

CP-symmetry

$$\mathcal{H}_{\chi_1\chi_2\chi_3\chi_4\chi_5}^{\text{hard}} = -\chi_1\chi_2\chi_3\chi_4\overline{\mathcal{H}}_{-\chi_1-\chi_2-\chi_3-\chi_4-\chi_5}^{\text{hard}}$$

SANC MONTE CARLO GENERATOR FOR BHABHA

We created a **Monte Carlo generator** of unweighted events for polarized Bhabha scattering $e^+e^- \rightarrow e^+e^-$ with complete one-loop EW corrections and with possibility to produce events in the standard Les Houches format.

This generator uses adaptive algorithm **mFOAM** ([CPC 177:441-458,2007](#)) which is part of the **ROOT** program

SETUP FOR TUNED COMPARISON

We performed a tuned comparison of **polarized** Born and hard Bremsstrahlung by **WHIZARD**. The **unpolarized** soft and virtual parts were compared with the results of **Aitalk**.

Initial parameters

$$\alpha^{-1}(0) = 137.03599976,$$

$$M_W = 80.451495 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.49977 \text{ GeV},$$

$$m_e = 0.5109990 \text{ MeV}, \quad m_\mu = 0.105658 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$

$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$

$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts

$$|\cos \theta| < 0.9,$$

$$E_\gamma > 1 \text{ GeV} \quad (\text{for comparison of hard Bremsstrahlung}).$$

$e^+e^- \rightarrow e^+e^-$: WHIZARD vs SANC (BORN)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma_{e^+e^-}^{\text{Born}}$, pb	56.677(1)	57.774(1)	56.272(1)	59.276(1)
$\sigma_{e^+e^-}^{\text{Born}}$, pb	56.677(1)	57.775(1)	56.272(1)	59.275(1)
$\sqrt{s} = 500$ GeV				
$\sigma_{e^+e^-}^{\text{Born}}$, pb	14.379(1)	15.030(1)	12.706(1)	17.355(1)
$\sigma_{e^+e^-}^{\text{Born}}$, pb	14.379(1)	15.030(1)	12.706(1)	17.354(1)
$\sqrt{s} = 1000$ GeV				
$\sigma_{e^+e^-}^{\text{Born}}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7756(1)
$\sigma_{e^+e^-}^{\text{Born}}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7755(1)

$e^+e^- \rightarrow e^+e^-$: WHIZARD vs SANC (HARD)

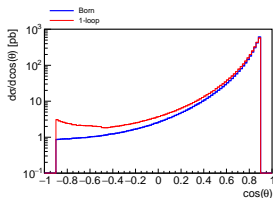
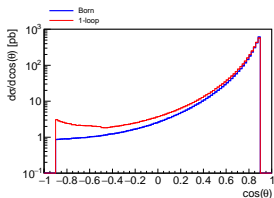
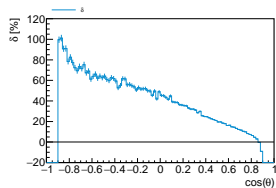
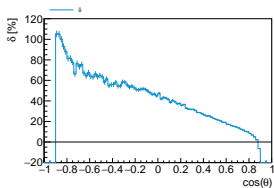
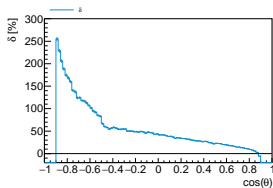
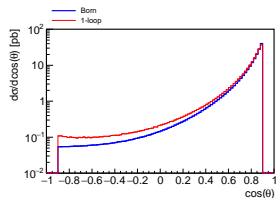
P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{hard}}, \text{pb}$	48.62(1)	49.58(1)	48.74(1)	50.40(1)
$\sigma_{e^+e^-}^{\text{hard}}, \text{pb}$	48.65(1)	49.56(1)	48.78(1)	50.44(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{hard}}, \text{pb}$	15.14(1)	15.81(1)	13.54(1)	18.07(1)
$\sigma_{e^+e^-}^{\text{hard}}, \text{pb}$	15.12(1)	15.79(1)	13.55(1)	18.11(2)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{hard}}, \text{pb}$	4.693(1)	4.976(1)	3.912(1)	6.041(1)
$\sigma_{e^+e^-}^{\text{hard}}, \text{pb}$	4.694(1)	4.975(1)	3.913(1)	6.043(1)

$e^+e^- \rightarrow e^+e^-$: **AiTALK** VS **SANC** $\sqrt{s} = 500\text{GeV}$

$\cos\theta$	$\sigma_{e^+e^-}^{\text{Born}}, \text{pb}$	$\sigma_{e^+e^-}^{\text{Born+virt+soft}}, \text{pb}$
-0.9	$2.16999 \cdot 10^{-1}$ $2.16999 \cdot 10^{-1}$	$1.93445 \cdot 10^{-1}$ $1.93445 \cdot 10^{-1}$
-0.5	$2.61360 \cdot 10^{-1}$ $2.61360 \cdot 10^{-1}$	$2.38707 \cdot 10^{-1}$ $2.38707 \cdot 10^{-1}$
0	$5.98142 \cdot 10^{-1}$ $5.98142 \cdot 10^{-1}$	$5.46677 \cdot 10^{-1}$ $5.46677 \cdot 10^{-1}$
+0.5	$4.21273 \cdot 10^0$ $4.21273 \cdot 10^0$	$3.81301 \cdot 10^0$ $3.81301 \cdot 10^0$
+0.9	$1.89160 \cdot 10^2$ $1.89160 \cdot 10^2$	$1.72928 \cdot 10^2$ $1.72928 \cdot 10^2$
+0.99	$2.06556 \cdot 10^4$ $2.06555 \cdot 10^4$	$1.90607 \cdot 10^4$ $1.90607 \cdot 10^4$
+0.999	$2.08236 \cdot 10^6$ $2.08236 \cdot 10^6$	$1.91624 \cdot 10^6$ $1.91624 \cdot 10^6$
+0.9999	$2.08429 \cdot 10^8$ $2.08429 \cdot 10^8$	$1.91402 \cdot 10^8$ $1.91402 \cdot 10^8$

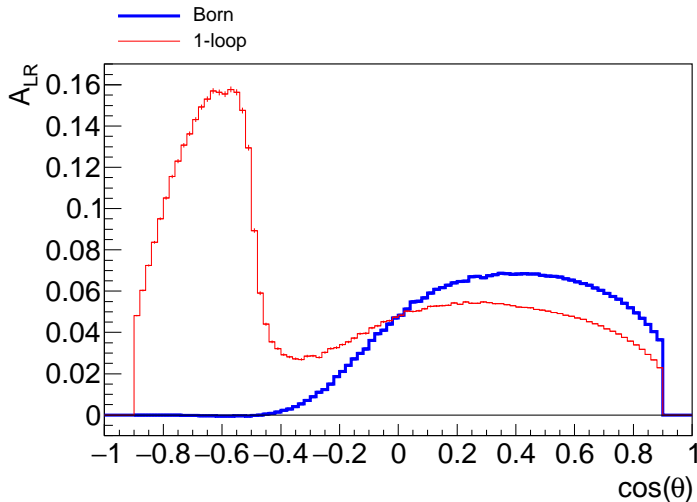
BORN VS 1-LOOP

$P_{e^-, P_{e^+}}$	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{Born}}, \text{ pb}$	56.677(1)	57.775(1)	56.272(1)	59.275(1)
$\sigma_{e^+e^-}^{1\text{-loop}}, \text{ pb}$	61.55(1)	59.72(3)	61.02(3)	58.44(3)
$\delta, \%$	8.59(2)	3.37(5)	8.45(5)	-1.42(5)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{Born}}, \text{ pb}$	14.379(1)	15.030(1)	12.706(1)	17.354(1)
$\sigma_{e^+e^-}^{1\text{-loop}}, \text{ pb}$	15.436(7)	14.441(7)	13.501(6)	15.40(1)
$\delta, \%$	7.35(5)	-3.92(5)	6.26(5)	-11.29(5)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{Born}}, \text{ pb}$	3.6792(1)	3.9057(1)	3.0358(1)	4.7755(1)
$\sigma_{e^+e^-}^{1\text{-loop}}, \text{ pb}$	3.862(2)	3.609(2)	3.148(1)	4.067(3)
$\delta, \%$	4.98(5)	-7.60(5)	3.70(5)	-14.84(6)

DISTRIBUTIONS IN $\cos\theta$ $\sqrt{s} = 250$ GeV $\sqrt{s} = 500$ GeV $\sqrt{s} = 1000$ GeV

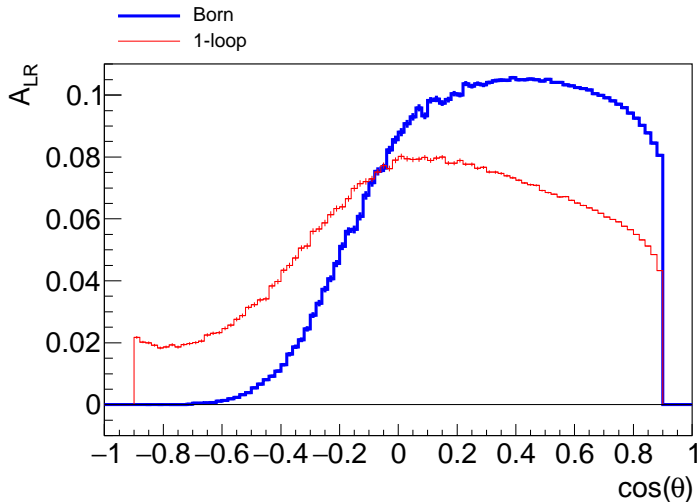
LEFT-RIGHT ASYMMETRY (250 GEV)

$$A_{LR} \equiv \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$



LEFT-RIGHT ASYMMETRY (500 GeV)

$$A_{LR} \equiv \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$



$e^+e^- \rightarrow \mu^+\mu^-$: SANC VS WHIZARD (BORN & HARD)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, 0.6	-0.8, -0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma^{\text{Born}}, \text{pb}$ [SANC]	1.6537(1)	1.8040(1)	2.7105(1)	0.89749(1)
$\sigma^{\text{Born}}, \text{pb}$ [WHIZARD]	1.6537(1)	1.8039(1)	2.7102(1)	0.89744(1)
$\sigma^{\text{Hard}}, \text{pb}$ [SANC]	1.822(1)	2.034(1)	3.068(1)	1.001(1)
$\sigma^{\text{Hard}}, \text{pb}$ [WHIZARD]	1.822(1)	2.034(1)	3.048(1)	1.018(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma^{\text{Hard}}, \text{pb}$ [SANC]	0.393(1)	0.426(1)	0.641(1)	0.213(1)
$\sigma^{\text{Hard}}, \text{pb}$ [WHIZARD]	0.394(1)	0.428(1)	0.641(1)	0.214(1)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma^{\text{Hard}}, \text{pb}$ [SANC]	0.1155(1)	0.1247(1)	0.1872(1)	0.0623(1)
$\sigma^{\text{Hard}}, \text{pb}$ [WHIZARD]	0.1153(2)	0.1245(2)	0.1874(2)	0.0626(1)

$e^+e^- \rightarrow \mu^+\mu^-$: AITALK VS SANC, $\sqrt{s} = 500\text{GeV}$

	$\sigma_{\mu^+\mu^-}^{\text{BORN}}$, PB	$\sigma_{\mu^+\mu^-}^{\text{BORN+VIRT+SOFT}}$, PB
$\cos\vartheta = -0.9$		
[AITalk]	0.09458936	0.09028587
[SANC]	0.09458937	0.09028587
$\cos\vartheta = -0.5$		
[AITalk]	0.08929449	0.08468314
[SANC]	0.08929448	0.08468313
$\cos\vartheta = 0.0$		
[AITalk]	0.1503198	0.1402075
[SANC]	0.1503199	0.1402075
$\cos\vartheta = 0.5$		
[AITalk]	0.2865049	0.2761361
[SANC]	0.2865049	0.2761361
$\cos\vartheta = 0.9$		
[AITalk]	0.4495681	0.4663674
[SANC]	0.4495682	0.4663675

$e^+e^- \rightarrow \mu^+\mu^-$: PRELIMINARY SANC RESULTS

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, 0.6	-0.8, -0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma_{\mu^+\mu^-}^{\text{Born}}, \text{pb}$	1.4174(1)	1.5462(1)	2.3231(2)	0.7690(2)
$\sigma_{\mu^+\mu^-}^{1\text{-loop}}, \text{pb}$	2.397(1)	2.614(1)	3.927(1)	1.301(1)
$\delta, \%$	69.1(1)	69.1(1)	69.1(1)	69.2(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma_{\mu^+\mu^-}^{\text{Born}}, \text{pb}$	0.34361(1)	0.37159(1)	0.55751(1)	0.18567(1)
$\sigma_{\mu^+\mu^-}^{1\text{-loop}}, \text{pb}$	0.4696(1)	0.4953(1)	0.7399(1)	0.2506(1)
$\delta, \%$	36.67(3)	33.30(2)	32.71(2)	34.98(2)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma_{\mu^+\mu^-}^{\text{Born}}, \text{pb}$	0.085354(1)	0.09213(1)	0.13818(1)	0.04608(1)
$\sigma_{\mu^+\mu^-}^{1\text{-loop}}, \text{pb}$	0.11627(2)	0.12119(2)	0.18069(3)	0.61694(1)
$\delta, \%$	36.22(2)	31.55(2)	30.78(2)	33.90(2)

OUTLOOK

- Having high theoretical precision for the normalization processes $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$, and $e^+e^- \rightarrow 2\gamma$ with *polarized* beams is crucial for future e^+e^- colliders
- There are several two-loop QED results, but most of them are *w/o polarization* yet
- Two-loop EW RC are in progress, polarization to be foreseen from the beginning
- Complete one-loop EW RC the polarized Bhabha and $e^+e^- \rightarrow \mu^+\mu^-$ were computed for the first time
- New *Monte Carlo* codes are required